Distributed event-triggered formation control for multi-agent systems in presence of packet losses

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Abstract

This paper proposes an event-triggered approach for the distributed formation control problem of an Euler-Lagrange multi-agent system with state perturbations, when communications between agents are prone to losses. To evaluate its control input, each agent maintains estimators of its own state and of the states of its neighbors. Each estimator accounts for a different packet-loss hypothesis. Each agent is then able to compute the expected value of the estimation error of its own state as evaluated by its neighbors. A communication triggering condition (CTC) exploiting this expected error is then proposed. An analysis of the behavior of the system with this CTC is performed using stochastic Lyapunov functions. Simulations confirm the effectiveness of the proposed approach.

Key words: Multi-agent systems, event-triggered control, packet losses, formation stabilization.

1 Introduction

Distributed control with event-triggered communication is an efficient method to coordinate Multi-Agent Systems (MAS) with a reduced amount of communications between agents. The Communication Triggering Condition (CTC) is instrumental in these approaches to limit communications, while allowing enough information to be exchanged between agents to complete the task assigned to the MAS [12, 13, 16, 19, 24]. Designing a suitable CTC when communications between agents are prone to packet losses is challenging. With event-triggered control, a message is transmitted only when required. A loss of information may thus have a critical impact on the performance and even stability of the MAS.

Packet losses may result from collisions between packets simultaneously transmitted from different agents, from occlusions by obstacles, or from interference with other communications systems. Considering two packet-loss models, [7] has shown that event-triggered control schemes are more vulnerable to packet losses than time-triggered control strategies. Acknowledgment mechanisms are helpful to detect and possibly re-transmit lost messages. Nevertheless, acknowledgments or re-transmitted messages may also be lost, which increases communication delays, risk of packet collisions, and may lead to desynchronization between agents. In [2, 6, 7, 20, 23] packet losses are addressed by combining an H_∞ control and event-triggered communications. For agents with linear dynamics, sufficient conditions are established to ensure the global exponential stability of the system. In [6], communication delays and packet losses are considered simultaneously. In [2], the focus is on a MAS where agents follow several leaders. Each agent maintains observers of the state of other agents. These observers account for the last received message from the other agents and for models of their dynamics. In [21], two types of networked controller design methods are proposed. The first one ensures that the system is asymptotically stable in the presence of an arbitrary bounded number of packet losses. The second one provides mean square stability in presence of Markovian packet losses. In [5], communication delays are also considered. Two communication protocols are proposed, and the convergence of the MAS is guaranteed if the delay and the number of consecutive of packet losses are bounded. All previous works consider only linear dynamics.

Nonlinear dynamics are studied in [3, 4]. In [3], packet losses are taken into account in the estimator models but not in the CTC: new distributed estimators are designed to guarantee the exponential stability of the estimation errors. To update the estimate of the state of other agents, each agent uses its own innovation and the innovation of its neighbors obtained from received packets. This improves the accuracy of the estimates at the cost of an increased sensitivity to losses. The control of a *single* agent in presence of measurements losses is considered in [4]. An event-triggered strategy is proposed along with two communication protocols, with and without acknowledgments. With acknowledgments, the most recently received measurement can be identified. Without acknowledgment, this information is no longer available. A set of estimators is used to estimate the measurement lost during transmission. Each estimator uses a different hypothesis of the last packet received. As previously stated, the case of a MAS is not considered.

This paper addresses the distributed formation control of a MAS consisting of agents with nonlinear Euler-Lagrange dynamics, affected by state perturbations, and communications with losses. An event-triggered control strategy is proposed extending that presented in [17] to account for packet losses. Each agent maintains several estimators of its *own* state to mimic the estimators of its state maintained by its neighbors. Each estimator considers a different hypothesis of packet reception by these neighbors. This extends the idea of [4], where only two estimators are maintained. Contrary to most studies accounting for packet losses, no explicit feedback mechanism is considered here. Nevertheless, packets received from neighbors provide some (usually delayed) implicit feedback which is exploited to reduce the number of considered loss hypotheses, without requiring additional communications. This reduces the amount of estimators of its own state maintained by each agent. The CTC proposed in [17] is then updated to explicitly account for the potential loss of transmitted packets. The asymptotic convergence of the MAS to the target formation, as well as the absence of Zeno behavior have been proved.

Notations, assumptions, and the formation control problem are introduced in Section 2. The distributed control law used to drive each agent of the MAS is described in Section 3. State estimators required for the evaluation of the distributed control law are detailed in Section 4. The impact of packet losses on these estimators is analyzed. The CTC used by each agent is presented in Section 5. A simulation example is presented in Section 6. Section 7 concludes this work.

2 Notations and hypotheses

For a vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $x \geqslant 0$ indicates that $x_i \geqslant 0$, $\forall i \in \{1, \ldots, n\}$. The absolute value of the i-th component of x is $|x_i|$ and $|x| = [|x_1|, \ldots, |x_n|]^T$. Table 1 gathers the main notations used in this paper.

N	number of agents
q_i	coordinates vector of Agent i
q	configuration vector, $q = \left[q_1^T, \ldots, q_N^T\right]^T$
$\dot{q}_i^*$	target reference velocity of Agent i
r_{ij}	relative coordinate vector between Agents i and j, with $r_{ij} = q_i - q_j$
r_{ij}^*	target relative coordinate vector between Agents i and j
r^*	target relative configuration vector with $r^* = [r_{11}^*, r_{12}^*, ..., r_{1N}^*]^T$
x_i	state of Agent i, $x_i = \left[q_i^T, \dot{q}_i^T\right]^T$
$\hat{x}_i^j$	estimate of x_i by Agent j with $\left(\hat{x}_i^j\right)^T = \left[\left(\hat{q}_i^j\right)^T, \left(\dot{\hat{q}}_i^j\right)^T\right]$
$\hat{q}_i^{i,\ell}$	estimate of q_i performed by Agent i using the information in its ℓ-th transmitted message and not in the following one
e_i^j	error between q_i and $\hat{q}_i^j$
$\dot{e}_i^j$	error between $\dot{q}_i$ and $\dot{\hat{q}}_i^j$
$\bar{r}_{ij}$	estimated relative coordinate vector between Agents i and j as evaluated by Agent i with $\bar{r}_{ij} = q_i - \hat{q}_j^i$
k_i	index of k_i-th message sent by Agent i
t_{i,k_i}	transmission time of the k_i-th message sent by Agent i
π	packet loss probability
κ	maximum number of estimators of its own state maintained by each agent
δ_{i,k_i}^j	indicates whether the k_i-th message sent by Agent i has been received by Agent j ($\delta_{i,k_i}^j = 1$) or lost ($\delta_{i,k_i}^j = 0$)
k_i^j	index of the last message received by Agent j among those sent by Agent i
$k_i^{j,i}$	index managed by Agent i of the last message received by Agent j among those sent by Agent i
m_{ij}	potential energy coefficient between Agents i and j
α_i	sum of coefficients m_{ij} for $j \in \mathcal{N}_i$

Table 1
Main notations

2.1 Multi-agent system

Consider a system of N agents with indexes in $\mathcal{N} = \{1, \ldots, N\}$. In a fixed reference frame $\mathcal{F}$, let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i and $q = \left[q_1^T, \ldots, q_N^T\right]^T \in \mathbb{R}^{Nn}$ be the *configuration* of the MAS. The relative coordinate vector between two agents i and j is $r_{ij} = q_i - q_j$. The evolution of the state $x_i = \left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i is assumed to be described by the Euler-Lagrange model

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G = u_i + d_i, \tag{1}$$

where $u_i \in \mathbb{R}^n$ is the control input, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ is the matrix of the Coriolis and centripetal terms for Agent i, G accounts for gravitational acceleration supposed to be known and constant, and $d_i \in \mathbb{R}^n$ is a time-varying state perturbation satisfying $||d_i(t)|| \leqslant D_{\max}$ with known $D_{\max}$. In what follows, the notations M_i and C_i are used in place of $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

For each Agent i of the MAS, we consider the following assumptions.

A1) $M_i(q_i)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x \in \mathbb{R}^n$, $x^T M_i x \leqslant k_M x^T x$.

A2) $\dot{M}_i - 2C_i$ is skew symmetric or negative definite and $\exists k_C > 0$ satisfying $\forall x \in \mathbb{R}^n$, $x^T C_i x \leqslant k_C ||\dot{q}_i|| x^T x$.

