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Collaborative multi-vehicle localization with respect to static/dynamic target from range and velocity measurements

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Abstract— **We treat the problem of collaborative multi-vehicle localization using time-varying range and relative velocity measurements. The proposed solution combines local nonlinear observers that estimate the relative positions between agents and their neighbors, and cooperative filters that fuse each agent's local estimates to globally localize them with respect to a target (and therefore to each other). Furthermore, we explicitly introduce an estimator that filters the noisy measured signals and feeds the aforementioned observers. Both scenarios of a static as well as a dynamic target are considered. The overall architecture is proved to provide a uniformly globally exponentially converging localization under the assumptions of persistently exciting motion and of a communication topology that contains a directed spanning tree. The efficiency of the results is illustrated through detailed numerical simulations.**

I. Introduction

The last two decades have witnessed the explosion of applications incorporating multiple agents. Inspired by the behavior of animals in nature and motivated by the fact that a variety of objectives can be more efficiently, rapidly and robustly accomplished collaboratively rather than independently, multi-agent systems have been in the core of attention from both theoreticians and practicioners. Of particular interest have been applications involving multiple (aerial, ground, marine) vehicles that need to collaborate to achieve a common goal such as to ensure the exploration of unknown environments, to follow targets, to seek dangerous emitting sources or to ensure high-precision photography.

For most of these applications, the location of the vehicles is an information of paramount importance since it is exploited in the guidance, control and estimation algorithms that ensure the succesful undertaking of the mission scenario. However, such global information, as obtained for example by GPS receivers, is not available in indoor environments and in general, due to hardware malfunction or unavailability of the minimum number of GPS satellites. Instead local, low-cost sensors (cameras, infrared sensors, sonars) are usually incorporated to provide a sufficient localization. We can distinguish two large types of localization scenarios [14], [18]; a) Mutual localization, referring to the scenario where each agent needs to find its own (static) position in a reference frame common to the entire network; and b) Collaborative localization referring to the localization of a (dynamic) target using an already mutually localized network.

This work is concerned with the second type of localization problems. Depending on the community (control, robotics, sensors) and the mission objective, we can have 2D or 3D models, centralized or distributed algorithms, a variety of available measurements, e.g. absolute position (GPS), relative positions, distances, bearings or IMU measurements, and additional known points (anchors, markers). Additionally, the solutions can be signal/information-based or model-based which are essentially divided into optimization-based and observer-based. Observer-based, distributed estimation algorithms have recently been shown to present some rather interesting robust characteristics. In particular, it was established that distributed observers can enhance the quality of estimation by eliminating noise, see [17], [15], which is of great interest in all applications.

Hence motivated by these recent developments and unlike the probabilistic and Kalman-filter-based approaches [3], [7], [16], which cannot in general guarantee analytical global convergence, we adopt an observer-based approach to treat the problem of multi-vehicle collaborative localization using time-varying range and relative velocity measurements without requiring any global positioning information. The range measurements can be obtained using a variety of sensors such as stero-vision systems that typically equip robotic vehicles. This measurement scenario renders our obtained algorithm applicable to GPS-denied environments as well. We consider that the graph topology defining the communication interconnection between agents contains a directed spanning tree. We show that each agent can localize itself with respect to the target by the combination of local estimates of his neighbors' relative positions and the fusion with the neighbors' own estimates.

As opposed to other works, e.g. [2], [4], [5], our algorithm does not require global information (absolute position) but rather local measurements. Compared to the relevant work in [14] that treats the collaborative localization problem with respect to a static target, instead of single integrators we consider double integrator dynamics to model the agents' translational dynamics and require no knowledge on the rate-of-change of the distances. Futhermore we extend these results to the scenario of a dynamic target and show that by adopting an approach inspired by the recent developments on dynamically scaled Lyapunov functions [6], [13] we are able to prove uniform global exponential relative localization using a strict Lyapunov function.

The structure of the paper is as follows. In Section II we present the dynamic model of the agents, the network topology characteristics as well as the available measurements. Section III follows with the two main results. First, a global localization algorithm, combining local nonlinear observers and fusion algorithms, is presented for the case of unfiltered

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measurements along with a classic Lyapunov stability proof. Then, we extend this algorithm to the case where noisy measurements are considered by including a nonlinear pre-filter and present a thorough stability proof hinging upon the recent developments of dynamically scaled Lyapunov functions. We conclude the exposition with detailed numerical simulations given in Section IV and some concluding remarks.

II. Model and Problem Formulation

A. Network topology

The interconnection graph (directed or undirected) describing the communication between the $N+1$ agents forming the multi-agent system, target included, can be modeled using the Laplacian matrix $\mathcal{L} := [l_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$, $i, j \in \{0, \dots, N\}$, whose elements are defined as

$$l_{ij} = \begin{cases} \sum\limits_{j\in\mathcal{N}_i} w_{ij} & i = j \\ -w_{ij} & i \neq j \end{cases}, \tag{1}$$

where $w_{ij} = 0$ if $i = j$, $w_{ij} > 0$ if $j \in \mathcal{N}_i$ and $w_{ij} = 0$ otherwise. In this case, $\mathcal{N}_i$ stands for the set of agents transmitting information to the i-th agent. Note that, by construction, $\mathcal{L}$ has zero row sum, i.e., $\mathcal{L}\mathbf{1}_{N+1} = \mathbf{0}$, where $\mathbf{1}_{N+1}$ is a column vector of size N filled with ones or, equivalently, $l_{ii} - \sum\limits_{j\in\mathcal{N}_i} l_{ij} = 0$. In particular, when the graph topology is directed we assume that it includes a directed spanning tree graph, which ensures that the flow of information along the whole network. For more details on network topologies refer for example to [19].

B. Dynamic model

We consider that the dynamics of each of the $N+1$ identical agents composing the multi-vehicle system of interest can be described by the double integrator model

$$\dot{x}_i = v_i \tag{2}$$
$$\dot{v}_i = u_i, \quad i = \{0, \dots, N\} \tag{3}$$

with $x_i, v_i \in \mathbb{R}^3$ denoting the position and velocity vectors of the i-th vehicle in the inertial frame, while $u_i \in \mathbb{R}^3$ is the applied acceleration.

