Distributed Full-Consensus Control of Multi-Robot Systems with Range and Field-of-View Constraints

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***Abstract*— In this paper we solve the full-consensus problem for multiple nonholonomic vehicles interacting over a directed leader-follower topology and subject to sensing constraints in the form of limited range and limited field-of-view. Remarkably, based on a polar-coordinates model transformation, the designed controller is time-invariant and smooth (in the domain of definition). Moreover, the control laws rely only on local measurements, making it well suited for implementation. The asymptotic convergence to the consensus manifold as well as the respect of the constraints is established using Lyapunov's first method and cascaded systems theory. Realistic simulations in the Gazebo-ROS environment, which illustrate the effectiveness of our theoretical contributions, are shown in an accompanying video.**

I. Introduction

The distributed consensus problem, being the basis of most multi-vehicle interactions, has received considerable attention in recent years [1], [2]. For multi-agent systems of nonholonomic vehicles, two main problems are addressed in the literature, position consensus [1], [3] and full consensus [4], [5]. In the first case, all agents converge to the same position with an arbitrary or predetermined orientation. In the second case, agreement on both position and orientation is sought. Furthermore, these protocols may be implemented using absolute position and orientation values [3], [5] or rather, using relative measurements [2], [6], [7]. In most practical scenarios, however, protocols using absolute measurements may not be implementable since only the relative measurements from embedded sensing devices are available to the vehicles. This, in turn, presents an additional challenge, when the sensing devices have a limited range or a limited field of view. Indeed, in such scenarios assuming that each agent has access to its neighbors' information at all times, although necessary from a theoretical viewpoint, might be conservative in practice.

Consequently, considerable attention has been focused on the study of coordination strategies of multi-agent systems subject to distance and field-of-view constraints. In [8]–[10] coordination protocols with field-of-view-based connectivity are considered, albeit for linear integrator models. For nonholonomic systems, most works in the literature address only the problem of position consensus. Moreover, the solutions are based, for the most part, on non-smooth or time-varying control laws. The latter have the advantage to guarantee uniform forms of asymptotic stability in closed loop, which guarantees robustness, but, in general, they are difficult to tune and may give rise to undesirable behavior, such as oscillations [4], [5]).

In [11] using relative information, a navigation-function-based controller with distance-based connectivity maintenance is proposed for multi-agent systems interconnected over directed graphs; nevertheless, the controller is non-smooth and it presents some problems inherent to the navigation-function framework such as local minima and the need to have a bounded workspace. In [12] distance constraints are considered for leader-follower topologies based on barrier functions. However, only position consensus is achieved and the controller requires the knowledge of absolute positions. In [13] distance constraints are addressed and practical stability of a position-consensus-based formation is achieved, but the estimation of global parameters is required. In [6], [7] the authors develop time-varying control laws with prescribed performance considering both distance and field-of-view constraints. Nevertheless, full consensus is not achieved and only the platooning problem, with interaction topologies consisting of single directed chains, is addressed.

In this paper we extend the results obtained in [14], where a distributed controller achieving full consensus of multiple vehicles in a leader-follower configuration is proposed, albeit considering only relative-distance constraints. More precisely, herein the full-consensus problem is solved for multi-agent systems subject to both distance and field-of-view constraints and interacting over an arbitrary leader-follower topology, that is, a directed spanning tree graph. The control design is based on a polar-coordinates model for unicycles proposed in [15]. The advantages of using this model are twofold. First, we are able to propose a controller which is time-invariant and smooth in the domain of definition. Second, since the polar-coordinates model relies only on relative variables of distances between agents and line-of-sight angles, the proposed controller may be implemented without inter-agent information exchange. These two characteristics not only facilitate the analysis of

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the closed-loop system, but they also render the control law more suited to implementation.

The paper is organized as follows. In Section II the model and the problem statement are presented. The main results are presented in Section III and are illustrated via simulations in Section IV. Concluding remarks are given in Section V.

