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Cooperative guidance of a fleet of UAVs for multi-target discovery and tracking in presence of obstacles using a set membership approach ⋆

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Abstract: This paper presents a set-membership approach for the coordinated control of a fleet of UAVs aiming to search and track an *a priori* unknown number of targets spread over some area of interest. The originality of the approach lies in the description of the perturbations and measurement uncertainties via bounded sets. Sets guaranteed to contain the actual state of already detected targets are provided using the set-membership approach. A specific criterion evaluating the uncertainty of the detected and still to be detected target has been defined, which is used to determine the trajectories of the UAVs so as to decrease the global estimation uncertainty. The criterion accounts for potential occlusions in the field of view of UAVs due to obstacles located on the zone. Simulations show that the proposed control input optimization is able to provide good localization and tracking performance for multiple targets.

Keywords: Cooperative guidance, membership-set estimation, target search and tracking

Observation of multiple targets by a fleet of cooperative UAVs constitutes a challenging task involved in a wide range of applications including crowd monitoring, search operations, surveillance, or tactical patrolling. The determination of the UAV trajectories must fulfill the two following requirements. All the targets contained within the search area must be detected and the detected target trajectories must be tracked with sufficient precision. These usually time-critical missions may be more efficiently fulfilled when the agents are cooperating. Fleets of UAVs have already demonstrated their flexibility to mission requirements, robustness to faults, and ability to achieve the global objective in a shorter time.

In this paper, our aim is to control a fleet of UAVs so as to search and track an *a priori* unknown number of targets moving in an urban-type environment. Most of the methods presented in the surveys Robin and Lacroix (2016); Khan et al. (2018), addressed either the target detection or the target tracking problem, but seldom tackled simultaneously the two aspects. Examples of methods addressing both aspects can be found, *e.g.*, in Adamey et al. (2017); Dames (2017). In Adamey et al. (2017), a grid in the configuration space is considered. A Bayesian filter updates the probability attached to each grid point to be a target location. An allocation of the UAVs to the region of interest is performed using a binary tree search. The approach in Dames (2017) defines search patterns using Lloyd's algorithm, combined with a probability hypothesis density filter to evaluate the occurrence probability of targets, allowed to enter or leave the search area. Target discovery and tracking is performed combining information from sensors mounted on several UAVs. This problem is formulated as a path planning problem to determine the future UAV sensor locations that are reachable and decrease the target state estimation uncertainty. Various methods have been considered in this context, see, *e.g.*, Yu et al. (2015); Capitan et al. (2013); Morbidi and Mariottini (2013).

In all of the aforementioned approaches, a stochastic framework is used to describe the measurement errors and model uncertainties. As pointed out in Gu et al. (2015), the resulting performance may prove sensitive to the *a priori* assumptions on the probability density functions (pdfs) describing the process and measurement noises. As the definition of suitable pdfs might prove tedious, a set-membership description of uncertainties is suggested in Gu et al. (2015). The only assumption made on the noises and uncertainties is that their realizations remain within known bounds. Using this description, one no longer searches for a single point estimate associated with a *posterior* density function but for sets guaranteed to contain the target states at each time step.

In this paper, we assume that each UAV is equipped with a sensor able to detect and localize targets in some compact subset of the search area. A distributed set-membership estimator has been presented in Reynaud et al. (2018) that accounts for information provided by each UAV, but also for the absence of detected target in the sensed subset. Compared to Reynaud et al. (2018), this paper addresses

⋆ This work has been performed with DGA support.

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the target discovery and tracking problem in urban-type environment, where buildings and obstacles are described by convex polytopes. The information provided by UAV sensors has thus to account for visibility issues. A control input for each UAV is derived by predicting the impact of future measurements on the set estimates of target states. The control inputs minimizing the estimation uncertainty are selected.

The paper is organized as follows. Section 1 describes the multi-target multi-UAV detection and tracking problem. In Section 2, the set membership estimator introduced in Reynaud et al. (2018) is recalled. Section 3 describes the way obstacles modify the field of view of UAVs. Section 4 presents a criterion to evaluate the impact of future measurements on target state estimates and the cooperative guidance scheme that defines for each UAV the trajectory to follow so as to minimize an upper bound of the estimation uncertainty. In Section 5, the control input design algorithms are evaluated on simulations. Section 6 concludes this paper.

1. PROBLEM FORMULATION

We consider N_u identical UAVs flying over some area of interest, assumed to be a compact subset of a plane. Buildings and obstacles are present in the environment of the UAVs. They are represented by a set of convex polytopes $\mathcal{P}_\ell$, $\ell = 1, \ldots, N_\mathrm{o}$, each of which is defined by its set of vertices $\mathcal{V}_\ell = \{\mathbf{z}_{\ell,1}, \ldots, \mathbf{z}_{\ell,n_\ell}\}$, where $\mathbf{z}_{\ell,m} \in \mathbb{R}^3$. N_t targets are potentially moving within the area of interest. They are assumed not being able to enter the obstacles. N_t is fixed but not known *a priori*. Each UAV is equipped with a sensor providing observations of some subset of the area of interest. The objective of the fleet of UAVs is to detect and track the targets using these observations.