By the index $i = 0$ we denote the (static or dynamic) target with respect to which the localization will be referred. As is evident, the static scenario corresponds to a target's dynamics

$$\dot{x}_0 = 0 \tag{4}$$
$$\dot{v}_0 = 0. \tag{5}$$

Now, we naturally define the relative position, velocity and acceleration between two agents as

$$x_{ij} = x_i - x_j \tag{6}$$
$$v_{ij} = v_i - v_j \tag{7}$$
$$u_{ij} = u_i - u_j \tag{8}$$

that yield the required relative dynamics

$$\dot{x}_{ij} = v_{ij} \tag{9}$$
$$\dot{v}_{ij} = u_{ij}. \tag{10}$$

For our localization problem, we consider that the available measurements consist of the relative velocities and distances [1]

$$y_i = \mathtt{col}(v_{ij}^T, d_{ij}^T) \tag{11}$$

with the distance d_{ij} between agent i and its neighbor j defined as

$$d_{ij} := |x_i - x_j| = |x_{ij}|. \tag{12}$$

A simple derivation provides

$$\dot{d}_{ij} = \frac{x_{ij}^T v_{ij}}{d_{ij}} = \frac{v_{ij}^T x_{ij}}{d_{ij}}. \tag{13}$$

In conclusion, the complete model on which our design will be based is summarized as

$$\dot{x}_{ij} = v_{ij} \tag{14}$$
$$\dot{v}_{ij} = u_{ij} \tag{15}$$
$$\dot{d}_{ij} = \frac{v_{ij}^T x_{ij}}{d_{ij}}. \tag{16}$$

III. Cooperative localization

Before presenting our main results, we define some additional notation and then remind the definition of a persistently-exciting function. The notation for a matrix A being positive (semi-)definite is expressed by $A \succ 0 (\succeq 0)$, while for the case of a positive scalar a we write instead $a > 0$. The notation $|\cdot|$ will refer depending on its argument either to the absolute value of a scalar function, to the Euclidean norm of a vector or to the induced 2-norm of a matrix.

Definition 1: Let the function $v_{ij} : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$ be continuous. It is persistently exciting (PE) if there exist some $T > 0$ and $\mu > 0$ such that

$$\int_t^{t+T} v_{ij}(\tau) v_{ij}^T(\tau) d\tau \succeq \mu I \succ 0, \quad \forall t. \tag{17}$$

For the distance-based localization scenario at hand, we require that certain relative velocities are persistently-exciting which means that in order for an agent to be able to reconstruct a relative position with respect to a neighboring agent, it is necessary to move out of the line-of-sight for some time which in fact is required for the relative position to be observable. In practice, this condition imposes a requirement on the applied accelerations (controls) which can always be ensured for each agent by including an excitation term but however, might complicate the stability analysis.

[1]With some slight abuse of notation we denote the relative measurements for each agent as y_i instead of the more correct y_{ij}. Similarly, in what follows we define the state of the observer as ξ_i instead of the more appropriate ξ_{ij} that would be coherent also with the notation of the corresponding vector x_{ij}. The same notation will be adopted for the estimation error z_i and the mapping β_i.

A. Single vehicle localization from direct local measurements: Static Target

In this subsection we will consider the problem of localization of each agent with respect to its neighbors by incorporating local, noiseless measurements, and considering a static target. This will be achieved by means of a carefully designed nonlinear observer that is based on the invariant-manifold observer methodology, see [1], [6] for the general setting and [10]–[12] for recent applications on UAVs.

Proposition 1: Consider the dynamical system defined in (14)-(16) and assume that v_{ij} is persistently exciting. Then, the dynamical system

$$\dot{\xi}_i := -\frac{K_{ij}d_{ij}^2}{2}u_{ij} - K_{ij}v_{ij}v_{ij}^T\hat{x}_{ij} + v_{ij} \tag{18}$$

$$\hat{x}_{ij} := \xi_i + \frac{d_{ij}^2}{2}K_{ij}v_{ij} \tag{19}$$

is a globally exponential observer with gain $K_{ij} > 0$.

Proof: First, let us define the relative position estimation error

$$z_i := \xi_i + \beta_i(y_i) - x_{ij} =: \hat{x}_{ij} - x_{ij}. \tag{20}$$

Then, the general form of the z_i-dynamics gives

$$\begin{aligned}\dot{z}_i &:= \dot{\xi}_i + \partial_{y_i}\beta_i\dot{y}_i - \dot{x}_{ij} = \dot{\xi}_i + \partial_{d_{ij}}\beta_i\dot{d}_{ij} + \partial_{v_{ij}}\beta_i\dot{v}_{ij} - \dot{x}_{ij}\\ &= \dot{\xi}_i + \partial_{d_{ij}}\beta_i\frac{v_{ij}^T x_{ij}}{d_{ij}} + \partial_{v_{ij}}\beta_i u_{ij} - v_{ij}.\end{aligned}$$

With the choice

$$\dot{\xi}_i := -\partial_{d_{ij}}\beta_i\frac{v_{ij}^T\hat{x}_{ij}}{d_{ij}} - \partial_{v_{ij}}\beta_i u_{ij} + v_{ij}$$

and the β mapping as

$$\beta_i(y_i) := \frac{d_{ij}^2}{2}K_{ij}v_{ij}, \tag{21}$$

the z_i-dynamics obtains the more explicit form

$$\dot{z}_i = -K_{ij}v_{ij}v_{ij}^T z_i. \tag{22}$$

From Lemma 5 of [8] we know that the nominal system

$$\dot{z}_i = -K_{ij}v_{ij}v_{ij}^T z_i, \tag{23}$$

has a uniformly global exponentially stable (UGES) equilibrium at the origin for a persistently-exciting (PE) and uniformly bounded v_{ij}. ■

Remark 1: From the converse Lyapunov lemma (Lemma 1 of [8]) we know that there exists a quadratic Lyapunov function

$$V_{z_i} := \frac{1}{2}z_i^T P(t) z_i, \tag{24}$$

with $P(t)$ such that $0 \prec c_1 I \preceq P(t) = P^T(t) \preceq c_2 I$, the unique solution of the equation

$$\dot{P} - PK_{ij}v_{ij}v_{ij}^T - v_{ij}v_{ij}^T K_{ij}P = -Q, \tag{25}$$

with $Q(t) = Q^T(t)$ such that $0 \prec c_3 I \preceq Q(t) \preceq c_4 I$.

This lemma will be exploited in the construction of a strict, dynamically scaled Lyapunov function of the more general solution that follows in the next subsection.

Remark 2: Notice that in our algorithm, we further require that the relative acceleration between neighboring agents be either available or can be reconstructed. As is common in the literature for example, the agents might transmit their respective control actions (accelerations) to their neighbors. Alternatively, and under the assumption that relative motion is not too aggressive, we can consider that the relative acceleration is reconstructed by numerical differentiation of the available relative velocities.

B. Single vehicle localization from filtered local measurements: Dynamic Target

In continuation of the previous scenario, we proceed to extend the localization algorithm to the case of a dynamic target in the presence of noisy velocity measurements, without assuming any particular noise characteristics.