Notation. The real n-coordinate space, with $n \in \mathbb{N}$, is denoted as $\mathbb{R}^n$; $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements non-negative and positive, respectively. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$, if it is continuous, strictly increasing and zero at zero. If moreover $\gamma(s) \to \infty$ as $s \to \infty$, we say that $\gamma \in \mathcal{K}_\infty$. A digraph, denoted $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, is defined by a set of nodes, $\mathcal{V} := \{1, 2, \ldots, n\}$ whose elements correspond to the labels of the agents' states and a set of edges, $\mathcal{E} \subseteq \mathcal{V}^2$ of cardinality m, that represents the communication between a pair of nodes. A directed edge e_k, with $k \leq m$, is an ordered pair $(i, j) \in \mathcal{E}$ if and only if a connection exists from node i to node j. The distance $d(i, j)$ between nodes $i, j \in \mathcal{V}$ is the number of edges in the shortest path from i to j. A directed *spanning tree* $\mathcal{G}_\mathcal{T}$ is a graph consisting of a root node, with no parent, and a set of nodes reachable from the root.

II. Model and problem statement

We consider a multi-agent system composed of n unicycles. To facilitate the control design and the analysis, the multi-vehicle system is represented using an alternative polar-coordinates-based model. Such model has the advantage that equilibria are reachable via smooth time-invariant feedback. Moreover it naturally leads to the design of controllers that rely only on local relative measurements –*cf.* [14], [15].

For every pair of leader and follower vehicles in the system, labeled i and j respectively, let $\rho_k \in \mathbb{R}_{\geq 0}$ denote the distance separating them, $\beta_k \in (-\pi, \pi]$ denote the angle between the line of sight and the leader's direction of motion, and $\alpha_k \in (-\pi, \pi]$ denote the angle between the line of sight and the follower's direction of motion. See Figure 1 for a graphical representation.

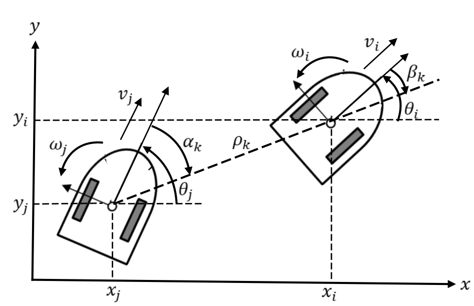

Fig. 1. Leader-follower scheme and polar-coordinates variables

Then, for a network of agents interacting in a leader-follower configuration, we have m interconnected dynamical systems described by the following equations:

$$\dot{\rho}_k = v_i \cos\beta_k - v_j \cos\alpha_k \tag{1a}$$

$$\dot{\beta}_k = \frac{1}{\rho_k}\left[-v_i \sin\beta_k + v_j \sin\alpha_k\right] - \omega_i \tag{1b}$$

$$\dot{\alpha}_k = \frac{1}{\rho_k}\left[-v_i \sin\beta_k + v_j \sin\alpha_k\right] - \omega_j, \tag{1c}$$

where v_j and ω_j are the control inputs, and v_i and ω_i, which are respectively the leader's velocity and angular rate, are considered as external signals.

The objective is to design distributed control laws that achieve full consensus for the multi-agent system. From a dynamical systems viewpoint, the solutions to Eqs. (1) correspond to the leader-follower relative error trajectories for the pair of index k. Therefore, the full consensus control goal is reached if $(\rho_k, \beta_k, \alpha_k) \to (0, 0, 0)$ for all $k \leq m$.

In addition to the distributed nature of the information in the system, we also assume that the agents cannot communicate with each other. Instead, we consider that each agent is only able to measure or estimate information from its immediate leader. Thus, the leader-follower configuration, may be represented by an arbitrary directed spanning tree $\mathcal{G}_\mathcal{T}$ graph as in Figure 2, where the triple $(\rho_k, \beta_k, \alpha_k)$, corresponds to the state of an edge e_k in the tree $\mathcal{G}_\mathcal{T}$. Herein assume to have perfect knowledge of the estimates of the relative state $(\rho_k, \beta_k, \alpha_k)$. The estimation of these variables from the on-board measurements is out of the scope of this paper.

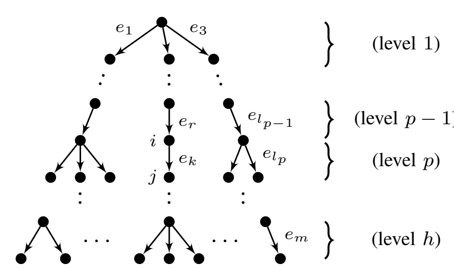

Fig. 2. Directed spanning tree $\mathcal{G}_\mathcal{T}$.