A3) For all $\xi_1 \in \mathbb{R}^n$ and $\xi_2 \in \mathbb{R}^n$, the left side of (1) can be linearly parametrized as

$$M_i(q_i)\,\xi_1 + C_i(q_i,\,\dot{q}_i)\,\xi_2 = Y_i(q_i,\,\dot{q}_i,\,\xi_1,\,\xi_2)\,\theta_i, \qquad (2)$$

see [14]. $Y_i(q_i,\,\dot{q}_i,\,\xi_1,\,\xi_2)$ is a regressor matrix with known structure, identical for all agents, and $\theta_i \in \mathbb{R}^p$ is a vector of constant parameters known by Agent i via, *e.g.*, an offline identification. Modeling and estimation errors for Y_i and θ_i may be incorporated in $d_i(t)$.

A4) x_i is measured without error.

Assumptions A1, A2, A3 and A4 have been previously considered in [9–11, 14, 15]. The following assumptions are considered for each Agent i:

A5) An estimate $\hat{x}_i^j(0) = \left[\hat{q}_i^j(0)^T, \dot{\hat{q}}_i^j(0)^T\right]$ of the state $x_i(0)$ is known by all its neighbors $j \in \mathcal{N}_i$ and the squared norm of the estimation errors $||q_i(0) - \hat{q}_i^j(0)||^2$ and $||\dot{q}_i(0) - \dot{\hat{q}}_i^j(0)||^2$ are bounded with bounds described in Proposition 8, see Section 5.

A6) The velocity $\dot{q}_i$ and acceleration $\ddot{q}_i$ are bounded,

$$||\dot{q}_i(t)|| \leqslant \dot{q}_{\max}, \qquad (3)$$
$$||\ddot{q}_i(t)|| \leqslant \ddot{q}_{\max}. \qquad (4)$$

Moreover, $\dot{q}_i$ is Lipschitz, *i.e.*, there exists $K_d > 0$ such that $\forall t, \forall \Delta t$,

$$||\dot{q}_i(t+\Delta t) - \dot{q}_i(t)|| \leqslant K_d|\Delta t|. \qquad (5)$$

2.2 Communication model

When its CTC is satisfied, Agent i broadcasts a message to its neighbors. The packet containing the message is either received without error or is lost. Usually, packet losses are due *i*) to collisions (packets are transmitted at the same time instants by different agents), *ii*) to occlusions by obstacles (two agents are not in line of sight), *iii*) to a signal-to-noise ratio below a certain threshold (agents are too far away).

The packet loss probability between Agents i and j is denoted $\pi_{ij} = \pi_{ji}$. One considers that a communication link exists between two agents i and j if π_{ij} is less than $\pi \leqslant 0.5$. From this hypothesis, the communication topology of the MAS can be described by a undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of edges of the graph. Agent i can communicate with its N_i one-hop neighbors with indexes in $\mathcal{N}_i = \{j \in \mathcal{N} |\, (i,j) \in \mathcal{E},\, i \neq j\}$. For each Agent $j \in \mathcal{N}_i$, one has therefore $\pi_{ij} \leqslant \pi$.

In this paper, we assume that $\mathcal{G}$ is connected and invariant with the time. Moreover, to simplify analysis, we assume that $\forall (i,j) \in \mathcal{E}$, $\pi_{ij} = \pi$.

Communication delays are neglected: When Agent i broadcasts its k_i-th message at time t_{i,k_i}, Agent $j \in \mathcal{N}_i$ either receives this message without error at time t_{i,k_i} or does not receive it.

Consider a pairs of neighboring agents $(i,j) \in \mathcal{E}$. Let $\{\delta_{i,k_i}^j\}_{k_i \geqslant 1}$ be a sequence of binary variables such that $\delta_{i,k_i}^j = 1$ if the k_i-th message sent by Agent i has been received by Agent j and $\delta_{i,k_i}^j = 0$ else. The δ_{i,k_i}^js are modeled as realizations of time-invariant memoryless Bernoulli processes with

$$\Pr\left(\delta_{i,k_i}^j = 1\right) = 1 - \pi \qquad (6)$$
$$\Pr\left(\delta_{i,k_i}^j = 0\right) = \pi. \qquad (7)$$

The model (6)-(7) captures relatively accurately situation *i*). Packet loss events due to collisions are often independent from one communication trial to the next one, provided that there is no synchronization between agents (as in the ALOHA protocol [1]). The considered packet loss model can also represent situation *ii*) provided that obstacles are small or agents move fast enough to experience only very short occlusions. Situation *iii*) is more difficult to represent. Adjusting the transmission power periodically, so as to reach farther agents (even less frequently), may partly address the problem. Nevertheless, this would lead to a time-varying probability π of packet loss. For situations *ii*) and *iii*), one may alternatively consider a modification of the agent communication topology, which is out of the scope of this paper. In [6, 7], feedback is used to partially solve the problem, but feedback requires extra communications and so increases the risk of collision between packets, as described in situation *i*). This is why, here, the only feedback information considered is that received from packets sent by other agents, when this own CTC is satisfied.

2.3 Message content

Let $k_j^i \leqslant k_j$ be the index of the last message Agent i has received from its neighbor j. When a communication is triggered at time t_{i,k_i}, Agent i broadcasts a message containing k_i, t_{i,k_i}, $x_i(t_{i,k_i})$, θ_i, and $\{k_j^i\}, j \in \mathcal{N}_i$. By sending $k_j^i \leqslant k_j$ for all $j \in \mathcal{N}_i$, Agent i indicates the index of the last message received from each of its neighbors. This can be considered as an implicit acknowledgment mechanism for the neighbors $j \in \mathcal{N}_i$.

When Agent j receives a message from Agent i, it updates k_i^j to k_i. Moreover, $x_i(t_{i,k_i})$ and θ_i are used to update its estimator of the state of Agent i, as detailed in Section 4.1. Finally, Agent j keeps track in the variables $k_j^{i,j}$ of the value of k_j^i which represents the index of the last message sent by Agent j and which has been actually received by Agent i. The index $k_j^{i,j}$ is used by Agent j to evaluate the knowledge Agent i has about x_j (see the example in Figure 1).

2.4 Target formation

A potentially time-varying target formation is defined by the set $\mathcal{R}(t) = \left\{r_{ij}^*(t), (i,j) \in \mathcal{N} \times \mathcal{N}\right\}$, where $r_{ij}^*(t)$ is the target relative coordinate vector between Agents i and j. Without loss of generality, the first agent is considered as the reference agent. Any target relative coordinate vector r_{ij}^* can be expressed as $r_{ij}^*(t) = r_{i1}^*(t) - r_{j1}^*(t)$. The target relative *configuration* vector is $r^*(t) = [\,r_{11}^{*T}(t)\;\ldots\;r_{1N}^{*T}(t)\,]^T$. Each Agent i is assumed to only know the relative coordinate vector with its direct neighbors $r_{ij}^*(t), j \in \mathcal{N}_i$. Additionally, a constant reference velocity $\dot{q}_1^*$ known by all agents is imposed

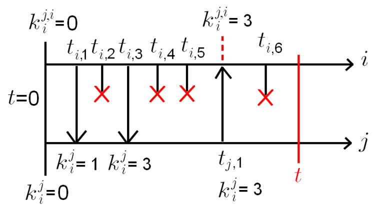


Fig. 1. Communication instants between Agents i and j and evolution of the indexes k_i^j and $k_i^{j,i}$ of last message received; from the packet received at time $t_{j,1}$, Agent i can deduce that Agent j has received the packet sent at time $t_{i,3}$ from the content of the packet it receives from Agent j at time $t_{j,1}$.

to the MAS. The reference velocities $\dot{q}_i^*$ are expressed as $\dot{q}_i^* = \dot{q}_1^* + \dot{r}_{i1}^*$ and are assumed to satisfy

A7) For all agents, the reference velocity $\dot{q}_i^*$ and acceleration $\ddot{q}_i^*$ are bounded

$$||\dot{q}_i^*(t)|| < \dot{q}_{\max}, \tag{8}$$
$$||\ddot{q}_i^*(t)|| < \ddot{q}_{\max}, \tag{9}$$

for $i = 1, \ldots, N$. Moreover, $\dot{q}_i^*$ is Lipschitz with constant $K_d^* \leqslant K_d$, *i.e.*, $\forall t$, $\forall \Delta t$,

$$||\dot{q}_i^*(t+\Delta t) - \dot{q}_i^*(t)|| \leqslant K_d^* |\Delta t|. \tag{10}$$

Our aim is to evaluate the control input for each agent in a distributed way so that the MAS converges to $\mathcal{R}(t)$, while limiting the number of communications between agents and accounting for losses. For that purpose, the control input of each agent has to provide an asymptotic convergence of the MAS to the target configuration vector with a bounded Mean-Square Error (MSE).

