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Design and flight experiments of a Tube-Based Model Predictive Controller for the AR.Drone 2.0 quadrotor

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Abstract: This paper focuses on the design, implementation and experimental validation of a Tube-Based Model Predictive Control (TBMPC) law for the stabilization of the horizontal dynamics of an Unmanned Aerial Vehicle (UAV) quadrotor. These dynamics are modelled by a discrete-time linear system subject to additive disturbance and polytopic constraints, which model is derived through an identification strategy from experimental flight data that is adapted to the subsequent design of invariant sets. The results obtained from a validation flight with the TBMPC law are presented to illustrate the robust state and control input constraints satisfaction.

Keywords: Tube-Based MPC, quadrotor UAV, robust control, constrained control.

1. INTRODUCTION

The diversity of tasks performed by UAVs has significantly increased the last decade. Among these we can cite commercial tasks such as terrain mapping (Tahar et al. (2011)) and building facade assessment (Choi and Kim (2015)). The design of controllers for UAVs to perform autonomously these tasks should take into account safety and technological constraints, such as distance to obstacles or actuator limitations. Moreover, UAVs are subject to disturbances such as ground effect or aerodynamic perturbations when flying close to obstacles (McKinnon (2015)).

The quadrotor UAV used in the experiments presented here is the Parrot AR.Drone 2.0. This drone is affordable and is equipped with a suitable software interface (Bristeau et al. (2011); Krajník et al. (2011)). Its dynamics and control have been largely studied (Prayitno et al. (2014); Santana et al. (2015)). However, these approaches suffer from the lack of constraints handling. The constraint handling by the control law can benefit from the time-domain formulation of Model Predictive Control (MPC) strategies. Recent results report the use of predictive control law on UAVs (Alexis et al. (2011); Liu et al. (2012)), but these works do not consider constraints satisfaction in the presence of disturbance.

The ability to handle bounded disturbances on the system dynamics at the control design stage has been extensively studied in the literature and current MPC strategies allow for improved robustness (Scokaert and Mayne (1998); Langson et al. (2004)). Tube-Based MPC is a strategy that has been successfully applied for constraint handling to face disturbances (Mayne et al. (2005)). It consists in computing a trajectory for a system without disturbance while maintaining the error between the uncertain and the disturbance-free system in an invariant set. This method has been particularly studied in the context of linear discrete-time systems (Mayne et al. (2006)). The position stabilization of an AR.Drone 2.0 fits in this framework. There exist several methods in the literature regarding the computation of invariant sets for linear discrete-time systems subject to bounded additive disturbance (Kolmanovsky and Gilbert (1998); Rakoviċ et al. (2005); Olaru et al. (2010)).

In this paper we propose an online implementation of a TBMPC controller for the stabilization of a commercial quadrotor UAV. Several results have focused on the design in simulation of Tube-Based MPC controllers for mini-UAVs (Hu et al. (2018); Mammarella and Capello (2018); Köhler et al. (2019)). These papers present the framework and parameter tuning of the predictive controller without experimental results. The practical implementation raises additional issues, such as the evaluation of the disturbance due to communication delay, model mismatch, and the presence of external aerodynamic forces. Tube-Based MPC has already been implemented in other areas of research, such as mobile robots (González et al. (2011)) and tractor-trailer systems (Kayacan et al. (2015)). In our knowledge it has yet to be implemented on an UAV, and this paper aims at answering this desideratum.

This paper is structured as follow. Section 2 presents the experimental setup and the modelling of the horizontal dynamics of the Parrot AR.Drone 2.0. The robust Tube-Based MPC control law is stated in Section 3. Section 4

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discusses the identification of the linearized model and the disturbance estimation from experimental data. Section 5 presents the experimental results of the practical implementation of the TBMPC law. Finally, Section 6 draws conclusions and perspectives for further researches.