In graph theory, it is well-known that a directed spanning tree is the minimal configuration necessary to achieve consensus [16]. However, in this paper we also consider that the vehicles are subject to sensing constraints in the form of a limited field-of-view, described by an angle of view, and limited range described by a maximal detection distance. Therefore, the existence of this minimal topology cannot be assumed, but rather has to be enforced by the control law. Thus, besides achieving full consensus, in this paper we design controllers that ensure the following properties.

Definition 1 (Distance-based connectivity maintenance): For each $k \leq m$, let $\Delta_{\rho,k} > 0$ denote the maximum distance between the leader-follower pair (i, j), *i.e.,* the edge e_k, such that the measurements of the follower j are reliable. We say that the distance constraints are respected if the set

$$\mathcal{J}_\rho := \{\rho_k \in \mathbb{R}_{\geq 0} \,:\, \rho_k < \Delta_{\rho,k},\ \forall\, k \leq m\} \qquad (2)$$

is forward invariant. That is, $\rho_k(0) < \Delta_{\rho,k}$ implies that $\rho_k(t) \in \mathcal{J}_\rho$ for all $t \geq 0$.

Definition 2 (Field-of-view connectivity maintenance): For each $k \leq m$, let $\Delta_{\alpha,k} > 0$ denote the maximum line-of-sight angle, corresponding to half the angle of view, such that for the leader-follower pair (i,j), *i.e.*, the edge e_k, the leader is inside the follower's field of view. We say that the field-of-view constraints are respected if the set

$$\mathcal{J}_\alpha := \{\alpha_k \in (-\pi, \pi] \,:\, |\alpha_k| < \Delta_{\alpha,k},\ \forall\, k \leq m\} \quad (3)$$

is forward invariant. That is, if $\alpha_k(0) < \Delta_{\alpha,k}$ implies that $\alpha_k(t) \in \mathcal{J}_\alpha$ for all $t \geq 0$.

Thus, the problem considered herein is to define distributed smooth time-invariant control laws v_j and ω_j, $j \leq n$, such that, the relative variables $(\rho_k, \beta_k, \alpha_k)$ converge to zero for all $k \leq m$ and that connectivity, according to Definition 1 and Definition 2, is maintained for all initial conditions satisfying $\rho_k(0) \in \mathcal{J}_\rho \backslash \{0\}$ and $\alpha_k(0) \in \mathcal{J}_\alpha$, and for all $t \geq 0$.

III. Main results

A. Controller design

The control approach is based on the classical backstepping method [17]. The main idea for system (1) is to first design a virtual input α_k^* along with v_j in order to stabilize the origin of the subsystem (1a)-(1b). Then, the angular rate ω_j is designed so that $\alpha_k(t) \to \alpha_k^*(t)$, or equivalently, defining the error variable $\tilde{\alpha}_k := \alpha_k - \alpha_k^*$, so that $\tilde{\alpha}_k \to 0$.

Furthermore, in order to guarantee connectivity and field-of-view requirements, the controller design relies on the concept of *barrier* Lyapunov functions. These functions are reminiscent of artificial potential fields, which have been extensively used in robotics to enforce constraints [18], [19]. Here, we encode the distance and field-of-view constraints using the barrier Lyapunov function candidates

$$U_{\zeta,k}(\zeta_k) := \frac{1}{2}\left[\zeta_k^2 + \ln\left(\frac{\Delta_{\zeta,k}^2}{\Delta_{\zeta,k}^2 - \zeta_k^2}\right)\right], \quad \zeta \in \{\rho, \alpha\}. \qquad (4)$$

Unlike quadratic Lyapunov functions, the barrier Lyapunov functions have the property of growing unbounded as the state approaches the imposed limits, *i.e.*, $U_{\zeta,k}(\zeta_k) \to \infty$ as $\zeta_k \to \Delta_{\zeta,k}$. Therefore, by guaranteeing that $U_{\zeta,k}(\zeta_k(t))$ remains bounded along the trajectories of the closed-loop system, the respect of the state constraints is guaranteed.