Proposition 1 *The MAS asymptotically converges to the target formation with a bounded MSE (bounded average asymptotic convergence) iff there exists $\varepsilon_1 > 0$ such that*

$$\forall (i,j) \in \mathcal{N}^2, \lim_{t\to\infty} \mathbb{E}\left(||r_{ij}(t) - r_{ij}^*(t)||^2\right) \leqslant \varepsilon_1, \tag{11}$$

where the expectation is over all packet loss events.

2.5 Overview of the proposed approach

A control law is introduced in Section 3 to drive the MAS to its target formation and reference speed in a distributed way. This requires the knowledge by each agent of the state vector of its neighbors. Since the state vector of a neighbor Agent j is only available at Agent i when Agent j broadcasts its state, Agent i has to maintain an estimator of the state of each of its neighbors. This estimator is described in Section 4.

To determine the quality of the estimate of x_i evaluated by its neighbors, Agent i has also to estimate its own state x_i with the information it has transmitted to its neighbors. As soon as a function of the error between this estimate and x_i reaches some threshold, Agent i triggers a communication to allow its neighbors to refresh their estimate of x_i. The main difficulty, compared to [15, 17], lies in the fact that estimators have to account for packet losses. In the solution proposed here, each agent maintains several estimates of its own state accounting for different packet loss hypotheses, and an estimate of the state of its neighbors with the last information received. As will be seen in Section 4.4, the number of hypotheses can be limited to a manageable amount determined by the last received packet from Agent i.

Usually, a CTC relies on the error between the states of agents and the state estimates evaluated by neighboring agents. Here, since this error cannot be exactly obtained due to packet losses, the CTC involves the MSE between the state of an agent and its estimate evaluated by its neighbors, see Section 5. This paper proposes different methods to evaluate or upper-bound this MSE, which is then used to analyze the convergence and the stability of the MAS.

3 Distributed control inputs

Section 3.1 introduces the potential energy $P(q, t)$ of the MAS to quantify the discrepancy between the current and target formations. A control input minimising $P(q, t)$ by exploiting the agent state estimators is presented in Section 3.2.

3.1 Potential energy of the MAS

As in [22], consider the *potential energy* of the MAS

$$P(q, t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}||r_{ij} - r_{ij}^*||^2, \tag{12}$$

where $m_{ij} = m_{ji}$ are some positive or null coefficients. $P(q,t)$ quantifies the discrepancy between the actual and target relative coordinate vectors. We take $m_{ii} = 0$, $m_{ij} = 0$ if $(i,j) \notin \mathcal{E}$, and $m_{ij} > 0$ if $(i,j) \in \mathcal{E}$. Since $\mathcal{G}$ is connected, the minimum number of non-zero coefficients m_{ij} to properly define a target formation is $N-1$.

Proposition 2 *The MAS asymptotically converges to the target formation with a bounded MSE iff there exists some $\varepsilon_2 > 0$ such that $\lim_{t\to\infty} \mathbb{E}(P(q,t)) \leqslant \varepsilon_2$, where the expectation is over all packet loss events.*

The proof of Proposition 2 is provided in [18, Appendix A.1].

3.2 Control input with agent state estimators

In what follows, a control law is designed for each agent so that the MAS asymptotically converges to the target formation with a bounded MSE. The control law requires only local knowledge of the agent and can therefore be implemented in a distributed way. It has to make $P(q, t)$ decrease. One introduces, as in [22],

$$g_i = \frac{\partial P(q,t)}{\partial q_i} = \sum_{j\in\mathcal{N}_i} m_{ij}\left(r_{ij} - r_{ij}^*\right), \tag{13}$$
$$\dot{g}_i = \sum_{j\in\mathcal{N}_i} m_{ij}\left(\dot{r}_{ij} - \dot{r}_{ij}^*\right), \tag{14}$$
$$s_i = \dot{q}_i - \dot{q}_i^* + k_p g_i, \tag{15}$$

where $\dot{q}_i^* = \dot{q}_1^* - \dot{r}_{1i}^*$ is the reference velocity of Agent i. The vectors g_i and $\dot{g}_i$ characterize the evolution with q_i and $\dot{q}_i$ of the discrepancy between the actual and target relative coordinate vectors. In (15), $k_p > 0$ is a scalar design parameter. When the agents are far from the target formation, g_i is large and determines the direction for Agent i to get closer to the target formation.

To make $P(q, t)$ decrease, Agent i has to evaluate (13). The control input of Agent i requires r_{ij}, and thus q_j, $j \in \mathcal{N}_i$. Nevertheless, q_j is only available to Agent i when it receives a packet from Agent j containing q_j, see Section 2.2. Between the reception of two packets from Agent j, an estimate $\hat{q}_j^i$ of $q_j, j \in \mathcal{N}_i$ needs to be evaluated, see Section 4.1. Using estimates $\hat{q}_j^i$ and $\dot{\hat{q}}_j^i$ of q_j and $\dot{q}_j$ for all $j \in \mathcal{N}_i$, Agent i is able to evaluate the discrepancies $\bar{r}_{ij} = q_i - \hat{q}_j^i$, $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$ between its own state and the estimate of the state of its neighbors, as well as

$$\bar{g}_i = \sum_{j \in \mathcal{N}_i} m_{ij} \left(\bar{r}_{ij} - r_{ij}^* \right), \tag{16}$$

$$\dot{\bar{g}}_i = \sum_{j \in \mathcal{N}_i} m_{ij} \left(\dot{\bar{r}}_{ij} - \dot{r}_{ij}^* \right), \tag{17}$$

$$\bar{s}_i = \dot{q}_i - \dot{q}_i^* + k_p \bar{g}_i. \tag{18}$$

Then, the following control input (to be used in (1)) can be evaluated in a distributed way by each Agent $i \in \mathcal{N}$, *i.e.*, using only vectors available locally

$$u_i = -k_s \bar{s}_i - k_g \bar{g}_i + G - Y_i \left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i \right) \theta_i, \tag{19}$$

where $\bar{p}_i = k_p \bar{g}_i - \dot{q}_i^*$ and $\dot{\bar{p}}_i = k_p \dot{\bar{g}}_i - \ddot{q}_i^*$ with the additional design parameters $k_g > 0$ and $k_s \geqslant 1 + k_p (k_M + 1)$. In (19), $\bar{s}_i$ maintain the formation at the reference velocity, $\bar{g}_i$ drives the agent to the target formation, G compensates the action of the gravity, and $Y_i \left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i \right) \theta_i$ compensates the inertia, Coriolis and centripetal terms of the dynamic of the agents. The convergence properties of the MAS when each agent applies (19) is analyzed in Section 5.

4 State estimators and packet losses

Section 4.1 introduces the estimators involved in the control input (19) applied by each agent. Section 4.2 describes the way Agent i estimates its own state x_i, with the information transmitted to its neighbors, to determine the quality of their estimates of x_i. In Section 4.3, the MSE between the current state x_i and its remote estimates $\hat{x}_i^j, j \in \mathcal{N}_i$ is evaluated. In Section 4.4, packets received from other agents are exploited to improve the evaluation of the MSE of the estimate $\hat{x}_i^j$ of x_i.

