# Robust Rendezvous Control of UAVs with Collision Avoidance and Connectivity Maintenance

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*Abstract*— **We address the rendezvous control problem of a group of thrust-propelled Unmanned Aerial Vehicles (UAVs) interconnected over an undirected graph and subject to inter-agent constraints. The proposed distributed control law achieves the desired formation using only local information and guarantees inter-agent collision-avoidance as well as connectivity maintenance. Relying on the edge-agreement framework and on singular-perturbation theory of multi-stable systems we establish almost-everywhere practical input-to-state stability of the desired formation with respect to disturbances. In the absence of perturbations, asymptotic convergence to the consensus manifold is ensured. A numerical simulation illustrates the effectiveness of the proposed approach.**

## I. Introduction

The rendezvous problem for autonomous vehicles [1] consists in making them move towards an undetermined meeting point and acquire a desired formation. It is part of a more complex maneuvering task in which, also, the vehicles may be required to advance in formation [2]. To approach realistic scenarios, however, certain assumptions are important. First, the systems' dynamics are nonlinear and underactuated [3], so consensus controllers tailored for linear systems [4], [5] do not directly apply. Second, the measurements often come from embedded relative-measurement sensors. The use of such devices implies that each agent has access only to local information about the relative state of its neighbors, imposing the need of distributed controllers for the coordination of the multi-vehicle system. Moreover, the on-board devices, are reliable only if used within a limited range. This translates into guaranteeing that the UAVs do not drift "too far" apart from their neighbors. A third difficulty stems from the fact that the autonomous vehicles moving "freely" in space are prone to undesired collisions among themselves, thereby preventing a successful realization of the task. Hence, guaranteeing the safety of the system in the sense of inter-agent collision avoidance is a major feature to be considered. Finally, in realistic settings, UAVs operate under disturbances in the form of wind gusts, aerodynamic effects or unmodeled dynamics, that may destabilize the system. Therefore, the controllers must ensure robustness with respect to disturbances.

The difficulties described above are not artificially cobbled to set up an academic problem. They coin a realistic scenario of automatic control and, to the best of our knowledge, have never been addressed simultaneously for thrust-propelled UAVs; only individually.

In order to deal with the underactuation, some hierarchical control approaches that use the natural cascaded structure of the UAVs' dynamics have been reported. They rely on dynamic feedback [3], attitude and thrust extraction [6], backstepping [7] or singular-perturbations theory [8]. The controller proposed in [7], e.g., applies to consensus and formation tracking over undirected and balanced directed graphs whereas the controller from [6] applies to directed graphs with dynamically-changing topologies. In [9] a distributed estimation technique is proposed for formation tracking over directed graphs. In [10] switching communication topologies in which the directed graph may be disconnected over intervals is considered. In [1] for directed graphs, a backstepping-based general framework is proposed achieving (almost) global asymptotic full consensus. Based on an input-output perspective, in [11] a dynamic controller to solve the consensus problem over digraphs is proposed. Nonetheless, none of the previous works addresses the limited-range and collision avoidance constraints mentioned above.

Relatively few works consider the problem for multiple thrust-propelled UAVs under such constraints. A distributed formation-tracking controller is proposed in [12] based on prescribed-performance and guaranteeing collision avoidance. In [13] the formation problem for multiple thrust-propelled UAVs subject to inter-agent distance (connectivity) constraints is solved. However, collision-avoidance restrictions are not considered and only non-robust convergence to the consensus manifold is concluded. In [14] and [15] robust formation controllers are proposed based on edge-agreement frameworks [16], guaranteeing also collision avoidance and connectivity maintenance. However, in these references only fully-actuated Lagrangian systems interconnected over undirected-tree topologies are considered.

In this paper we consider the rendezvous problem as a position consensus problem. Our main result is to propose a dynamic controller that guarantees the achievement of a desired formation and ensures that the inter-agent constraints are always respected, even in the presence of disturbances. Compared to the works in the literature, our contributions are twofold. We address the rendezvous problem simultaneously

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with all the aspects evoked above, that is, collision avoidance, connectivity maintenance, and disturbances. On the other hand we guarantee stronger results on stability and robustness. Indeed, we establish almost-everywhere practical-input-to-state stability of the desired formation with respect to disturbances while guaranteeing connectivity maintenance and collision avoidance; in the absence of disturbances, we ensure asymptotic attractivity of the consensus manifold and respect of the constraints.

