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Global exponential estimation of rigid body angular velocity directly from multiple vector measurements

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Abstract— **The problem of angular velocity estimation for a rigid body directly from body-referenced vector measurements is addressed. The case of multiple (at least a pair of) vector measurements is treated without requiring the knowledge of the corresponding inertial vectors or assuming any additional information on the attitude and the angular velocity. The proposed solution ensures uniform global exponential stability with respect to null estimation error through a thorough strict Lyapunov analysis. The efficiency of the proposed nonlinear observer is illustrated by numerical simulations.**

I. Introduction

The fast technological developments related to rigid body systems such as satellites, drones and underwater vehicles have increased the demand for even more efficient, and robust, control and estimation algorithms that are able to respond successfully to various mission scenarios. Typically, the principal objectives that need to be attained by a rigid body are precise attitude stabilization and tracking.

Throughout the years a variety of control algorithms have been proposed that can in one way be classified into velocity-independent and velocity-dependent based on whether they require measurements of angular velocity or not. The first category exploits, the well-known by now, passivity properties of the rigid-body dynamics to use only orientation information, see e.g. [1], [2], while the second category either exploits direct velocity information or adopts an observer-controller structure with the observer providing an estimate of the angular velocity by exploiting direct attitude measurements, see [3], [4] and references therein for an exhaustive list of references.

In practice direct angular velocity information is usually obtained in two ways [5]. First, from a dedicated embedded sensor that is commonly referred to as strap-down rate gyroscope. However, it is well known that gyros are quite expensive, fragile and prone to failure [4], [6]–[10]. The second alternative follows a cascaded approach since it requires to first obtain an estimate of the body's orientation from (at least two linearly independent) body-referenced vector measurements, for example from accelerometers, magnetometers or Sun sensors, and then generate an angular velocity estimate. See [11], [12] for detailed surveys on the different approaches on attitude determination. Among the various approaches, it has been established that static attitude determination algorithms are very noise-sensitive while dynamic attitude observers require the incorporation of gyro and vector measurements to produce a robust attitude estimate.

To remedy the deficiencies of the two aforementioned approaches, and in particular to obviate the use of direct gyro measurements and the necessity to derive an estimate of the rigid body's orientation, a third alternative has been recently proposed [8], [9]. This approach rather proposes a direct manipulation of body-referenced vector measurements, that are obtained from cheaper sensors, and the use of Euler's equations to design a nonlinear time-varying observer for the angular velocity. When at least two vector measurements are available the design framework requires that these are not collinear [9] while in the single measurement case the above working assumption is naturally replaced, from an observability point of view, with a persistency-of-excitation (PE) condition [8].

As mentioned previously, the first works that considered the problem of dynamically estimating the angular velocities directly from body-referenced vector measurements are [8], [9]. Under standard assumptions for both cases of single and multiple measurements, a very simple, high-gain observer was shown to ensure uniform local exponential stability (ULES) of zero estimation error through a non-trivial Linear Time-Varying (LTV) system analysis. The present work was motivated and inspired by these recent developments. The contribution of this work is a nonlinear observer that provides a *uniformly globally exponentially converging* estimate of the angular velocity with measurements from at least two triaxial sensors. To the author's knowledge this is the first such result in the literature. The proposed observer has a classic structure that consists of a copy of the system plus some nonlinear correction terms, with the addition of a (scalar) dynamic extension. The observer design framework is based on the invariant–manifold approach [13], [14] and its recent extensions that hinge upon the notion of dynamic scaling, that allows in particular to explicitly construct a strict Lyapunov function.

The organization of the paper is as follows. Section II introduces the dynamic model of the rigid body, the available measurements and the working assumptions. Section III presents the proposed observer and the corresponding Lyapunov-based stability analysis. Numerical simulations of the presented estimation schemes then follow in Section IV. The article is concluded with some remarks and future perspectives.