4.1 Estimation of the state of other agents

To evaluate (19), Agent i has to maintain an estimate $\hat{x}_j^i$ of the state x_j of all its neighbors $j \in \mathcal{N}_i$. Assume that Agent j broadcasts its k-th message at time $t_{j,k}$. Then, since communication delays are neglected, depending on whether this message has been received by Agent i, $\hat{x}_j^i$ is updated as

$$\hat{x}_j^i \left(t_{j,k}^+ \right) = \delta_{j,k}^i x_j \left(t_{j,k} \right) + \left(1 - \delta_{j,k}^i \right) \hat{x}_j^i \left(t_{j,k}^- \right), \tag{20}$$

where $x_j(t_{j,k})$ is obtained from the received packet, where $\hat{x}_j^i \left(t_{j,k}^- \right)$ is the value of the state estimate at t_{i,k_i} before the update and $\hat{x}_j^i \left(t_{j,k}^+ \right)$ is its value after the update. For all $t \geqslant t_{j,k}$ and up to the reception of the next packet sent by Agent j, the components $\hat{q}_j^i$ and $\dot{\hat{q}}_j^i$ of $\hat{x}_j^i$ evolve as

$$M_j \left(\hat{q}_j^i \right) \ddot{\hat{q}}_j^i + C_j \left(\hat{q}_j^i, \dot{\hat{q}}_j^i \right) \dot{\hat{q}}_j^i + G = \hat{u}_j^i. \tag{21}$$

where M_j and C_j are evaluated using (2) with Y_j and $\hat{\theta}_j^i = \theta_j$, since the structure of Y_j and θ_j are initially known by Agent i or have been transmitted by Agent j at time $t = 0$. The estimator (21) maintained by Agent i itself requires an estimate $\hat{u}_j^i$ of the control input u_j evaluated by Agent j. This estimate $\hat{u}_j^i$, used by Agent i, is chosen as

$$\hat{u}_j^i = -k_s \dot{\hat{\varepsilon}}_j^i + G - Y_j \left(\hat{q}_j^i, \dot{\hat{q}}_j^i, -\ddot{q}_j^*, -\dot{q}_j^* \right) \hat{\theta}_j^i, \tag{22}$$

with $\dot{\hat{\varepsilon}}_j^i = \dot{\hat{q}}_j^i - \dot{q}_j^*$. The control input (22) only depends on information available to Agent i. Therefore, (22) has been built from (19) by removing all terms unknown by the neighbors of Agent j. Consequently $\bar{g}_j$ and $\dot{\bar{g}}_j$ cannot be used, and x_j is replaced by $\hat{x}_j^i$. Since (22) differs from (19), $\hat{x}_j^i$ will progressively diverge from x_j. Since each agent runs an estimator of its own state with the information available to its neighbors, it can trigger a communication when the discrepancy is too large. When the MAS is close to the target formation, the term $\dot{\bar{g}}_j$ becomes negligible and (19) and (22) get closer. This choice limits the number of state estimate hypotheses to consider, see Section 4.2.

We consider Assumptions A8 and A9 on the components of $\hat{x}_j^i$.

A8) The velocity $\dot{\hat{q}}_j^i$ and acceleration $\ddot{\hat{q}}_j^i$ are bounded

$$||\dot{\hat{q}}_j^i|| \leqslant \dot{q}_{\max} \tag{23}$$

$$||\ddot{\hat{q}}_j^i|| \leqslant \ddot{q}_{\max}. \tag{24}$$

Moreover, $\dot{\hat{q}}_j^i$ is Lipschitz on all intervals $[t_{j,k}, t_{j,k+1}[$, *i.e.*, there exists $\hat{K}_d > 0$ such that $\forall t \in [t_{j,k}, t_{j,k+1}[$ and $\forall (t + \Delta t) \in [t_{j,k}, t_{j,k+1}[$ one has

$$||\dot{\hat{q}}_j^i (t + \Delta t) - \dot{\hat{q}}_j^i (t)|| \leq \hat{K}_d |\Delta t|. \tag{25}$$

This assumption is consistent with that considered for $\dot{q}_j$, *i.e.*, Assumption A6, since between two communication time instants, (21) is similar to (1).

A9) There exists $e_{\max} > 0$ such that the norm of the estimation error satisfies

$$||q_i(t) - \hat{q}_i^j(t)|| \leqslant e_{\max}. \tag{26}$$

This assumption is reasonable for MAS evolving in some limited geographical area.

4.2 Multi-hypothesis state estimates

The estimate $\hat{q}_i^j$ of the state of Agent i, evaluated by Agent j, only depends on the information provided by Agent i. The estimate $\hat{q}_i^j$ is reset to q_i as soon as a message sent by Agent i is received by Agent j, see (20). Consequently, when Agent i has sent k_i messages, and wants to evaluate an image of its own state as computed by one of its neighbors, k_i different hypotheses have to be considered, each of which is associated to a different estimator of q_i at time $t \in [t_{i,k_i}, t_{i,k_i+1}[$:

- A first estimator considers the k_i-th packet as received,

- A second estimator considers the k_i-th packet as lost, but the $k_i - 1$-th packet as received,
- ...
- A last estimator considers that no packet has been received, and uses the initial state estimate $\hat{x}_i^j(0)$.

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, the state estimates corresponding to these hypotheses are denoted as

$$\hat{x}_i^{i,\ell}(t) = \left[\hat{q}_i^{i,\ell}(t), \dot{\hat{q}}_i^{i,\ell}(t)\right], \tag{27}$$

with $\ell = 0, \dots, k_i$ and $\hat{x}_i^{i,k_i} = \hat{x}_i^i$.

Since k_i can become very large, we impose that Agent i maintains at most κ estimates of x_i, denoted $\hat{x}_i^{(1)}(t), ..., \hat{x}_i^{(\kappa)}(t)$. For all $t \in [t_{i,k_i}, t_{i,k_i+1}[$, $k_i \geqslant \kappa$, one has $\hat{x}_i^{(1)}(t) = \hat{x}_i^{i,k_i}(t), \dots, \hat{x}_i^{(\kappa)}(t) = \hat{x}_i^{i,k_i-\kappa+1}(t)$. These estimates evolve according to the dynamics (21)-(22) introduced in Section 4.1. When a new packet is sent by Agent i at time t_{i,k_i+1}, the estimates are updated as

$$\hat{x}_i^{(1)}\left(t_{i,k_i+1}^{+}\right) = x_i\left(t_{i,k_i+1}^{-}\right). \tag{28}$$

$$\hat{x}_i^{(\ell+1)}\left(t_{i,k_i+1}^{+}\right) = \hat{x}_i^{(\ell)}\left(t_{i,k_i+1}^{-}\right),\ \ell = 1, \dots, \kappa - 1. \tag{29}$$

4.3 Expected value of the estimation error of $x_i(t)$

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, Agent i has sent k_i packets. Let

$$p_{k_i,\ell}^j = \Pr\left(\delta_{i,\ell}^j = 1, \delta_{i,\ell+1}^j = 0, \dots, \delta_{i,k_i}^j = 0\right) \tag{30}$$

with $0 \leqslant \ell \leqslant k_i$, be the probability that the ℓ-th packet has been received by a given neighbor j and that all packets from the $\ell+1$-th to the k_i-th have been lost. By convention,

$$p_{k_i,0}^j = \Pr\left(\delta_{i,1}^j = 0, \dots, \delta_{i,k_i}^j = 0\right)$$

and

$$p_{k_i,k_i}^j = \Pr\left(\delta_{i,k_i}^j = 1\right). \tag{31}$$

Note that $p_{k_i,\ell}^j$ only depends on the packet loss probability π of the packet loss model (6)-(7), and does not depend on the neighbor index j, which is omitted in what follows.

Proposition 3 *One has*

$$p_{1,1} = 1 - \pi \tag{32}$$

$$p_{1,0} = \pi. \tag{33}$$

For all $k_i > 0$ and $\ell < k_i$,

$$p_{k_i,\ell} = \pi p_{k_i-1,\ell}. \tag{34}$$

Moreover

$$p_{k_i,k_i} = 1 - \pi. \tag{35}$$

The proof of Proposition 3 is in Appendix A.

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, the estimation error of the coordinates of Agent i, as evaluated by Agent j, is

$$e_i^j(t) = \hat{q}_i^j(t) - q_i(t). \tag{36}$$

Since Agent i does not know the index of the last packet received by Agent j among those it has sent, Agent i cannot evaluate (36). Alternatively, Agent i can evaluate the mean square value of $e_i^j(t)$ considering the estimates $\hat{q}_i^{i,\ell}$ and the associated probabilities $p_{k_i,\ell}$

$$\mathbb{E}\left(||e_i^j(t)||^2\right) = \sum_{\ell=0}^{k_i} p_{k_i,\ell}||\hat{q}_i^{i,\ell}(t) - q_i(t)||^2. \tag{37}$$

Since Agent i maintains only κ estimators of x_i, it cannot evaluate (37) when $k_i > \kappa$. Nevertheless, using Assumption A9, the MSE (37) can be upper-bounded.