Proposition 2: Consider the dynamical system defined in (14)-(16) and assume that v_{ij} is persistently exciting. Then, the dynamical system

$$\begin{aligned}\dot{\xi}_i :=& -\frac{K_{ij}d_{ij}^2}{2}(u_{ij} - K_{v_i}(\hat{x}_{ij},\hat{v}_{ij},v_{ij},r)(\hat{v}_{ij} - v_{ij}))\\ &- K_{ij}\hat{v}_{ij}\hat{v}_{ij}^T\hat{x}_{ij} + \hat{v}_{ij}\end{aligned} \tag{26}$$

$$\hat{x}_{ij} := \xi_i + \frac{d_{ij}^2}{2}K_{ij}\hat{v}_{ij} \tag{27}$$

$$\dot{r} := -c_7(r-1) + \frac{c_2^2K_{ij}^2}{c_1c_5}|v_{ij}|^2|\hat{v}_{ij} - v_{ij}|^2 \tag{28}$$

$$\dot{\hat{v}}_{ij} := u_{ij} - K_{v_i}(\hat{x}_{ij},\hat{v}_{ij},v_{ij},r)(\hat{v}_{ij} - v_{ij}) \tag{29}$$

is a globally exponential observer, for some $c_i > 0$, with $r(0) \geq 1$ and gains

$$K_{ij} := c_8 + \frac{c_5 + c_6 + c_7c_2}{c_3}$$

$$\begin{aligned}K_{v_i}(\hat{x}_{ij},\hat{v}_{ij},v_{ij},r) :=& c_9I + (r-1)\frac{c_2^2}{c_1c_5}K_{ij}^2|v_{ij}|^2 I\\ &+ \frac{c_2^2}{c_6 r}(K_{ij}^2|\hat{v}_{ij}|^2|\hat{x}_{ij}|^2 + 1)I.\end{aligned}$$

Proof: First, let us define the relative position estimation error

$$z_i := \xi_i + \beta_i(y_i,\hat{y}_i) - x_{ij} =: \hat{x}_{ij} - x_{ij}. \tag{30}$$

Then, the general form of the z_i-dynamics gives

$$\begin{aligned}\dot{z}_i &:= \dot{\xi}_i + \partial_{\hat{y}_i}\beta_i\dot{\hat{y}}_i + \partial_{y_i}\beta_i\dot{y}_i - \dot{x}_{ij}\\ &= \dot{\xi}_i + \partial_{\hat{d}_{ij}}\beta_i\dot{\hat{d}}_{ij} + \partial_{\hat{v}_{ij}}\beta_i\dot{\hat{v}}_{ij} + \partial_{d_{ij}}\beta_i\dot{d}_{ij} + \partial_{v_{ij}}\beta_i\dot{v}_{ij}\\ &\quad - \dot{x}_{ij}\\ &= \dot{\xi}_i + \partial_{\hat{d}_{ij}}\beta_i\dot{\hat{d}}_{ij} + \partial_{\hat{v}_{ij}}\beta_i\dot{\hat{v}}_{ij} + \partial_{d_{ij}}\beta_i\frac{v_{ij}^T x_{ij}}{d_{ij}} + \partial_{v_{ij}}\beta_i u_{ij}\\ &\quad - v_{ij},\end{aligned}$$

which with the choice

$$\begin{aligned}\dot{\xi}_i :=& -\partial_{\hat{d}_{ij}}\beta_i\dot{\hat{d}}_{ij} - \partial_{\hat{v}_{ij}}\beta_i\dot{\hat{v}}_{ij} - \partial_{d_{ij}}\beta_i\frac{\hat{v}_{ij}^T\hat{x}_{ij}}{d_{ij}}\\ &- \partial_{v_{ij}}\beta_i u_{ij} + \hat{v}_{ij}\end{aligned}$$

reduces, after defining $e_{v_{ij}} := \hat{v}_{ij} - v_{ij}$, to

$$\begin{aligned}\dot{z}_i &= -\partial_{d_{ij}}\beta_i(\frac{\hat{v}_{ij}^T\hat{x}_{ij}}{d_{ij}} - \frac{v_{ij}^T x_{ij}}{d_{ij}}) + \hat{v}_{ij} - v_{ij} \\ &= -\partial_{d_{ij}}\beta_i\frac{v_{ij}^T}{d_{ij}}z - \partial_{d_{ij}}\beta\frac{\hat{x}_{ij}^T}{d_{ij}}e_{v_{ij}} + e_{v_{ij}}.\end{aligned}$$

Selecting further the β mapping as

$$\begin{aligned}\beta_i(y_i, \hat{y}_i) &:= \frac{d_{ij}^2}{2}K_{ij}\hat{v}_{ij} = \frac{d_{ij}^2}{2}K_{ij}(v_{ij} + e_{v_{ij}}) \\ \partial_{d_{ij}}\beta_i &= d_{ij}K_{ij}(v_{ij} + e_{v_{ij}}),\end{aligned}$$

the z_i-dynamics obtains the more explicit form

$$\begin{aligned}\dot{z}_i &= -K_{ij}v_{ij}v_{ij}^T z_i - K_{ij}e_{v_{ij}}v_{ij}^T z_i - K_{ij}(v_{ij} + e_{v_{ij}})\hat{x}_{ij}^T e_{v_{ij}} \\ &\quad + e_{v_{ij}} \\ &= -K_{ij}v_{ij}v_{ij}^T z_i - K_{ij}e_{v_{ij}}v_{ij}^T z_i - (K_{ij}\hat{v}_{ij}\hat{x}_{ij}^T - I)e_{v_{ij}}.\end{aligned}$$

Taking the function V_{z_i} defined in (24) and computing its time derivative along trajectories of the z_i-dynamics yields

$$\begin{aligned}\dot{V}_{z_i} &:= \frac{1}{2}z_i^T(\dot{P}(t) - P(t)K_{ij}v_{ij}v_{ij}^T - v_{ij}v_{ij}^T K_{ij}P(t))z_i \\ &\quad - z_i^T P(t)K_{ij}e_{v_{ij}}v_{ij}^T z_i - z_i^T P(t)(K_{ij}\hat{v}_{ij}\hat{x}_{ij}^T - I)e_{v_{ij}} \\ &\leq -\frac{c_3}{2}|z_i|^2 + c_2|z_i|^2 K_{ij}|e_{v_{ij}}||v_{ij}| \\ &\quad + c_2|z_i|(K_{ij}|\hat{v}_{ij}||\hat{x}_{ij}| + 1)|e_{v_{ij}}| \\ &\leq -(\frac{c_3}{2} - \frac{c_5 + c_6}{2})|z_i|^2 + \frac{c_2^2}{2c_5}K_{ij}^2|v_{ij}|^2|e_{v_{ij}}|^2|z_i|^2 \\ &\quad + \frac{c_2^2}{c_6}(K_{ij}^2|\hat{v}_{ij}|^2|\hat{x}_{ij}|^2 + 1)|e_{v_{ij}}|^2,\end{aligned}$$

where we applied Young's inequality to the two cross-terms of the first inequality. In order to handle the last two cross-terms in the above right handside we employ a dynamic scaling of the form

$$W_{z_i} := \frac{V_{z_i}}{r}, \tag{31}$$

with

$$\dot{r} := -c_7(r-1) + \frac{c_2^2}{c_1 c_5}K_{ij}^2|v_{ij}|^2|e_{v_{ij}}|^2, r(0) \geq 1. \tag{32}$$

