Distributed event-triggered control
for multi-agent formation stabilization

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Abstract: This paper addesses the problem of formation control in multi-agent systems (MAS) and adopts an event-triggered strategy to reduce the number of communications between agents. For that purpose, to evaluate its control input, each agent maintains estimators of the states of the other agents. Communication is triggered when the discrepancy between the actual state of an agent and its estimate reaches some threshold. The impact of additive state perturbations is studied. A condition for the convergence of the MAS to a stable formation is studied. Simulations show the effectiveness of the proposed approach.

Keywords: Communication constraints, event-triggered control, formation stabilization, MAS.

1. INTRODUCTION

Distributed cooperative control of a MAS usually requires significant exchange of information between agents. In early contributions, see, *e.g.*, Olfati-Saber et al. (2007); Do (2008), communication was considered permanent. Recently, more practical approaches have been proposed like intermittent communication Wen et al. (2012, 2013), discrete or periodical communication Garcia et al. (2014), or communications triggered by some event, as in Garcia et al. (2015); Viel et al. (2016); Seyboth et al. (2013).

Event-triggered communication is a promising approach to save energy. The main difficulty consists in determining the communication triggering condition (CTC) that will ensure the completion of the task assigned to the MAS, *e.g.*, reaching some consensus, maintaining a formation, *etc.* In a distributed strategy, the states of the other agents are not permanently available, thus each agent usually maintains estimators of the state of its neighbors to estimate their control laws. Nevertheless, without permanent communication, the quality of the state estimates is difficult to evaluate. As a consequence, each agent has to maintain an estimate of its own state only using the information it has shared with its neighbours. When the discrepancy between this own state estimate and its actual state reaches some threshold, the agent triggers a communication. This is the approach considered, *e.g.*, in Seyboth et al. (2013); Garcia et al. (2015); Viel et al. (2016); Sun et al. (2015); Liu et al. (2015); Zhu et al. (2014).

Most of the event-triggered approaches have been applied in the context of consensus in MAS Seyboth et al. (2013); Garcia et al. (2015); Viel et al. (2016); Zhu et al. (2014). This paper focuses on distributed formation control, which has been considered in Liu et al. (2015); Sun et al. (2015); Tang et al. (2011). Formation control consists in driving and maintaining all agents of a MAS to

some reference, *e.g.*, their relative position, orientation, and speed. Various approaches have been considered. For example, displacement-based control Wang et al. (2014), virtual structure Ren and Beard (2004), tensegrity Qingkai et al. (2015), leader-follower Do (2008), *etc.* Most of these formation control approaches assume permanent communication between agents.

Some recent works consider event-triggered approaches for distance-based or displacement-based formation control Liu et al. (2015); Sun et al. (2015); Tang et al. (2011). In these works, the dynamics of the agents are described by a simple integrator, with control inputs considered constant between two successive communications. The proposed CTCs are all centralized, considering different threshold formulations. A constant threshold is considered in Sun et al. (2015) and a time-varying threshold in Liu et al. (2015); Tang et al. (2011). The CTC depends then on the relative positions between agents and the relative discrepancy between actual and estimated agent states. These CTCs reduce the number of triggered communications when the system converges to the desired formation. A minimal time between two communications is also defined. Finally, in all these works, no perturbations are considered. Similar problems have been considered using Logic-Based Communications (LBCs) to reduce the number of communications in Rego et al. (2013); Aguiar and Pascoal (2007). These papers consider MAS with decoupled nonlinear agent dynamics. Each agent has to follow a parametrized path. The paths are designed in a centralized way. LBC allows agents to follow the paths in a synchronized way to reach a desired formation pattern. Communication delays, as well as packet losses are considered.

This paper proposes a strategy to reduce the number of communications for displacement-based formation control, where agent dynamics are described by an Euler-Lagrange system including perturbations. Contrary to LBC techni-

ques, no *a priori* trajectory for agents has to be designed. This work both extends results presented in Qingkai et al. (2015) by introducing an event-triggered strategy, and results of Liu et al. (2015); Sun et al. (2015); Tang et al. (2011) by addressing systems with more complex dynamics than simple integrators. To obtain distributed control laws, estimators of other agents' states are introduced. The proposed distributed CTC involves the relative discrepancy between the actual and estimated agent states. The impact of state perturbations on the formation and on the communications is analyzed.