Notation: For a matrix $A \in \mathbb{R}^{n\times n}$, denote $A^\top$ its transpose. For a positive definite matrix $Q \in \mathbb{R}^{n\times n}$ and a vector $x \in \mathbb{R}^n$, denote $||h||_Q = h^\top Qh$. For a matrix $A \in \mathbb{R}^{n\times n}$ and a set $\mathcal{X} \subset \mathbb{R}^n$, define the set $A\mathcal{X} = \{y \in \mathbb{R}^n \mid y = Ax, x \in \mathcal{X}\}$. We denote $\mathbb{R}_+^{m\times n}$ the set of matrices of dimension $m\times n$ with positive elements. Given two sets $\mathcal{A} \subset \mathbb{R}^n, \mathcal{B} \subset \mathbb{R}^n$, then $\mathcal{A} \oplus \mathcal{B} = \{a + b \mid a \in \mathcal{A}, b \in \mathcal{B}\}$ (set addition) and $\mathcal{A} \ominus \mathcal{B} = \{x \mid \{x\} \oplus \mathcal{B} \subseteq \mathcal{A}\}$ (set substraction). A polytope is a closed and bounded intersection of a finite number of closed half-spaces.

2. EXPERIMENTAL SETUP AND SYSTEM MODELLING

This section briefly describes the experimental setup and presents a model for the horizontal position control of a Parrot AR.Drone 2.0.

2.1 Experimental Setup

The AR.Drone 2.0 is a low-cost quadrotor UAV with embedded control laws allowing to regulate roll angle, pitch angle, yaw rate and vertical speed. We denote the reference values, the inputs to the UAV, by $\begin{bmatrix}\theta_r & \phi_r & \dot{\psi}_r & v_{zr}\end{bmatrix}^\top$. Each component of this vector has values within $[-1, 1]$ that can be interpreted as percentages of maximal configured values.

The measurements of the UAV position ($[x\ y\ z]$), velocity ($[v_x\ v_y\ v_z]$) and roll, pitch and yaw angles ($[\theta\ \psi\ \phi]$) with respect to an inertial reference frame are obtained from a motion capture system with OptiTrack cameras.

We use CVXGEN to generate the code for QP-representable convex optimization problems (Mattingley and Boyd (2012)). The obtained QP corresponds to an MPC tube-based strategy that computes the reference values θ_r, ϕ_r to be transmitted to the drone for horizontal control, as described in the next section.

A ground PC station running ROS is used to compute the control values by the TBMPC algorithm. These values are sent by WiFi to the UAV, hence inducing delays in the transmission of the control signal.

2.2 Model for horizontal position regulation

In this section we present a model for the horizontal position control. We assume that the yaw angle and vertical speed are zero. Moreover, assuming small pitch and roll angles, the dynamics in the directions x and y of the horizontal movement can be considered as identical and decoupled.

Below we focus on a model for the position x. Thanks to the internal regulation, we can assume that the pitch angle dynamics can be described as a first order system (Krajník et al. (2011))

$$\dot{\phi} = -C(\phi - \phi^r) + \delta,$$

where $C > 0$ and δ represents external disturbance and errors in the model. The force generated by the rotors in the horizontal direction x is considered to be linear on the pitch angle, yielding an acceleration which is linear on ϕ, as $m\dot{v}_x = F\phi + f$ where f is a disturbance corresponding to the error in the model and exogenous disturbance (such as aerodynamical forces) and F is the magnitude of the force generated by the four propellers.

From the above considerations we propose the following continuous-time linear system

$$\dot{X} = A_cX + B_cU + \Delta_x, \tag{1}$$

$$X = \begin{bmatrix} x \\ v_x \\ \phi \end{bmatrix}, A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{F}{m} \\ 0 & 0 & -C \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ C \end{bmatrix}, \Delta_x = \begin{bmatrix} 0 \\ \dfrac{f}{m} \\ \delta \end{bmatrix},$$

with $U = \phi^r$. A similar model can be obtained for the dynamics in the direction y. The theoretical developments of this paper will be presented for the direction x. The TBMPC is validated in flight for both directions (x, y).

A discretized model of (1) is considered for the design of the TBMPC law,

$$X_{k+1} = AX_k + BU_k + W_k, \tag{2}$$

where $X_k = X(k\tau)$ is the state, $U_k = U(k\tau)$ is the control input, $W_k \in \mathbb{R}^3$ is the additive disturbance, and $\tau > 0$ is a chosen sampling period. The disturbance W_k is the discrete-time representation of Δ_x, the discretization error, the measurement error from the Motion Capture, and the error induced by the communication delay.