The control inputs are chosen in terms of the gradient of the barrier function (4), which is given by

$$\nabla_\zeta U_{\zeta,k} \triangleq \frac{\partial U_{\zeta,k}}{\partial \zeta_k}, \quad \zeta \in \{\rho, \alpha\}.$$

After a direct computation we obtain

$$\nabla_\zeta U_{\zeta,k} = p_{\zeta,k}(\zeta_k)\zeta_k \qquad (5)$$

where

$$p_{\zeta,k}(\zeta_k) = 1 + \frac{1}{(\Delta_{\zeta,k}^2 - \zeta_k^2)}.$$

For further analysis we stress that $p_{\zeta,k}(\zeta_k) \geq p_0 > 0$ for all $|\zeta_k| < \Delta_{\zeta,k}$ with p_0 a positive constant, and $p_{\zeta,k}(\zeta_k) \to \infty$ as $\zeta_k \to \Delta_{\zeta,k}$.

Remark 1: Notice that when the sensing limits $\Delta_{\zeta,k}$ tend to infinity, that is, there are no constraints on the state, the second term in the right-hand-side of (4) vanishes and the barrier Lyapunov function becomes a quadratic Lyapunov function. Thus, the barrier function (4) may be used for the design of consensus protocols with or without sensing constraints.

From a control theory viewpoint, we want to use the barrier functions as Lyapunov functions for the analysis of the closed-loop system. However, from the backstepping method, the closed-loop system depends on the error variable $\tilde{\alpha}_k$ rather than on the constrained variable α_k. Therefore, in order to be able to use the barrier function encoding the field-of-view constraints as a Lyapunov function, $U_{\alpha,k}$ has to be modified so that it is made positive definite in $\tilde{\alpha}_k$. For this purpose, we rely on the concept of *recentered barrier function* introduced in [20], and exploited in multi-robot coordination in [21] among others. The recentered barrier function $\tilde{U}_{\alpha,k}$ is defined as

$$\tilde{U}_{\alpha,k}(\tilde{\alpha}_k) := U_{\alpha,k}(\tilde{\alpha}_k + \alpha_k^*) - U_{\alpha,k}(\alpha_k^*) - \frac{\partial U_{\alpha,k}(\alpha_k^*)}{\partial \alpha_k^*}\tilde{\alpha}_k. \qquad (6)$$

Remark 2: The recentered barrier function $\tilde{U}_{\alpha,k}$ is positive definite, that is, $\tilde{U}_{\alpha,k}(\tilde{\alpha}_k) > 0$ for all $\tilde{\alpha}_k \neq 0$ and $\tilde{U}_{\alpha,k}(0) = 0$, and it tends to $+\infty$ as $|\alpha_k| \to \Delta_{\alpha,k}$, or equivalently, as $|\tilde{\alpha}_k + \alpha_k^*| \to \Delta_{\alpha,k}$. Therefore it is a valid barrier Lyapunov function for the closed-loop system.

Now, following the backstepping approach, the virtual input is designed as [14],

$$\alpha_k^* := \arctan(-c_3\beta_k), \qquad (7)$$

with c_3 a design constant satisfying

$$0 < c_3 < \min_{k \leq m}\{\Delta_{\alpha,k}/\pi\}.$$

Remark 3: Note that the bound on c_3 comes from the design of the recentered barrier function (6). Indeed, in order for the term $U_{\alpha,k}(\alpha_k^*)$ to be well defined, we need to guarantee $|\alpha_k^*| < \Delta_{\alpha,k}$. Therefore, since $|\alpha_k^*| \leq c_3|\beta_k|$ and $\beta_k \in (-\pi, \pi]$ by definition, choosing $c_3 < \min_{k\leq m}\{\Delta_{\alpha,k}/\pi\}$, the terms $U_{\alpha,k}(\alpha_k^*)$ and $[\partial U_{\alpha,k}/\partial \alpha_k^*](\alpha_k^*)$ are well defined and upper bounded by some positive constants.

