Symmetry breaking and propulsion of vertically flapping foils

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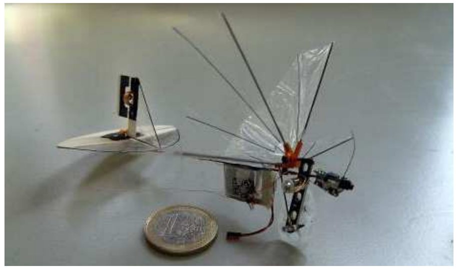
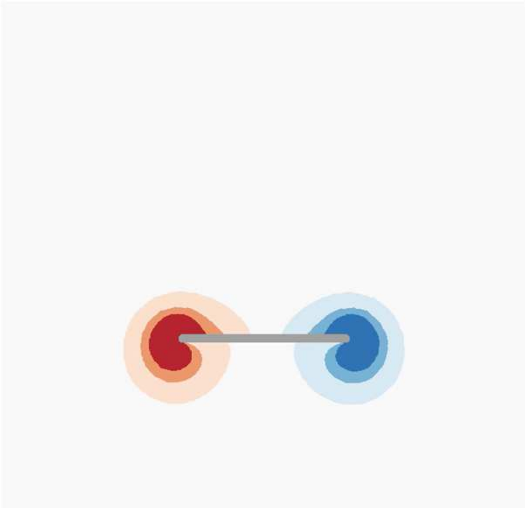
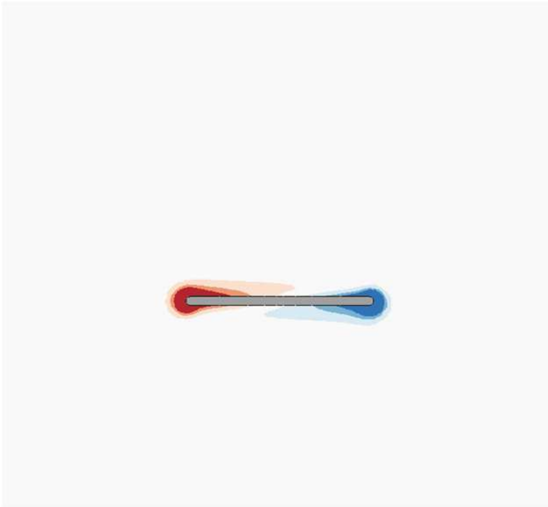
Flapping propulsion

Flapping wing propulsion:

- Symmetry breaking and threshold – Time reciprocal movements and propulsion *[Purcell 1977, Lauga 2007]*





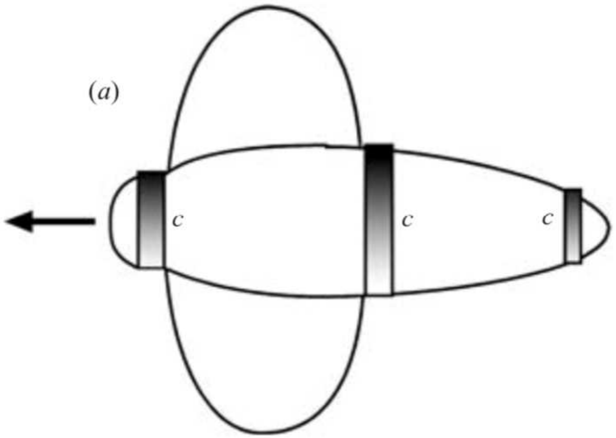


$Freq$ →

- How does horizontal propulsion appear and evolve while increasing frequency?