The dynamics of targets and UAVs are described by discrete-time dynamical systems of the form

$$\mathbf{x}^\mathrm{T}_{j,k+1} = \mathbf{g}^\mathrm{T}_k \left(\mathbf{x}^\mathrm{T}_{j,k}, \mathbf{v}_{j,k}\right) \quad (1)$$

$$\mathbf{x}^\mathrm{U}_{i,k+1} = \mathbf{f}^\mathrm{U}_k \left(\mathbf{x}^\mathrm{U}_{i,k}, \mathbf{u}_{i,k}\right) \quad (2)$$

where $\mathbf{x}^\mathrm{T}_{j,k} \in \mathbb{R}^{n_\mathrm{T}}$ and $\mathbf{x}^\mathrm{U}_{i,k} \in \mathbb{R}^{n_\mathrm{U}}$ are the state vectors of the jth target and the ith UAV, respectively. Time is sampled with a constant period T and k represents the discrete time instant ($t = kT$). The inputs $\mathbf{v}_{j,k}$ are unknown target state perturbations belonging to some known box common to all targets $[\mathbf{v}_k]$ and $\mathbf{u}_{i,k}$ is the control input for UAV i, whose values belong to the set $\mathbb{U}$ of admissible control inputs. At time $k = 0$, all initial target states $\mathbf{x}^\mathrm{T}_{j,0}$ are only assumed to belong to some *a priori* known compact set $\mathbb{Z}_0 \subset \mathbb{R}^{n_\mathrm{T}}$.

1.1 Representation of information

While evolving along their trajectories, at each time instant k, the UAVs observe subsets $\mathbb{F}_i(\mathbf{x}^\mathrm{U}_{i,k}) \subset \mathbb{R}^{n_\mathrm{T}}$, $i = 1, \ldots, N_\mathrm{u}$ of the area of interest. $\mathbb{F}_i(\mathbf{x}^\mathrm{U}_{i,k})$ is called the *field of view* (FoV) of UAV i at time k. More details about $\mathbb{F}_i(\mathbf{x}^\mathrm{U}_{i,k})$ are provided in Section 3. Let $\mathcal{L}_{i,k}$ be the list of the targets detected by UAV i at time k. Assuming that the probabilities of non-detection and of false alarm are null, one has

$$\mathbf{x}^\mathrm{T}_{j,k} \in \mathbb{F}_i\left(\mathbf{x}^\mathrm{U}_{i,k}\right) \Leftrightarrow j \in \mathcal{L}_{i,k}. \quad (3)$$

If the jth target state belongs to $\mathbb{F}_i(\mathbf{x}^\mathrm{U}_{i,k})$, the ith UAV collects a noisy observation of $\mathbf{x}^\mathrm{T}_{j,k}$ described as

$$\mathbf{y}_{i,j,k} = \mathbf{h}_i\left(\mathbf{x}^\mathrm{U}_{i,k}, \mathbf{x}^\mathrm{T}_{j,k}\right) + \mathbf{w}_{i,j,k} \quad (4)$$

where $\mathbf{h}_i$ is the observation equation and $\mathbf{w}_{i,j,k}$ is the measurement noise, bounded in the known box $[\mathbf{w}_k]$.

Each UAV i maintains a set $\mathcal{D}_{i,k}$ of indices of targets already detected at time k. The availability of $\mathbb{F}_i(\mathbf{x}^\mathrm{U}_{i,k})$ and of the detection or non-detection information are used to evaluate a set of *set estimates* $\mathcal{Z}_{i,k} = \{\mathbb{Z}_{i,j,k}\}_{j\in\mathcal{D}_{i,k}}$ containing the state values of the already detected targets. Each set $\mathbb{Z}_{i,j,k}$ has to contain all possible values of $\mathbf{x}^\mathrm{T}_{j,k}$ that are consistent with the information available to UAV i up to time k. Moreover, UAV i also maintains a set $\overline{\mathbb{Z}}_{i,k}$ containing the possible state values of not detected targets.

1.2 Target state estimation uncertainty

The target state estimation uncertainty can be evaluated using the previously introduced set estimates. The choice considered in Reynaud et al. (2018) is

$$\Phi\left(\mathcal{Z}_{i,k}, \overline{\mathbb{Z}}_{i,k}\right) = \frac{1}{\max\{1, |\mathcal{D}_{i,k}|\}} \sum_{j\in\mathcal{D}_{i,k}} \phi\left(\mathbb{Z}_{i,j,k}\right) + \alpha\phi\left(\overline{\mathbb{Z}}_{i,k}\right)$$

where $\phi\left(\mathbb{Z}_{i,j,k}\right)$ is the volume of the set $\mathbb{Z}_{i,j,k}$, $|\mathcal{D}_{i,k}|$ is the cardinal number of $\mathcal{D}_{i,k}$, and α some parameter to adjust the relative importance of the state estimation uncertainties of detected and of not yet detected targets. The (target state) estimation uncertainty at time k is defined as the average of $\Phi\left(\mathcal{Z}_{i,k}, \overline{\mathbb{Z}}_{i,k}\right)$ over all UAVs

$$\Phi_k = \frac{1}{N_\mathrm{u}} \sum_{i=1}^{N_\mathrm{u}} \Phi\left(\mathcal{Z}_{i,k}, \overline{\mathbb{Z}}_{i,k}\right). \quad (5)$$

1.3 Communications

Assume that the topology of the fleet is described by an undirected graph $\mathcal{G} = (\mathcal{N}_\mathrm{U}, \mathcal{E})$. $\mathcal{N}_\mathrm{U} = \{1, 2, ..., N_\mathrm{U}\}$ is the set of nodes and $\mathcal{E} \subset \mathcal{N}_\mathrm{U} \times \mathcal{N}_\mathrm{U}$ the set of edges of the network. The set of neighbors of UAV i is $\mathcal{N}_i = \{j \in \mathcal{N}_\mathrm{U} \,|\, (i,j) \in \mathcal{E},\ i \neq j\}$. UAV i may exchange information with all its neighbors, which allows each of the neighboring UAVs to update their set estimates.