Let the coefficients

$$a_{jk} := \begin{cases} 1 & \text{if edge } e_k \text{ is incident on node } j \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

represent the available information for each agent j based on the interaction topology $\mathcal{G}_\mathcal{T}$. Then, based on the latter, the distributed control inputs v_j and ω_j are taken proportional to the gradient of the barrier functions $U_{\rho,k}$ and $\tilde{U}_{\alpha,k}$, respectively. More precisely, the controller is given as follows:

$$v_j := c_1 \sum_{k\leq m} a_{jk}\eta_k \; \nabla_\rho U_{\rho,k}(\rho_k), \tag{9}$$

$$\eta_k := \sqrt{1+(c_3\beta_k)^2} \tag{10}$$

$$\omega_j := \sum_{k\leq m} a_{jk} \Bigg[c_2 \nabla_{\tilde{\alpha}} \tilde{U}_{\alpha,k}(\tilde{\alpha}_k) + \left[\psi_k + \left[1+\frac{c_3}{\eta_k^2}\tilde{\omega}_k\right]\frac{\sin(\alpha_k)}{\rho_k}\right]\sum_{i\leq n} a_{ik} v_i \Bigg] \tag{11}$$

where $c_1, c_2 > 0$ are design constants. In turn, the term

$$\psi_k := -\nabla_\rho U_{\rho,k} \frac{[\cos(\alpha_k) - \cos(\alpha_k^*)]}{\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k}} + \frac{\beta_k}{\rho_k}\frac{[\sin(\alpha_k)-\sin(\alpha_k^*)]}{\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k}} \tag{12}$$

compensates for the error dynamics coming from the tracking problem of the backstepping method. The term

$$\frac{c_3}{\eta_k^2}\tilde{\omega}_k \frac{\sin(\alpha_k)}{\rho_k}\sum_{i\leq n} a_{ik}v_i,$$

where

$$\tilde{\omega}_k := \left[\frac{\partial^2 U_{\alpha,k}(\alpha_k^*)}{\partial \alpha_k^{*2}}\right]\frac{\tilde{\alpha}_k}{\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k}}, \tag{13}$$

is used to dominate the additional terms coming from the derivative of the recentered barrier function (6).

Remark 4: Note that (13) is well posed for all $\rho_k \in \mathcal{J}_\rho \backslash \{0\}$ and all $\alpha_k \in \mathcal{J}_\alpha$. Indeed, using (5), (6), and L'Hôpital's rule, we have

$$\lim_{\tilde{\alpha}_k\to 0}\frac{\tilde{\alpha}_k}{\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k}} = \lim_{\tilde{\alpha}_k\to 0}\frac{1}{p_{\alpha,k}(\tilde{\alpha}_k)+\frac{\partial^2 p_{\alpha,k}(\tilde{\alpha}_k)}{\partial\tilde{\alpha}_k^2}\tilde{\alpha}_k} = \varpi,$$

where ϖ is a positive constant. Moreover

$$\lim_{|\tilde{\alpha}_k|\to\Delta_{\alpha,k}}\frac{\tilde{\alpha}_k}{\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k}} = 0.$$

Furthermore, noting that $|\sin(\tilde{\alpha}_k+\alpha_k^*)-\sin(\alpha_k^*)| \leq |\tilde{\alpha}_k|$ and $|\cos(\tilde{\alpha}_k+\alpha_k^*)-\cos(\alpha_k^*)| \leq |\tilde{\alpha}_k|$, the previous statement holds also for (12).

Since the root node (which represents the leader agent) has no incoming edges, $a_{1k} = 0$ for all $k \leq m$ —*cf.* Eq. (8). Therefore, with the full consensus protocol (9)-(11), the inputs of the leader are set equal to zero, *i.e.*, $v_1(t) = \omega_1(t) = 0$. This case is analyzed next. It may be shown, however, that with the proposed controller, the range and field-of-view requirements are satisfied, even when the leader has non-zero inputs which might come from additional objectives like trajectory tracking.

B. Multi-agent system with static leader

Consider a multi-agent system composed of n unicycles with interactions represented through an arbitrary directed spanning tree. The multi-agent system in a leader-follower topology may be represented as a cascaded system. To see this, notice that the tree representing the leader-follower topology may be divided into $h \leq n-1$ levels based on the distance to the root node —see Figure 2. Denote $\mathcal{E}_p \subset \mathcal{E}$ the set of edges such that the distance from its terminal node to the root of the tree, labeled $i=1$, is equal to $p \leq h$, *i.e.*, $\mathcal{E}_p := \{e_k = (i,j) \in \mathcal{E} : d(1,j) = p\}$. Without loss of generality, assume that each level p of the tree contains an l_p number of edges, such that $1 \leq l_p \leq m$, $\sum_{p=1}^h l_p = m$. Then, for each level p having l_p arcs labeled e_k with $k \in [l_{p-1}+1, l_p]$ we define the *closed-loop* state variables

$$\xi_p^\top := \left[\xi_{p,1}^\top \cdots \xi_{p,l_p}^\top\right] \in \mathbb{R}^{3l_p}, \quad \xi_{p,k}^\top := [\rho_k \; \beta_k \; \tilde{\alpha}_k] \in \mathbb{R}^3.$$