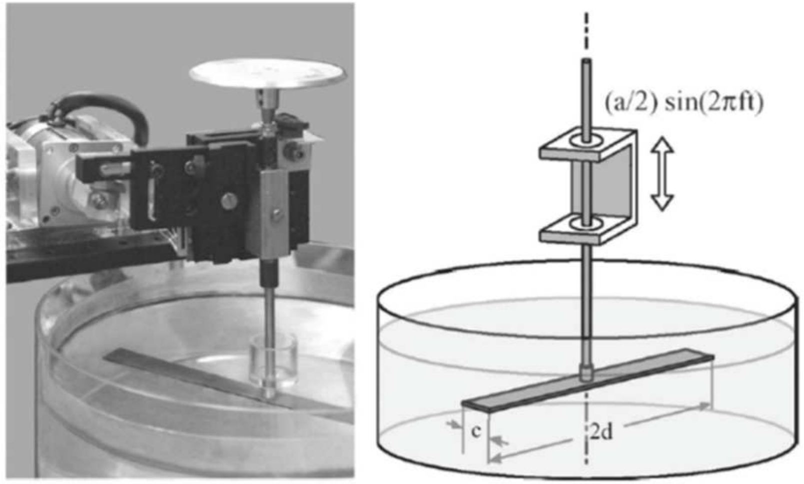
When is flapping propulsion possible?

When is such flapping wing strategy possible?



[Childress et al 2004]

Ciliary + Flapping propulsion



[Vandenberghe et al 2004]

Fluid non-linearities and propulsion

Followed by several numerical studies :

- Flow symmetry breaking only [*Elston et al 2004,2005; Deng & Caulfield 2016]*
- Horizontal locomotion *[Deng & Caulfield 2018; Alben & Shelley 2005; Lu & Liao 2006]*

Outline

➢Different propulsive regimes and their connection with linear stability of the coupled system

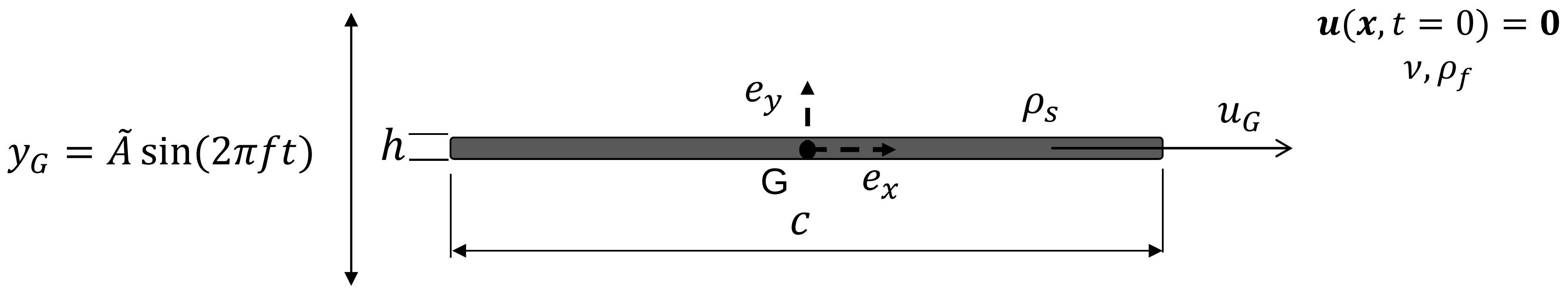
I. Unsteady non-linear simulations – Horizontal locomotion

II. Floquet stability analysis – Coupled system

III. Conclusions

Configuration

Flow generated and locomotion of a wing under imposed vertical motion



Control parameters :

$$\beta = \frac{fc^2}{\nu}, A = \frac{\tilde{A}}{c}, \rho = \frac{\rho_s}{\rho_f}$$

$$A = 0.5, \rho_s = 100\rho_f, h = 0.05c, \beta \in [1,35]$$

Model and numerical method

Flow and locomotion equations:

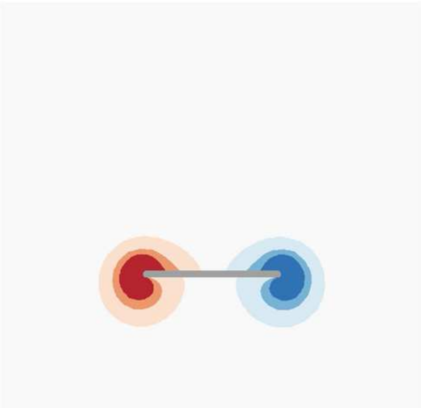
$$\begin{cases} \partial_t \boldsymbol{u} + \nabla \boldsymbol{u} \cdot (\boldsymbol{u} - \boldsymbol{u_G}) = -\boldsymbol{\nabla}\mathrm{p} + \beta^{-1} \boldsymbol{\Delta u} \quad ; \nabla \cdot \boldsymbol{u} = 0 \\ \rho S \dfrac{du_G}{dt} = F_x \\ \boldsymbol{u}(Wing) = \boldsymbol{u_g} \\ \boldsymbol{u}(|\boldsymbol{x}| \to \infty) = \boldsymbol{0} \end{cases}$$

- Fluid/structure coupling: Non-linear volumic terms [*Jallas et al 2017]*
- Unsteady non-linear simulations (BDF2) semi-implicit scheme;
- FE discretization: P2/P1 Elements – FreeFem *[Hecht 2012].*