We assume that the disturbance W_k is bounded by a set $\mathcal{W} \subset \mathbb{R}^3$. The identification of $A \in \mathbb{R}^{3\times 3}$ and $B \in \mathbb{R}^{3\times 1}$ with the structural constraints inherited from (1) and the determination of $\mathcal{W}$ are discussed in Section 4.

The system is subject to physical constraints, such as presence of obstacles or actuators saturation, and to constraints related to the mission requirements, such as speed or angle limitations. These constraints are represented by

$$X_k \in \mathcal{X} \subset \mathbb{R}^3, U_k \in \mathcal{U}. \tag{3}$$

This model is used for the design of a TBMPC law for the stabilization of the horizontal position around successive waypoints, or references, $X_r = [x_r\ 0\ 0]^\top, Y_r = [y_r\ 0\ 0]^\top$.

3. TUBE-BASED MODEL PREDICTIVE CONTROL

This section recalls the principle of TBMPC as presented in Mayne et al. (2006) for the stabilization around the origin of a linear discrete-time system subject to bounded additive disturbance. The results presented in this section are obtained under the following assumption.

Assumption 1. The sets $\mathcal{X}, \mathcal{U}$ and $\mathcal{W}$ are polytopes containing the origin in their interior.

Let us consider the disturbance-free system from (2),

$$\bar{X}_{k+1} = A\bar{X}_k + B\bar{U}_k, \tag{4}$$

which we refer to as the *nominal system*. Assume that a control sequence $\{\bar{U}_{k|k}, ..., \bar{U}_{k+N|k}\}, N \in \mathbb{N}$, has been computed. The associated predicted trajectory of the nominal system is given by

$$\bar{X}_{k+i+1|k} = A\bar{X}_{k+i|k} + B\bar{U}_{k+i|k}, i = \{0, .., N\}. \tag{5}$$

Clearly, the trajectory of the system (2) differs due to the presence of disturbance. The following control action is chosen to counteract its impact,

$$U_{k+i} = \bar{U}_{k+i} + K(X_{k+i} - \bar{X}_{k+i}), \tag{6}$$

with K computed such that $A_K = A + BK$ is Schur. The error dynamics is

$$Z_{k+1} = X_{k+1} - \bar{X}_{k+1} = A_K Z_k + W_k, W_k \in \mathcal{W}. \tag{7}$$

Since A_K is Schur, and the set $\mathcal{W}$ is bounded, Z_k is bounded. This bound can be characterized by an invariant set $\mathcal{Z}$ (Blanchini (1999)).

Definition 1. (Mayne et al. (2005)) The set $\mathcal{Z}$ is said Robustly Positively Invariant (RPI) for the system (7) if for all $Z_k \in \mathcal{Z}$ and all $W_k \in \mathcal{W}$, $Z_{k+1} \in \mathcal{Z}$.

Proposition 1. (Mayne et al. (2005)) Let $\mathcal{Z}$ be an RPI set for the error system (7), and let $X_0 - \bar{X}_0 \in \mathcal{Z}$. Then, the trajectory of (2) with the control action (6) is such that $X_k \in \{\bar{X}_k\} \oplus \mathcal{Z}, \forall k \in \mathbb{N}$ for any possible realization of the disturbance $W_k \in \mathcal{W}$.

The trajectory of the uncertain system lies in a neighborhood, or *tube*, of the predicted trajectory of the nominal system (5). Moreover, if the nominal system state and control input are such that $\bar{X}_k \in \bar{\mathcal{X}} = \mathcal{X} \ominus \mathcal{Z}, \bar{U}_k \in \bar{\mathcal{U}} = \mathcal{U} \ominus K\mathcal{Z}, \forall k \in \mathbb{N}$, then $X_k \in \mathcal{X}, U_k \in \mathcal{U}, \forall k \in \mathbb{N}$. By introducing a tighter set of constraints on the nominal system (4), it is possible to enforce the satisfaction of the constraints (3).