Using this notation, the systems (1) in closed loop with the inputs (9)-(11), with $k \leq m$ and for the considered graph, can be written in the compact cascaded-system form,

$$\begin{aligned} \dot{\xi}_p &= f_p(\xi_p) + g_p(\xi_p, \xi_{p-1}), \quad p \in [2,h] \\ \dot{\xi}_1 &= f_1(\xi_1) \end{aligned} \tag{14}$$

where, for each $p \leq h$,

$$\begin{aligned} f_p(\xi_p) &:= \left[f_{p,1}(\xi_{p,1})^\top \ldots \; f_{p,l_p}(\xi_{p,l_p})^\top\right]^\top, \\ g_p(\xi_p,\xi_{p-1}) &:= \left[g_{p,1}(\xi_{p,1},\, \xi_{p-1})^\top \ldots \; g_{p,l_p}(\xi_{p,l_p},\, \xi_{p-1})^\top\right]^\top \end{aligned}$$

and the nominal systems, $\dot{\xi}_{p,k} = f_{p,k}(\xi_{p,k})$, correspond to

$$\dot{\rho}_k = -c_1\nabla_\rho U_{\rho,k}\Big[1+[\cos(\alpha_k)-\cos(\alpha_k^*)]\eta_k\Big] \tag{15a}$$

$$\dot{\beta}_k = -2c_1 p_{\rho,k}(\rho_k)\Big[c_3\beta_k - [\sin(\alpha_k)-\sin(\alpha_k^*)]\eta_k\Big] \tag{15b}$$

$$\dot{\tilde{\alpha}}_k = -c_2\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,k} - c_1\Big[\psi_k\eta_k - (1-\tilde{\omega}_k)\frac{c_3}{\eta_k}\frac{\sin(\alpha_k)}{\rho_k}\Big]\nabla_\rho U_{\rho,k} \tag{15c}$$

where we recall that $\alpha_k = \tilde{\alpha}_k + \alpha_k^*$.

The interconnection term $g_p(\xi_p, \xi_{p-1})$ depends on states relative to the p-th level in the tree and to the previous one in the following way. Fix $k \in [l_{p-1}+1, l_p]$ and $r \in [l_{p-2}+1, l_{p-1}]$ in a manner that the edge $e_r \in \mathcal{E}_{p-1}$ is incident on $e_k \in \mathcal{E}_p$, that is, so that the terminal node of e_r is the initial node of e_k —see Figure 2. Let $\xi_{p-1,r} := [\rho_r \ \beta_r \ \tilde{\alpha}_r]^\top$ be the state associated to e_r. Then,

$$g_{p,k}\left(\xi_{p,k}, \xi_{p-1}\right) = \begin{bmatrix} c_1 \cos(\beta_k)\eta_r \ \nabla_\rho U_{\rho,r} \\ \tilde{g}_\beta\left(\xi_{p,k}, \xi_{p-1,r}\right) \\ \tilde{g}_\alpha\left(\xi_{p,k}, \xi_{p-1,r}\right) \end{bmatrix}, \qquad (16)$$

where $\eta_r := \sqrt{1 + (c_3\beta_r)^2}$,

$$\tilde{g}_\beta := -c_1\eta_r\nabla_\rho U_{\rho,r}\left[\frac{\sin(\beta_k)}{\rho_k} + \left[1 + \frac{c_3}{\eta_r^2}\tilde{\omega}_r\right]\frac{\sin(\alpha_r)}{\rho_r} + \psi_r\right] - c_2\nabla_\alpha\tilde{U}_{\alpha,r}$$

and

$$\tilde{g}_\alpha := -\frac{c_2c_3}{\eta_k^2}\nabla_\alpha\tilde{U}_{\alpha,r} - c_1\eta_r\nabla_\rho U_{\rho,r}\left[\left(1 + \frac{c_3}{\eta_k^2}\right)\frac{\sin(\beta_k)}{\rho_k} + \frac{c_3}{\eta_k^2}\left[\psi_r + \left(1 + \frac{c_3}{\eta_r^2}\tilde{\omega}_r\right)\frac{\sin(\alpha_r)}{\rho_r}\right]\right] \qquad (17)$$

where $\alpha_r = \tilde{\alpha}_r + \alpha_r^*$.