Hypotheses are introduced in Section 2. The considered formation parametrization is presented in Section 3 and the new decentralized control law is proposed in Section 3.2, based on estimates of the agents' states described Section 3.3. The CTC is presented in Section 3.4. A simulation example is considered in Section 4 to illustrate the reduction in communications obtained. Finally, conclusions are drawn in Section 5.

2. NOTATIONS AND HYPOTHESES

Consider a MAS consisting of a network of N agents which topology is described by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. $\mathcal{N} = \{1, 2, ..., N\}$ is the set of nodes and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ the set of edges of the network. The set of neighbours of Agent i is $\mathcal{N}_i = \{j \in \mathcal{N} \,|\, (i, j) \in \mathcal{E}, \, i \neq j\}$. N_i is the cardinal number of $\mathcal{N}_i$. Let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$ and let $q = \begin{bmatrix} q_1^T & q_2^T & ... & q_N^T \end{bmatrix}^T \in \mathbb{R}^{N.n}$ be the *configuration* of the MAS. The dynamics of each agent is described by the Euler-Lagrange system

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i + d_i(t) \tag{1}$$

where $\tau_i \in \mathbb{R}^n$ is some control input described in Section 3.2, $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix of Agent i, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n}$ is the matrix of the Coriolis and centripetal term on Agent i, and $d_i(t)$ is the additive external state perturbation satisfying $\|d_i(t)\| < D_{\max}$. The state vector of Agent i is $x_i^T = \begin{bmatrix} q_i^T, \dot{q}_i^T \end{bmatrix}$. Assume that the dynamics satisfy the following assumptions:

- $M_i(q_i)$ is symmetric positive and such that there exists $k_M > 0$ satisfying $\forall x, \, x^T M_i(q_i) x \leqslant k_M x^T x$,
- $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite, and such that there exists $k_C > 0$ satisfying $\forall x, \, x^T C_i(q_i, \dot{q}_i) x \leqslant k_C \|\dot{q}_i\| x^T x$,
- For all $(i, j) \in \mathcal{N}$, if Agent j knows q_i and $\dot{q}_i$, it can evaluate $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

In what follows, the notations M_i and C_i are used to replace $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$. One also assumes that each Agent i has access to its own state x_i without error.

3. FORMATION CONTROL PROBLEM

This paper aims at designing a decentralized control strategy to drive a MAS to a desired target formation in some global reference frame $\mathcal{R}$, while reducing as much as possible the communications between agents. The target formation is first described in Section 3.1. The potential energy of a MAS is introduced to quantify the discrepancy between the target and current formations. The distributed control introduced in Section 3.2 tries to minimize this

potential energy. To evaluate the control input, estimators of the coordinate vectors of all agents are managed by each agent, as presented in Section 3.3. A CTC is designed to limit this discrepancy by updating the estimators as described in Section 3.4.

3.1 Formation parametrization

Consider the relative coordinate vector $r_{ij} = q_i - q_j$ between two agents i and j. A target formation is obtained using some control input which ensures that r_{ij} converges to a desired vector r_{ij}^* for all $(i, j) \in \mathcal{N}$:

Définition 1. The MAS converges to the target formation iff

$$\forall (i, j) \in \mathcal{N}, \quad \lim_{t \to \infty} (q_i(t) - q_j(t)) = r_{ij}^*.$$

Consider without loss of generality the first agent as reference and introduce the target relative configuration vector $r^* = \begin{bmatrix} r_{11}^{*T} & ... & r_{1N}^{*T} \end{bmatrix}^T$. Any target relative configuration vector r_{ij}^* can be expressed as $r_{ij}^* = r_{1i}^* - r_{1j}^*$. In this paper, one assumes that the target relative configuration r^* is constant.

As defined for tensegrety formations in Qingkai et al. (2015), the potential energy $P(q, t)$ of the formation related to the disagreement between r_{ij} and r_{ij}^* is expressed as

$$P(q, t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \|r_{ij} - r_{ij}^*\|^2 \tag{2}$$

where $k_{ij} = k_{ji}$ is some spring coefficient, which can be positive, negative, or null. The minimum number of non-zero coefficients $k_{ij} \; i, j \in \mathcal{N}$ to properly define a target formation is $N-1$. Indeed, for a given r^*, all target relative coordinate vectors r_{ij}^* between any agents i and j can be expressed from components of r^*.