Then, the time-derivative of W_{z_i} can be shown to be

$$\begin{aligned}\dot{W}_{z_i} &= \frac{\dot{V}_{z_i}}{r} - W_{z_i}\frac{\dot{r}}{r} \\ &\leq \frac{\dot{V}_{z_i}}{r} + c_2|z_i|^2 c_7\frac{(r-1)}{r} - c_1\frac{|z_i|^2}{r}\frac{c_2^2}{c_1 c_5}K_{ij}^2|v_{ij}|^2|e_{v_{ij}}|^2 \\ &\leq -(\frac{c_3}{2} - \frac{c_5 + c_6 + c_7 c_2}{2})\frac{|z_i|^2}{r} \\ &\quad + \frac{c_2^2}{c_6}(K_{ij}^2|\hat{v}_{ij}|^2|\hat{x}_{ij}|^2 + 1)\frac{|e_{v_{ij}}|^2}{r},\end{aligned}$$

with the last right handside term depending on the error between the filtered $\hat{v}_{ij}$ and the true measurements v_{ij}.

Choosing

$$\dot{\hat{v}}_{ij} := u_{ij} - K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r)e_{v_{ij}}, \tag{33}$$

with K_{v_i} a (free) positive gain function of $\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r$, yields the dynamics of the filtering error $e_{v_{ij}} := \hat{v}_{ij} - v_{ij}$

$$\dot{e}_{v_{ij}} := -K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r)e_{v_{ij}}. \tag{34}$$

By simple derivations one can show that the following function

$$V_{e_v} := \frac{1}{2}|e_{v_{ij}}|^2, \tag{35}$$

is a Lyapunov function for the $e_{v_{ij}}$-dynamics since it satisfies

$$\dot{V}_{e_v} = -e_{v_{ij}}^T K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r)e_{v_{ij}},$$

and hence, ensuring global exponential convergence of the estimate $\hat{v}_{ij}$ to v_{ij}. Similarly, for the r-dynamics we take the function

$$V_r := \frac{1}{2}(r-1)^2, \tag{36}$$

that gives

$$\dot{V}_r = -c_7(r-1)^2 + (r-1)\frac{c_2^2}{c_1 c_5}K_{ij}^2|v_{ij}|^2|e_{v_{ij}}|^2.$$

Selecting then the functions

$$K_{ij} := c_8 I, \quad c_8 > c_3 - c_5 + c_6 + c_7 c_2$$

and

$$\begin{aligned}K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r) &:= c_9 I + (r-1)\frac{c_2^2}{c_1 c_5}K_{ij}^2|v_{ij}|^2 I \\ &\quad + \frac{c_2^2}{c_6 r}(K_{ij}^2|\hat{v}_{ij}|^2|\hat{x}_{ij}|^2 + 1)I, \quad c_9 > 0,\end{aligned}$$

we can finally establish that the composite function $W_{z_i} + V_{e_v} + V_r$ serves as a Lyapunov function for the complete dynamics with

$$\dot{\overbrace{W_{z_i} + V_{e_v} + V_r}} \leq -c_8\frac{|z_i|^2}{r} - c_7(r-1)^2 - c_9|e_{v_{ij}}|^2,$$

which establishes UGES of the origin. ■

Remark 3: Notice that in the case where the mapping β_i is simply defined as

$$\beta_i(y_i) := \frac{d_{ij}^2}{2}K_{ij}v_{ij},$$

then the resulting error dynamics is described as

$$\dot{z}_i = -K_{ij}v_{ij}v_{ij}^T z_i - e_{v_{ij}}.$$

Then, using the PE condition, UGES of the nominal z_i-system with respect to the origin, and UGES of the origin for the $e_{v_{ij}}$-system we can immediately conclude, e.g. from cascaded systems [9] or Input-to-State (ISS) arguments [14], UGES of the interconnected system.

C. Collaborative localization from fusion of local estimates and measurements

In this subsection, we take advantage of the collaborative setting between the agents, that is the information sharing with their local neighbors, in order to enhance the localization capabilities of the agents, in particular, that do not have direct relative measurements with respect to the target.

To this end, define the fused estimate of the relative coordinates between agent j and the target as

$$\hat{x}_{i0}^{j} := \rho_j - \hat{x}_{ij} \tag{37}$$

$$\rho_0 := 0 \tag{38}$$

Then, the proposed consensus-based estimation mechanism for agent i is given by

$$\dot{\rho}_i := \hat{v}_{i0} + \sum_{j \in \mathcal{N}_i} (\hat{x}_{i0}^{j} - \rho_i), \tag{39}$$

with $\hat{v}_{i0}$ an estimation of the relative velocity v_{i0} when not available to be defined, that exploits the fusion of its own estimate with the ones of its neighbors to produce a more accurate fused-estimate.

Proposition 3: Consider the dynamical system defined in (14)-(16) and assume that v_{ij} is persistently exciting. Then, the dynamical system

$$\dot{\xi}_i := -\frac{K_{ij} d_{ij}^2}{2}(u_{ij} - K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r)(\hat{v}_{ij} - v_{ij})) - K_{ij} \hat{v}_{ij} \hat{v}_{ij}^T \hat{x}_{ij} + \hat{v}_{ij} \tag{40}$$

$$\hat{x}_{ij} := \xi_i + \frac{d_{ij}^2}{2} K_{ij} \hat{v}_{ij} \tag{41}$$

$$\dot{r} := -c_7(r-1) + \frac{c_2^2}{c_1 c_5} K_{ij}^2 |v_{ij}|^2 |\hat{v}_{ij} - v_{ij}|^2 \tag{42}$$

$$\dot{\hat{v}}_{ij} := u_{ij} - K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r)(\hat{v}_{ij} - v_{ij}) \tag{43}$$

$$\dot{\rho}_i := \hat{v}_{i0} + \sum_{j \in \mathcal{N}_i} (\hat{x}_{i0}^{j} - \rho_i) \tag{44}$$