## II. Model and Problem Formulation

Consider a swarm of $n$ thrust-propelled UAVs; each agent's motion being described by a so-called "mixed" model, which consists in a second-order Cartesian dynamics equation on E(3) and a first-order attitude kinematics equation on SO(3) —see e.g., [3], [7], and [17]. This *underactuated* model is a common representation of the UAV dynamics, used for the hierarchical control framework [3].

Let $\mathcal{I} := \{e_1, e_2, e_3\}$ denote an inertial frame in North-East-Down coordinates, and $\mathcal{B}_i$ denote the body-fixed frame. The orientation of the $i$th agent with respect to the inertial frame $\mathcal{I}$ is represented by a rotation matrix $\mathfrak{R}_i \in \mathrm{SO}(3)$. Let $p_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ be respectively the inertial position and inertial velocity. Then, the model for the $i$th vehicle is

$$\dot{p}_i = v_i \tag{1a}$$

$$\dot{v}_i = -\frac{T_i}{m_i}\mathfrak{R}_i e_3 + g e_3 + \theta_i(t) \tag{1b}$$

$$\dot{\mathfrak{R}}_i = \mathfrak{R}_i S(\omega_i), \tag{2}$$

where $m_i$ is the mass of the quadrotor, $g$ is the gravitational acceleration, $S(\cdot)$ is the skew-symmetric matrix associated with the cross product, i.e., such that $S(u)v = u \times v$, $\forall u, v \in \mathbb{R}^3$), and $\theta_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$, is an essentially bounded disturbance (e.g. aerodynamic forces). The inputs are the thrust force, $T_i \in \mathbb{R}$, and the angular rates $\omega_i \in \mathbb{R}^3$, expressed in the body-fixed frame $\mathcal{B}_i$.

It is assumed that each agent relies only on embedded limited-range measurement/communication devices to obtain information from its neighbors. This implies that each vehicle has access only to local information from a limited number of neighbors. This local interaction is represented by an undirected graph, denoted $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the set of nodes, $\mathcal{V} := \{1, 2, \ldots, n\}$, corresponds to the labels of the agents and the set of edges, $\mathcal{E} \subseteq \mathcal{V}^2$ of cardinality $m$, represents the communication between a pair of nodes. An edge $e_k := (i, j) \in \mathcal{E}$, $k \leq m$, exists if and only if agent $i$ is within the measurement/communication range of agent $j$.

From a graph-theory perspective, it is well established that a fundamental requirement for the agreement of the agents is that the fixed undirected graph representing the interconnections remains connected [4]. However, the use of limited-range devices imposes a maximal inter-agent distance so that the information exchange can be established. Therefore, to guarantee that the connectivity properties of the graph are maintained and to achieve the desired task, it must be ensured that the agents do not drift "too far" apart from each other. Additionally, autonomous agents moving "freely" in space are prone to undesired collisions among themselves. These inter-agent restrictions are represented in terms of the relative positions of the interconnected agents as follows.

Define the relative positions as

$$z_k := p_i - p_j \quad \forall k \leq m, \quad e_k \in \mathcal{E}. \tag{3}$$

For each $k \leq m$, let $\Delta_k$ be the maximal distance so that the communication through the arc $e_k$ is reliable. Similarly, let $\delta_k$ be the minimal distance that guarantees the avoidance of collisions between agents $i$ and $j$. Let $z^\top = [z_1^\top \ \cdots \ z_m^\top] \in \mathbb{R}^{3m}$ be the collection of edge states. Then, the set of inter-agent constraints is defined as

$$\mathcal{D} := \big\{ z \in \mathbb{R}^{3m} : \delta_k < |z_k| < \Delta_k, \ \forall k \leq m \big\}. \tag{4}$$

The control goal is for the robots to achieve a predetermined static formation in the three-dimensional space, using only local information and considering the inter-agent constraints given by the set $\mathcal{D}$ in (4). Mathematically, the formation problem translates into making $p_i - p_j \to z_k^d$, where $z_k^d \in \mathbb{R}^3$ denotes the desired relative state between a pair of neighboring agents $i$ and $j$. We can now state the problem as follows.