Définition 2. The MAS converges to the formation shape with a bounded error iff there exists some $\varepsilon > 0$ such as

$$\lim_{t \to \infty} P(q, t) \leqslant \varepsilon. \tag{3}$$

From Definition 1 and 2, the control law should be designed to reduce the potential energy $P(q, t)$. To describe the evolution of $P(q, t)$, one introduces

$$g_i = \frac{\partial P(q, t)}{\partial q_i} = \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \tag{4}$$

$$s_i = \dot{q}_i + k_p g_i \tag{5}$$

where g_i characterizes the evolution of the discrepancy between the current and target formations and k_p is a positive scalar design parameter.

3.2 Distributed control

The control law proposed in Qingkai et al. (2015) is defined as $\tau_i = \tau_i(q_i, \dot{q}_i, q)$ and aims at reducing $P(q, t)$, thus making the MAS converge to the target formation in case of permanent communication. In this approach, each agent evaluates its control using the state vectors of others agents obtained via permanent communication. Here, in a decentralized context with limited communications between agents, agents cannot have permanent access to q. Thus,

one introduces the estimate $\hat{q}_j^i$ of q_j performed by Agent i to replace the missing information in the control law. The MAS configuration estimated by Agent i is denoted $\hat{q}^i = \begin{bmatrix} \hat{q}_1^{iT} & \dots & \hat{q}_N^{iT} \end{bmatrix}^T \in \mathbb{R}^{N.n}$. The evaluation of $\hat{q}_j^i$ is described in Section 3.3.

In a decentralized context with limited communications, Agent i is able to evaluate

$$\bar{g}_i = \sum_{j=1}^N k_{ij} \left(\bar{r}_{ij} - r_{ij}^* \right) \tag{6}$$

$$\bar{s}_i = \dot{q}_i + k_p \bar{g}_i \tag{7}$$

with $\bar{r}_{ij} = q_i - \hat{q}_j^i$ and $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$. From these terms, it is able to evaluate the decentralized control input

$$\tau_i \left(q_i, \dot{q}_i, \hat{q}^i, \dot{\hat{q}}^i \right) = -k_s \bar{s}_i - k_g \bar{g}_i - k_p \left(M_i \left(q_i \right) \dot{\bar{g}}_i + C_i \left(q_i, \dot{q}_i \right) \bar{g}_i \right). \tag{8}$$

for some $k_g > 0$ and $k_s \geq 1 + k_p \left(k_M + 1 \right)$. Section 3.3 introduces the estimator $\hat{q}_j^i$ of q_j needed in the control (8).

3.3 Estimator dynamics and control law

In what follows, let $t_{j,k}^i$ be the time at which the k-th message sent by Agent j has been received by Agent i. Assuming that there is no communication delay between agents, $t_{j,k}^i = t_{j,k}^p$ for all $(i, p) \in \mathcal{N}_j$. Due to the presence of perturbations and of communications occurring at discrete time instants, errors

$$e_j^i = \hat{q}_j^i - q_j \tag{9}$$

appear between q_j and its estimate $\hat{q}_j^i$ obtained by an other Agent j, which are used in Section 3.4 to trigger communications when e_i^i becomes too large. Thus the estimator will be designed so as to keep e_i^i small.

To stay close to the agent behavior, the dynamics of the estimate is expressed as

$$M_j \left(\hat{q}_j^i \right) \ddot{\hat{q}}_j^i + C_j \left(\hat{q}_j^i, \dot{\hat{q}}_j^i \right) \dot{\hat{q}}_j^i = \hat{\tau}_j^i, \ \forall t \in \left[t_{j,k}^i, \ t_{j,k+1}^i \right[\tag{10}$$

$$\hat{x}_j^i \left(t_{j,k}^i \right) = x_j \left(t_{j,k}^i \right) \tag{11}$$

where $x_j = \begin{bmatrix} q_j^T, \ \dot{q}_j^T \end{bmatrix}^T$ and $\hat{x}_j^i = \begin{bmatrix} \hat{q}_j^{iT}, \ \dot{\hat{q}}_j^{iT} \end{bmatrix}^T$. The control $\hat{\tau}_j^i$ can be evaluated with one of the two following proposed methods.

Basic control:
$$\hat{\tau}_j^i = -k_s \dot{\hat{q}}_j^i \tag{12}$$
In this case, for all i, j such that $k_{ij} \neq 0$, Agents i and j must be connected in the communication graph. The main advantage of this control input is that the estimates of other agent state are not required.