$$\hat{v}_{i0} := \hat{v}_{ij} + \sum_{k \in \mathcal{M}_j, l \in \mathcal{N}_0} \hat{v}_{kl} + \hat{v}_{l0}, \tag{45}$$

for some l, with $\mathcal{M}_j$ defining a directed path from the target to the jth agent, and with $r(0) \geq 1$, ensures that every agent is globally exponentially localized with respect to the target, for some $c_i > 0$ and with gains

$$K_{ij} := c_8 + \frac{c_5 + c_6 + c_7 c_2}{c_3}$$

$$K_{v_i}(\hat{x}_{ij}, \hat{v}_{ij}, v_{ij}, r) := c_9 I + (r-1)\frac{c_2^2}{c_1 c_5} K_{ij}^2 |v_{ij}|^2 I + \frac{c_2^2}{c_6 r}(K_{ij}^2 |\hat{v}_{ij}|^2 |\hat{x}_{ij}|^2 + 1) I.$$

Proof: For $i = 1, \ldots, N$, we define

$$\sigma_i := \rho_i - x_{i0} \tag{46}$$

$$\sigma_0 := 0, \qquad \dot{\sigma}_0 = 0. \tag{47}$$

Then we obtain the consensus system

$$\begin{aligned} \dot{\sigma}_i :=& -\sum_{j \in \mathcal{N}_i} (\sigma_i - \sigma_j) + \sum_{j \in \mathcal{N}_i} (\hat{x}_{ij} - x_{ij}) + e_{v_{ij}} \\ &+ \sum_{k \in \mathcal{M}_j, l \in \mathcal{N}_0} e_{v_{kl}} + e_{v_{l0}} \\ =& -\sum_{j \in \mathcal{N}_i} (\sigma_i - \sigma_j) + \sum_{j \in \mathcal{N}_i} z_i + e_{v_{ij}} \\ &+ \sum_{k \in \mathcal{M}_j, l \in \mathcal{N}_0} e_{v_{kl}} + e_{v_{l0}}, \end{aligned}$$

with σ_i seen as the individual states of the N agents and σ_0 the state of a leader, while the last 4 terms are seen as external signals. Defining the stacked variables

$$\sigma := \texttt{col}(\sigma_0, \ldots, \sigma_N)$$

$$\tau_i := \sum_{j \in \mathcal{N}_i} z_i$$

$$\tau := \texttt{col}(\tau_0, \ldots, \tau_N),$$

and a stacked vector ψ containing all the linear terms in velocity errors, we obtain the dynamics

$$\dot{\sigma} := -(\mathcal{L} \otimes I_3)\sigma + \tau + \psi. \tag{48}$$

As is well known, from the properties of the assumed underlying graph topology, we have that the nominal system $\dot{\sigma} = -(\mathcal{L} \otimes I_3)\sigma$ has a uniformly global exponentially stable equilibrium at the origin. The claim is established by standard arguments on cascaded systems (see for example Lemma 2.1 or Proposition 2.3 of [9]) since the complete error system consists of two nominal UGES subsystems interconnected through the terms τ, ψ that satisfy a linear growth condition. ■

Remark 4: Although not presented here, notice that our results are also applicable for switched communication graphs (due e.g. to loss of communication link or measurements) under the additional assumption of uniform connectivity as is done e.g. for the single-landmark multi-agent localization in the recent work [14]. We stress again that in our setting however the derivative of the relative distances is not required and furthermore, measurement noise is explicitly treated by means of additional filters.

IV. Simulations

In this section we study the efficiency of the obtained algorithms by means of detailed numerical simulations that serve as proof–of–concept. We will consider two two-dimensional scenarios with bidirectional communication topologies and continuous measurements. First, we consider a simple scenario where two dynamic agents are localizing themselves with respect to a static target. We consider that the measurements are perfect and thus, incorporate the algorithm of subsection III-A along with the fusion scheme of subsection III-C. Then, we proceed with a more complex localization scenario with three agents and a dynamic target. For this scenario we further consider that the relative velocity measurements are corrupted by white Gaussian noise and apply

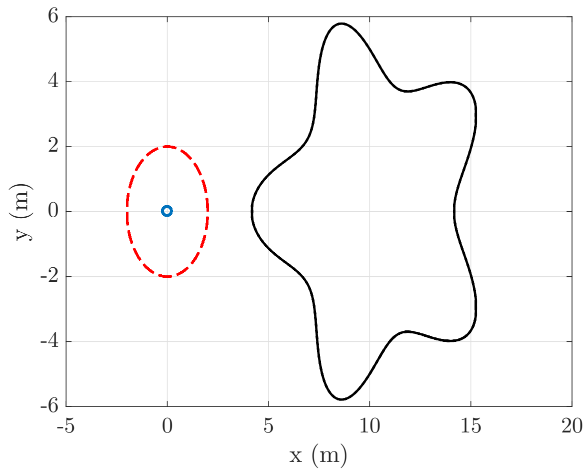

Fig. 1. Scenario 1: Positions of target (blue), vehicle 1 (red) and vehicle 2 (black).

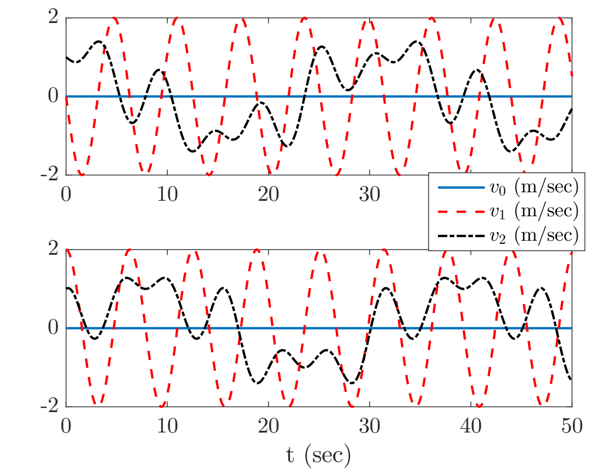

Fig. 2. Scenario 1: Agents' true velocities.

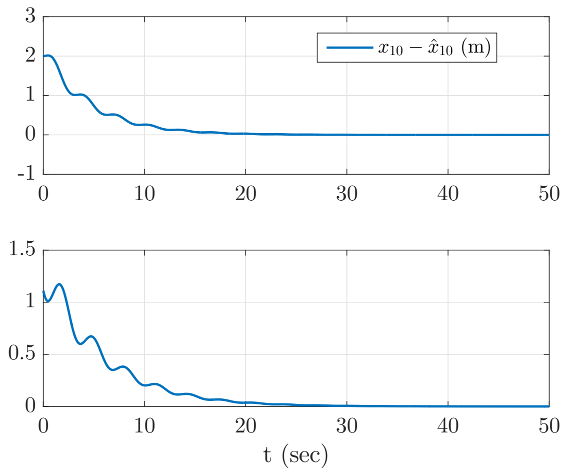

Fig. 3. Scenario 1: Estimation error for x_{10}.