*Robust formation problem with inter-agent constraints:* Consider a multi-agent system composed of $n$ thrust-propelled UAVs with underactuated dynamics described by (1)-(2). Let the interactions of the vehicles be given by a connected undirected graph. Moreover, let the inter-agent constraints be given by the set (4) and define the formation error $\tilde{z}_k := z_k - z_k^d$. Then, find distributed controllers $T_i$ and $\omega_i$, $i \leq n$, that, in the absence of disturbances, that is, $\theta_i(t) \equiv 0$, $i \leq n$, achieve the desired static formation, i.e.

$$\lim_{t\to\infty} \tilde{z}_k(t) = 0 \quad \forall k \leq m \tag{5}$$

$$\lim_{t\to\infty} v_i(t) = 0 \quad \forall i \leq n. \tag{6}$$

and render the constraints set (4) forward invariant, i.e. $z(0) \in \mathcal{D}$ implies that $z(t) \in \mathcal{D}$ for all $t \geq 0$. Furthermore, in the presence of disturbances, that is, if $\theta_i(t) \not\equiv 0$ and bounded, the control law renders the formation practically input-to-state stable with respect to the disturbances, whereas the set defined in (4) is made forward-invariant. •

## III. Main result

The proposed solution consists in first using a dynamic input transformation, inspires by the hierarchical approach in [7], and then designing a distributed rendezvous controller based on the command-filtered-backstepping methodology [18]. We start by expressing the system in feedback form.

### A. Feedback transformation

The translational dynamics (1) may be assimilated to a second-order integrator

$$\dot{p}_i = v_i \tag{7a}$$

$$\dot{v}_i = \zeta_i + \theta_i(t), \tag{7b}$$

considering

$$\zeta_i := -\frac{T_i}{m_i}\mathfrak{R}_i e_3 + g e_3. \tag{8}$$

as a virtual input. However, because of the underactuation of (1), the virtual input $\zeta_i \in \mathbb{R}^3$ cannot take an arbitrary value since $T_i \in \mathbb{R}$ and its direction is determined by the vehicle's orientation, $\mathfrak{R}_i$. Therefore, to overcome the underactuation, we solve equation (8) dynamically. More precisely, we design the actual inputs $\omega_i$ and $T_i$, so that $\zeta_i$ in (8) satisfies the dynamic equation

$$\dot{\zeta}_i = u_i, \quad i \leq n, \tag{9}$$

where $u_i \in \mathbb{R}^3$ is a new input to be defined.

Differentiating (8) with respect to time, and using (2), the left-hand side of (9) becomes

$$-\frac{\dot{T}_i}{m_i}\mathfrak{R}_i e_3 - \frac{T_i}{m_i}\mathfrak{R}_i S(\omega_i) e_3 = u_i. \tag{10}$$

Then, for a given $u_i$, we define $\nu_i \in \mathbb{R}^3$ as

$$\nu_i := u_i - \frac{c_3}{m_i} T_i \mathfrak{R}_i e_3, \tag{11}$$

where $c_3$ is a positive control gain. Then, replacing (11) into (10), we obtain

$$-\frac{1}{m_i}\left[\dot{T}_i\mathfrak{R}_i + T_i\mathfrak{R}_i S(\omega_i)\right] e_3 = \nu_i + \frac{c_3}{m_i}T_i\mathfrak{R}_i e_3$$

$$\iff \left[(\dot{T}_i + c_3\, T_i)\mathfrak{R}_i + T_i\mathfrak{R}_i S(\omega_i)\right] e_3 = -m_i\nu_i \tag{12}$$

Left-multiplying by $\mathfrak{R}_i^\top$, the dynamic equation (12) is equivalent to

$$\left[T_i\omega_{yi},\ -T_i\omega_{xi},\ \dot{T}_i + c_3\, T_i\right]^\top = -m_i\mathfrak{R}_i^\top \nu_i. \tag{13}$$

Now, denote $\tilde{\nu}_i := [\tilde{\nu}_{i,x}\ \tilde{\nu}_{i,y}\ \tilde{\nu}_{i,z}]^\top = \mathfrak{R}_i^\top \nu_i$. Then, (13) holds if the angular rates are set to

$$\omega_i = \left[\frac{m_i\tilde{\nu}_{i,y}}{T_i},\ -\frac{m_i\tilde{\nu}_{i,x}}{T_i},\ \omega_{zi}\right]^\top, \tag{14}$$

and the thrust is given by the update law

$$\dot{T}_i = -c_3\, T_i - m_i\tilde{\nu}_{i,z}. \tag{15}$$

*Remark 1:* Note that (14) is valid only if $T_i \neq 0$. This requirement is not assumed but ensured by our control design as explained below. •

### B. Formation control

With the system's dynamics modeled in feedback form and the consensus problem expressed as that of steering $\tilde{z}_k \to 0$, the latter can be formulated as one of stabilization at the origin. With this in mind, we rely on the edge-agreement perspective [16] to model the multi-agent system dynamics.