2. SET ESTIMATION

At time k, the information available at UAV i consists in the sets $\mathcal{D}_{i,k}$, $\mathcal{Z}_{i,k} = \{\mathbb{Z}_{i,j,k}\}_{j\in\mathcal{D}_{i,k}}$ and $\overline{\mathbb{Z}}_{i,k}$. The way these sets are updated at time $k+1$ considering additional information available has been detailed in Reynaud et al. (2018). The update rules alternating a prediction step followed by a correction step are briefly recalled here.

2.1 Prediction step

The prediction step is only used under the hypothesis of moving targets. The predicted set of detected targets is simply evaluated as $\mathcal{D}_{i,k+1|k} = \mathcal{D}_{i,k}$. For each target $j \in \mathcal{D}_{i,k+1|k}$, the set of possible target state values at time

$k+1$ consistent with $\mathbb{Z}_{i,j,k}$, with the dynamics (1), and the bounded state perturbation is

$$\mathbb{Z}_{i,j,k+1|k} = \mathbf{g}_k^{\mathrm{Z}}\left(\mathbb{Z}_{i,j,k}, [\mathbf{v}_k]\right), \text{ for all } j \in \mathcal{D}_{i,k+1|k} \quad (6)$$

Similarly, the predicted set $\overline{\mathbb{Z}}_{i,k+1|k}$ has to contain all possible state values of targets not already detected and evolving with dynamics (1)

$$\overline{\mathbb{Z}}_{i,k+1|k} = \mathbf{g}_k^{\mathrm{Z}}\left(\overline{\mathbb{Z}}_{i,k}, [\mathbf{v}_k]\right). \quad (7)$$

2.2 Correction step from measurements

Assume that at time $k+1$, UAV i has evaluated $\mathcal{L}_{i,k+1}$ from the observation of $\mathbb{F}_i(\mathbf{x}^{\mathrm{U}}_{i,k+1})$ and, for each $j \in \mathcal{L}_{i,k+1}$, has access to $\mathbf{y}_{i,j,k+1}$ obtained using (4). The set of detected targets is then updated as $\mathcal{D}_{i,k+1|k+1} = \mathcal{D}_{i,k+1|k} \cup \mathcal{L}_{i,k+1}$. Then, three cases have to be considered.

If Target j has been previously detected, *i.e.*, if $j \in \mathcal{D}_{i,k}$ and if $\mathbf{x}^{\mathrm{T}}_{j,k+1} \in \mathbb{F}_i(\mathbf{x}^{\mathrm{U}}_{i,k+1})$, only $\mathbb{Z}_{i,j,k+1|k}$ is updated using the new measurement $\mathbf{y}_{i,j,k+1}$ and the measurement noise bound $[\mathbf{w}_{k+1}]$ by

$$\mathbb{Z}_{i,j,k+|k+1} = \mathbb{Z}_{i,j,k+1|k} \cap \mathbf{h}^{-1}_{i,k+1}\left(\mathbf{y}_{i,j,k+1} - [\mathbf{w}_{k+1}]\right), \quad (8)$$

where $\mathbf{h}^{-1}_{i,k+1}\left(\mathbf{y}_{i,j,k+1} - [\mathbf{w}_k]\right)$ is the pre-image of the box $\mathbf{y}_{i,j,k+1} - [\mathbf{w}_k]$ by the function $\mathbf{h}_{i,k+1}(\mathbf{z}) = \mathbf{h}_{k+1}(\mathbf{r}_{i,k+1}, \mathbf{z})$.

If Target j has just been detected at time $k+1$, *i.e.*, if $j \notin \mathcal{D}_{i,k}$, but $\mathbf{x}^{\mathrm{T}}_{j,k+1} \in \mathbb{F}_i(\mathbf{x}^{\mathrm{U}}_{i,k+1})$, one knows that $\mathbf{x}^{\mathrm{T}}_{j,k+1} \in \overline{\mathbb{Z}}_{i,k+1|k}$ and that $\mathbf{x}^{\mathrm{T}}_{j,k+1}$ has to be consistent with $\overline{\mathbb{Z}}_{i,k+1|k}$, $\mathbf{y}_{i,j,k+1}$, the measurement equation (4), and the measurement noise bound $[\mathbf{w}_{k+1}]$, thus

$$\mathbb{Z}_{i,j,k+1|k+1} = \overline{\mathbb{Z}}_{i,k+1|k} \cap \mathbf{h}^{-1}_{i,k+1}\left(\mathbf{y}_{i,j,k+1} - [\mathbf{w}_{k+1}]\right). \quad (9)$$

If Target j has been already detected, but is not detected at time $k+1$, *i.e.*, $j \in \mathcal{D}_{i,k}$, but $\mathbf{x}^{\mathrm{T}}_{j,k+1} \notin \mathbb{F}_i(\mathbf{x}^{\mathrm{U}}_{i,k+1})$,

$$\mathbb{Z}_{i,j,k+1|k+1} = \mathbb{Z}_{i,j,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{r}_{i,k+1}\right), \quad (10)$$

where $\mathbb{B} \setminus \mathbb{A} = \{x \in \mathbb{B} \,|\, x \notin \mathbb{A}\}$. To evaluate the set containing the state of targets still to be detected, one has to account for the fact that all targets in $\mathbb{F}_i(\mathbf{x}^{\mathrm{U}}_{i,k+1})$ have been processed. Thus, one has

$$\overline{\mathbb{Z}}_{i,k+1|k+1} = \overline{\mathbb{Z}}_{i,k+1|k} \setminus \mathbb{F}_i\left(\mathbf{x}^{\mathrm{U}}_{i,k+1}\right). \quad (11)$$

2.3 Correction step after communications

During each time step, UAV i communicates to its neighbors the sets $\mathcal{D}_{i,k+1|k+1}$, $\mathcal{Z}_{i,k+1|k+1}$, and $\overline{\mathbb{Z}}_{i,k+1|k+1}$ and receives the corresponding sets from its neighbors. Using this additional information, UAV i updates its estimates.