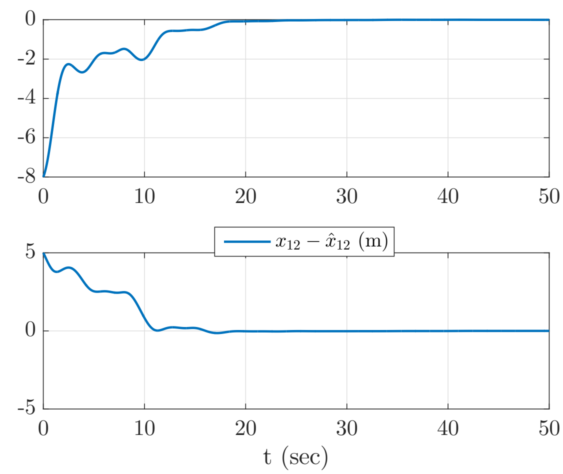

Fig. 4. Scenario 1: Estimation error for x_{12}.

the observer of subsection III-B and the consensus-based scheme of subsection of III-C.

A. Two–agent localization with respect to a static target with noise-free measurements

We consider a target that is static while the two agents move along a circular path and a star-like path respectively, see Fig. 1. The initial positions (in m) and velocities (in m/s) of the agents are respectively given as $x_0(0) = [0, 0]^T$, $x_1(0) = [2, 0]^T$, $x_2(0) = [10, -5]^T$ and $v_0(0) = [0, 0]^T$, $v_1(0) = [0, 2]^T$, $v_2(0) = [1, 1]^T$. The parameters related to the observer and the stability analysis are selected as $c_1 = c_3 = 0.9, c_2 = c_4 = c_9 = 1$, $c_5 = c_6 = c_8 = 0.1$, $c_7 = 0.05$, while the observer gains are chosen as $K_{10} = c_8 + (c_5 + c_6 + c_7 c_2)/c_3$, $K_{12} = K_{13} = K_{21} = 0.1$. There is of course always a compromise between convergence rate and robustness (related e.g. to high-gain effects or noise) and as such we have selected small but reasonable (from the convergence viewpoint) values of the gains. Furthermore, we assume that we do not have any prior knowledge on the relative positions and thus, choose the estimates as $\hat{x}_{ij}(0) = 0$ which translates to initial observer states given by $\xi_i(0) = -\frac{d_{ij}^2(0)}{2} K_{ij} v_{ij}(0)$. Finally, the initial conditions for the fused estimates are taken as $\rho_1(0) = [0, 0]^T$, $\rho_2(0) = [0, 0]^T$.

As it can be observed from Fig. 2, where the velocities of each agent are depicted, the chosen motions are sufficiently rich for observing the relative positions and as such the relative velocities satisfy the persistence-of-excitation condition. The comparison of the errors between the true and estimated relative positions are shown in Figs. 3-5. We can observe a fast, smooth convergence to zero with an exponential rate of convergence.

Finally, we illustrate the transient performances of the fusion schemes for agents 1 and 2 in Fig. 6. We can establish that the two agents are succesfully localized with respect to the target.

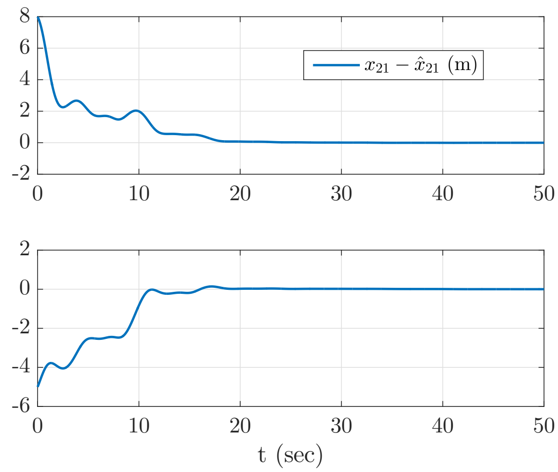

Fig. 5. Scenario 1: Estimation error for x_{21}.

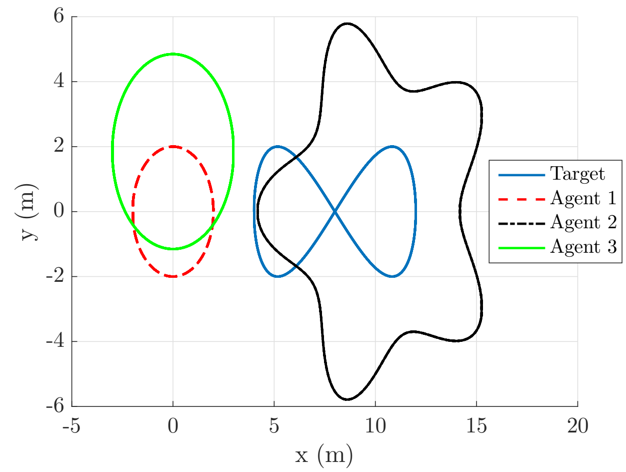

Fig. 7. Scenario 2: Positions of all agents.

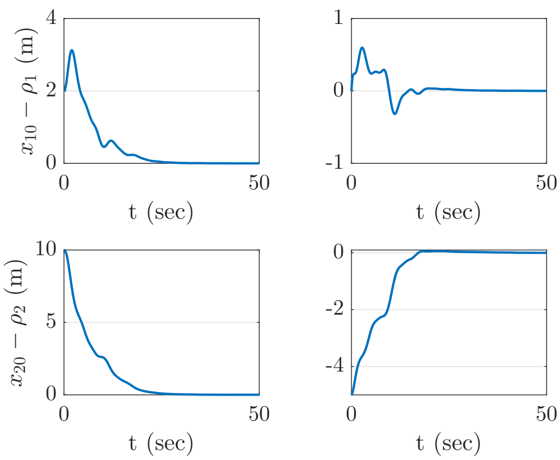

Fig. 6. Scenario 1: Error between fused estimates ρ_i and true relative positions x_{i0}.