Let us denote the so-called incidence matrix of a graph, $E \in \mathbb{R}^{n\times m}$, which is a matrix with rows indexed by the nodes and columns indexed by the edges. Its $(i,k)$th entry is defined as follows: $[E]_{ik} := -1$ if $i$ is the terminal node of edge $e_k$, $[E]_{ik} := 1$ if $i$ is the initial node of edge $e_k$, and $[E]_{ik} := 0$ otherwise. Then, the edge states in (3) satisfy

$$z = [E^\top \otimes I_3]p \tag{16}$$

and the formation errors

$$\tilde{z} = [E^\top \otimes I_3]p - z^d, \tag{17}$$

with $p^\top = [p_1^\top \cdots p_n^\top] \in \mathbb{R}^{3n}$ and $z^{d\top} = [z_1^{d\top} \cdots z_m^{d\top}] \in \mathbb{R}^{3m}$.

An advantage of considering the edge states rather than the node states is that it is possible to obtain an equivalent reduced system, easier to analyze using stability theory. After [16], using an appropriate labeling of the edges, the incidence matrix and the edge states may be expressed as

$$E = [\ E_t \quad E_c\ ], \quad \tilde{z} = \left[\tilde{z}_t^\top \ \ \tilde{z}_c^\top\right]^\top \tag{18}$$

where $E_t \in \mathbb{R}^{n\times(n-1)}$ and $\tilde{z}_t \in \mathbb{R}^{3(n-1)}$ denote, respectively, the full-column-rank incidence matrix and the edge states corresponding to an arbitrary spanning tree $\mathcal{G}_t \subset \mathcal{G}$; $E_c \in \mathbb{R}^{n\times(m-n+1)}$ and $\tilde{z}_c \in \mathbb{R}^{3(m-n+1)}$ represent, respectively, the incidence matrix and the states corresponding to the remaining edges not contained in $\mathcal{G}_t$. Moreover, defining

$$R := [\ I_{n-1} \quad T\ ], \quad T := \left(E_t^\top E_t\right)^{-1} E_t^\top E_c \tag{19}$$

with $I_{n-1}$ denoting the $n-1$ identity matrix, one obtains the following identities

$$E = E_t R, \quad \tilde{z} = \left[R^\top \otimes I_3\right]\tilde{z}_t. \tag{20}$$

The identities (20) are useful to derive a reduced-order dynamic model —cf. [16]. Indeed, denoting $v^\top = [v_1^\top \ \cdots \ v_n^\top] \in \mathbb{R}^{3n}$, $\zeta^\top = [\zeta_1^\top \ \cdots \ \zeta_n^\top] \in \mathbb{R}^{3n}$, $\theta^\top = [\theta_1^\top \ \cdots \ \theta_n^\top] \in \mathbb{R}^{3n}$, and using the edge transformation (17), the third-order multi-agent system (7)-(9), for all $i \leq m$, in the reduced error-edge coordinates becomes

$$\dot{\tilde{z}}_t = \left[E_t^\top \otimes I_3\right] v \tag{21a}$$

$$\dot{v} = \zeta + \theta(t) \tag{21b}$$

$$\dot{\zeta} = u. \tag{21c}$$

Thus, the formation objective (5), is achieved by designing a controller to stabilize the origin for (21).

We start by defining a virtual control law for (21a), using $v$ as an input. The virtual control is based on the gradient of a barrier Lyapunov function [19] encoding the inter-agent constraints. Barrier Lyapunov functions are reminiscent of Lyapunov functions, so they are positive definite, but their domain of definition is restricted by design to open subsets of the Euclidean space. Furthermore, they grow unbounded as $z_k$ approaches the border of the open set.

To encode the inter-agent constraints, we first define a weight recentered barrier function [20] given by

$$\bar{B}_k(z_k) = \kappa_{1,k}\left[\ln\left(\frac{\Delta_k^2}{\Delta_k^2 - |z_k|^2}\right) - \ln\left(\frac{\Delta_k^2}{\Delta_k^2 - |z_k^d|^2}\right)\right] + \kappa_{2,k}\left[\ln\left(\frac{|z_k|^2}{|z_k|^2 - \delta_k^2}\right) - \ln\left(\frac{|z_k^d|^2}{|z_k^d|^2 - \delta_k^2}\right)\right]$$

where

$$\kappa_{1,k} := \frac{\delta_k^2}{|z_k^d|^2(|z_k^d|^2 - \delta_k^2)} \quad \kappa_{2,k} := \frac{1}{\Delta_k^2 - |z_k^d|^2}. \tag{22}$$