A key property that the pair $(K, \mathcal{Z})$ has to satisfy is that the sets $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$ have to be non-empty, which means, following the definition of $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$, the set constraint

$$\mathcal{Z} \subseteq \mathcal{X}, K\mathcal{Z} \subseteq \mathcal{U} \tag{8}$$

has to hold. The larger the set $\mathcal{Z}$, the smaller the nominal system constraints $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$ are. Indeed, the set $\mathcal{Z}$ bounds the difference between the nominal system state and the uncertain system state. Therefore, the control law benefits from having a set $\mathcal{Z}$ as *small* as possible. For given $(A, B, K, \mathcal{W})$, the minimal RPI (mRPI) set in terms of set inclusion is given by (Mayne et al. (2005))

$$\mathcal{Z}_\infty(A, B, K, \mathcal{W}) = \bigoplus_{i=0}^{\infty} (A + BK)^i \mathcal{W}. \tag{9}$$

An MPC strategy is chosen to control the nominal system to enforce nominal constraints satisfaction. For a given nominal state $\bar{X}_k$, we consider the following optimization problem

$$\begin{aligned} P_N(\bar{X}_k) \; = & \\ \underset{\bar{U}_0,\dots,\bar{U}_{N-1}}{\text{minimize}} & \sum_{i=0}^{N-1} (||\bar{X}_i||_Q + ||\bar{U}_i||_R) + ||\bar{X}_N||_P, \\ \text{subject to } & \bar{X}_{i+1} = A\bar{X}_i + B\bar{U}_i, i = \{0, ..., N-1\}, \\ & \bar{X}_i \in \bar{\mathcal{X}}, i = \{1, ..., N-1\}, \\ & \bar{U}_i \in \bar{\mathcal{U}}, i = \{0, ..., N-1\}, \\ & \bar{X}_0 = \bar{X}_k, \bar{X}_N \in \bar{\mathcal{X}}_f. \end{aligned} \tag{10}$$

The weight matrices Q and R are definite positive. The terminal weight matrix P and set $\bar{\mathcal{X}}_f$ are chosen to ensure the stability of the MPC (Mayne et al. (2005)). Their choice is discussed in Section 5.

We denote $\bar{U}_0^*(\bar{X}_k)$ the first element of the solution of $P_N(\bar{X}_k)$. The control law (6) leads to

$$U_k = \bar{U}_0^*(\bar{X}_k) + K(X_k - \bar{X}_k).$$

This control law ensures recursive feasibility of (10) and recursive satisfaction of the constraints (3). Note that the nominal control action $\bar{U}_0^*(\bar{X}_k)$ is independent of the current uncertain state X_k. The disturbance realizations W_k do not impact the evolution of the nominal system (4).

Similar results are obtained for the stabilization around a reference (X_r, U_r) such that $X_r = AX_r + BU_r, X_r \in \bar{\mathcal{X}}, U_r \in \bar{\mathcal{U}}$ by considering the translated state and control input $\tilde{X} = X - X_r, \tilde{U} = U - U_r$, and constraints sets $\tilde{X} = \mathcal{X} \ominus \{X_r\}, \tilde{U} = \mathcal{U} \ominus \{U_r\}$. Note that the computation of the terminal set for a reference is more complex than a translation of the terminal set for the origin. Its design and computation are discussed in Section 5.

Our goal is to experimentally validate a TBMPC law for the stabilization of the horizontal dynamics of the AR.Drone 2.0. The critical aspect of the implementation and validation of this control law is the design of an RPI set $\mathcal{Z}$ satisfying (8) as detailed next.

4. SYSTEM IDENTIFICATION AND INVARIANT SET DESIGN

The computation of an invariant set $\mathcal{Z}$ depends on the matrices A, B, and K, and the disturbance set $\mathcal{W}$.

The linear state feedback gain K has to stabilize the quadrotor. Indeed, once the nominal state $\bar{X}_k$ has converged toward the reference, the control law (6) is equivalent to $U(X) = K(X - X_r)$. We have tuned and validated in flight tests a linear state feedback gain K leading to satisfying behavior of the quadrotor in the flight zone for stationary flights around chosen waypoints.