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II. Problem Formulation

A. Notation

We denote by $|x|$ the Euclidean norm of the vector x and similarly, $||A||$ represents the matrix induced 2-norm for any matrix A. Also, I will represent the identity matrix of appropriate dimensions and $\lambda(A)$ will denote the spectrum of the matrix A. Throughout, for any $x \in \mathbb{R}^3$ we will define by

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \quad (1)$$

the matrix representation of the linear map $y \mapsto x \times y$, with $y \in \mathbb{R}^3$, and ' $\times$ ' denoting the usual cross product in $\mathbb{R}^3$. We will interchangeably use both notations, *i.e.* $x \times y$ and $S(x)y$, to simplify the presentation. Finally, the special orthogonal group is defined as $\mathrm{SO}(3) = \{A \in \mathbb{R}^{3\times 3} \mid A^T A = I, \det(A) = 1\}$.

B. Dynamic Model

We consider a moving rigid body subjected to the angular velocity $\omega \in \mathbb{R}^3$ (in body axes). Its orientation, from body to inertial axes, matrix $R \in \mathrm{SO}(3)$ is related to ω through the orientation kinematics as

$$\dot{R} = RS(\omega), \quad (2)$$

We consider that the sensors placed on the rigid body provide us with the (unit) body vectors a, b that correspond to two constant, but unknown, reference vectors $\mathring{a}$, $\mathring{b}$, expressed in the inertial frame. Then we can write the following relationships

$$\mathring{a} = Ra \quad (3)$$
$$\mathring{b} = Rb. \quad (4)$$

The model describing the dynamic evolution of the angular velocity and the vector measurements a, b, and which will be adopted for the observer design in the following section, is given as

$$J\dot{\omega} = (J\omega) \times \omega + \tau \quad (5)$$
$$\dot{a} = a \times \omega \quad (6)$$
$$\dot{b} = b \times \omega. \quad (7)$$

Similarly to [9], we consider the following working assumptions.

Assumption 1 (A.1): The angular velocity is bounded, i.e, $|\omega(t)| \leq \bar{\omega}$.

Assumption 2 (A.2): The inertia matrix J and the torque τ are known.

In order to be able to succesfully design an angular velocity observer, we require the following additional assumption.

Assumption 3 (A.3): The (unit) body vector measurements, a and b, are linearly independent.

Assumption A.3 gives rise to the following fact.

Fact 1 ([7], [15]): Under assumption A.3, the matrix

$$M(a, b) := -K_{10}S^2(a) - K_{20}S^2(b), \quad (8)$$

is symmetric and positive definite. Furthermore, the positive scalars K_{10}, K_{20} can arbitrarily increase the (simple) eigenvalues of $M(a, b)$.

Problem Statement: Design a continuous observer based on the model (5)-(7), measurements a, b and under assumptions A.1-A.2, that provides a *uniformly globally exponentially convergent* angular velocity estimate.

III. Observer

Under assumptions A.1-A.3, an observer for system (5)-(7) is the dynamical system

$$\begin{aligned} J\dot{\xi} &= (J\hat{\omega}) \times \hat{\omega} + \tau + K_1((\hat{a} \times a) \times \hat{\omega}) + K_2((\hat{b} \times b) \times \hat{\omega}) \\ &\quad + K_1 K_a(\hat{a}, r)(\hat{a} - a) + K_2 K_b(\hat{b}, r)(\hat{b} - b) \quad (9) \\ \hat{\omega} &:= \xi - J^{-1}K_1(\hat{a} \times a) - J^{-1}K_2(\hat{b} \times b) \quad (10) \\ \dot{\hat{a}} &= \hat{a} \times \hat{\omega} - K_a(\hat{a}, r)(\hat{a} - a) \quad (11) \\ \dot{\hat{b}} &= \hat{b} \times \hat{\omega} - K_b(\hat{b}, r)(\hat{b} - b) \quad (12) \\ \dot{r} &= -2\psi_1(r - 1) + 2r(K_1|\hat{a} - a| + K_2|\hat{b} - b|), \quad (13) \\ & r(0) \geq 1, \end{aligned}$$

with the positive scalars K_1,K_2, ψ_1 and the mappings $K_a(\hat{a}, r)$, $K_b(\hat{b}, r)$ serving as gain functions to be properly chosen. Notice that the proposed observer, with state $\xi \in \mathbb{R}^3$ has the classical structure consisting of a copy of the original system, some correction terms with nonlinear gains and an additional dynamic extension $r \in \mathbb{R}$.