The set of detected targets is evaluated taking the unions of all detected targets

$$\mathcal{D}_{i,k+1} = \mathcal{D}_{i,k+1|k+1} \bigcup_{\ell \in \mathcal{N}_i} \mathcal{D}_{\ell,k+1|k+1}. \quad (12)$$

Consider the subset $\mathcal{N}^j_{i,k+1}$ of neighbors of UAV i which have already detected target j at time $k+1$ and the subset $\overline{\mathcal{N}}^j_{i,k+1}$ of neighbors which have not detected target j. For all targets j which have been already detected by UAV i, the update equation is

$$\mathbb{Z}_{i,j,k+1} = \mathbb{Z}_{i,j,k+1|k+1} \bigcap_{\ell \in \mathcal{N}^j_{i,k+1}} \mathbb{Z}_{\ell,j,k+1|k+1} \bigcap_{\ell \in \overline{\mathcal{N}}^j_{i,k+1}} \overline{\mathbb{Z}}_{\ell,k+1|k+1}.$$

For all targets j which have not yet been detected by UAV i, the update equation is

$$\mathbb{Z}_{i,j,k+1} = \overline{\mathbb{Z}}_{i,k+1|k+1} \bigcap_{\ell \in \mathcal{N}^j_{i,k+1}} \mathbb{Z}_{\ell,j,k+1|k+1} \bigcap_{\ell \in \overline{\mathcal{N}}^j_{i,k+1}} \overline{\mathbb{Z}}_{\ell,k+1|k+1}.$$

The subset of the state space still to be explored becomes

$$\overline{\mathbb{Z}}_{i,k+1} = \overline{\mathbb{Z}}_{i,k+1|k+1} \bigcap_{\ell \in \mathcal{N}_i} \overline{\mathbb{Z}}_{\ell,k+1|k+1}. \quad (13)$$

3. EVALUATION OF THE FIELD OF VIEW

In the frame $\mathcal{F}_i$ attached to the ith UAV, one assumes that the FoV is defined as a conical combination of N_{f} vectors $\mathbf{x}^{\mathrm{F}}_{i,1}, \ldots, \mathbf{x}^{\mathrm{F}}_{i,N_{\mathrm{f}}}$. When expressed in the state space of the UAV, in absence of obstacles, the FoV is

$$\overline{\mathbb{F}}_i(\mathbf{x}^{\mathrm{U}}_{i,k}) = \{\alpha_{i,1}\mathbf{x}^{\mathrm{F}}_{i,1}\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right) + \cdots + \alpha_{i,N_{\mathrm{f}}}\mathbf{x}^{\mathrm{F}}_{i,N_{\mathrm{f}}}\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right) \\ \mid \alpha_{i,1} \geqslant 0, \ldots, \alpha_{i,N_{\mathrm{f}}} \geqslant 0\}, \quad (14)$$

where $\mathbf{x}^{\mathrm{F}}_{i,n}(\mathbf{x}^{\mathrm{U}}_{i,k})$ are the coordinates of $\mathbf{x}^{\mathrm{F}}_{i,n}$ expressed in the global frame.

The area of interest contains obstacles. Due to occlusions, these obstacles will limit the FoV $\overline{\mathbb{F}}_i(\mathbf{x}^{\mathrm{U}}_{i,k})$ of UAV i. To simplify the analysis, UAVs are assumed to be represented by points in $\mathbb{R}^{n_{\mathrm{U}}}$ and one only evaluates the impact of obstacles on the intersection of the FoV with the area of interest, which is also a compact subset of a plane. A point $\mathbf{z}$ in the area of interest will be visible by UAV i if and only if it belongs to the intersection of $\overline{\mathbb{F}}_i(\mathbf{x}^{\mathrm{U}}_{i,k})$ with the plane containing the area of interest, and the segment linking $\mathbf{z}$ and $\mathbf{x}^{\mathrm{U}}_{i,k}$ does not intersect any obstacle polytope. If the segment linking $\mathbf{z}$ and $\mathbf{x}^{\mathrm{U}}_{i,k}$ intersects the ℓth polytope $\mathcal{P}_\ell$, it can be seen to belong to the shadow of $\mathcal{P}_\ell$, assuming that UAV i behaves as a point light source.

The impact of obstacles on the FoV requires thus to evaluate the shadow on the area of interest created by the obstacles when illuminated by UAV i. For that purpose, one adopts the approach introduced in Blinn (1988). The shadowed areas have then to be removed from the intersection of $\overline{\mathbb{F}}_i(\mathbf{x}^{\mathrm{U}}_{i,k})$ with the area of interest.