4.1 Disturbance evaluation and model structure

The disturbance set is defined from flight tests, where the states and control inputs are measured over $L + 1$ points,

$$\{X_k^m, U_k^m\}, k = \{1..., L + 1\}.$$

For a given pair of matrices (A, B), the disturbance realizations are defined as the mismatch between the one-step state prediction of model (2) and the measured system state and control input,

$$W_k^m = X_{k+1}^m - AX_k^m - BU_k^m, k = \{1, ..., L\}. \tag{11}$$

The disturbance set $\mathcal{W}$ is then chosen as the convex-hull of this disturbance sequence, $\mathcal{W} = \text{conv}\{W_k^m, k = \{1, ..., L\}\}$. Hence, the identification of the matrices A and B fully determines the disturbance set $\mathcal{W}$.

We impose the following model structure to have matrices A and B in relationship with the continuous-time model (1),

$$A = \begin{bmatrix} 1 & a_{xv} & a_{x\phi} \\ 0 & a_v & a_{v\phi} \\ 0 & 0 & a_\phi \end{bmatrix} \in \mathbb{R}_+^{3\times3}, B = \begin{bmatrix} b_x \\ b_v \\ b_\phi \end{bmatrix} \in \mathbb{R}_+^{3\times1}.$$

The diagonal terms a_v and a_ϕ allow additional degrees of freedom with regards to the external forces and torques not taken into account in the modelling. The other coefficients are chosen positive to ensure that the model is physically realistic.

4.2 System identification and invariant set design

Some results have been established in the literature on the design of a control law, for a given pair (A, B), leading to invariant sets with remarkable properties, such as constraints satisfaction (Raković et al. (2005)).

Instead, we are interested here in the slightly different problem of the design of the matrices A and B for a given gain K leading to invariant sets with similar remarkable properties.

Recall that for given $A, B, K, \mathcal{W}$, the mRPI set is given by

$$\mathcal{Z}_\infty(A, B, K, \mathcal{W}) = \bigoplus_{i=0}^{\infty} (A + BK)^i \mathcal{W}.$$

In general it is not possible to have an explicit characterization of this set.

The mRPI depends on $(A + BK)^i, i \in \mathbb{N}$ and the disturbance set $\mathcal{W}$. The identification method proposed here consists in identifying matrices A and B that minimize the disturbance affecting our system while constraining the eigenvalues of the matrix $A + BK$. The choice of such a criterion is motivated by the fact that both the disturbance realizations $\{W_k^m\}$ and the eigenvalues of $A+BK$ depend linearly on the elements of A and B.

The studied system is similar to a triple integrator, with real eigenvalues that are close to 1. We force the eigenvalues of A to be smaller than arbitrary values to limit the eigenvalues of $A + BK$ for the chosen K. Meanwhile, we minimize the quadratic norm of the disturbance sequence to decrease the influence of the set $\mathcal{W}$. The resulting optimization problem can be written as

$$\begin{aligned} (A, B) = \arg \underset{A,B}{\text{minimize}} & \sum_{k=0}^{L} ||W_k^m||_2^2, \qquad (12) \\ \text{subject to } & W_k^m = X_{k+1}^m - AX_k^m - BU_k^m, \\ & A = \begin{bmatrix} 1 & a_{xv} & a_{x\phi} \\ 0 & a_v & a_{v\phi} \\ 0 & 0 & a_\phi \end{bmatrix}, B = \begin{bmatrix} b_x \\ b_v \\ b_\phi \end{bmatrix}, \\ & a_{xv}, a_{x\phi}, a_{v\phi}, b_x, b_v, b_\phi \geq 0, \\ & 0 \leq a_v \leq \alpha, 0 \leq a_\phi \leq \beta. \end{aligned}$$

where α and β are parameters to be tuned. The choice of α and β leads to an identification of A and B from (12), to a set $\mathcal{W}$ from (11), and to an mRPI set $\mathcal{Z}_\infty(A, B, K, \mathcal{W})$ from (9). The tuning of α and β consists in the comparison of the associated mRPI sets, and thus requires the computation of RPI outer approximations. These approximations are obtained using the method presented in Raković et al. (2005). This method relies on the computation of partial sums $\mathcal{Z}(A, B, K, i) = \bigoplus_{j=0}^{i} (A+BK)^j \mathcal{W}$. For each partial sum, it is possible to compute a scaling factor $\mu_i \geq 0$ such that the set $\mu_i \mathcal{Z}(A, B, K, i)$ is an RPI set. Each iteration further approximates the mRPI at the expense of increasing its complexity, hence computational efforts.