Proposition 1: Consider the system (5)-(7). Then, the dynamical system (9)-(13) ensures the *uniform global exponential stability (*__UGES__*)* of null estimation error $\hat{\omega} - \omega$ provided the gains satisfy the conditions

$$\min(\lambda(-K_1S^2(a) - K_2S^2(b))) > (\psi_1 + \bar{\omega}||J|| + 1) \quad (14)$$
$$\psi_1 > \frac{1}{2} \quad (15)$$
$$K_a(\hat{a}, r) := K_{a0} + (2r^2K_1^2 + \frac{r|\hat{a}|^2}{2})I \quad (16)$$
$$K_b(\hat{a}, r) := K_{b0} + (2r^2K_2^2 + \frac{r|\hat{b}|^2}{2})I \quad (17)$$

with $K_{a0} = K_{a0}^\top \succ 0$, $K_{b0} = K_{b0}^\top \succ 0$, and with the estimate for the angular velocity ω given by

$$\hat{\omega} := \xi - J^{-1}K_1(\hat{a} \times a) - J^{-1}K_2(\hat{b} \times b). \quad (18)$$

Proof: Let us define the errors

$$\begin{aligned} z &:= \xi - J^{-1}K_1(\hat{a} \times a) - J^{-1}K_2(\hat{b} \times b) - \omega \\ \tilde{a} &:= \hat{a} - a \\ \tilde{b} &:= \hat{b} - b. \end{aligned} \quad (19)$$

Using the expressions in (5)-(7),(9)-(13), the z-dynamics is expressed as

$$\begin{aligned} \dot{z} &= J^{-1}(K_1S^2(a) + K_2S^2(b))z - J^{-1}(K_1S(\tilde{a})S(a) \\ &\quad + K_2S(\tilde{b})S(b))z + J^{-1}((Jz) \times z + (J\omega) \times z) \\ &\quad + J^{-1}(Jz) \times \omega. \end{aligned}$$

Consider now the candidate Lyapunov function

$$V(z) = \frac{1}{2}z^\top Jz, \quad (20)$$

whose time-derivative along trajectories of (20) yields

$$\begin{aligned}\dot{V} &= z^\top(K_1S^2(a) + K_2S^2(b))z \\ &\quad - z^\top(K_1S(\tilde{a})S(a) + K_2S(\tilde{b})S(b))z + z^\top((Jz)\times\omega) \\ &\leq z^\top(K_1S^2(a) + K_2S^2(b))z + (K_1|\tilde{a}| + K_2|\tilde{b}|)|z|^2 \\ &\quad + \bar{\omega}||J|||z|^2 \\ &= z^\top(K_1S^2(a) + K_2S^2(b) + \bar{\omega}||J||I)z \\ &\quad + z^\top((||K_1|||\tilde{a}| + ||K_2|||\tilde{b}|)I)z,\end{aligned}$$

using the properties of the triple scalar product and the boundedness of ω from A.1.

Now, in order to be able to dominate the cross-terms in $\dot{V}$ we introduce a dynamic scaling function r, with dynamics given in (13), and define the candidate (dynamic) Lyapunov function

$$V_s(z,r) := \frac{V(z)}{r}. \tag{21}$$

Its time-derivative along the (z,r)-trajectories gives

$$\begin{aligned}\dot{V}_s &= \frac{\dot{V}}{r} - V_s\frac{\dot{r}}{r} \\ &\leq z^\top(K_1S^2(a) + K_2S^2(b) + (\psi_1 + \bar{\omega}||J||)I)\frac{z}{r} \\ &\leq -\kappa V_s(z,r),\end{aligned}$$

with $\kappa > 0$, after using the property that $\frac{r-1}{r} \leq 1$ and from the Fact 1 that the matrix $K_{10}S^2(a) + K_{20}S^2(b)$ can be chosen as

$$-K_1S^2(a) - K_2S^2(b) \succ (\psi_1 + \bar{\omega}||J||)I \succeq \kappa I \succ 0, \tag{22}$$

since a,b are non-collinear, and with eigenvalues that can be arbitrarily increased by increasing the gains K_1, K_2.