B. Three–agent localization with respect to a dynamic target with noisy measurements

For the second scenario, we consider a network of 3 moving agents along with a dynamic target with their positions shown in Fig. 7. The initial positions (in m) and velocities (in m/s) of the agents are respectively given as $x_0(0) = [4, 0]^T$, $x_1(0) = [2, 0]^T$, $x_2(0) = [10, -5]^T$, $x_3(0) = [3\sin(\pi/8), 5\cos(\pi/8)]^T$, $v_0(0) = [0, 4]^T$, $v_1(0) = [0, 2]^T$, $v_2(0) = [1, 1]^T$, $v_3(0) = [6\cos(\pi/8), -6\sin(\pi/8)]^T$. The parameters related to the observer are chosen as $c_1 = c_3 = 0.9$, $c_2 = c_4 = c_9 = 1$, $c_5 = c_6 = c_8 = 0.01$, $c_7 = 0.005$ and the observer gains as $K_{10} = c_8 + (c_5 + c_6 + c_7 c_2)/c_3$, $K_{12} = K_{13} = K_{21} = K_{23} = K_{31} = K_{32} = 0.03$. Some of the gains were given smaller values with respect to the previous scenario in order to reduce the effect of noise and avoid unwanted phenomena such as overshooting. As in the previous scenario, we assume that we do not have any prior knowledge on the relative positions and thus, choose the estimates as $\hat{x}_{ij}(0) = 0$ which translates to initial observer states given by $\xi_i(0) = -\frac{d_{ij}^2(0)}{2} K_{ij} v_{ij}(0)$. In addition, the initial conditions for the fused estimates are again taken as $\rho_1(0) = [0, 0]^T$, $\rho_2(0) = [0, 0]^T$ while the initial condition for the dynamic scaling $r(t)$ is selected as $r(0) = 1$. Furthermore, we consider the standard scenario where relative velocity measurements are corrupted by band-limited white Gaussian noises n_{ij} (although any type of noise can be considered) with noise power intensity $\sigma_m = 10^{-4}/5\ (m/s)^2/Hz$ and a sampling period of $T_s = 10^{-3}\ (s)$.

From Figs. 8, 9 (and although not depicted here, similarly for all relative velocities) we see that the relative velocities are persistently-exciting and thus, we can obtain converging estimates of the relative positions. Furthermore, by zooming

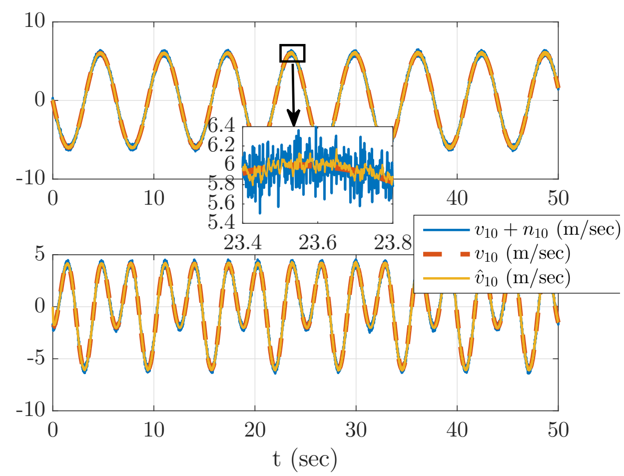

Fig. 8. Scenario 2: Relative velocity v_{10} (noisy, estimated, true).

on a particular time interval we observe the effect of the noise as well as the result of the filtering. Of course, the former can be further adjusted by proper selection of the filter gains. In addition Figs. 10-15 show the errors between the estimated and true relative positions. In all these figures we see that the estimation algorithms have successfully filtered

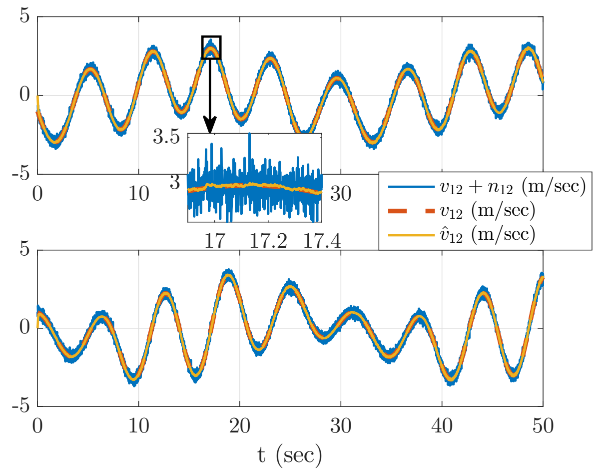

Fig. 9. Scenario 2: Relative velocity v_{12} (noisy, estimated, true).

the measurement noise and the convergence of the estimates is smooth and exponential as expected.

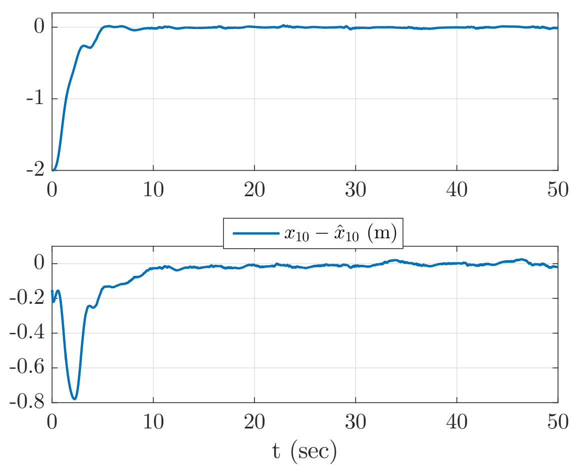

Fig. 10. Scenario 2: Estimation error for x_{10}.

Finally, we can see the fused estimates for the three agents in Fig. 17. We can observe that all agents are succesfully localized with respect to the target and furthermore, that the effect of the noisy measurements has been significantly removed (although some slight oscillations do appear). Hence, the transient behavior is quite smooth and the convergence is exponential as was proposed by the theoretical analysis.

V. Conclusions

We have proposed a global solution to the problem of multi-vehicle localization based on continuous measurements of ranges and relative velocities. Under the standard assumption of persistent relative inter-agent motion related to distance-based multi-agent scenarios, we have presented an algorithm that produces a uniformly globally convergent localization that is established analytically through a novel Lyapunov-based stability analysis. The localization

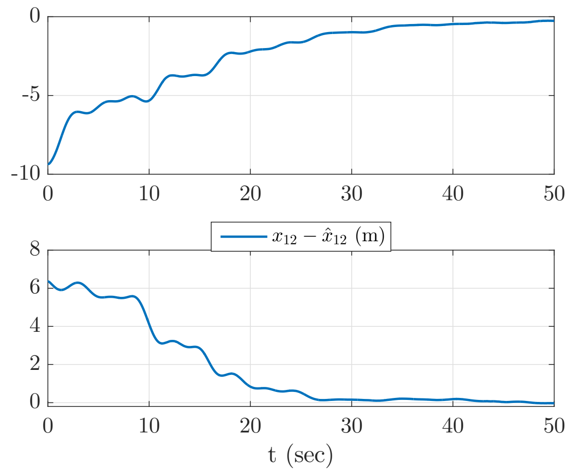

Fig. 11. Scenario 2: Estimation error for x_{12}.