4.3 Identification flights and experimental results

In the results presented below, we fix $K = -[0.5\ 0.4\ 1]$, and $\tau = 0.05s$.

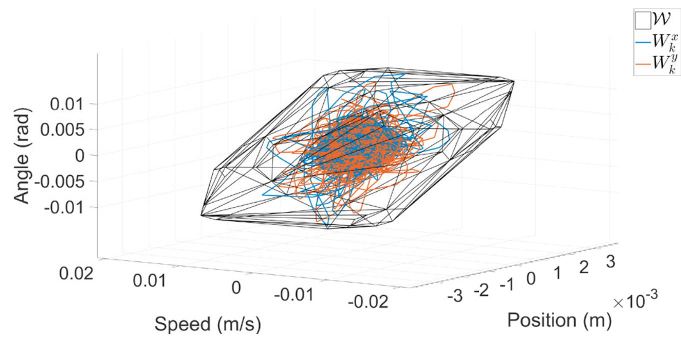

Fig. 1. Disturbance set $\mathcal{W}$ identified and further used for the TBMPC design, and disturbance realizations during the validation flight regarding both directions x and y in the state-space position-speed-angle.

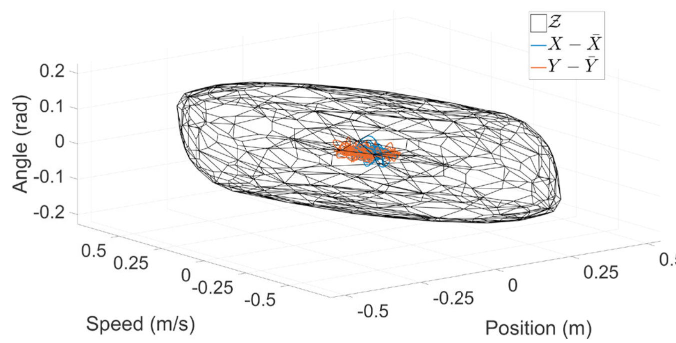

Fig. 2. Invariant set $\mathcal{Z}$ and the trajectory of the system error for the directions x and y during the validation flight in the state-space position-speed-angle.

The experimental validation of the TBMPC law requires that the disturbance realizations encountered in flight are contained in the disturbance set used for the design of its invariant set. However, the set $\mathcal{W}$ has to be as small as possible to have an invariant set $\mathcal{Z}$ with reduced influence. Therefore, the identification requires a large set of experimental data from identification flights with similar flight conditions, in particular the control law.

First, we performed identification flights with the linear control law $U(X) = K(X-X_r)$ and waypoints as shown in Figure 3, leading to an initial identification of $(A, B, \mathcal{W})$. Then, we refined the identification of $(A, B, \mathcal{W})$ by performing identification flights with TBMPC laws until one of these control laws was validated in flight, as presented in the next section.

Regarding the tuning of (α, β), we computed and compared the 100^{th} mRPI outer approximation using the method discussed above for a range of parameters (α, β) as in (12). We chose the invariant set whose projection on the first state (i.e. the position) is minimal. This leads to $\alpha = \beta = 0.97$,

$$A = \begin{bmatrix} 1 & 0.049 & 0.0524 \\ 0 & 0.97 & 0.4294 \\ 0 & 0 & 0.9389 \end{bmatrix}, B = \begin{bmatrix} 0.005 \\ 0.01 \\ 0.0695 \end{bmatrix},$$

the disturbance set $\mathcal{W}$ presented in Figure 1, and the invariant set $\mathcal{Z}$ presented in Figure 2. This identification method used data from the directions x and y and thus applies for both directions.

We have identified the matrices A and B, the disturbance set $\mathcal{W}$ and computed an RPI set $\mathcal{Z}$. In the following, we provide the results obtained from the validation flight with the TBMPC law.