Unsteady simulations – Observed regimes

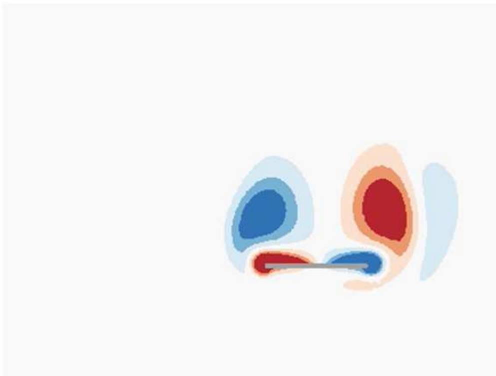
Different regimes - Vorticity

Non-propulsive



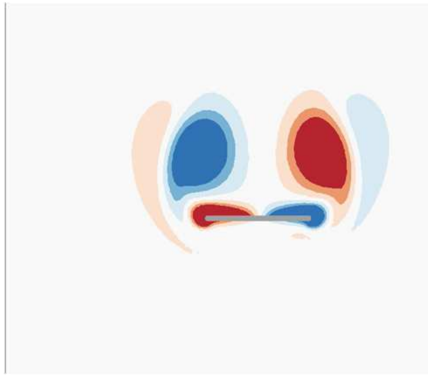
Spatial symmetry
No instantaneous speed

Propulsive



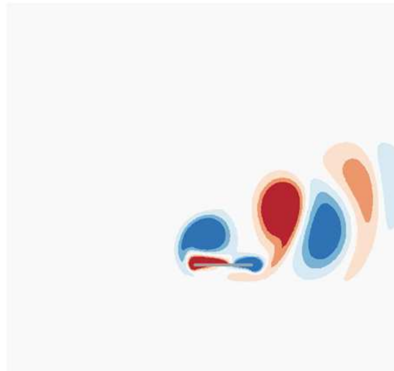
Unidirectional
Net average speed

Back & Forth



Direction switches
Zero average speed

Oscillating Propulsive



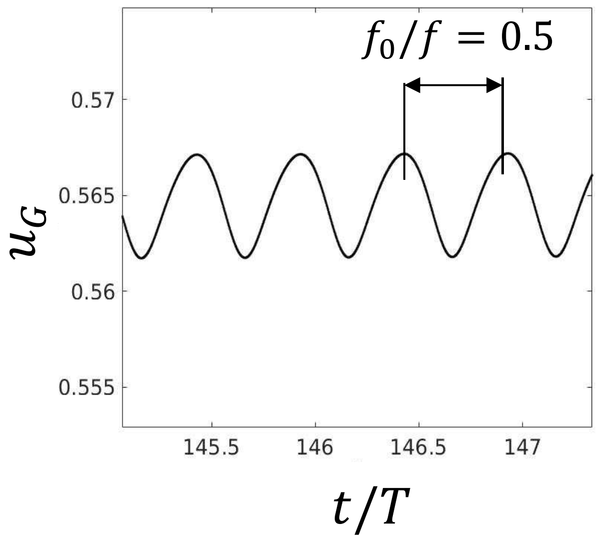
Unidirectional
Net average speed

→ β

Regimes characterization

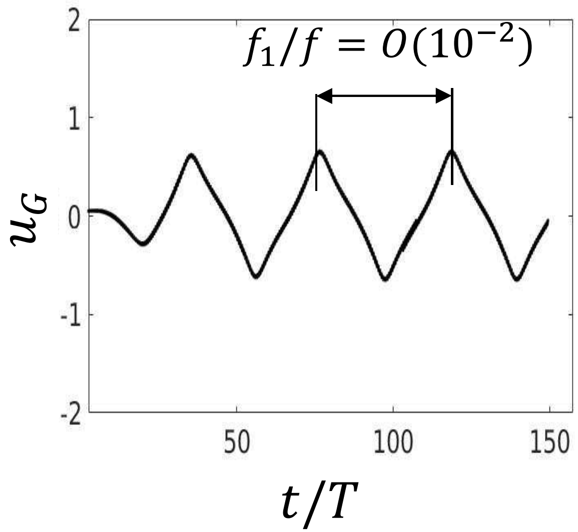
Different propulsive regimes – Instantaneous speed