The function $\bar{B}_k$ is non-negative, and satisfies: $\bar{B}_k(z_k^d) = 0$, $\nabla\bar{B}_k(z_k^d) = 0$, and $\bar{B}_k(z_k) \to \infty$ as either $|z_k| \to \Delta_k$ or

$|z_k| \to \delta_k$, or equivalently in the error coordinates, as either $|\tilde{z}_k + z_k^d| \to \Delta_k$ or $|\tilde{z}_k + z_k^d| \to \delta_k$. Then, in the error coordinates, a barrier Lyapunov function is given by

$$W_k(\tilde{z}_k) := \frac{1}{2}\left[|\tilde{z}_k|^2 + \bar{B}_k(\tilde{z}_k + z_k^d)\right]. \tag{23}$$

The barrier Lyapunov function (23) is positive definite in $\tilde{z}_k$ and grows unbounded as $z_k$ approaches the distance constraints, that is, $W_k(\tilde{z}_k) \to \infty$ as either $|\tilde{z}_k + z_k^d| \to \Delta_k$ or $|\tilde{z}_k + z_k^d| \to \delta_k$. Therefore, to guarantee the respect of the constraints, the control must ensure the boundedness of $W_k(\tilde{z}_k)$. We follow [21], where the consensus problem with connectivity constraints was considered for first and second-order integrators, and define the virtual input for the relative-position subsystem (21a) as

$$v^* := -c_1[E_t \otimes I_3]\nabla W(\tilde{z}_t) \tag{24}$$

where $c_1$ is a positive constant,

$$\nabla W(\tilde{z}_t) := \frac{\partial W(\tilde{z}_t)}{\partial \tilde{z}_t}, \quad W(\tilde{z}_t) = \sum_{k \le m} W_k(\tilde{z}_k),$$

and $W_k(\tilde{z}_k)$ is given in (23) for all $k \le m$.

*Remark 2:* The functions (23) are reminiscent of scalar potential functions in constrained environments. Hence, as stated in [22], the appearance of multiple critical points is inevitable. Therefore, the gradient $\nabla W_k(\tilde{z}_k)$, vanishes at the origin and at an isolated saddle point away from the origin. This implies that the closed-loop system has multiple equilibria, preventing us from using the classical stability tools for the analysis. This major difficulty is addressed using tools tailored for multi-stable systems—see [23], [24]. •

Now following a backstepping design, we need to design a virtual control law $\zeta^*$ so that $v \to v^*$, or equivalently $v - v^* \to 0$. Owing to the fact that the system (21) is in normal form, a natural virtual control law is given by

$$\zeta^* := -c_2(v - v^*) + \dot{v}^*, \quad c_2 > 0. \tag{25}$$

It is well known, however, as evidenced by (25) that the backstepping approach may lead to an increase of complexity of the control law due to the successive differentiation of the virtual controllers. This problem is emphasized by the fact that the said virtual control is designed as the gradient of a barrier function, which has multiple local minima and is defined only in open subsets of the state space. Therefore, to avoid the use of successive derivatives of $\nabla W$, inspired by the command filtered backstepping approach [18], we approximate the virtual controls $v^*$ and $\zeta^*$ and their derivatives by means of second-order command filters defined as:

$$\xrightarrow{\chi^*} \boxed{H_1(s)} \xrightarrow{\chi_f} \qquad H_1(s) := \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

Note that the dirty derivative of $\chi^* \in \{v^*, \zeta^*\}$ may be obtained using $\dot{\chi}_f := sH_1(s)\chi^*$ which is equivalent to $\dot{\chi}_f = H_1(s)\dot{\chi}^*$. Alternatively, in state space form, defining $\alpha_{\chi,1} := \chi_f$ and $\alpha_{\chi,2} := \dot{\chi}_f/\omega_n$, we have

$$\dot{\alpha}_\chi = \omega_n [A \otimes I_{3n}]\, \alpha_\chi + \omega_n [B \otimes I_{3n}]\, \chi^* \tag{26a}$$

$$\begin{bmatrix}\chi_f^\top & \dot{\chi}_f^\top\end{bmatrix}^\top = [C \otimes I_{3n}]\, \alpha_\chi, \quad \chi \in \{v, \zeta\}, \tag{26b}$$

$$A := \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C := \begin{bmatrix} 1 & 0 \\ 0 & \omega_n \end{bmatrix} \tag{26c}$$

and the initial conditions are set to $\alpha_{\chi,1}(0) = \chi^*(0)$ and $\alpha_{\chi,2}(0) = 0$, $\chi \in \{v, \zeta\}$.