Accurate control:

$$\hat{\tau}_j^i = -k_s \hat{s}_j^i - k_g \hat{g}_j^i \left(\hat{q}^i \right) - k_p \left(M_j \left(\hat{q}_j^i \right) \dot{\hat{g}}_j^i + C_j \left(\hat{q}_j^i, \dot{\hat{q}}_j^i \right) \hat{g}_j^i \right) \tag{13}$$

where $\hat{s}_j^i = \dot{\hat{q}}_j^i + k_p \hat{g}_j^i$, $\hat{g}_j^i \left(\hat{q}^i \right) = \sum_{k=1}^N k_{jk} \left(\hat{r}_{jk} - r_{jk}^* \right)$, $\dot{\hat{g}}_j^i \left(\hat{q}^i \right) = \sum_{k=1}^N k_{jk} \left(\dot{\hat{r}}_{jk} - \dot{r}_{jk}^* \right)$, and $\hat{r}_{jk} = \hat{q}_j^i - \hat{q}_k^i$. This expression of the control input makes the estimator more accurate than (12) and so helps to keep e_i^i small. Note

that if there is no perturbation, i.e., $D_{\max} = 0$, the error e_i^i vanishes. The price to be paid for this method is that every agent needs to maintain an estimator of the state of all other agents, and a fully-connected communication graph is hence required.

Note that, since one assumed that there is no communication delay, these estimators satisfy $\hat{q}_j^i = \hat{q}_j^j$, $\forall (i, j) \in \mathcal{N}$. Estimates are used in the evaluation of the agents control law, but are also used in the evaluation of the CTC presented in what follows.

3.4 Event-triggered communications

Theorem 3 introduces a CTC used to trigger communications to ensure a bounded convergence of the MAS to the target formation. A message broadcast by an Agent i contains the state x_i. The initial value of the state vectors are considered to be known by all agents. In practice, this condition can be satisfied by triggering a communication from all agents at time $t = 0$ to initialize the estimates of its neighbours. Let $\alpha_i = \sum_{j=1}^N k_{ij}$ and $\alpha_M = \max_{i=1:N} \{ \alpha_i \}$.

Theorem 3. Consider a MAS with agent dynamics given by (1) and the control law (8). Consider some design parameters $\eta \geq 0$, $\eta_2 \geq 0$ $\eta_3 \geq 0$, and $b_i > 0$. In absence of communication delays, the agents can be driven to some target formation such that

$$\lim_{t \to \infty} P \left(q, t \right) \leq \eta_2 \tag{14}$$

if the communications are triggered when one of the following conditions is satisfied

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \leq \alpha_M^2 \left(k_e e_i^{iT} e_i^i + k_p k_M \dot{e}_i^{iT} \dot{e}_i^i \right)$$
$$+ \alpha_M k_C^2 k_p \left\| e_i^i \right\|^2 \sum_{j=1}^N k_{ij} \left(\left\| \dot{\hat{q}}_j^i \right\| + \eta_3 \right)^2 + k_g b_i \left\| \dot{q}_i \right\|^2 \tag{15}$$

$$\left\| \dot{q}_i \right\| \geq \left\| \dot{\hat{q}}_j^i \right\| + \eta_3 \tag{16}$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$ and if the bound on the perturbation satisfies

$$D_{\max}^2 \leq \frac{4 k_g k_p \alpha_M \eta_2}{N} - \eta \tag{17}$$

and $\eta \leq \frac{4 k_g k_p \alpha_M \eta_2}{N}$.

The proof of Theorem 3 is given in Appendix 6.

From (14) and (15), one sees that η and σ can be used to adjust the trade-off between the bound η_2 on the potential energy and the amount of triggered communications. If $\eta_2 = 0$ and if there is no perturbation, the system achieves an asymptotic convergence.

The CTC (15) mainly depends on e_i^i and $\dot{e}_i^i$. A communication is triggered by Agent i when e_i^i and $\dot{e}_i^i$ becomes large. To reduce the number of triggered communications, one has to keep e_i^i and $\dot{e}_i^i$ as small as possible, which can be achieved by increasing the accuracy of the estimator, as proposed in Section 3.3, but possibly at the price of a more complex structure for the estimator. The choice of the estimator should hence be driven by a trade-off between complexity and amount of triggered communications.

The perturbations have a direct impact on e_i^i and $\dot{e}_i^i$, thus on the frequency of communications. The sufficient

condition (17) on $D_{\max}$ to have a formation convergence depends on η and on the desired bound η_2 on the potential energy. Nevertheless, we were not able to prove the absence of Zeno behavior.