In order to conclude uniform local (global) asymptotic (exponential) stability of $z = 0$, we need to further show that $r(t)$ stays bounded for all times. To this end, let us first express the dynamics of $\tilde{a}$, $\tilde{b}$ as

$$\dot{\tilde{a}} = -(S(\omega) + K_a)\tilde{a} - S(z)\hat{a} \tag{23}$$
$$\dot{\tilde{b}} = -(S(\omega) + K_b)\tilde{b} - S(z)\hat{b}. \tag{24}$$

Considering the candidate Lyapunov function for these subsystems

$$V_{ab}(\tilde{a},\tilde{b}) := \frac{1}{2}(|\tilde{a}|^2 + |\tilde{b}|^2),$$

and taking its time-derivative results in

$$\begin{aligned}\dot{V}_{ab}(\tilde{a},\tilde{b}) &\leq -\tilde{a}^\top K_a\tilde{a} + |\hat{a}||\tilde{a}||z| - \tilde{b}^\top K_b\tilde{b} + |\hat{b}||\tilde{b}||z| \\ &\leq -\tilde{a}^\top(K_a - r\frac{|\hat{a}|^2}{2}I)\tilde{a} - \tilde{b}^\top(K_b - r\frac{|\hat{b}|^2}{2}I)\tilde{b} + |z|^2,\end{aligned}$$

with the last inequality obtained after applying Young's inequality as $xy = xy\frac{\sqrt{r}}{\sqrt{r}} \leq \frac{r|x|^2}{2} + \frac{|y|^2}{2r}$ and observing from $\dot{r}$ that $r(t) \geq 1$.

Similarly, we take the time-derivative of the function

$$V_r(r) = \frac{1}{2}(r-1)^2,$$

that gives the expression

$$\begin{aligned}\dot{V}_r &= -2\psi_1(r-1)^2 + 2(r-1)r(K_1|\tilde{a}| + K_2|\tilde{b}|) \\ &\leq -(2\psi_1 - 1)(r-1)^2 + 2r^2(K_1^2|\tilde{a}|^2 + K_2^2|\tilde{b}|^2),\end{aligned}$$

where we applied Young's inequality and used the fact that $(x+y)^2 \leq 2(x^2 + y^2)$.

Finally, defining the composite Lyapunov function as

$$W(z,\tilde{a},\tilde{b},r) := V_s(z,r) + V_{ab}(\tilde{a},\tilde{b}) + V_r(r), \tag{25}$$

and taking its derivative along trajectories of the $(z,\tilde{a},\tilde{b},r)$-dynamics results in the expressions

$$\begin{aligned}\dot{W} &\leq -(\kappa - 1)V_s - \lambda_m(K_{a0})|\tilde{a}|^2 - \lambda_m(K_{b0})|\tilde{b}|^2 \\ &\quad - (2\psi_1 - 1)(r-1)^2 \\ &\leq -\min(\kappa - 1, \lambda_m(K_{a0}), \lambda_m(K_{b0}), (2\psi_1 - 1))W < 0,\end{aligned}$$

$\forall(z,\tilde{a},\tilde{b},r) \neq (0,0,0,1)$, and with $\kappa > 1$, $\psi_1 > 0.5$, $K_{a0} = K_{a0}^\top \succ 0$, $K_{b0} = K_{b0}^\top \succ 0$,

$$K_a(\hat{a},r) := K_{a0} + (2r^2||K_1||^2 + \frac{r|\hat{a}|^2}{2})I \tag{26}$$

$$K_b(\hat{b},r) := K_{b0} + (2r^2||K_2||^2 + \frac{r|\hat{b}|^2}{2})I. \tag{27}$$