Consider the set of vertices $\mathcal{V}_\ell = \{\mathbf{z}_{\ell,1}, \ldots, \mathbf{z}_{\ell,n_\ell}\}$ of the ℓth obstacle $\mathcal{P}_\ell$. Let $\mathbf{z}^{\mathrm{U}}_{i,k}$ denote the projection of the state $\mathbf{x}^{\mathrm{U}}_{i,k}$ of the ith UAV in the space $\mathbb{R}^3$ of obstacles. Since $\mathcal{P}_\ell$ is convex, the shadow $\mathcal{S}_\ell(\mathbf{x}^{\mathrm{U}}_{i,k})$ on the area of interest of $\mathcal{P}_\ell$ for an UAV with state $\mathbf{x}^{\mathrm{U}}_{i,k}$ is the convex hull of the intersections of the lines $(\mathbf{z}^{\mathrm{U}}_{i,k}, \mathbf{z}_{\ell,m})$, $m = 1, \ldots, n_\ell$ with the plane containing the area of interest. These lines are defined as

$$\mathbf{z} = \mathbf{z}_{\ell,m} + \alpha\left(\mathbf{z}^{\mathrm{U}}_{i,k} - \mathbf{z}_{\ell,m}\right), \quad (15)$$

with $\alpha \in \mathbb{R}$. Let $(z_1, z_2, z_3)^T$ denote the components of the vector $\mathbf{z}$. Assuming that the area of interest belongs to the plane defined by $z_3 = 0$, the intersection of $(\mathbf{z}^{\mathrm{U}}_{i,k}, \mathbf{z}_{\ell,m})$ with this plane is obtained when $\alpha = -z_{\ell,m,3}/(z_{\ell,m,3} - z^{\mathrm{U}}_{i,k,3})$ in (15). The intersection $\mathbb{F}^{z_3=0}_i(\mathbf{x}^{\mathrm{U}}_{i,k})$ of the FoV with the area of interest, accounting for all obstacles is then obtained as

$$\mathbb{F}^{z_3=0}_i\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right) = \overline{\mathbb{F}}^{z_3=0}_i\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right) \setminus \bigcup_{\ell=1}^{N_{\mathrm{o}}} \left(\overline{\mathbb{F}}^{z_3=0}_i\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right) \cap \mathcal{S}_\ell\left(\mathbf{x}^{\mathrm{U}}_{i,k}\right)\right).$$

Figure 1 illustrates the effect of an obstacle on $\mathbb{F}_i^{z_3=0}(\mathbf{x}_{i,k}^{\mathrm{U}})$.

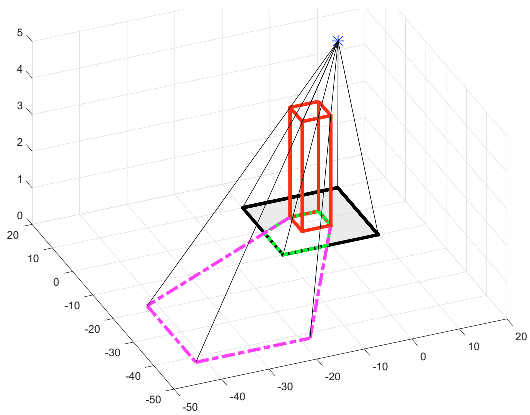

Fig. 1. Intersection $\mathbb{F}_i^{z_3=0}(\mathbf{x}_{i,k}^{\mathrm{U}})$ (in grey) of the FoV with the area of interest in presence of an obstacle (in red); the location of an UAV (blue star); intersection of the FoV with the area of interest (in black); shadow of the obstacle (in pink); occluded area (in green).

4. COOPERATIVE CONTROL DESIGN

Our aim is to determine for each UAV i, the sequence of control inputs that minimizes the predicted estimation uncertainty knowing the sets $\mathcal{D}_{i,k}$, $\mathcal{Z}_{i,k}$, and $\overline{\mathbb{Z}}_{i,k}$. This determination is performed adopting the distributed Model Predictive Control (MPC) formalism introduced, *e.g.*, in Morari and Lee (1999); Rochefort et al. (2014).

At time k, once the communications have been performed with its neighbors, each UAV i has access to $\mathcal{D}_{i,k}$, $\mathcal{Z}_{i,k}$, and $\overline{\mathbb{Z}}_{i,k}$. If $T_h = hT$ denotes the time horizon of the MPC scheme, the sequence of control inputs should be designed for each UAV so as to minimize

$$\Phi_{k+h} = \frac{1}{N_{\mathrm{u}}} \sum_{i=1}^{N_{\mathrm{u}}} \Phi\left(\mathcal{Z}_{i,k+h}, \overline{\mathbb{Z}}_{i,k+h}\right). \tag{16}$$

Nevertheless, this is very difficult to evaluate in a decentralized manner. Several simplifications will thus be considered. First, one assumes that the communication graph does not change over the MPC time horizon. Second, UAV i assumes that the estimates at time $k+\kappa$ of all its neighbors in $\mathcal{N}_i$ are equal to $\mathcal{D}_{i,k+\kappa}$, $\mathcal{Z}_{i,k+\kappa}$, and $\overline{\mathbb{Z}}_{i,k+\kappa}$, $\kappa = 0, \dots, h$. Then, instead of minimizing (16), UAV i will try to design its control inputs so as to minimize

$$\begin{aligned} \widetilde{\Phi}_{i,k+h} &= \frac{1}{N_i + 1} \sum_{\ell \in \overline{\mathcal{N}}_i \cup \{i\}} \Phi\left(\mathcal{Z}_{\ell,k+h}, \overline{\mathbb{Z}}_{\ell,k+h}\right) \\ &= \Phi\left(\mathcal{Z}_{i,k+h}, \overline{\mathbb{Z}}_{i,k+h}\right), \end{aligned} \tag{17}$$

where $\overline{\mathcal{N}}_i = \mathcal{N}_i \cup \{i\}$, since the estimates have been assumed to be equal. The design of the control inputs for UAV i has to account for the control inputs that will be evaluated by its neighbors. If the control inputs evaluated by UAV i at time $k+\kappa$ are denoted $\mathbf{u}_{\ell,k+\kappa}$, $\kappa = 1, \dots, h$, $\ell \in \overline{\mathcal{N}}_i$, then the problem to be solved by UAV i is

$$\left(\widehat{\mathbf{u}}_{\ell,k+1}, \dots, \widehat{\mathbf{u}}_{\ell,k+h}\right)_{\ell \in \overline{\mathcal{N}}_i} = \arg\min \Phi\left(\mathcal{Z}_{i,k+h}, \overline{\mathbb{Z}}_{i,k+h}\right), \tag{18}$$

where the minimization is performed with respect to $(\mathbf{u}_{\ell,k+1}, \dots, \mathbf{u}_{\ell,k+h})$ for all $\ell \in \overline{\mathcal{N}}_i$.