Proposition 4 *The MSE (37) can be expressed or upper-bounded as*

$$\begin{cases}\mathbb{E}\left(||e_i^j(t)||^2\right) = \sum_{\ell=0}^{k_i} p_{k_i,\ell}||\hat{q}_i^{i,\ell}(t) - q_i(t)||^2 & \text{if } k_i \leqslant \kappa\\ \mathbb{E}\left(||e_i^j(t)||^2\right) \leqslant \overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2\right) & \text{else,}\end{cases} \tag{38}$$

where

$$\overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2\right) = \sum_{\ell=k_i-\kappa+1}^{k_i} p_{k_i,\ell}\left\|\hat{q}_i^{i,\ell}(t) - q_i(t)\right\|^2 + \min\left(\left\{\overline{e}_{i,k_i}^j(t),\, \pi^\kappa e_{\max}^2\right\}\right), \tag{39}$$

with

$$\overline{e}_{i,k_i}^j(t) = 4\dot{q}_{\max}e_{\max}\pi^\kappa(t - t_{i,k_i}) + \pi\overline{\mathbb{E}}\left(\left\|e_i^j\left(t_{i,k_i}^-\right)\right\|^2\right) - \sum_{\ell=k_i-\kappa+1}^{k_i} p_{k_i,\ell}^j\left\|\hat{q}_i^{i,\ell}\left(t_{i,k_i}^+\right) - q_i\left(t_{i,k_i}^+\right)\right\|^2. \tag{40}$$

Similar bounds can be obtained for $\mathbb{E}\left(\left\|\dot{e}_i^j(t)\right\|^2\right)$, $\mathbb{E}\left(\left\|e_i^j(t)\right\|^4\right)$, and $\mathbb{E}\left(\left\|\dot{e}_i^j(t)\right\|^4\right)$, see Appendix B.

The proofs of Proposition 4 is provided in [18, Appendix A.6].

Using (38), Agent i is able to determine the quality of the estimate of x_i evaluated by its neighbors. The choice of κ impacts the upper bounds of $\mathbb{E}\left(||e_i^j(t)||^2\right)$ and $\mathbb{E}\left(||\dot{e}_i^j(t)||^2\right)$. A large value of κ reduces the influence of $e_{\max}$ and $\dot{q}_{\max}$ on the MSE. Nevertheless, κ should not be chosen too large to limit the number of estimators of its own state maintained by Agent i.

4.4 Estimates accounting for received packets (implicit acknowledgement)

Consider Agent i, the time interval $[t_{i,k_i}, t_{i,k_i+1}[$, and assume that $[t_{i,\rho_i^j}, t_{i,\rho_i^j+1}[$ is the time interval during which the last packet has been received from Agent j. This packet contains the index k_i^j of the last message received by Agent j and sent by Agent i, as illustrated by Figure 1. This index is kept by Agent i in $k_i^{j,i}$, see Section 2.2. This implicit acknowledgment can significantly improve the evaluation of the mean-square values of $e_i^j(t)$ and $\dot{e}_i^j(t)$. From this message, Agent i knows that all packets sent in the time interval $[t_{i,k_i^j+1}, t_{i,\rho_i^j+1}[$ have not been received by Agent j.

Consider again the example in Figure 1. The packet received in $[t_{i,k_i}, t_{i,k_i+1}[$ with $k_i^j = k_i - 2$ indicates that packet $k_i - 2$ has been received, but neither packet $k_i - 1$ nor k_i.

Using this knowledge, Agent i can evaluate the probability

$$p^j_{k_i,\ell|k_i^{j,i},\rho_i^j} = \Pr\left(\delta^j_{i,\ell} = 1, \sum\nolimits_{m=\ell+1}^{k_i} \delta^j_{i,m} = 0 \;\middle|\; \delta^j_{i,k_i^j} = 1, \sum\nolimits_{m=k_i^{j,i}+1}^{\rho_i^j} \delta^j_{i,m} = 0\right) \quad (41)$$

that the ℓ-th message sent by Agent i (with $k_i^j \leqslant \ell \leqslant k_i$) has been received by Agent j and that all following messages, including the k_i-th have been lost. By convention,

$$p^j_{k_i,k_i|k_i^{j,i},\rho_i^j} = \Pr\left(\delta^j_{i,k_i} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum\nolimits_{m=k_i^{j,i}+1}^{\rho_i^j} \delta^j_{i,m} = 0\right). \quad (42)$$

Proposition 5 *As long as Agent i has not received any message from Agent j, $p^j_{k_i,\ell|0,0}$ is evaluated for all $k_i > 0$ and $\ell \leqslant k_i$ as*

$$p^j_{k_i,\ell|0,0} = (1-\pi)\,\pi^{k_i-\ell} \quad \text{if } \ell > 0, \quad (43)$$
$$p^j_{k_i,0|0,0} = \pi^{k_i} \quad \text{else.} \quad (44)$$

If Agent i receives a message from Agent j at $t_{j,k_j} \in [t_{i,k_i}, t_{i,k_i+1}[$ containing k_i^j, then $k_i^{j,i} = k_i^j$, $\rho_i^j = k_i$, and

$$p^j_{k_i,k_i^{j,i}|k_i^{j,i},k_i} = 1 \quad (45)$$
$$p^j_{k_i,\ell|k_i^{j,i},k_i} = 0 \quad \forall \ell \leqslant k_i, \ell \neq k_i^{j,i}. \quad (46)$$

Consider $t \in [t_{i,k_i+n}, t_{i,k_i+n+1}[$ with $n > 0$ and assume that the last message received by Agent i from Agent j has been at time $t_{j,k_j} \in [t_{i,k_i}, t_{i,k_i+1}[$. Consequently, $k_i^{j,i} \leqslant k_i$, and one has still $\rho_i^j = k_i$. Then $p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ is evaluated recursively for all $\ell = 0, \dots, k_i + n$ as

$$p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i} = \pi p^j_{k_i+n-1,k_i^{j,i}|k_i^{j,i},k_i} \quad (47)$$
$$p^j_{k_i+n,\ell|k_i^{j,i},k_i} = \pi p^j_{k_i+n-1,\ell|k_i^{j,i},k_i} \quad \text{if } k_i < \ell < k_i + n$$
$$= 0 \quad \text{if } \ell < k_i \text{ and } \ell \neq k_i^{j,i} \quad (48)$$
$$p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} = 1 - \pi. \quad (49)$$

The proof of Proposition 5 is in [18, Appendix A.4].

Table 2 illustrates the evolution of $p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ as a function of n when $\kappa = 3$, $k_i = 5$, and $k_i^j = 3$.

Proposition 5 can be used by Agent i to evaluate $\mathbb{E}\left(||e_i^j(t)||^2\right)$, taking into account the implicit acknowledgement provided by neighbors as follows.

Proposition 6 *Consider some Agent i and $k_i > 0$. Assume that Agent i knows the index k_i^j of the last message sent by Agent i and received by some neighbor Agent j. At time $t \in [t_{i,k_i+n}, t_{i,k_i+n+1}[$, one has*

$$\mathbb{E}\left(\left\|e_i^j(t)\right\|^2 |k_i^{j,i}\right) \leqslant \overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2 |k_i^{j,i}\right), \quad (50)$$

where

$$\overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2 |k_i^{j,i}\right) = \sum_{\ell=\ell_{\min}}^{k_i+n} p^j_{k_i+n,\ell|k_i^{j,i},k_i} \left\|\hat{q}_i^{i,\ell}(t) - q_i(t)\right\|^2$$
$$+ 1_{k_i>\kappa} 1_{k_i - k_i^{j,i} > \kappa}$$
$$\times \min\left(\left\{\overline{e}^j_{i,k_i+n}(t), \sigma_i^j\left(k_i+n, \kappa|k_i^{j,i}, k_i\right) e^2_{\max}\right\}\right) \quad (51)$$

with $\ell_{\min} = \max(\{0, k_i + n - \kappa + 1\})$, $\sigma_i^j\left(k_i+n, \kappa|k_i^{j,i}, k_i\right) = 1 - \sum_{\ell=k_i-\kappa+1}^{k_i} p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ *and*

$$1_{k_i>\kappa} = \begin{cases} 1 & \text{if } k_i > \kappa \\ 0 & \text{else,} \end{cases} \quad (52)$$

$$\overline{e}^j_{i,k_i+n}(t) = 4\dot{q}_{\max} e_{\max} \sigma_i^j\left(k_i+n, \kappa|k_i^{j,i}, k_i\right)(t - t_{i,k_i+n})$$
$$+ \pi\overline{\mathbb{E}}\left(\left\|e_i^j\left(t^-_{i,k_i+n}\right)\right\|^2 |k_i^{j,i}\right)$$
$$- \sum_{\ell=\ell_{\min}}^{k_i+n} p^j_{k_i+n,\ell|k_i^{j,i},k_i} \left\|\hat{q}_i^{i,\ell}\left(t^+_{i,k_i+n}\right) - q_i\left(t^+_{i,k_i+n}\right)\right\|^2. \quad (53)$$

Contrary to (38), (51) depends now on the index of the neighbor Agent j via $k_i^{j,i}$, and so is updated by its neighbor when Agent i receives a packet, in addition to the update made each time Agent i broadcast a message as in (38). Note that the value of σ_i^j tends to π^κ like in Proposition 4 if Agent i does not received implicit acknowledgement from Agent j.