4. EXAMPLE

Consider a set of $N = 6$ agents with coordinate vector $q_i \in \mathbb{R}^2$. The performance of the proposed algorithm will be evaluated considering the following two dynamical models, assumed identical for all the agents. For Model 1, one has

$$M_i^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C_i^1(\dot{q}_i) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \|\dot{q}_i\|, \qquad (18)$$

with $k_g = 15$, $k_p = 1$, $k_M = 1$, $k_C = 0.1$ and $k_s = 3$. For Model 2, one considers

$$M_i^2 = \begin{bmatrix} 0.56 & -2.23 \\ -2.23 & 9.28 \end{bmatrix} \quad C_i^2(\dot{q}_i) = \begin{bmatrix} 1.40 & -1.76 \\ -1.76 & 2.99 \end{bmatrix} \|\dot{q}_i\|,$$

with $k_g = 15$, $k_p = 0.185$, $k_M = 9.81$, $k_C = 6.33$, and $k_s = 3$. The initial vector state $x(0)$ is such that

$$q(0) = \left[\begin{bmatrix} -0.35 \\ -1.11 \end{bmatrix}^T \begin{bmatrix} 4.59 \\ -4.59 \end{bmatrix}^T \begin{bmatrix} 4.72 \\ 2.42 \end{bmatrix}^T \cdots \right.$$

$$\left. \cdots \begin{bmatrix} 0.64 \\ 1.36 \end{bmatrix}^T \begin{bmatrix} 3.53 \\ 1.56 \end{bmatrix}^T \begin{bmatrix} -1.26 \\ 3.36 \end{bmatrix}^T \right]^T,$$

and $\dot{q}(0) = 0_{2N}$. The vector of relative configurations representing a hexagone

$$r^* = \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 3 \\ \sqrt{3} \end{bmatrix}^T \begin{bmatrix} 2 \\ 2\sqrt{3} \end{bmatrix}^T \begin{bmatrix} 0 \\ 2\sqrt{3} \end{bmatrix}^T \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}^T \right]^T.$$

A stress matrix has been computed using the approach in Qingkai et al. (2015). Its components are such that $k_{i(i+1)} = k_{i(i-1)} = 0.3$, $k_{ii} = 0$ and $k_{ij} = 0.1$ for all (i, j) such that $|i - j| > 1$.

A fully-connected communication graph is considered. The simulation duration is $T = 2.5\ s$ for Model 1 and $T = 6\ s$ for Model 2. Euler integration with a step size $\Delta t = 0.01$ s is used. As the system has been discretised, the minimum delay between the transmission of two messages by the same agent is set to Δt. The perturbation $d(t)$ is assumed constant over each interval of the form $[k\Delta t, (k+1)\Delta t[$. The components of $d(t)$ are independent realizations of zero-mean uniformly distributed noise $U\left(-\frac{D_{\max}}{\sqrt{2}}, \frac{D_{\max}}{\sqrt{2}}\right)$ and are thus such that $\|d(t)\|^2 \leq D_{\max}$. Let N_m be the total number of messages broadcast during a simulation. Performance are evaluated by comparing N_m to the maximum number of messages that can be broadcast $\overline{N}_m = NT/\Delta t \geq N_m$. The percentage of residual communications is defined as $R_{\mathrm{com}} = 100\frac{N_m}{\overline{N}_m}$ and expressed in %. R_{com} indicates the proportions of time slots during which a communication has been triggered.

Figure 1 shows the trajectories of the agents when the control (8) is applied along with the CTC defined in Theorem 3. It can be seen that agents converge to the desired formation with a limited number of communications. Figure 2 shows the evolution of the communication ratio R_{com} and of the potential energy once the system
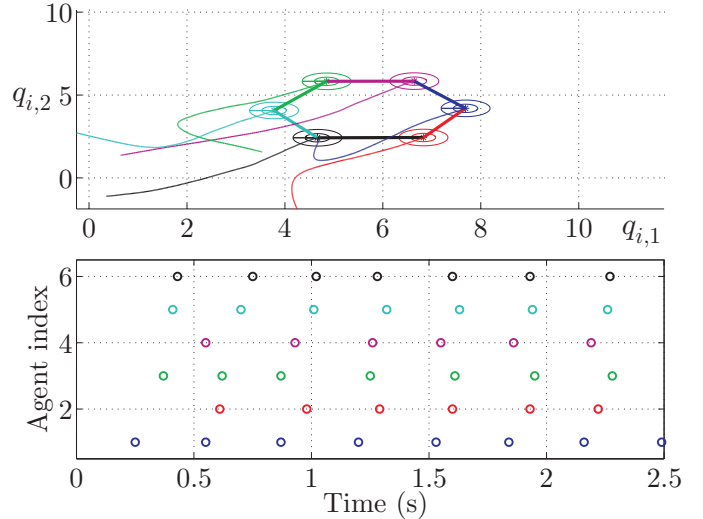