Propulsive



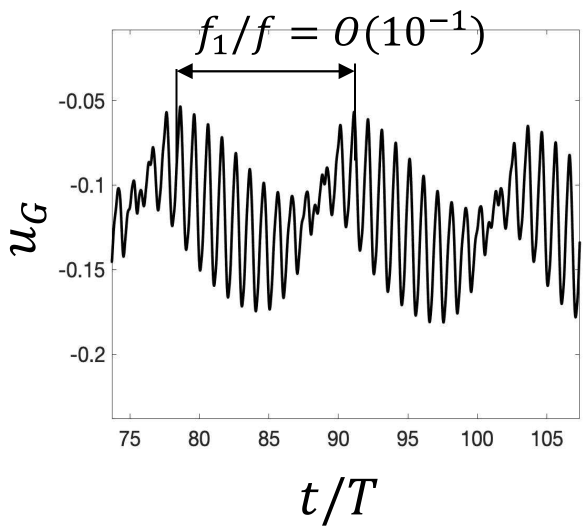
Net average speed
$f_0/f = 0.5$

Back & Forth



Zero average speed
$f_0/f = 0.5$
$f_1/f = O(10^{-2})$

Oscillating Propulsive



Net average speed
$f_0/f = 1.0$
$f_1/f = O(10^{-1})$

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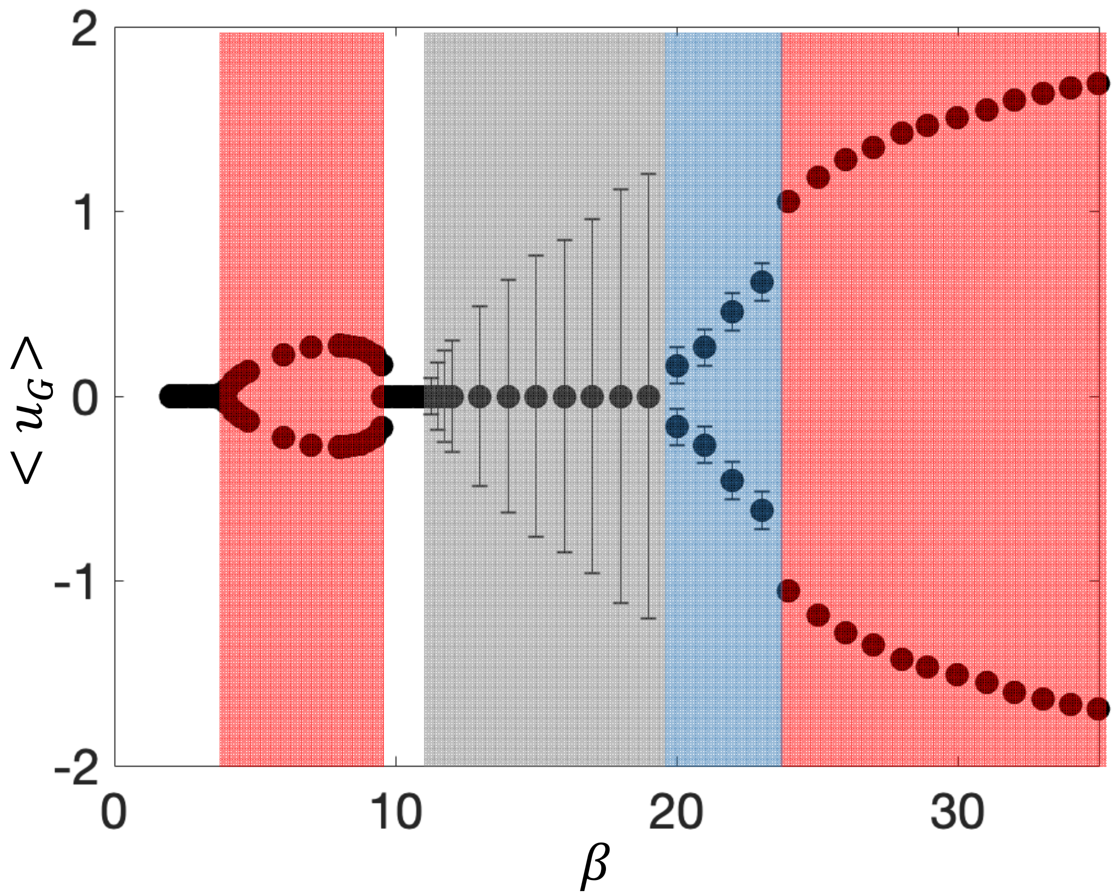
Bifurcation – Average speed x Frequency

Different regimes – Average speed



Propulsive	$f_0 = 0.5$	
B&F	$f_0 = 0.5$	$f_1 = O(10^{-2})$
Oscillating Propulsive	$f_0 = 1.0$	$f_1 = O(10^{-1})$
Non Propulsive		

- Can we explain these regimes through stability analysis of the non propulsive flow?

Stability analysis – Methods

- Base flow – Time-periodic symmetry preserved solutions $\begin{pmatrix} \boldsymbol{u_s} \\ p_s \\ 0 \end{pmatrix}(t)$:

  Time Spectral Method *[Sicot et al 2012];*
  Spatial symmetry imposition – Half plan with symmetry conditions

- Floquet stability analysis of the symmetrical solutions

$$\begin{pmatrix} \boldsymbol{u} \\ p \\ u_G \end{pmatrix}(t) = \begin{pmatrix} \boldsymbol{u_s} \\ p_s \\ 0 \end{pmatrix}(t) + \epsilon \begin{pmatrix} \hat{\boldsymbol{u}} \\ \hat{p} \\ \hat{u}_G \end{pmatrix}(t)\, e^{\alpha t}$$

$\boldsymbol{\alpha}$: Floquet exponent ($\Re \boldsymbol{e}(\boldsymbol{\alpha}) > \mathbf{0}$: unstable)
$f_{Floquet}$: $\mathrm{Im}(\boldsymbol{\alpha})/2\pi$
$\hat{\boldsymbol{q}}$: Floquet mode