Similar results can be obtained for $\mathbb{E}\left(||\dot{e}_i^j(t)||^2 |k_i^{j,i}\right)$. In what follows, the notation $\mathbb{E}(||e_i^j(t)||^2)$ is used in place of $\mathbb{E}(||e_i^j(t)||^2|k_i^{j,i})$.

5 Event-triggered communications accounting for packet losses

This section presents a CTC involving one of the state estimators introduced in Section 4. Let $m_{\min} = \min_{i,j=1,\dots,N} \{m_{ij} \neq 0\}$, $m_{\max} = \max_{i,j=1,\dots,N} \{m_{ij}\}$, $N_{\min} = \min_{i=1\dots N} \{N_i\}$, $\alpha_i = \sum_{j=1}^N m_{ij}$, $\alpha_{\mathrm{M}} = \max_{i=1,\dots,N} \alpha_i$. The CTC (54) presented in Theorem 7 is designed to ensure an asymptotic convergence of the MAS to the target formation with a bounded MSE.

Theorem 7 *Consider a MAS with agent dynamics given by (1), the communication protocol defined in Section 2.2, and the control law (19). In absence of communication delays,*

		$k_i + n$					
		5	message from Agent j	6	7	8	9
ℓ	0	π^5	0	0	0	0	0
	1	$(1-\pi)\,\pi^4$	0	0	0	0	0
	2	$(1-\pi)\,\pi^3$	0	0	0	0	0
	3	$(1-\pi)\,\pi^2$	1	π	π^2	π^3	π^4
	4	$(1-\pi)\,\pi$	0	0	0	0	0
	5	$1-\pi$	0	0	0	0	0
	6	$*$	$*$	$1-\pi$	$(1-\pi)\,\pi$	$(1-\pi)\,\pi^2$	$(1-\pi)\,\pi^3$
	8	$*$	$*$	$*$	$1-\pi$	$(1-\pi)\,\pi$	$(1-\pi)\,\pi^2$
	9	$*$	$*$	$*$	$*$	$1-\pi$	$(1-\pi)\,\pi$
	10	$*$	$*$	$*$	$*$	$*$	$1-\pi$

Table 2
Probabilities $p_{k_i+n,\ell|k_i^{j,i},k_i}$ that the ℓ-th message sent by Agent i has been received by Agent j and all following messages including the k_i-th one have been lost, for $n \in [0,\dots,5]$, $k_i = 5$; $*$ represents probabilities not defined. In the time interval $[t_{k_i,5},\ t_{k_i,6}[$, a message is received from Agent j indicating that the last message it has received from Agent i is message $k_i = 3$.

and with a packet loss model satisfying (6)-(7), if the communications are triggered by each Agent i of the MAS when the following condition is satisfied

$$\begin{aligned}\alpha_M \Bigg[\sum_{j=1}^{N} m_{ij}\left(k_e\overline{\mathbb{E}}\left(||e_i^j||^2\right) + k_p k_M \overline{\mathbb{E}}\left(||\dot{e}_i^j||^2\right)\right) + k_p k_C^2 \\ \times \sum_{j=1}^{N} m_{ij}\left(2\overline{\mathbb{E}}\left(||e_i^j||^2\right)||\dot{\hat{q}}_j^i||^2 + \mathbb{E}\left(||e_i^j||^4\right) + \mathbb{E}\left(||\dot{e}_i^j||^4\right)\right)\Bigg] \\ + k_g b_i ||\dot{q}_i - \dot{q}_i^*||^2 \geqslant k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \end{aligned} \tag{54}$$

where $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, η and b_i are design parameters such that

$$\eta > 4 k_g b_i \dot{q}_{\max}^2 \tag{55}$$

for some $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, then

(a) the MAS asymptotically converges to the target formation with a bounded MSE such that

$$\lim_{t\to\infty} \mathbb{E}\left(\frac{1}{2}P(q,t)\right) \leqslant \xi, \tag{56}$$

where $\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta\right]$,

$$c_3 = \frac{\min\{k_1, k_p\}\min\left(1, \frac{N_{\min} m_{\min}}{m_{\max}}\right)}{\max\{1, k_M\}} \tag{57}$$

and $k_1 = k_s - (1 + k_p(k_M + 1))$.

(b) one has $t_{i,k_i+1} - t_{i,k_i} > \tau_{\min}$ for some $\tau_{\min} > 0$.

The proof of Theorem 7(a) is given in [18, Appendix A]. The absence of Zeno behavior is shown by the existence of a minimum inter-event time $\tau_{\min}$, see Theorem 7(b), which proof is in [18, Appendix B]. Each Agent i has to evaluate the expected values of $||e_i^j||^2$, $||\dot{e}_i^j||^2$, $||e_i^j||^4$ and $||\dot{e}_i^j||^4$ for all $j \in \mathcal{N}_i$. This can be done using (38) or (51) as detailed in Sections 4.3 and 4.4.

The CTC proposed in Theorem 7 is analyzed considering that the state estimators and the communication protocol are such that for all $i \in \mathcal{N}$ and for all agent $j \in \mathcal{N}_i$ that has received its last message from Agent i at $t_\ell \in [t_{i,k_i-\kappa}, t_{i,k_i}[$ one has

$$\widehat{x}_i^{i,\ell}(t) = \widehat{x}_i^j(t) \tag{58}$$

all $t \in [t_\ell, t_{i,k_i}[$. If Agent j has received the ℓ-th message from Agent i, its estimation $\widehat{x}_i^j(t)$ of $x_i(t)$ is equal to $\widehat{x}_i^{i,\ell}(t)$, one of the κ estimators maintained by Agent i. This property is actually satisfied by the communication protocol described in Section 2.2 and the state estimator described in Section 4. Alternative estimators can be used.

The CTC (54) is satisfied for Agent i mainly when $\overline{\mathbb{E}}(||e_i^j||^2)$ and $\overline{\mathbb{E}}(||\dot{e}_i^j||^2)$ become large. Thus, it is preferable to use the knowledge of $k_i^{j,i}$ provided by the proposed implicit feedback mechanism to calculate (51) rather than using (38).

A large packet loss probability π results in large values of $\overline{\mathbb{E}}(||e_i^j(t)||^2)$ and $\overline{\mathbb{E}}(||\dot{e}_i^j(t)||^2)$, and therefore leads to an increase in the number of communications to compensate for the losses.

When π is a conservative upper-bound of the packet loss probabilities π_{ij}, $(i,j) \in \mathcal{E}$, the upper bounds of $\mathbb{E}(||e_i^j(t)||^2)$ and $\mathbb{E}(||\dot{e}_i^j(t)||^2)$ evaluated by Agent i will be conservative. This leads to more communications than necessary. This effect may be limited by exploiting the implicit acknowledgement.

The right hand side of the CTC (54) is proportional to $\bar{g}_i(t)$ and $\bar{s}_i(t)$, *i.e.*, to the potential energy of the formation $P(q,t)$, which is large when agents are far from the target formation. When agents are far from the target formation, the discrepancy between (19) and (22) is large. This lead to a fast increase of $\overline{\mathbb{E}}(||e_i^j(t)||^2)$ and $\overline{\mathbb{E}}(||\dot{e}_i^j(t)||^2)$. The fact that the right hand side of the CTC (54) is large too prevents the CTC from being satisfied too often. When agents are close to the target formation, even if the right hand side of the CTC (54) is small, the fact that (22) is close to (19) leads to $\overline{\mathbb{E}}(||e_i^j(t)||^2)$ and $\overline{\mathbb{E}}(||\dot{e}_i^j(t)||^2)$ increasing slowly. Consequently, less communications will be required. See Section 6 for an illustration.