Fig. 1. Results considering the second estimator with $D_{\max} = 20$. Top: Agent trajectory to an hexagonal formation; $\lim_{t \to \infty} P(q, t) = 0.026$. Bottom: communication time instants; $R_{\mathrm{com}} = 2.73\%$.
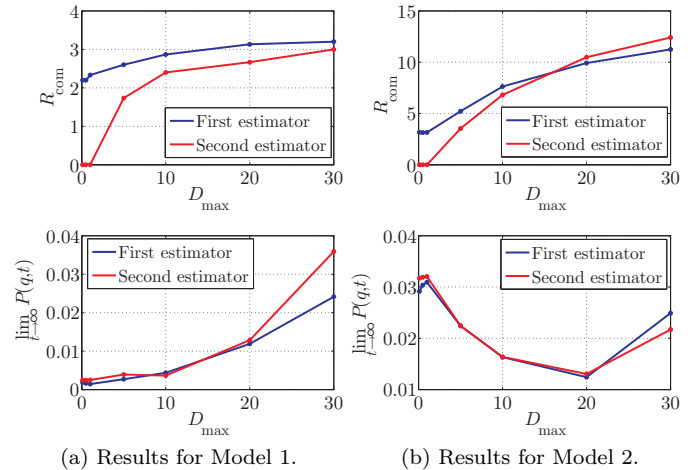

(a) Results for Model 1.   (b) Results for Model 2.

Fig. 2. Evolution of R_{com} and $P(q, t)$ for different values of $D_{\max} = \{\, 0.1, 0.5, 1, 5, 10, 20, 30 \,\}$.

has converged, for different values of $D_{\max}$. When $D_{\max}$ is small, the accurate estimator (13) provides better performance in terms of communication reduction than the basic estimator (12). As expected, the potential energy obtained once the system has converged increases for both estimators with the level of perturbations.

When $D_{\max}$ gets large, the performance of both estimators gets closer. In that case, the simpliest estimator should be preferred.

5. CONCLUSION

This paper presents an event-triggered communication strategy to reach a target formation for MAS with perturbed Euler-Lagrange dynamics. Two estimators of different complexity and accuracy have been considered to provide the missing information required by the control, allowing a trade-off between computation time and amount of triggered communications. Convergence to a desired formation

and influence of state perturbations on the convergence and on the amount of required communications have been studied. Simulations have shown the effectiveness of the proposed method.

In future work, the considered problem will be extended to time-varying formations and trajectory tracking. Proof of absence of Zeno behavior as well as communication delay will also be studied.

6. APPENDIX: PROOF OF CONVERGENCE

Consider the candidate Lyapunov function

$$V = \frac{1}{2} \sum_{i=1}^{N} \left(s_i^T M_i s_i \right) + \frac{k_g}{4} P\left(q, t \right)$$

Taking the time derivative of V leads to

$$\dot{V} = \sum_{i=1}^{N} \left[\frac{1}{2} s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i \right] + \frac{k_g}{4} \frac{d}{dt} P\left(q, t \right) \quad (19)$$

where $\dot{s}_i = \ddot{q}_i + k_p \dot{g}_i$. It can be shown that $\frac{1}{4} \frac{d}{dt} P\left(q, t \right) = \sum_{i=1}^{N} \left(\dot{q}_i - \dot{q}_i^* \right)^T g_i$. One focuses now on the term $M_i \dot{s}_i$ and tries to find an equivalent expression. Consider

$$M_i \dot{s}_i + C_i s_i = M_i \left[\ddot{q}_i + k_p \dot{g}_i \right] + C_i \left[\dot{q}_i + k_p g_i \right]$$
$$= \tau_i + k_p \left(M_i \dot{g}_i + C_i g_i \right) + d_i \quad (20)$$

Using (8), one gets

$$M_i \dot{s}_i + C_i s_i = -k_s \bar{s}_i - k_g \bar{g}_i - k_p \left(M_i \left(\dot{\bar{g}}_i - \dot{g}_i \right) + C_i \left(\bar{g}_i - g_i \right) \right) + d_i \quad (21)$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus $g_i = \bar{g}_i + E_j^i$ with $E_j^i = \sum_{i=1}^{N} k_{ij} e_j^i$. In the same way, one obtains $s_i = \bar{s}_i + k_p E_j^i$. One gets

$$M_i \dot{s}_i + C_i s_i = -k_s \bar{s}_i - k_g \bar{g}_i + k_p \left(M_i \dot{E}_j^i + C_i E_j^i \right) + d_i \quad (22)$$