Now, in light of the command filters in (26), let us define the backstepping error variables

$$\tilde{v} := v - v_f \quad \text{and} \quad \tilde{\zeta} := \zeta - \zeta_f \tag{27}$$

and, to ensure that $T_i \neq 0$ —see Remark 1— let the actual virtual control law be given by

$$\zeta^* := \text{sat}\left(-c_2\tilde{v} + \omega_n \alpha_{v,2}\right), \tag{28}$$

where $\text{sat}(s)$ is a saturation function defined element-wise, i.e., $\text{sat}(s) = \begin{bmatrix}\text{sat}(s_1)^\top \cdots \text{sat}(s_N)^\top\end{bmatrix}^\top$, $\text{sat}(s_i) = \text{sign}(s_i)\min\{|s_i|, \bar{\zeta}_M\}$, with $\bar{\zeta}_M < g$. Indeed, after (8), $T_i \neq 0$ if $\zeta_i^*$ satisfies $\zeta_i^* \neq ge_3$, for all $i \le n$, which holds by setting a proper saturation level. Finally, $u$ is set to

$$u := -c_3\tilde{\zeta} + \omega_n \alpha_{\zeta,2} - \tilde{v}. \tag{29}$$

*Proposition 1:* Consider the system (1)-(2). The dynamic controller defined by (15), (14), and (29) together with (24), (28), and (26) with a sufficiently large $\omega_n$, solves the robust formation problem with inter-agent constraints for almost all initial conditions satisfying $z(0) \in \mathcal{D}$, except for a set of measure zero. □

*Sketch of proof:*[1] The proof uses cascaded-systems and singular-perturbation theory tailored for multi-stable systems [23], [24]. Indeed, note that the system (21) in closed loop with (29), together with (24), (28), and (26) may be written in singular-perturbation form, with $\epsilon := 1/\omega_n$, as

$$\dot{\tilde{z}}_t = -c_1[E_t^\top E_t \otimes I_3]\nabla W(\tilde{z}_t) + [E_t^\top \otimes I_3]\left[\tilde{v} + \tilde{\alpha}_{v,1}\right] \tag{30a}$$

$$\dot{\tilde{v}} = \text{sat}(-c_2\tilde{v} + \omega_n\tilde{\alpha}_{v,2}) + \tilde{\zeta} + \tilde{\alpha}_{\zeta,1} - \omega_n\tilde{\alpha}_{v,2} + \theta(t) \tag{30b}$$

$$\dot{\tilde{\zeta}} = -c_3\,\tilde{\zeta} - \tilde{v} \tag{30c}$$

$$\epsilon\dot{\tilde{\alpha}} = \tilde{A}\tilde{\alpha} - \epsilon\frac{\partial h(\xi)}{\partial \xi}\dot{\xi}, \quad \tilde{A} := \text{blockdiag}\{[A \otimes I_{3n}]\}, \tag{30d}$$

where $\xi^\top := \begin{bmatrix}\tilde{z}_t^\top, \tilde{v}^\top, \tilde{\zeta}^\top\end{bmatrix}$ and $\tilde{\alpha}^\top := \begin{bmatrix}\tilde{\alpha}_v^\top\, \tilde{\alpha}_\zeta^\top\end{bmatrix} = \begin{bmatrix}\alpha_v^\top\, \alpha_\zeta^\top\end{bmatrix} - h(\xi)$, with

$$h(\xi) = \begin{bmatrix}(-c_1[E_t \otimes I_3]\nabla W(\tilde{z}_t))^\top & 0^\top & \text{sat}(-c_2\tilde{v})^\top & 0^\top\end{bmatrix}^\top \tag{31}$$

being the "quasi-steady-state" for the filter sub-system (26).

Then, the statement of the proposition may be established according the following ordered steps:

1) To show that the boundary layer system
$$\frac{d\tilde{\alpha}}{d\tau} = \tilde{A}\tilde{\alpha}, \quad \tau := \frac{t}{\epsilon}, \tag{32}$$
is exponentially stable at the origin. This holds since $\tilde{A}$ is Hurwitz by design.
2) Prove the reduced system given by (30) with $\epsilon = 0$, i.e.,

[1] Proposition 1 follows as a Corollary of [25, Theorem 1]. See the latter for a detailed proof.