We stress that $\nabla_\rho U_{\rho,r}$ is a function of ρ_r only and $[\nabla_\rho U_{\rho,r}](0) = 0$. Similarly, $[\nabla_{\tilde{\alpha}}\tilde{U}_{\alpha,r}](0) = 0$. Therefore, from the definition of g_p we have $g_p(\xi_p, 0) \equiv 0$. Hence, based on cascaded-systems theory [22], [23], one can assert that if for every $p \in [2, h+1]$ the solution of $\dot{\xi}_{p-1} = f_{p-1}(\xi_{p-1})$ converges to zero and if, for every $p \in [2, h]$, the solutions of $\dot{\xi}_p = f_p(\xi_p) + g_p(\xi_p, \xi_{p-1})$, denoted $\xi_p(t)$, remain bounded, we also have $\xi_p(t) \to 0$. This is established in our main statement.

Proposition 1 (Main result): Consider a multi-agent system composed of n unicycles, described by the m interconnected systems (1), interacting over a directed spanning tree $\mathcal{G}_\mathcal{T}$ and subject to distance and field-of-view constraints as defined by (2)-(3). The smooth time-invariant controller (9)-(11) asymptotically achieves full consensus with connectivity maintenance, *i.e.*, $(\rho_k, \beta_k, \alpha_k) \to (0, 0, 0)$, and $\rho_k(t) \in \mathcal{J}_\rho$ and $\alpha_k(t) \in \mathcal{J}_\alpha$, for all $k \le m$, all $t \ge 0$, and for all initial conditions $(\rho_k(0), \beta_k(0), \alpha_k(0))$ such that $\rho_k(0) \in \mathcal{J}_\rho\backslash\{0\}$ and $\alpha_k(0) \in \mathcal{J}_\alpha$.

The proof of this statement, which is omitted due to space constraints, follows arguments from cascaded systems' theory [22], [23], where convergence to the consensus manifold is concluded by establishing asymptotic stability for each nominal system and boundedness of the interconnections. Moreover, the connectivity and field-of-view constraints hold by showing that the barrier functions remain bounded along the trajectories of the system.

IV. Simulation results

To illustrate our theoretical results, some numerical simulations were performed in MATLAB. Furthermore, using the Gazebo-ROS simulator, we validated the connectivity-maintenance property of the proposed algorithm in a more realistic simulation scenario. A video with representative simulations accompanies this paper.

For the tests we consider six agents interconnected in a leader-follower topology (a directed spanning tree) as illustrated in Figure 3.

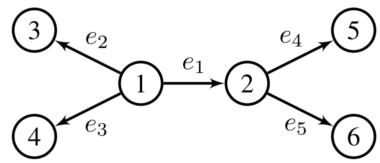

Fig. 3. Labeled directed spanning tree graph. The leader is labeled as “1”.

For all the simulations, the controller parameters are fixed to $c_1 = 0.2$, $c_2 = 1$ and $c_3 = 0.2$, and the initial conditions were set to

$$\begin{aligned}
[x_1(0), y_1(0), \theta_1(0)] &= [-4, 0, \pi/2] \\
[x_2(0), y_2(0), \theta_2(0)] &= [3.5, 0, 2.72] \\
[x_3(0), y_3(0), \theta_3(0)] &= [-11, 5, -0.21] \\
[x_4(0), y_4(0), \theta_4(0)] &= [-11, -5, 0.21] \\
[x_5(0), y_5(0), \theta_5(0)] &= [11, 5, -2.16] \\
[x_6(0), y_6(0), \theta_6(0)] &= [11, -5, 2.16].
\end{aligned}$$

The distance and field-of-view constraints are set, respectively, as $\Delta_{\rho,1} = 8\,\text{m}$, $\Delta_{\rho,2} = 8.8\,\text{m}$, $\Delta_{\rho,3} = 8.7\,\text{m}$, $\Delta_{\rho,4} = 9.3\,\text{m}$, $\Delta_{\rho,5} = 9.5\,\text{m}$, and $\Delta_{\alpha,k} = 25°$, $k \le 5$.