Put $\dot{V}_1 = \sum_{i=1}^{N} 2 k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i \right)$. Using (22) in (19), one obtains

$$\dot{V} = \sum_{i=1}^{N} \left[s_i^T \left[\frac{1}{2} \dot{M}_i - C_i \right] s_i - k_s s_i^T \bar{s}_i - k_g \left(\dot{q}_i + k_p g_i \right)^T \bar{g}_i \right.$$
$$\left. + k_g \dot{q}_i^T g_i + s_i^T d_i \right] + \frac{1}{2} \dot{V}_1$$

Since $\frac{1}{2} \dot{M}_i - C_i$ is skew symmetric or definite negative, $s_i^T \left[\frac{1}{2} \dot{M}_i - C_i \right] s_i \le 0$. Using $d_i^T s_i \le \frac{1}{2} \left(D_{\max}^2 + s_i^T s_i \right)$, one gets

$$\dot{V} \le \sum_{i=1}^{N} \left[-k_s s_i^T \bar{s}_i - g_i^T \bar{g}_i k_g k_p + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right.$$
$$\left. + k_g \dot{q}_i^T \left(g_i - \bar{g}_i \right) \right] + \frac{1}{2} \dot{V}_1 \quad (23)$$

Note that $\| s_i - \bar{s}_i \|^2 = \| k_p E_j^i \|^2$, thus

$$s_i^T \bar{s}_i = -\frac{1}{2} \left\| k_p E_j^i \right\|^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2} \bar{s}_i^T \bar{s}_i \quad (24)$$

In the same way, one obtains $g_i^T \bar{g}_i = -\frac{1}{2} \left\| E_j^i \right\|^2 + \frac{1}{2} g_i^T g_i + \frac{1}{2} \bar{g}_i^T \bar{g}_i$. Injecting it with (24) in (23)

$$\dot{V} \le \frac{1}{2} \sum_{i=1}^{N} \left[- \left(k_s - 1 \right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + \left(k_s k_p^2 + k_g k_p \right) \left\| E_j^i \right\|^2 \right.$$
$$\left. - k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + D_{\max}^2 + 2 k_g \dot{q}_i^T E_j^i \right] + \frac{1}{2} \dot{V}_1 \quad (25)$$

Using the property $x^T y \le \frac{1}{2} \left(b_i x^T x + \frac{1}{b_i} y^T y \right)$ for some $b_i > 0$, one shows that $2 \dot{q}_i^T \left(g_i - \bar{g}_i \right) \le b_i \| \dot{q}_i \|^2 + \frac{1}{b_i} \left\| E_j^i \right\|^2$. Injecting it in (25) and using $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$ one gets

$$\dot{V} \le \frac{1}{2} \dot{V}_1 + \frac{1}{2} \sum_{i=1}^{N} \left[- \left(k_s - 1 \right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + k_e \left\| E_j^i \right\|^2 \right.$$
$$\left. + b_i k_g \| \dot{q}_i \|^2 - k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + D_{\max}^2 \right] \quad (26)$$

Consider now $\dot{V}_1$. Using the property $x^T y \le \frac{1}{2} x^T x + \frac{1}{2} y^T y$, M_i is symmetric positive definite and the fact that $x^T M_i x \le k_M x^T x$, one shows that

$$\dot{V}_1 \le \sum_{i=1}^{N} k_p \left(\left(k_M + 1 \right) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i \right] \right)$$

The terms $\epsilon_i = E_j^{iT} C_i^T C_i E_j^i$ may be summed as

$$\sum_{i=1}^{N} \epsilon_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{ij} e_j^{iT} \| C_i \|^2 e_\ell^i \quad (27)$$

Remind $x^T C_i x \le k_c \| \dot{q}_i \| x^T x$ and using $x^T y \le \frac{1}{2} \left(x^T x + y^T y \right)$, one gets

$$\sum_{i=1}^{N} \epsilon_i \le \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left(e_j^{iT} \| C_i \|^2 e_j^i + e_\ell^{iT} \| C_i \|^2 e_\ell^i \right)$$
$$\le \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} k_{ij} \left(e_j^{iT} e_j^i k_C^2 \| \dot{q}_i \|^2 \right)$$