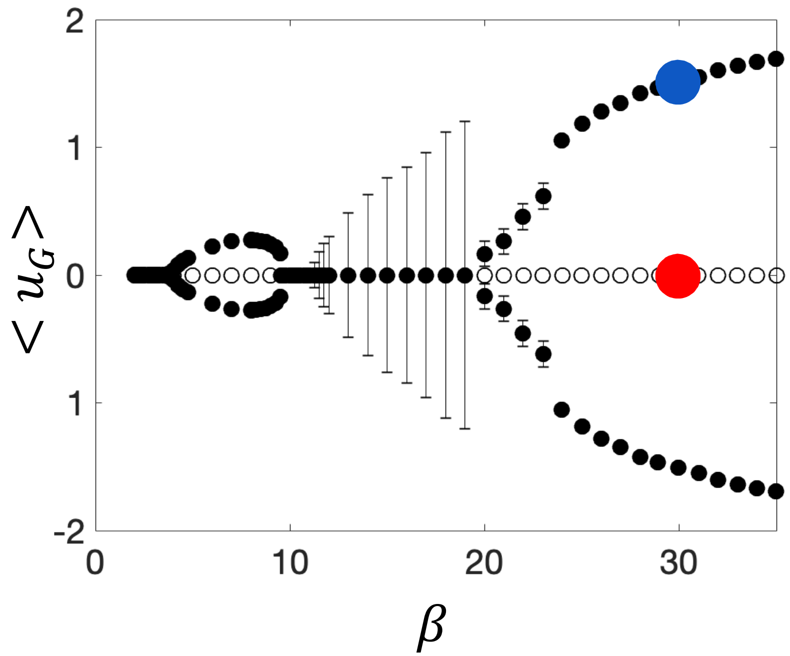
$$\boldsymbol{u_s}(t) = \boldsymbol{u_s}(t+T)$$
$$\hat{\boldsymbol{u}}(t) = \hat{\boldsymbol{u}}(t+T)$$

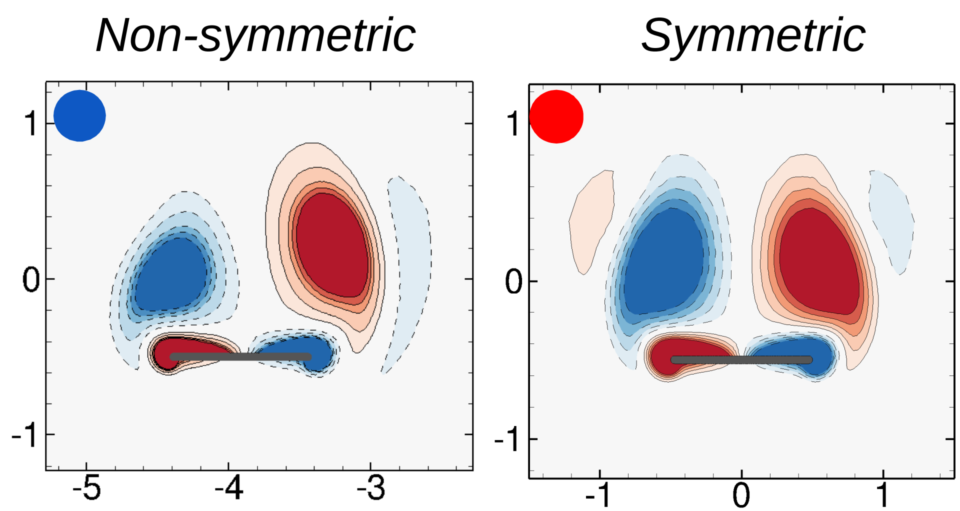
- Hill method - Jacobian of the TSM operator *[O. Thomas et al 2010]*
- FreeFem interface with PETSc/SLEPc – Krylov-Schur methods