In Figures 4-7 are presented the MATLAB simulation results. It is clear from the figures that, the leader being static, the triple $(\rho_k, \beta_k, \alpha_k)$ converges asymptotically to the origin, meaning that full-consensus is achieved for the group of agents. Moreover, as can be evidenced from Figures 5 and 6, both distance and field-of-view constraints are always respected.

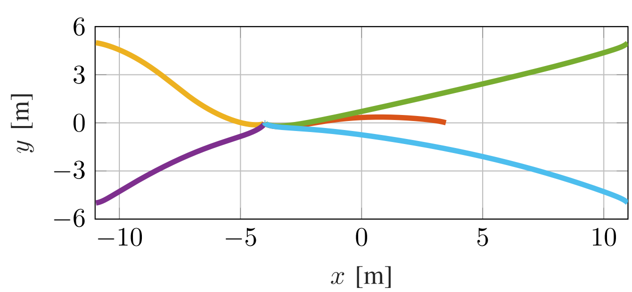

Fig. 4. MATLAB simulation - Full consensus with a static leader

V. Conclusions

A smooth time-invariant distributed feedback controller is proposed in this paper that solves the problem of full consensus of a multi-vehicle system subject to distance and field-of-view constraints and interconnected through an arbitrary leader-follower (directed spanning tree) topology.

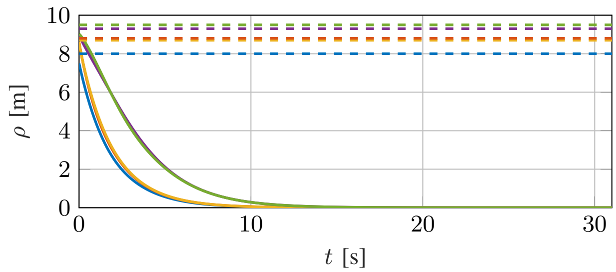

Fig. 5. MATLAB simulation - Inter-agent distances. Dashed lines: distance constraints.

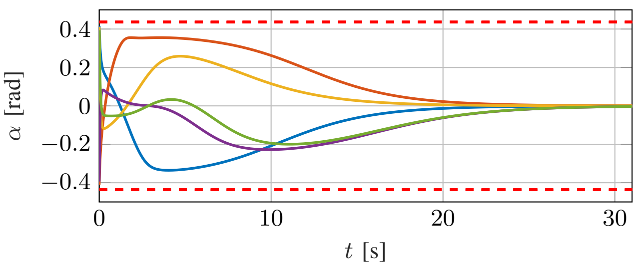

Fig. 6. MATLAB simulation - Followers' line-of-sight angle. Dashed lines: field-of-view constraints.

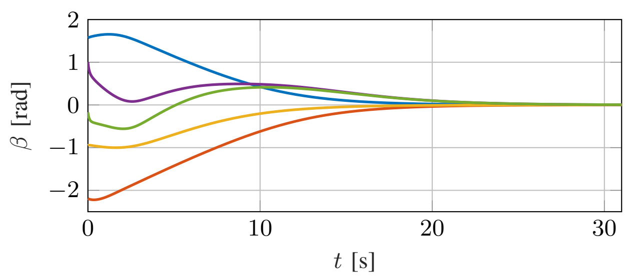

Fig. 7. MATLAB simulation - Angles β_k.

Moreover, we show that the solutions of the multi-vehicle system remain bounded and the constraints are respected for all time instants, even when the leader agent's inputs are non-zero. Remarkably, using a polar-coordinates-based model, the proposed control law is smooth time-invariant and relies only on relative inter-agent variables, which renders it more suited for practical implementation and facilitates the analysis through Lyapunov and cascaded systems' theory. The effectiveness of the controller was asserted with numerical simulations and, in an accompanying video, using a realistic Gazebo-ROS simulator.

Current and future work is focused on extending the results to 3D models and considering additional control objectives such as formation tracking and collision avoidance.

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