Since there is no communication delay, $e_j^i = e_j^j$. As a consequence,

$$\sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \| e_j^i \|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| e_j^j \right\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ji} \| e_i^i \|^2.$$
(28)

Since $k_{ij} = k_{ji}$, using the second CTC (16) leads to

$$\sum_{i=1}^{N} \epsilon_i \le \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} k_C^2 \| e_i^i \|^2 \sum_{j=1}^{N} k_{ij} \left(\left\| \dot{\hat{q}}_j^i \right\| + \eta_3 \right)^2 \right).$$

In the same way, one shows that

$\sum_{i=1}^{N} E_j^{iT} E_j^i \le \sum_{i=1}^{N} \alpha_{\mathrm{M}}^2 \| e_i^i \|^2$ and $\sum_{i=1}^{N} \dot{E}_j^{iT} \dot{E}_j^i \le \sum_{i=1}^{N} \alpha_{\mathrm{M}}^2 \dot{e}_i^{iT} \dot{e}_i^i$. Using this result in (26), one gets

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^{N} \left[-k_{s2} s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + D_{\max}^2 - k_p k_g g_i^T g_i \right.$$
$$-k_p k_g \bar{g}_i^T \bar{g}_i + k_g b_i \left\| \dot{q}_i \right\|^2 + k_p k_M \alpha_{\mathrm{M}}^2 \left\| \dot{e}_i^i \right\|^2$$
$$\left. + \alpha_{\mathrm{M}}^2 k_e \left\| e_i^i \right\|^2 + \alpha_{\mathrm{M}} k_p k_C^2 \left\| e_i^i \right\|^2 \sum_{j=1}^{N} k_{ji} \left[\left\| \dot{\dot{q}}_j^i \right\| + \eta_3 \right]^2 \right]$$

with $k_{s2} = (k_s - 1 - k_p (k_M + 1))$. The CTC (15) leads to

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^{N} \left[-k_{s2} s_i^T s_i - 2 k_g k_p g_i^T g_i + D_{\max}^2 + \eta \right]$$

One deduces that $\dot{V}$ is negative if

$$\|g_i\|^2 > \frac{D_{\max}^2 + \eta}{2 k_g k_p} \tag{29}$$

The candidate Lyapunov function V is lower-bounded by zero. $\dot{V}$ is continuous and negative semi-definite since (29) is respected. Using LaSalle principle, it can be concluded that $\dot{V} \to 0$ when $t \to \infty$ and $\lim_{t \to \infty} \|g_i\|^2 \to \frac{D_{\max}^2 + \eta}{2 k_g k_p}$. Thus V converges to a bounded value.

One will deduce from $\|g_i\|^2 \to \frac{D_{\max}^2 + \eta}{2 k_g k_p}$ some conditions on $D_{\max}$ and η to show the convergence bound η_2 on $P(q, t)$.

Conditions on $D_{\max}$ and η   One first try to express g_i as a function of $P(q, t)$:

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right)^T g_i \tag{30}$$

The inequality $\left| x^T y \right| \leq \frac{b_i}{2} x^T x + \frac{1}{2 b_i} y^T y$ is used to get, for some $b_i = \alpha_i$

$$\sum_{i=1}^{N} \|g_i\|^2 \leq \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left(\alpha_i \left\| r_{ij} - r_{ij}^* \right\|^2 + \frac{1}{\alpha_i} \|g_i\|^2 \right) \tag{31}$$

Since $\alpha_i = \sum_{j=1}^{N} k_{ij}$, one obtains

$$\frac{1}{2} \sum_{i=1}^{N} \|g_i\|^2 \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 = \alpha_{\mathrm{M}} P(q, t) \tag{32}$$

Then, it can be deduced that $P(q, t)$ converges within a domain bounded by η_2 provided that $\lim_{t \to \infty} \sum_{i=1}^{N} \|g_i\|^2 \leq \lim_{t \to \infty} 2 \alpha_{\mathrm{M}} P(q, t)$. Then, one should have

$$\sum_{i=1}^{N} \frac{D_{\max}^2 + \eta}{2 k_g k_p} \leq 2 \alpha_{\mathrm{M}} \eta_2 \tag{33}$$

$$D_{\max}^2 \leq \frac{4 k_g k_p \alpha_{\mathrm{M}} \eta_2}{N} - \eta \tag{34}$$

Since $D_{\max}^2$ has to be positive, this provides a condition on η.

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