5. EXPERIMENTAL VALIDATION OF THE TBMPC LAW

5.1 Parameters of the Model Predictive Control law

The prediction horizon is chosen as $N = 30$, which means a temporal prediction horizon of $N\tau = 1.5s$. We restricted the flight zone to $-0.6m \leq x, y \leq 1.8m$, the speed to $|v_x|, |v_y| \leq 1m.s^{-1}$, and the attitude to $|\phi|, |\theta| \leq 0.3rad$. We set a control input limit chosen as $|U| \leq 0.56$. The speed, attitude, and control input constraints are chosen arbitrary to illustrate the recursive constraints satisfaction of the control law.

This defines the nominal system constraints $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$,

$$\bar{\mathcal{X}} = \left\{\bar{X} \in \mathbb{R}^3 \mid H\bar{X} \leq \bar{g}\right\}, \bar{\mathcal{U}} = \left\{\bar{U} \in \mathbb{R} \mid |\bar{U}| \leq 0.332\right\}$$

$$\bar{g} = [1.3\ 0.321\ 0.094\ -0.1\ -0.321\ -0.094]^\top,$$

and $H = [I_3\ I_3]^\top$. The weight matrices Q and R as in (10) are chosen as $Q = \text{diag}(0.8, 1.2, 1), R = 5$. The terminal weight matrix P is chosen as the solution of the Riccati equation $A_K^\top PA_K - P + Q + K^\top RK = 0$. As mentioned in Section 3, the terminal set $\bar{\mathcal{X}}_f(X_r)$ is a function of the reference, and is to be designed as large as possible to increase the feasibility region. The largest admissible set is the Maximal Output Admissible Set (MOAS) for the system $\bar{X}^+ = (A+BK)\bar{X}$ and constraints $\bar{X} \in \bar{\mathcal{X}} \ominus \{X_r\}, K\bar{X} \in \bar{\mathcal{U}}$, as in Kolmanovsky and Gilbert (1998). The re-calculation of the MOAS along the change of waypoints is not recommended in the context of online implementation, as it can be too demanding in terms of computation time.

Our method consists in computing offline the MOAS for the origin, $\bar{\mathcal{X}}_f(0)$. Then, for a reference X_r we define $\bar{\mathcal{X}}_f(X_r) = \lambda\bar{\mathcal{X}}_f(0) \oplus \{X_r\}$, where λ is chosen as the largest positive real satisfying $\bar{\mathcal{X}}_f(X_r) \subseteq \bar{\mathcal{X}}$ and $\lambda K\bar{\mathcal{X}}_f(0) \subseteq \bar{\mathcal{U}}$. The computation of $\bar{\mathcal{X}}_f(X_r)$ requires to solve online a linear optimization problem.

During the identification and validation flights, the maximal computation time of the TBMPC law was $4ms$ for each direction, including the terminal set computation during a waypoint change.

5.2 Validation flight of the TBMPC

We present the results obtained from a validation flight of the TBMPC law for both directions x and y.

The trajectory of the quadrotor in the plane (x, y) and the successive waypoints are presented in Figure 3. The references are chosen close to one-another to ensure feasibility of the optimization problems.

Figure 1 presents the realizations of the disturbance during the validation flight in both directions. Note that all the disturbance realizations are contained in $\mathcal{W}$. As a consequence, the error between the uncertain system and the nominal system is contained in the RPI set $\mathcal{Z}$, as illustrated in Figure 2.

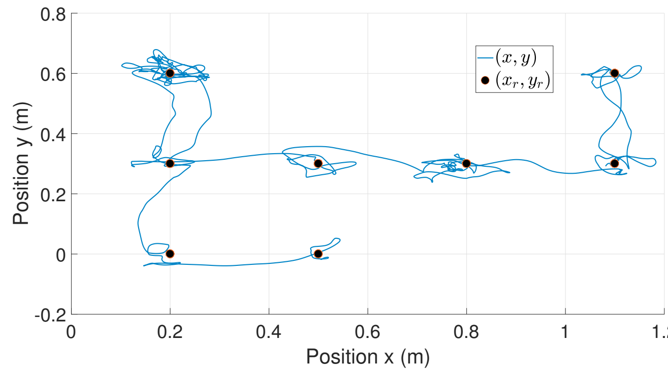

Fig. 3. Horizontal (x, y) trajectory of the uncertain system during the validation flight.