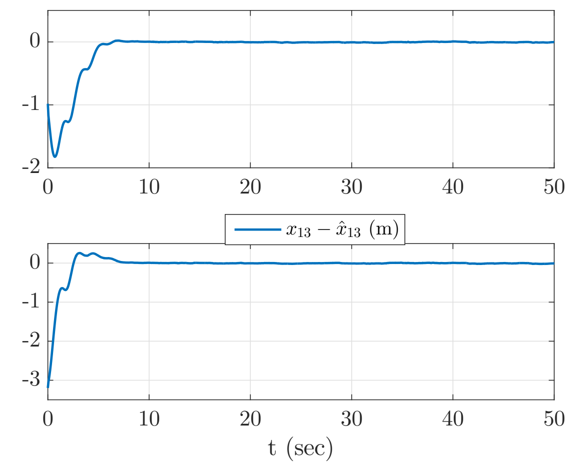

Fig. 12. Scenario 2: Estimation error for x_{13}.

algorithm is a combination of nonlinear observers providing local estimates of the neighbors' relative positions,for each vehicle, and consensus-based filters fusing local estimates with the neighbors' estimates. To support the theoretical developments simulations have been successfully carried out for 2D-scenarios of static and dynamic targets; the former one including also noise on the measured velocities.

Current work is focused on extending our algorithm to account for non-mutually localized networks, filtering of relative distances, measurement latency, field-of-view constraints and uncertainty in the communicated accelerations.

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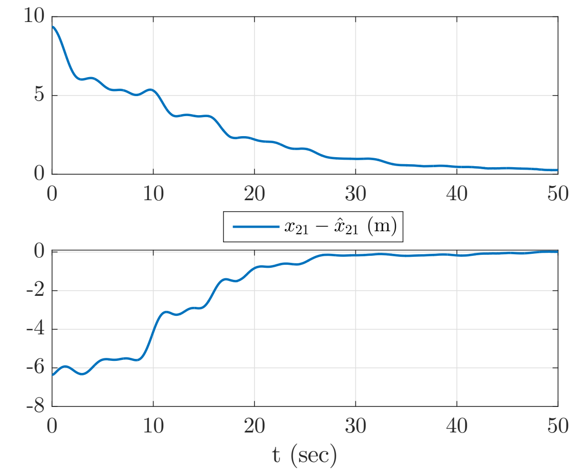

Fig. 13. Scenario 2: Estimation error for x_{21}.

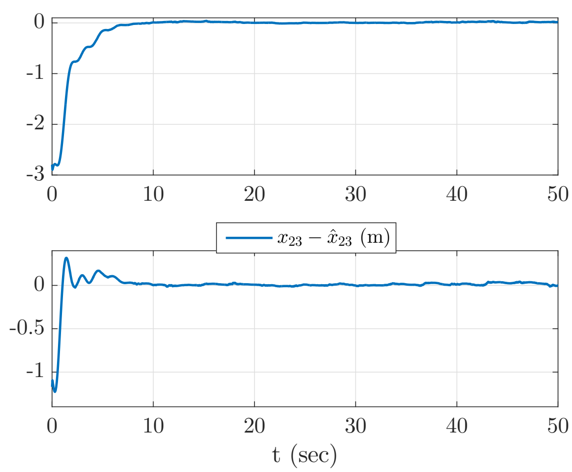

Fig. 14. Scenario 2: Estimation error for x_{23}.

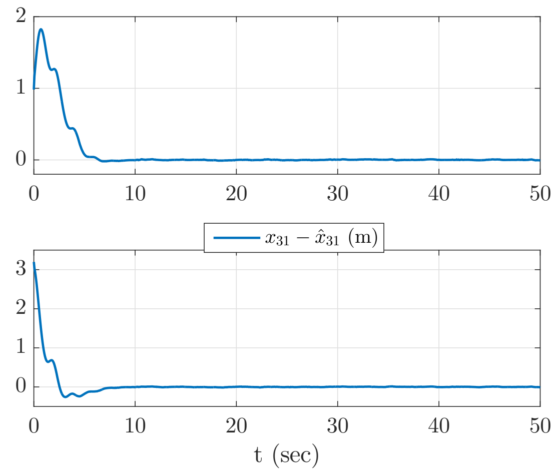

Fig. 15. Scenario 2: Estimation error for x_{31}.

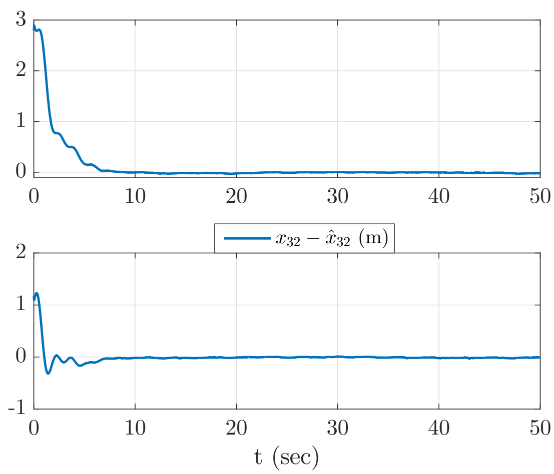

Fig. 16. Scenario 2: Estimation error for x_{32}.

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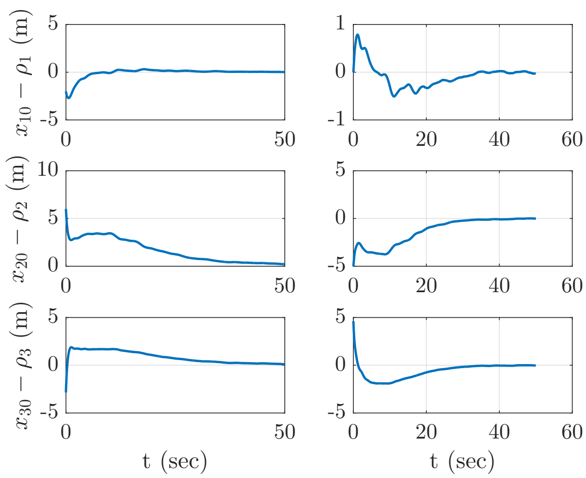

Fig. 17. Scenario 2: Error between fused estimates ρ_i and true relative positions x_{i0}.