$H_\infty$ Multi-objective and Multi-Model MIMO control design for Broadband noise attenuation in a 3D enclosure

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1 Introduction
   ■ General context
   ■ PhD objective
   ■ State of Art
   ■ Scope of the presentation

2 System to control

3 Control Strategy

4 Results

5 Conclusions and Perspectives
General Context
Brief ANC overview

Duct
- Propagative waves
- Feedforward + feedback

Headphone
- SISO control
- Co-located actuator and sensor

Headrest
- SISO control
- Co-located actuator and sensor
Characteristics of ANC in a cavity

- Stationary waves
- Actuators and sensors co-located or not
- feedback or feedback + feedforward
- $d$ narrow or broadband noise
- SISO or MIMO control
PhD objective
Active control of broadband low frequency noise in car cabin

Aeroacoustic noise
(Mainly in high frequency)

Engine noise
(Line spectrum)

ROAD noise
(Low frequency, Broadband spectrum)

- Passive treatments for low frequency noise ⇒ Addition of weight
- Active Noise Control (ANC) is a great opportunity to simultaneously:
  - Reduce road noise
  - Achieve car weight reduction
PhD objective
Active Noise Control of broadband noise

ANC problem characteristics
- 3D enclosure
- Actuators and sensors not co-located
- No measure of \( w \) is available
- \( d \) broadband low frequency noise

Limitations involved
- Waterbed effect (Bode integral)
- Non minimum phase zeros
State of Art
Adaptive feedforward control (FxLMS)

Fig. 2 – Control scheme for attenuation of interior noise in automobiles. Accelerometers attached to the bodywork provide reference signals for multiple-channel adaptive feedforward control.

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Figure 5.2: Mutli-input, multi-output feedback control system in the rectangular enclosure.

Scope of the presentation

Problem
- Attenuate broadband low frequency noise;
- In a closed cavity;
- by feedback.

Goal of the presentation
Compare SISO and MIMO achievable performances.
1 Introduction
   ■ General context
   ■ PhD objective
   ■ State of Art
   ■ Scope of the presentation

2 System to control
   ■ Experimental Set up
   ■ Identification

3 Control Strategy
   ■ Control problem formulation
   ■ Multi-objective optimization
   ■ Controller Structure
   ■ Initialization

4 Results

5 Conclusions and Perspectives
Experimental set up

Cavity characteristics

- One predominant dimension: 1D acoustic field in low frequency;
- One biased side: Attenuation of the first longitudinal mode;
- Frequency complexity: Similar to vehicle one.
MIMO Identification
Frequency Domain, Continuous time model

Identification
- Algorithm: Subspace;
- Model structure: Modal;
- Frequency range: [20-1000]Hz;
- Order: 80.

Fit indicator

<table>
<thead>
<tr>
<th></th>
<th>LS₁</th>
<th>LS₂</th>
<th>LS₃</th>
</tr>
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<td>M₁</td>
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<td>84.1038</td>
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Remark: SISO transfers contain RHP zeros.

Bode Diagram N = 80 (FIT : 84.1038)
From: LS₂ To: M₁
Control problem formulation

Optimization problem

\[ \min_K \left\| W_1 T_{w \rightarrow e_1} \right\|_{\infty} \quad \text{subject to} \quad \begin{cases} \left\| W_2 T_{w \rightarrow u_i} \right\|_{\infty} < 1 \\ \left\| W_3 T_{d' \rightarrow e_i} \right\|_{\infty} < 1 \\ |p_{iK}| < \frac{f_e}{N} \\ \text{Re}(p_{iK}) < 0 \end{cases} \]
Control problem formulation
Additional robustness needed

Environment conditions modify acoustic transfers

A multi-model approach was used to tackle system variations
Control problem formulation

Optimization problem

$$\min_{K} \max_{1, \ldots, N} \left\| W_{1} T_{w} \rightarrow e_{1} \right\|_{\infty} \quad \text{subject to} \quad \begin{cases} \max_{1, \ldots, N} \left\| W_{2} T_{w} \rightarrow u_{i} \right\|_{\infty} < 1 \\ \max_{1, \ldots, N} \left\| W_{3} T_{d'} \rightarrow e_{i} \right\|_{\infty} < 1 \\ \left| p_{iK} \right| < f_{e}/N \\ \text{Re}(p_{iK}) < 0 \end{cases} \quad i = 1, 2 \quad \text{and} \quad j = 1, 2$$
Motivations

▶ Be able to consider various constraints *without pessimism*;
▶ Clearly distinguish objective and constraints;
▶ Have the possibility to mix $H_2$ and $H_\infty$ objectives, if needed;
▶ Be able to structure the controller;
▶ Be able to consider reduce order controller.

Optimization tool: *systune*

▶ Specialized in tuning fixed-structure control systems;
▶ Based on non smooth optimization;
▶ *P. Apkarian*, “Tuning controllers against multiple design requirements”, in *American Control Conference (ACC)*, Washington, 2013, pp. 3888–3893

Drawback

▶ May lead to local optima;
▶ Necessity of ”good” initialization and controller structure.
Controller Structure
State feedback observer

Model of the system

- No real time measure of $w$
- $G_p$ is known

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bw w \\
 e &=Cx + Du + Dw w
\end{align*}
\]

Model of the controller

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + Bu + K_f (e - \hat{e}) \\
 u &= -K_c \hat{x}
\end{align*}
\]

Remarks

- $K_f$ : observation gain
- $K_c$ : state feedback gain
- full order controller
## Initialization

### LQG

### LQ criteria

\[ J_{LQ} = \min_{K_c} \| W_{LQ} e \|_2^2 + \rho \| u \|_2^2 \]

- \( W_{LQ} \) is a bandpass filter (attenuation frequency range)
- \( \rho \) manages trade-off between performances and control energy

### Kalman filter

\[
\begin{cases}
\dot{x}_a = A_a x_a + B_{u_a} u + B_{w_a} w \\
e = C_a x_a + D_{u_a} u + D_{w_a} w + v
\end{cases}
\]

- Tuning parameters are the covariances of noises \( v \) and \( w \)
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Results
Narrow attenuation: [190-220] Hz

![Graph showing transfer function for different configurations: Open loop, SISO (LS1), SISO (LS2), MISO, MIMO, with magnitude in dB on the y-axis and frequency in Hz on the x-axis.](image-url)
Results
Narrow attenuation: [190-300] Hz

Transfer $\frac{G_1}{W}$ [190-300] Hz (SIMULATION)
Results
Experimentation: 190-300 Hz (MIMO)
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Conclusions

▷ A general framework (for identification and control) was presented;
▷ It allows to quantify and compare SISO and MIMO achievable performances according to:
  ▷ Frequency range of attenuation;
  ▷ Actuators and sensors position;
  ▷ Cavity geometry
  ▷...

Ongoing work

▷ Compare feedback and feedforward control
▷ Apply methodology to the industrial problem where:
  ▷ $G_p$ is unknown
  ▷ System order and dimensions are higher