4.1 Prediction of Φ

The solution of (18) requires evaluations of future values $\widetilde{\Phi}_{i,k+\kappa}$ of $\widetilde{\Phi}_{i,k}$, knowing only $\mathcal{D}_{i,k}$, $\mathcal{Z}_{i,k}$, and $\overline{\mathbb{Z}}_{i,k}$. As the potential locations of undetected targets are unknown, this is still very difficult. A bound on $\widetilde{\Phi}_{i,k+1}$ has been introduced in Reynaud et al. (2018), which depends only on the prediction of sets $\mathcal{Z}_{i,k}$ and $\overline{\mathbb{Z}}_{i,k}$ from 6 and 7, and using the dynamics of the UAVs. This bound is adapted here, considering further approximations, to get approximations of $\widetilde{\Phi}_{i,k+h}$ in a recursive way.

At time k, determining whether future control inputs will help to identify new targets is very difficult. Thus, one assumes that the predicted values $\mathcal{D}_{i,k+\kappa}^{\mathrm{P}}$ of $\mathcal{D}_{i,k+\kappa}$ satisfy

$$\mathcal{D}_{i,k+\kappa}^{\mathrm{P}} = \mathcal{D}_{i,k}, \ \kappa = 1, \dots, h.$$

Assume that predicted values $\mathcal{Z}_{i,k+\kappa-1}^{\mathrm{P}}$ and $\overline{\mathbb{Z}}_{i,k+\kappa-1}^{\mathrm{P}}$ of $\mathcal{Z}_{i,k+\kappa-1}$ and $\overline{\mathbb{Z}}_{i,k+\kappa-1}$ have been evaluated, that the predicted states $\mathbf{x}_{\ell,k+\kappa-1}^{\mathrm{U,P}}$ of the UAVs in $\overline{\mathcal{N}}_i$ are known at time $k+\kappa-1$. Then, considering the control inputs $\mathbf{u}_{\ell,k+\kappa}$, $\ell \in \overline{\mathcal{N}}_i$, one is able to evaluate $\mathbf{x}_{\ell,k+\kappa}^{\mathrm{U,P}}$ using $\mathbf{x}_{\ell,k+\kappa-1}^{\mathrm{U,P}}$, $\mathbf{u}_{\ell,k+\kappa}$, and (2) for all $\ell \in \overline{\mathcal{N}}_i$. The evaluation of $\overline{\mathbb{Z}}_{i,k+\kappa|k+\kappa-1}^{\mathrm{P}}$ requires considering a prediction step similar to that in (7) starting from $\overline{\mathbb{Z}}_{i,k+\kappa-1}^{\mathrm{P}}$. One assumes to simplify that $\overline{\mathbb{Z}}_{i,k+\kappa|k+\kappa-1}^{\mathrm{P}} = \overline{\mathbb{Z}}_{i,k+\kappa-1}^{\mathrm{P}}$, neglecting in the MPC approach the potential increase of the uncertainty related to the targets still to be detected.

Then $\overline{\mathbb{Z}}_{i,k+\kappa}$ can be easily predicted combining (11) and (13) as

$$\overline{\mathbb{Z}}_{i,k+\kappa}^{\mathrm{P}} = \overline{\mathbb{Z}}_{i,k+\kappa|k+\kappa-1}^{\mathrm{P}} \setminus \bigcup_{\ell \in \overline{\mathcal{N}}_i} \mathbb{F}_\ell\left(\mathbf{x}_{\ell,k+\kappa}^{\mathrm{U,P}}\right)$$

and consequently,

$$\phi\left(\overline{\mathbb{Z}}_{i,k+\kappa}^{\mathrm{P}}\right) = \phi\left(\overline{\mathbb{Z}}_{i,k+\kappa-1}^{\mathrm{P}} \setminus \bigcup_{\ell \in \overline{\mathcal{N}}_i} \mathbb{F}_\ell\left(\mathbf{x}_{\ell,k+\kappa}^{\mathrm{U,P}}\right)\right). \tag{19}$$

The sets $\mathbb{Z}_{i,j,k+\kappa}^{\mathrm{P}}$ are much more complex to evaluate from $\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}$, since they will depend on the state of the UAVs and the actual state of Target j. Nevertheless, $\mathbb{Z}_{i,j,k+\kappa|k+\kappa-1}^{\mathrm{P}}$ can again be predicted from $\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}$ using (7). A coarse approximation then is to consider $\mathbb{Z}_{i,j,k+\kappa}^{\mathrm{P}} = \mathbb{Z}_{i,j,k+\kappa|k+\kappa-1}^{\mathrm{P}}$, neglecting information which may be provided by UAV measurements. With this approximation,

$$\phi\left(\mathbb{Z}_{i,j,k+\kappa}^{\mathrm{P}}\right) = \phi\left(\mathbf{g}_{k+\kappa-1}^{\mathrm{Z}}\left(\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}, [\mathbf{v}_{k+\kappa-1}]\right)\right), \tag{20}$$

which may be further approximated as

$$\phi\left(\mathbb{Z}_{i,j,k+\kappa}^{\mathrm{P}}\right) = (1+\mu)\,\phi\left(\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}\right), \tag{21}$$

where μ depends on $\mathbf{g}_k^{\mathrm{Z}}$ and $[\mathbf{v}_k]$, but can be taken as a constant. The main advantage of (21) compared to 20 is that it is not necessary to compute $\mathbf{g}_{k+\kappa-1}^{\mathrm{Z}}(\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}, [\mathbf{v}_{k+\kappa-1}])$, only the measures $\phi(\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}})$ of the sets $\mathbb{Z}_{i,j,k+\kappa-1}^{\mathrm{P}}$ are evaluated.