An analysis of the impact of the values of the parameters on the reduction of communications has been presented in [17] in absence of packet losses. These results can be extended to the case with packet losses. The choice of the parameters $\alpha_{\rm M}$, k_g, k_p and b_i also determines the number of broadcast messages. Choosing the coefficients m_{ij} such that $\alpha_i = \sum_{j=1}^{N} m_{ij}$ is small leads to a reduction in the number of communications triggered resulting from the satisfaction of (54), at the cost of a less accurate formation.

The following proposition introduces a condition on the initial estimate of the states of agents to guarantee that (54) in Theorem 7 is not satisfied at $t = 0$.

Proposition 8 *If a common initial estimate $\hat{x}_i^j(0)$ is known by all the neighbors $j \in \mathcal{N}_i$ of each Agent i such that*

$$||\dot{\hat{q}}_i^j(0)|| = 0,\ ||e_i^j(0)||^2 \leqslant H_i, ||\dot{e}_i^j(0)||^2 \leqslant H_i \tag{59}$$

where $H_i \geqslant 0$ is defined for each Agent i as

$$H_i = \frac{\sqrt{(k_e + k_p k_M)^2 + k_p k_C^2 \xi_i} - (k_e + k_p k_M)}{2 k_p k_C^2} \tag{60}$$

$$\xi_i = \frac{k_p (k_s + k_g)}{\alpha_M \alpha_i} \bar{g}_i^T \bar{g}_i(0) + \frac{\eta}{\alpha_M \alpha_i}, \tag{61}$$

then the condition (54) in Theorem 7 is guaranteed not to be satisfied at $t = 0$.

The proof of Proposition 8 is given in [18, Appendix A7]. $H_i = 0$ corresponds to the case where the initial state $x_i(0)$ is known by all neighbors of Agent i, *i.e.*, $\hat{x}_i^j(0) = x_i(0)$ $\forall j \in \mathcal{N}_i$. The value of the bound H_i is proportional to $\bar{g}_i(0)$, *i.e.*, the initial value of the potential energy of the formation. Thus, the most distant from the target formation agents are, the largest the initial error of the estimate $\hat{x}_i^j(0)$ can be tolerated. We have assumed in Proposition 8 that all neighbors of Agent i share the same estimate $\hat{x}_i^j(0)$ of $x_i(0)$. This allows Agent i to initialize the estimator of its own state by $\hat{x}_i^j(0)$ and avoids using a different estimator for each of its neighbors. When this hypothesis is not satisfied initially, in practice, the local estimators of x_i and those performed by neighbors are likely to coincide only after few communications. In practice, the formation can still be achieved even if the initial conditions do not satisfy Proposition 8.

6 Example

Consider a MAS of $N = 6$ identical surface ships with coordinate vectors $q_i = [\,x_i\ y_i\ \psi_i\,]^T \in \mathbb{R}^3$, $i = 1 \dots N$, in a local Earth-fixed frame. For Agent i, (x_i, y_i) represents its position and ψ_i its heading angle. The agent dynamics are expressed in the body frame as

$$M_{{\rm b},i}\dot{\rm v}_i + C_{{\rm b},i}({\rm v}_i)\,{\rm v}_i + D_{{\rm b},i}{\rm v}_i = u_{{\rm b},i} + d_{{\rm b},i}, \tag{62}$$

where ${\rm v}_i$ is the velocity vector in the body frame. The values of $M_{{\rm b},i}$, $D_{{\rm b},i}$, and $C_{{\rm b},i}({\rm v}_i)$ are found in [8]. The model (62) may be expressed as (1) with $G = 0$ using an appropriate change of variables detailed in [8]. The parameters of (19) are $k_M = ||M_i|| = 33.8$, $k_C = ||C_i(1_N)|| = 43.96$, $k_p = 2$, $k_g = 20$, $k_s = 1 + 6(k_M + 1)$, $b_i = 1/k_g$, $e_{\max} = 20$, $\dot{q}_{\max} = 2$ and $\ddot{q}_{\max} = 1$. In the simulations, the following state estimator $\widehat{x}_j^i$ is used for all $t \in [t_{j,k}, t_{j,k+1}[$ with components

$$\left[\dot{\hat{q}}_j^i(t), \ddot{\hat{q}}_j^i(t)\right] = \left[\dot{\hat{q}}_j^i(t_{i,k_i}), 0\right] \tag{63}$$

$$\hat{q}_j^i(t) = \hat{q}_j^i(t_{j,k}) + (t - t_{j,k})\,\dot{\hat{q}}_j^i(t_{j,k}), \tag{64}$$

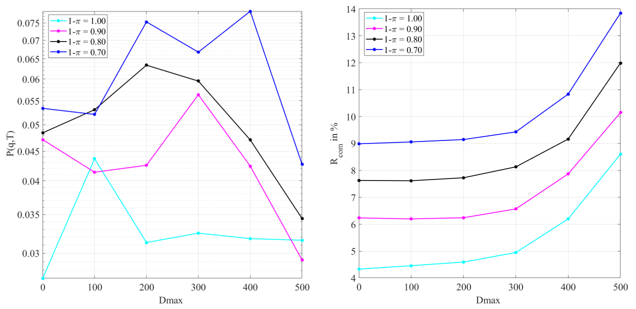


Fig. 2. Evolution of $P(q,T)$ and $R_{\rm com}$ for different values of $D_{\max}$ and π, when $\eta = 100$, the estimator (21) is considered for the dynamics of neighbors agents, as well as $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2 | k_j^{i,j}\right)$ from (51).

$\widehat{x}_j^i$ is updated using (20).

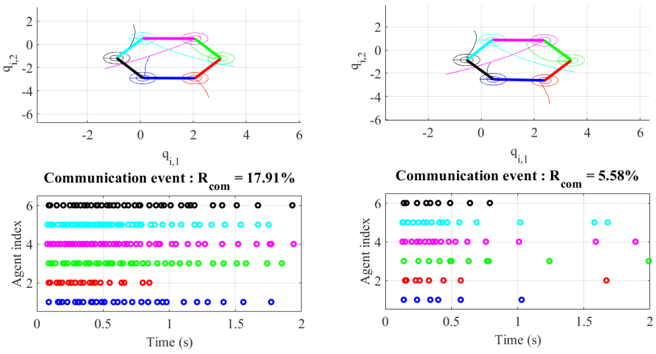
Starting from some initial configuration, the MAS has to reach an hexagonal target formation. From [22], one obtains $m_{ij} = 0\ \forall j$, except $m_{i,(i+1)} = m_{i,(i-1)} = 0.185$ and $m_{i,(i+3)} = 0.0926$. One has $\alpha_i = \sum_{j=1}^{N} m_{ij} = 0.463$, for $i = 1 \dots N$ and $\alpha_{\rm M} = 0.463$, see also [18] for more details. One takes $\kappa = 6$ for several value of $\pi \leqslant 0.3$. The simulation duration is $T = 4$ s with an integration step size $\Delta t = 0.01$ s. The perturbation $d_i(t)$ is assumed constant over each interval $[k\Delta t, (k+1)\Delta t[$. The components of $D_i(t)$ are independent realizations of zero-mean uniformly distributed noise $U\left(-D_{\max}/\sqrt{3}, D_{\max}/\sqrt{3}\right)$ and are thus such that $||d_i(t)|| \leqslant D_{\max}$. Let $N_{\rm m}$ be the total number of messages transmitted during a simulation. The performance of the proposed approach is evaluated with

$$R_{\rm com} = N_{\rm m}/\overline{N}_{\rm m} \tag{65}$$

where $\overline{N}_{\rm m} = NT/\Delta t \geqslant N_{\rm m}$. $R_{\rm com}$ is the ratio between the number of communications required using the proposed approach and the number of communications that would be obtained with a communication triggered at each sampling time instant.