Figure 4 shows the evolution of the uncertain and nominal systems states X and $\bar{X}$, the reference values X_r, and the control actions U and $\bar{U}$, in the direction x. The actual and nominal constraints are also depicted to illustrate the constraints satisfaction during the flight. Note that the constraints are satisfied during the flight despite the presence of disturbance, which is the main objective of the proposed control law. Similar results are obtained in the direction y.

6. CONCLUSION

We have designed, implemented and validated experimentally a Tube-Based Model Predictive Control law for the stabilization of the horizontal dynamics of the Parrot AR.Drone 2.0. A strategy is developed for the identification of the discretized and linearized dynamics in the goal of computing invariant sets as small as possible. Several test flights have been conducted beforehand to gather bounds for the disturbance. Satisfaction of the state and control input constraints are guaranteed as far as the disturbance encountered does not exceed the bounds. The design and implementation of a reference governor is to be sought to allow for greater distance between successive waypoints.

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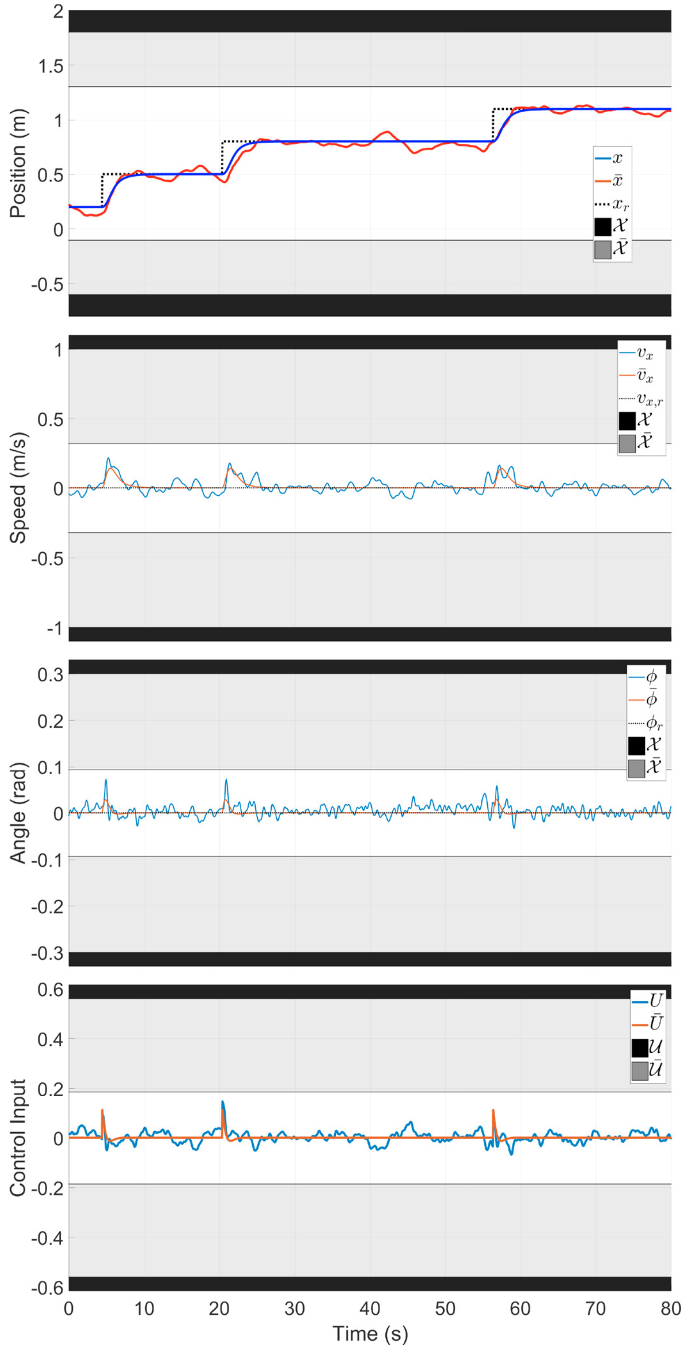

Fig. 4. Time-evolution of the position, speed, angle and control input (top to bottom) for the nominal and the uncertain systems, and the real and nominal constraints, during the validation flight.

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