Using (19) and (21), one is able to evaluate recursively (17) for a given sequence of control inputs $(\mathbf{u}_{\ell,k+1}, \ldots, \mathbf{u}_{\ell,k+h})$ for all $\ell \in \overline{\mathcal{N}}_i$, and to solve (18) considering any local minimization technique.

4.2 Analysis of uncertainty criterion

Figure 2 presents the variations of the criterion on a time horizon $T_h = 2T$ for two cooperating UAVs. The control inputs are limited to variation of heading angle. As illustrated, the uncertainty criterion presents several

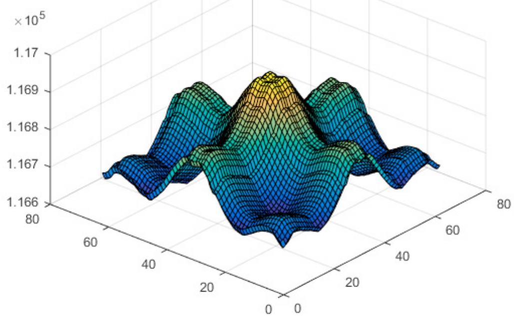

Fig. 2. Illustration of the variation of criterion for a two-step ahead MPC: two cooperative UAVs

local minima. As a result, it could be possible to obtain hippodrome type trajectories with UAVs circling above a detected target. To spur further explorations of the zone containing undetected targets, the criterion (18) is supplemented with

$$\Phi_{\text{add}} = \sum_{\ell \in \mathcal{N}_i} d_{\max}\left(\mathbf{x}_\ell^{\mathrm{U}}, \overline{\mathbb{Z}}_\ell\right)$$

where $d_{\max}$ is the maximum distance between the current positions of the UAVs and points in the set $\overline{\mathbb{Z}}_\ell$. This term allows to pull the trajectories out of the area corresponding to local minima.

5. SIMULATIONS

Two scenarii with static and moving targets are considered. ImageSp and SIVIA have been implemented using the Intlab library Rump (1999).

5.1 Scenario with static targets

The targets are located in a plane, $\mathbf{z}_{j,k} \in \mathbb{R}^3$, with $z_{j,k,3} = 0$, $j = 1, \ldots, 6$. The initial search area is a square of 500×500 m^2. Three UAVs fly at a constant height of 100 m above the terrain at a constant speed of 15ms^{-1}. Their control inputs consist in the heading angle. The UAVs are assumed to communicate at each time instant. They are equipped with an optical sensor able to detect targets within its FoV. Its opening angles are equal to $\frac{\pi}{4}$ in both azimuth and elevation. When a target is detected at time k, one assumes that the measurement equation provides its actual location with an uncertainty bounded in $[-10\,\text{m}, 10\,\text{m}]$ for both components.

Figure 3 presents the evolution of $\overline{\mathcal{Z}}$ from $k = 1$ (top left) to $k = 100$ (bottom left). Figure 3 also presents the corresponding $\mathcal{Z}$ at $k = 100$ (bottom right), when all targets have been detected. Figure 4 presents the resulting trajectories of the three UAVs.

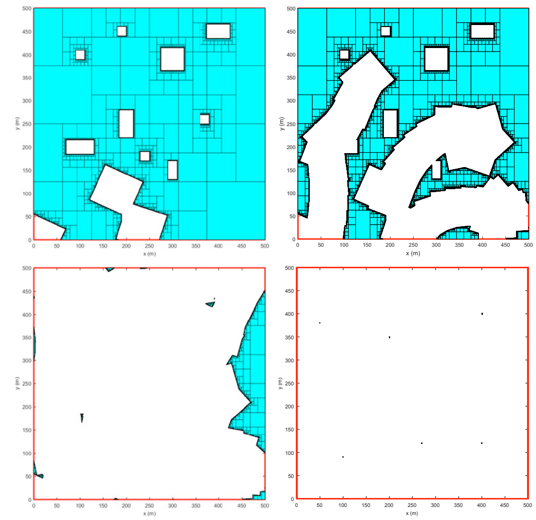

Fig. 3. Initial $\overline{\mathbb{Z}}_1$ (top left), $\overline{\mathbb{Z}}_{15}$ (top right), $\overline{\mathbb{Z}}_{100}$ (bottom left), and $\mathcal{Z}_{100}$ (bottom right)

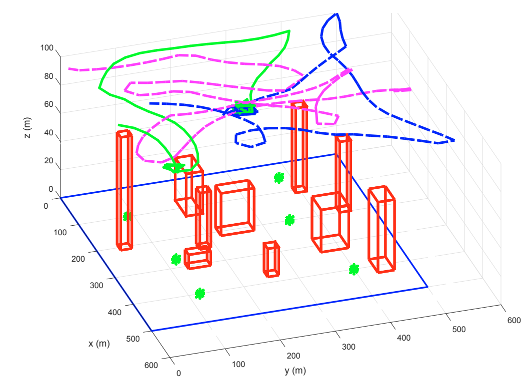

Fig. 4. Resulting trajectories for 3 UAVs; targets (static): green stars; obstacles: red polyhedrons