This concludes the uniformly global exponential stability (UGES) claim. ■

Remark 1: The proposed observer was derived using the observer methodology based on invariant manifolds, see [13], [14] and references therein for the original works. The general principle behind this technique is to estimate the unmeasured state η by rendering a certain manifold

$$\mathcal{M} = \{(\eta,y,\hat{y},\xi)|\beta(\xi,y,\hat{y}) = \varphi(\eta,y,\hat{y})\}$$

attractive and invariant for some ξ (the observer state), y the measured state, $\hat{y}$ a filtered version of y, and functions β, φ. The objective then is to stabilize to zero the dynamics of the "error" (usually called off-the-manifold coordinates)

$$z := \beta(\xi,y,\hat{y}) - \varphi(\eta,y,\hat{y}),$$

whose norm essentially captures the distance from the manifold $\mathcal{M}$. If this (non-standard) stabilization objective is achieved then an estimate of η is given by $\hat{\eta} = \varphi^{-1}(\beta(\xi,y,\hat{y}),y,\hat{y})$.

Apart from the applications in the above references, this methodology has been recently applied by the author and co-workers to succesfully solve a variety of problems such as global velocity estimation for Euler-Lagrange mechanical systems [16], [17], global exponential position-feedback tracking for fully-actuated mechanical systems [18], global asymptotic position-feedback synchronization for tele-operation systems [19], as well as for semi-global reduced attitude estimation for quadrotors [20] and full attitude estimation of rigid bodies for different measurement scenarios [21]–[23].

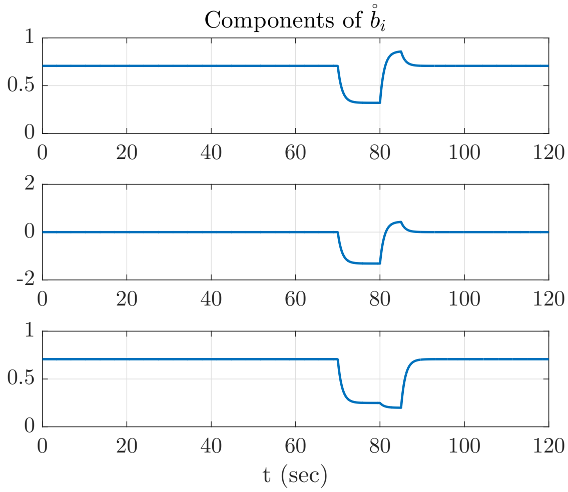

Fig. 1. Components of $\mathring{b}$.

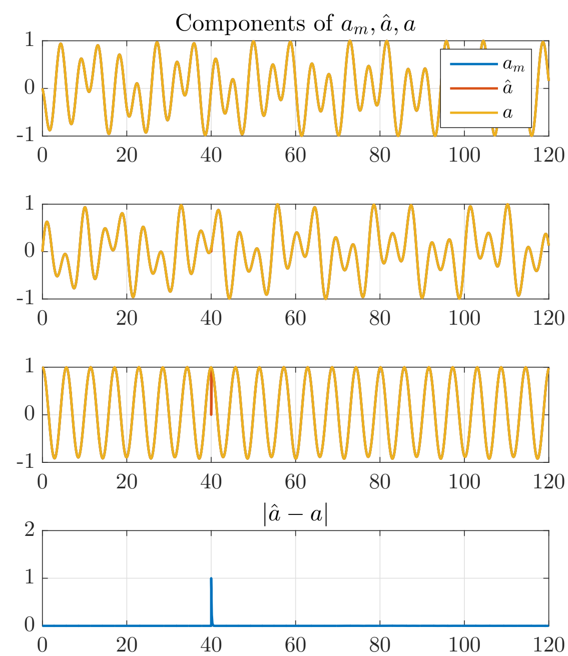

Fig. 2. Noiseless setting: Components of measured (a_m), true (a) and filtered ($\hat{a}$) vectors.