$$\dot{\tilde{z}}_t = -c_1[E_t^\top E_t \otimes I_3]\nabla W(\tilde{z}_t) + [E_t^\top \otimes I_3]\tilde{v} \quad (33a)$$
$$\dot{\tilde{v}} = -\operatorname{sat}(c_2\tilde{v}) + \tilde{\zeta} + \theta(t) \quad (33b)$$
$$\dot{\tilde{\zeta}} = -c_3\tilde{\zeta} - \tilde{v}, \quad (33c)$$

to be input-to-state stable with respect to the set $\mathcal{W}_\Theta := \mathcal{W}\times\{0\}^2$, where $\mathcal{W}$ denotes the set containing the origin and the saddle point of $W(\tilde{z}_t)$, and an external input $\theta$. Indeed, the subsystem (33a) is input-to-state stable with respect to $\mathcal{W}$ and to $\tilde{v}$ —cf. [21][2]; moreover, the subsystem (33b)-(33c) is input-to-state stable with respect to $\theta$. Hence, after [23, Theorem 3.1], the result follows.

3) To prove that, for a sufficiently small $\epsilon$, the system (30) is practically input-to-state stable with respect to the set $\mathcal{W}_\Theta \times \{0\}$ and a bounded external input $\theta$. This follows from the previous steps and from [24, Theorem 2]. Then, in the absence of disturbances, i.e. $\theta(t) \equiv 0$, we may deduce convergence to the set of equilibria.
4) Owing to the fact that the critical points of the barrier function $\mathcal{W}$ are a minimum (the origin) and a saddle point, after [26, Proposition 11], it follows that the region of attraction of the unstable saddle point has zero Lebesgue measure. Therefore, the properties evoked in the previous step hold almost-everywhere except for a set of measure zero.
5) Finally, to prove forward invariance of the output constraints set (4). This follows by showing that for any initial condition satisfying $z(0) \in \mathcal{D}$, the barrier Lyapunov function $W(\tilde{z}_t)$ remains bounded, implying that the trajectories $z(t)$ do not escape the set $\mathcal{D}$.

## IV. Simulation Results

To illustrate our theoretical findings, we present a numerical example consisting in the rendezvous of six quadrotors, subject to inter-agent collision avoidance and connectivity restrictions. At the initial time, the vehicles are interconnected over an undirected connected graph as in Fig. 1. The desired formation is determined by the desired relative position vector $z_{dk} = \left(z^d_{k,x}, z^d_{k,y}, z^d_{k,z}\right)$, for each $k \leq 7$, set to $(1, 0.5, 0)$, $(-1, 1.5, 0)$, $(-1, 0.5, 0)$, $(-2, 1, 0)$, $(-1, 0.5, 0)$, $(0, -1, 0)$, $(1, -0.5, 0)$. The initial conditions and constraint parameters are presented in Table I.

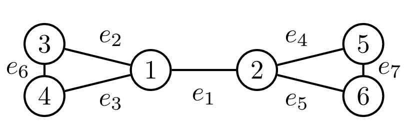


Fig. 1. Connected undirected graph

The maximal and minimal distances of the inter-agent constraint set (4) are, respectively, $\Delta_k = 4.3$m and $\delta = 0.2$m $\forall k \leq m$. The saturation limit for the desired controller of the translational dynamics was set to $\bar{\zeta}_M = 7$ m/s$^2$, the controller gains to $c_1 = 1$, $c_2 = 0.8$, $c_3 = 3$, and the filter natural frequency is set to $\omega_n = 350$ rad/s. We consider the mass of each drone to be $m_i = 0.4$ kg.

[2]In [21] only connectivity constraints are considered, hence the system has a single equilibrium. However, using the barrier function (23) the same result holds here in the context of multi-stability.

TABLE I
INITIAL CONDITIONS

| UAV index | $p_x$ [m] | $p_y$ [m] | $p_z$ [m] | $v_x$ [m/s] | $v_y$ [m/s] | $v_z$ [m/s] |
|---|---|---|---|---|---|---|
| 1 | 1.9 | 0 | -1 | 0.6 | -0.8 | 0 |
| 2 | -2 | 0 | 0 | -0.3 | 0 | 0 |
| 3 | 5.2 | 2 | 0 | 1.3 | 0.3 | 0 |
| 4 | 5.2 | -2 | 0 | 0.1 | 0 | 0 |
| 5 | -5.5 | 2 | 0 | 0 | 0 | 0 |
| 6 | -4.5 | 2 | 2 | -0.8 | 0 | 0 |

It is also assumed that the agents are subject to a vanishing disturbance defined as follows:

$$\theta_i(t) = -\sigma(t)\,[1\ 1\ 0]^\top$$
$$\sigma(t) = \begin{cases} -0.6\big[\tanh(2(t-15)) - 1\big], & i \in \{3,5\} \\ 0.6\big[\tanh(2(t-15)) - 1\big], & i = 2 \\ 0, & i \in \{1,4,6\}. \end{cases} \quad (34)$$

Fig. 2 illustrates the paths of each agent as well as the final desired formation for the multi-agent system.