5.2 Scenario with moving targets

The targets are now assumed to move with a maximum speed of $V_{max}^T = 5$m.s^{-1}. The other parameters are the same as those considered in the previous section. Figure 5 presents the evolution of $\overline{\mathbb{Z}}_k$ for $k = 10$ (top left) and $k = 15$ (top right) and of $\mathcal{Z}$ at $k = 15$ (bottom right) and $k = 40$ (bottom left). One observes that due to the evolution of the targets, the size of the components of $\mathcal{Z}$ increases with k, especially when the targets are not redetected. Videos presenting the evolution of $\overline{\mathbb{Z}}_k$ and $\mathcal{Z}$ with time are available on https://drive.google.com/drive/folders/10EzWYZVGu-8J2qhcv8XkcL1-P97ZpEgj?usp=sharing. The trajectories of the three UAVs are presented on Figure 6. The resulting performances are evaluated using the decrease of uncertainty function Φ_k. Figure 7 presents the evolution of the measures of Φ_k when the targets are static (left), and when the targets are moving for respectively 3 and 4 UAVs(right).

In the static case, the global uncertainty monotonically decreases. When the targets are moving, Φ_k appears to tend to a limit which is a compromise between the decrease of $\overline{\mathbb{Z}}_k$ due to the UAVs search and the growths of $\overline{\mathbb{Z}}_k$ and $\mathcal{Z}_k$ during the prediction step to account for the displacement of the detected and not yet detected targets

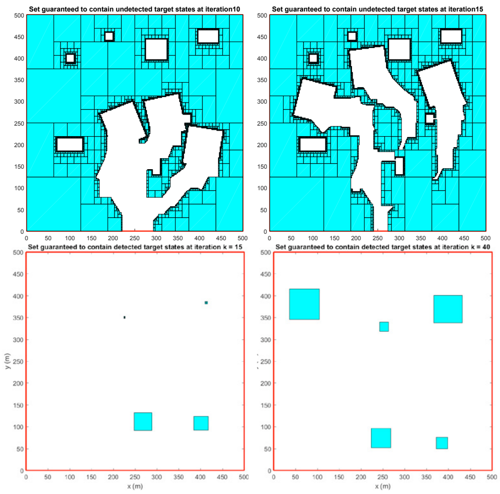

Fig. 5. $\overline{\mathbb{Z}}_{10}$ (top left), $\overline{\mathbb{Z}}_{15}$ (top right), $\mathcal{Z}_{15}$ (bottom left), and $\mathcal{Z}_{40}$ (bottom right)

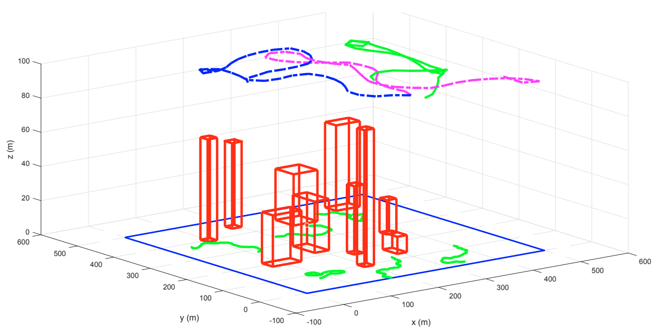

Fig. 6. Resulting trajectories for 3 UAVs; target trajectories indicated by green lines

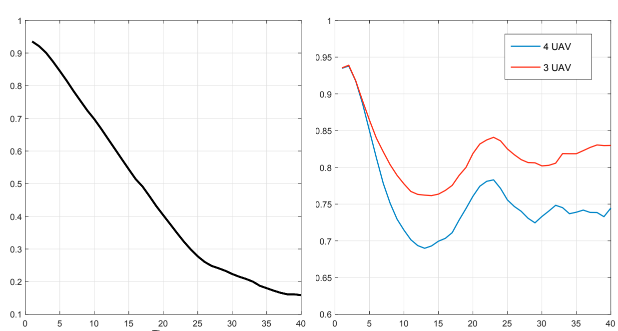

Fig. 7. Φ_k for static (left) and dynamic (right) targets.

with the only knowledge of their maximal speed. As illustrated in Figure 7 (right), the localization uncertainty depends on the number of UAVs involved. This evaluation could provide a mean to determine the lower limit of the number of UAVs required to guarantee a certain level of uncertainty, given the size of the search zone, the characteristics of the UAVs in terms of speed, the size of the FoV, and the bound on the target speed.

6. CONCLUSIONS

A cooperative distributed guidance scheme has been presented for the coordinated control of a fleet of UAVs aiming to search and track an *a priori* unknown number of targets spread over a delimited geographical area. The trajectories of the UAVs are evaluated so as to minimize a criterion combining a measure of the uncertainty on the locations of detected targets and of the potential targets not yet detected. This criterion relies on the evaluation of a measure of the sets guaranteed to contain the targets, which are recursively updated while the UAVs are scanning the search zone. Evaluation of the variations of these sets accounts for potential obscuring of the UAVs FoV by obstacles located on the zone. Simulations considering several UAVs show that the proposed control input optimization is able to provide good localization and tracking performance for multiple targets. Further extensions of the proposed approach include taking into account false-alarm and non-detection in the estimation scheme and evaluating the search behavior when the connectivity graph evolves when the UAVs move.

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