IV. Simulations

The excellent behavior of the observer is now illustrated in a simple simulation scenario. Simulations were run for the model of a CubeSat used in [8], [9]. This CubeSat is a rectangular parallelepiped of dimensions 20 (cm)×10 (cm)×10 (cm) and mass 2 (kg) assumed to be homogeneously distributed. As such the inertia matrix is given as $J = \mathtt{diag}(87, 83, 37)(\text{kg/cm}^2)$.

The constant vectors $\mathring{a}$ and $\mathring{b}$, corresponding to the measured vectors a, b, are respectively set to the nominal values $(0, 0, 1)^T$ and $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})^T$, in order to mimic the gravity and magnetic vectors.

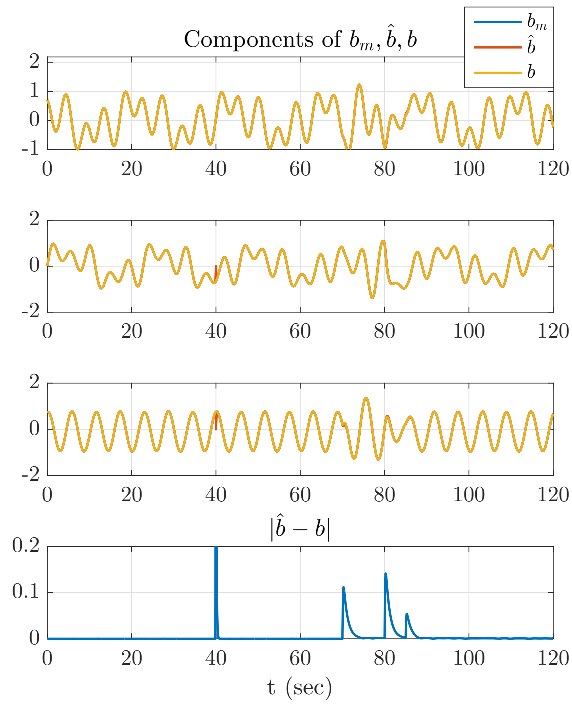

Fig. 3. Noiseless setting: Components of measured (b_m), true (b) and filtered ($\hat{b}$) vectors.

The initial conditions on the angular velocity are taken as $\omega_0 = [1, 0.3, -0.6]^T$ (rad/sec) and we further consider the upper bound on the angular velocity $\bar{\omega} = 1.2042$ (rad/sec). The inital state of the observer has been set in such a way that the initial velocity estimate $\hat{\omega}_0 = 0$ by selecting $\xi_0 = K_1 J^{-1}(\hat{a}_0 \times \mathring{a}) + K_2 J^{-1}(\hat{b} \times \mathring{b})$.

We will consider two scenarios. In the first we consider that measurements are noiseless while in the second measurements are affected by additive noise. To keep notation consistent for both scenarios, we will refer to the measurements as a_m, b_m which in the presence of noise do not identically correspond to the vectors a, b. In both cases, we test the robustness of the proposed algorithm in two different ways; a sudden re-initialization of the observer at $t = 40$ (sec) and a perturbation of the b vector, through a perturbation of its corresponding inertial vector $\mathring{b}$ in the time interval $70 - 90$ (sec), which can be visualized in Fig. 1.

A. *Perfect setting*

We first consider the ideal setting where the measurements are noiseless. The initial conditions and the gains are selected as follows $\hat{a}_0 = [0, 0, 0]^T, \hat{b}_0 = [0, 0, 0]^T, \bar{\omega} = |\omega_0|, \psi = 1, K_1 = 1/2(\psi + \bar{\omega}||J|| + 1) + 0.5, K_2 = 1/2(\psi + \bar{\omega}||J|| + 1) + 0.5, K_{a0} = 0.5, K_{b0} = 0.5$.