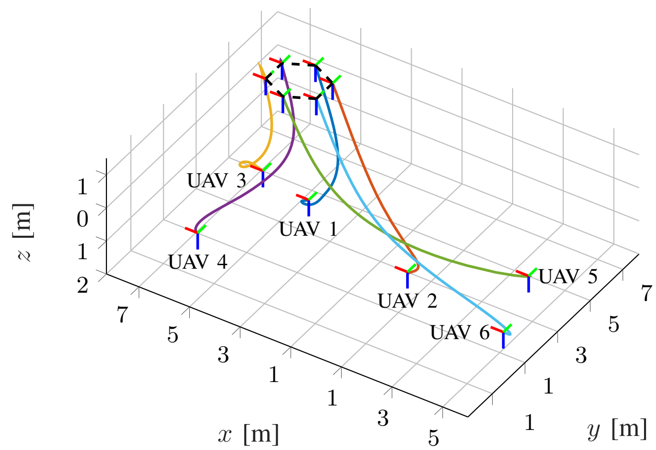


Fig. 2. Paths of the agents. The dashed hexagon represents the final formation.

Fig. 3 depicts the inter-agent distances. Note that both connectivity and collision avoidance constraints (dashed lines) are respected, even in the presence of the disturbance. Then, as soon as the disturbance vanishes, after 15 seconds, the agents converge to the desired static formation. This is also appreciated from Fig. 4 where the velocities are shown.

The thrust control inputs are shown in Fig. 5 —one may appreciate that the non-crossing of zero condition for the thrust is respected.

## V. Conclusions

We presented a solution to the formation control problem of cooperative thrust-propelled UAVs under a set of realistic assumptions and not at the expense of formal analysis. Building upon a feedback transformation design we proposed a distributed controller, using the gradient of a barrier Lyapunov function and the framework of command-filtered backstepping, that achieves the desired formation

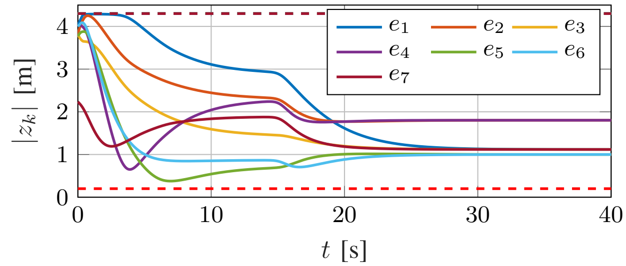


Fig. 3. Distances between connected UAVs ($e_i$ denoting the corresponding edges). The dashed lines represent the connectivity and collision avoidance constraints.

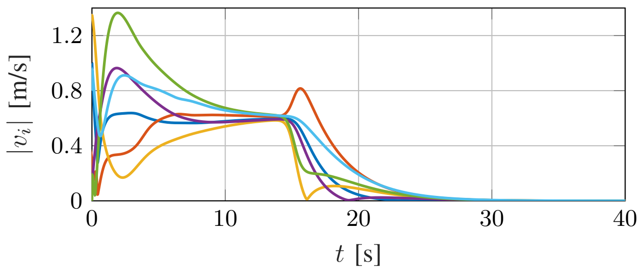


Fig. 4. Inertial velocities of the UAVs.

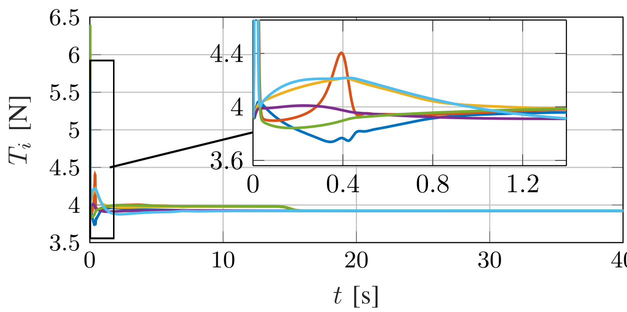


Fig. 5. Thrust inputs of the agents.

with guaranteed connectivity maintenance and inter-agent collision avoidance even in the presence of disturbances. We established almost-everywhere practical input-to-state stability of the desired formation with respect to bounded disturbances, as well as asymptotic convergence in the absence of disturbances. Current research focuses on solving the formation-tracking problem for UAVs interconnected over directed topologies and under state constraints.

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