Figure 2 shows the performance of the proposed approach with the CTC (54) for different values of the packet loss probability π and disturbance bound $D_{\max}$. Results are averaged over 50 independent realizations of the noise and of the packet loss events. As expected, the number of communications required for the MAS to converge increases with π and $D_{\max}$. The influence of η on the number of communication is detailed in [18]. Increasing η leads to a reduction of $R_{\rm com}$ but increases the potential energy $P(q,T)$, and thus the discrepancy with respect to the target formation at $t = T$. Figure 3 compares results of the proposed approach obtained without (a) and with (b) the exploitation of the index $k_i^{j,i}$ of the last message sent by Agent i and received by some neighbor Agent j. Using this implicit acknowledgement from neighbors, and thus $\overline{\mathbb{E}}(||e_i^j(t)||^2 | k_j^i)$ instead of $\overline{\mathbb{E}}(||e_i^j(t)||^2)$ in the CTC, convergence is obtained with 75% less messages.

Figure 4 shows the influence of κ on the number of communication $R_{\rm com}$. One observes that increasing κ reduces $R_{\rm com}$, until $R_{\rm com}$ reaches a minimum value (when $k = 3$). Increasing further κ does not reduce $R_{\rm com}$.



(a) without acknowledgement (b) with acknowledgement

Fig. 3. Results of the proposed method using an estimated of the state error (a) via (38) (implicit acknowledgement not used), (b) via (51) (implicit acknowledgement used) when $D_{\max} = 200$ and $\pi = 0.2$

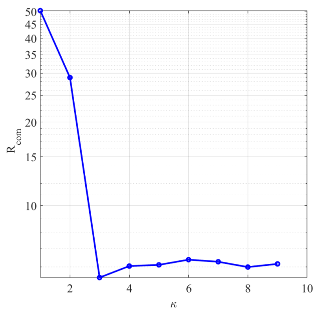


Fig. 4. Evolution of R_{com} as a function of κ when $\eta = 100$, $\pi = 0.2$ and $D_{\max} = 100$. $\overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2 |k_j^{i,j}\right)$ given by (51) is considered in the CTC.

When κ is small, $\overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2 |k_j^{i,j}\right)$ and $\overline{\mathbb{E}}\left(\left\|\dot{e}_i^j(t)\right\|^2 |k_j^{i,j}\right)$ are conservative, which leads to more communications than necessary. When $\kappa \geqslant 3$, the additional terms in the upper bound have a negligible impact. R_{com} is large due to $e_{\max}$, $\dot{q}_{\max}$ans $\ddot{q}_{\max}$ which influence the value of $\overline{\mathbb{E}}\left(\left\|e_i^j(t)\right\|^2 |k_j^{i,j}\right)$.

7 Conclusions

This paper addresses the problem of communication reduction in distributed formation control of a MAS with Euler-Lagrange dynamics in presence of packet losses and state perturbations.

To evaluate its control input, each agent maintains estimators of the states of its neighbors as well as multiple estimators of its own state accounting for different packet loss hypotheses in the communications with its neighbors. Using these estimators, each agent is then able to compute the expected value of the estimation error of its own state as evaluated by its neighbors. An implicit acknowledgement from other agents may be used to evaluate more accurately the estimation error. A distributed CTC is then proposed, involving these estimation errors. The behavior of the MAS is analyzed using stochastic Lyapunov functions in [18]. The convergence to the target formation and the absence of Zeno behavior have been proven. Simulations illustrate the effectiveness of the proposed approach. In future work, communication delays will also be considered along with packet losses.

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A Proof of Proposition 3

Consider first $p_{k_i,\ell}$:

$$\begin{aligned} p^j_{k_i,\ell} &= \Pr\left(\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i}=0\right)\\ &= \Pr\left(\delta^j_{i,k_i}=0|\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i-1}=0\right)\\ &\times \Pr\left(\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i-1}=0\right) \end{aligned} \tag{A.1}$$

Then, using (6)-(7), one gets

$$p^j_{k_i,\ell} = \pi p^j_{k_i-1,\ell}. \tag{A.2}$$

Consider now $p^j_{k_i+n,k_i+n}$. Using (6)-(7), one gets

$$\begin{aligned} p^j_{k_i+n,k_i+n} &= 1-\Pr\left(\delta^j_{i,k_i+n}=0\right)\\ &= 1-\pi. \end{aligned} \tag{A.3}$$

Note that $p^j_{k_i+n,k_i+n}$ is independent of j because no implicit acknowledgment is exploited in Proposition 3.

B Upper-bounds of $\mathbb{E}\left(||\dot{e}^j_i(t)||^2\right)$, $\mathbb{E}\left(||e^j_i(t)||^4\right)$ and $\mathbb{E}\left(||\dot{e}^j_i(t)||^4\right)$

As for $\overline{\mathbb{E}}\left(\left\|e^j_i(t)\right\|^2\right)$, one gets

$$\begin{aligned} \overline{\mathbb{E}}\left(\left\|\dot{e}^j_i(t)\right\|^2\right) &= \sum_{\ell=\ell_{\min}}^{k_i} p_{k_i,\ell}\left\|\dot{\hat{q}}^{i,\ell}_i(t)-\dot{q}_i(t)\right\|^2\\ &+1_{k_i>\kappa}\min\left(\left\{\dot{\overline{e}}^j_{i,k_i}(t),\,4\sigma^j_i(t_{i,k_i})\dot{q}^2_{\max}\right\}\right) \end{aligned} \tag{B.1}$$

$$\begin{aligned} \overline{\mathbb{E}}\left(\left\|e^j_i(t)\right\|^4\right) &= \sum_{\ell=\ell_{\min}}^{k_i} p_{k_i,\ell}\left\|\hat{q}^{i,\ell}_i(t)-q_i(t)\right\|^4\\ &+1_{k_i>\kappa}\min\left(\left\{\overline{e}^{j,2}_{i,k_i}(t),\,\sigma^j_i(t_{i,k_i})e^4_{\max}\right\}\right) \end{aligned} \tag{B.2}$$

$$\begin{aligned} \overline{\mathbb{E}}\left(\left\|\dot{e}^j_i(t)\right\|^4\right) &= \sum_{\ell=\ell_{\min}}^{k_i} p_{k_i,\ell}\left\|\dot{\hat{q}}^{i,\ell}_i(t)-\dot{q}_i(t)\right\|^4\\ &+1_{k_i>\kappa}\min\left(\left\{\dot{\overline{e}}^{j,2}_{i,k_i}(t),\,16\sigma^j_i(t_{i,k_i})\dot{q}^4_{\max}\right\}\right) \end{aligned} \tag{B.3}$$

with $\ell_{\min}=\max\left(\{0,k_i+n-\kappa+1\}\right)$ and

$$\begin{aligned} \dot{\overline{e}}^j_{i,k_i}(t) &= 8\dot{q}_{\max}\ddot{q}_{\max}\sigma^j_i(t_{i,k_i})\left(t-t_{i,k_i}\right)\\ &+\pi\overline{\mathbb{E}}\left(\left\|\dot{e}^j_i\left(t^-_{i,k_i}\right)\right\|^2\right)-\sum_{\ell=\ell_{\min}}^{k_i} p^j_{k_i,\ell}\left\|\dot{\hat{q}}^{i,\ell}_i\left(t^+_{i,k_i}\right)-\dot{q}_i\left(t^+_{i,k_i}\right)\right\|^2 \end{aligned} \tag{B.4}$$

$$\begin{aligned} \overline{e}^{j,2}_{i,k_i}(t) &= 8\dot{q}_{\max}e^3_{\max}\sigma^j_i(t_{i,k_i})\left(t-t_{i,k_i}\right)\\ &+\pi\overline{\mathbb{E}}\left(\left\|e^j_i\left(t^-_{i,k_i}\right)\right\|^4\right)-\sum_{\ell=\ell_{\min}}^{k_i} p^j_{k_i,\ell}\left\|\hat{q}^{i,\ell}_i\left(t^+_{i,k_i}\right)-q_i\left(t^+_{i,k_i}\right)\right\|^4 \end{aligned} \tag{B.5}$$

$$\begin{aligned} \dot{\overline{e}}^{j,2}_{i,k_i}(t) &= 32\ddot{q}_{\max}\dot{q}^3_{\max}\sigma^j_i(t_{i,k_i})\left(t-t_{i,k_i}\right) &(\text{B.6})\\ &+\pi\overline{\mathbb{E}}\left(\left\|\dot{e}^j_i\left(t^-_{i,k_i}\right)\right\|^4\right)-\sum_{\ell=\ell_{\min}}^{k_i} p^j_{k_i,\ell}\left\|\dot{\hat{q}}^{i,\ell}_i\left(t^+_{i,k_i}\right)-\dot{q}_i\left(t^+_{i,k_i}\right)\right\|^4. \end{aligned} \tag{B.7}$$