The behavior of the measurement vectors a, b are depicted in Figs. 2 and 3. In both figures we can observe the sudden peak due to the sudden reinitialization of the observer that does not however hamper the behavior of the estimator that

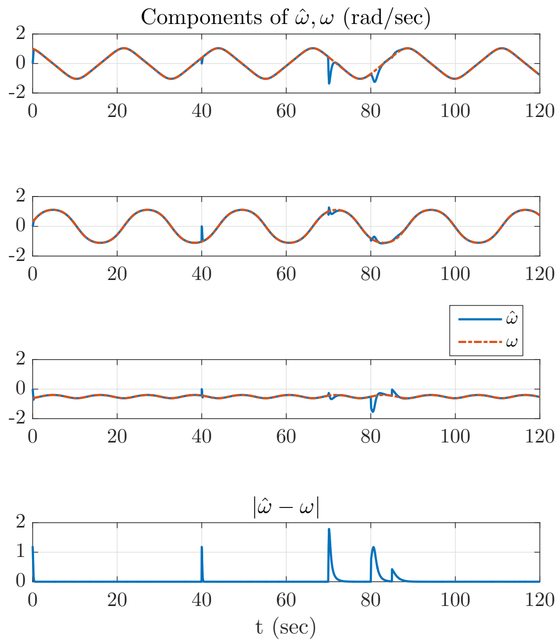

Fig. 4. Noiseless setting: Components of the true ω (dotted red) and its estimate $\hat{\omega}$ (blue).

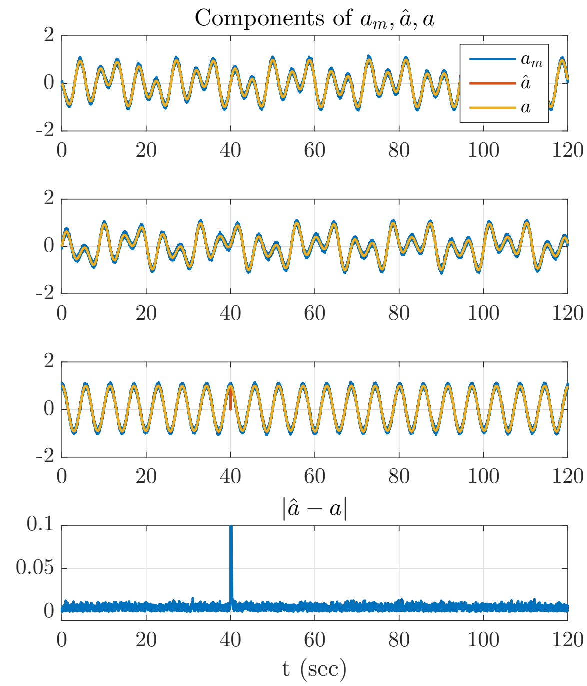

Fig. 5. Noisy setting: Components of measured (a_m), true (a) and filtered ($\hat{a}$) vectors.

recovers fast. In the second, we can additionally visualize the effect of the induced perturbation which, similarly to the reinitialization, is attenuated quite fast.

From the above, it is expected that the angular velocity estimate will inherit a similar behavior and this is confirmed from Fig. 4. In fact, one can hardly observe the effect of the abrupt reset of the observer while during the perturbation of b the estimate converges fast to the true value.

B. Vector measurement noise

We now consider the case where both vector measurements are affected by noise. All the measurement signals are corrupted by band-limited independent gaussian white noises (sample time 10^{-3}, noise powers 10^{-6} for the components of a and b) $\omega_0 = [1, 0.3, -0.6]^T, \bar{\omega} = |\omega_0|, \hat{a}_0 = [0, 0, 0]^T, \hat{b}_0 = [0, 0, 0]^T, \psi = 1, K_1 = 1/500(\psi + \bar{\omega}||J|| + 1) + 0.001, K_2 = 1/500(\psi + \bar{\omega}||J|| + 1) + 0.001, K_{a0} = 0.5, k_{b0} = 0.5$

Similarly to the ideal measurement setting, the transient behavior of the observer is excellent despite the presence of additive noise. This can be established by looking at the time evolution of the measurement vectors a, b, depicted in Figs. 5 and 6, and the comparison of the true and estimated angular velocities, depicted in Fig. 7.

V. Conclusions–Perspectives

A global solution has been proposed to the problem of angular velocity estimation for a rigid body. Under the main realistic assumption of linear independence of two body measurements, the proposed observer ensures the *uniform global exponential stability* of zero estimation error. This is established through a thorough stability analysis based on a strict Lyapunov function. The theoretical results are further supported by extensive numerical simulations.

Given the nice properties of the proposed observer, it is expected that its use can be exploited among others for the problems of output-feedback attitude stabilization and tracking control. Although not presented here, the proposed observer can be shown to work well in simulations even if only one available vector measurement is available, in the expense of it being persistently exciting. As such, a future direction is to provide a detailed proof for this case.

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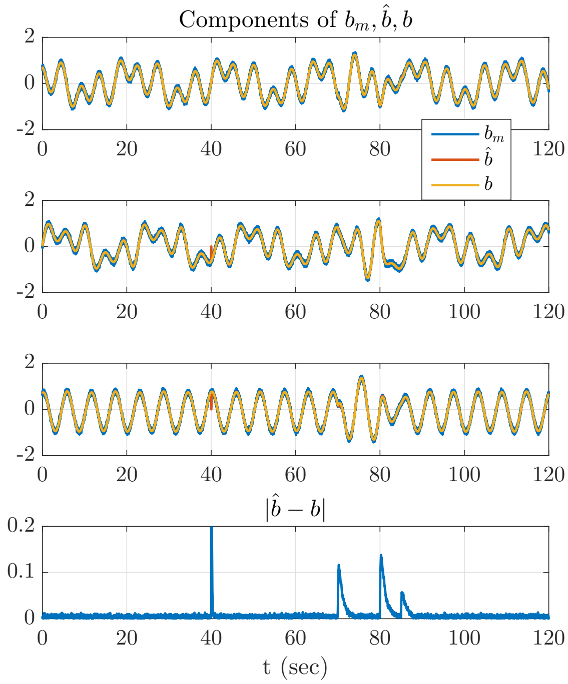

Fig. 6. Noisy setting: Components of measured (b_m), true (b) and filtered ($\hat{b}$) vectors.

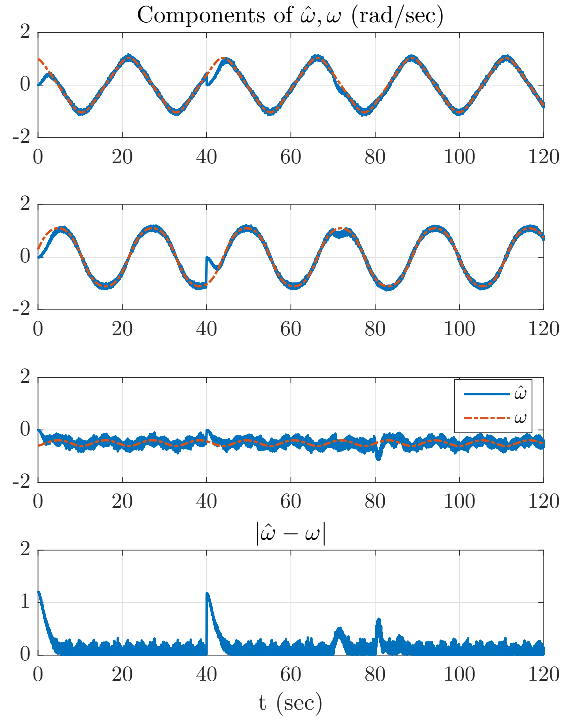

Fig. 7. Nosiy setting: Components of the true ω (dotted red) and its estimate $\hat{\omega}$ (blue).

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