Symmetry preserving method

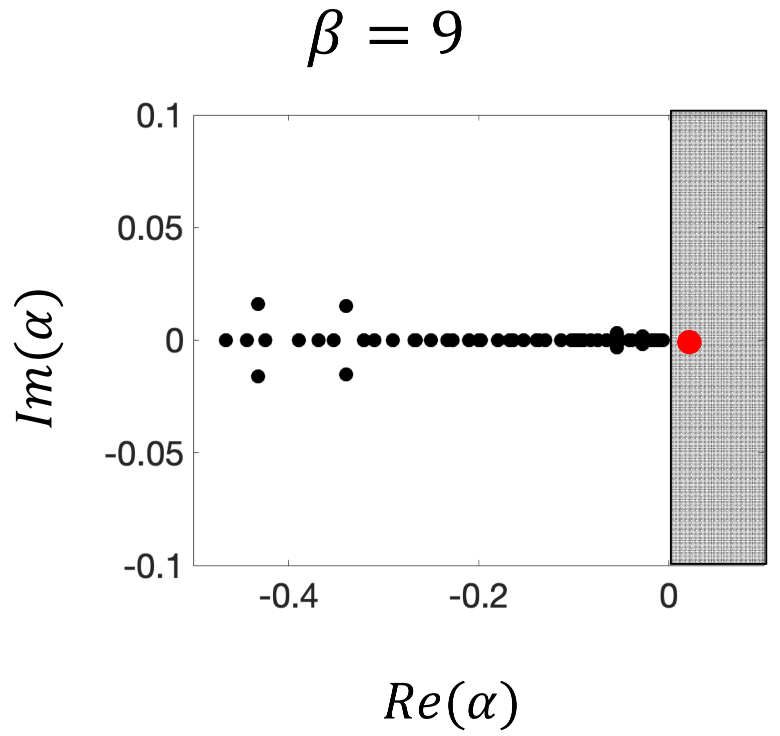
Base flow – Time-periodic and symmetry preserved solutions:

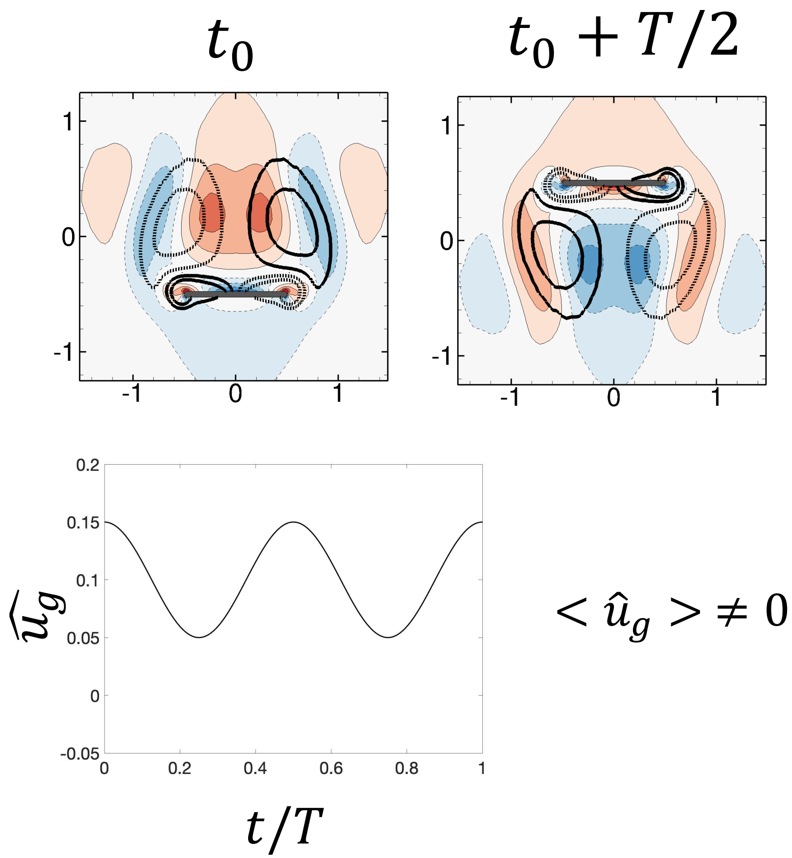
- Time Spectral Method *[Sicot et al 2012];*
- Spatial symmetry imposition – Half plan with symmetry conditions.





Propulsive regime

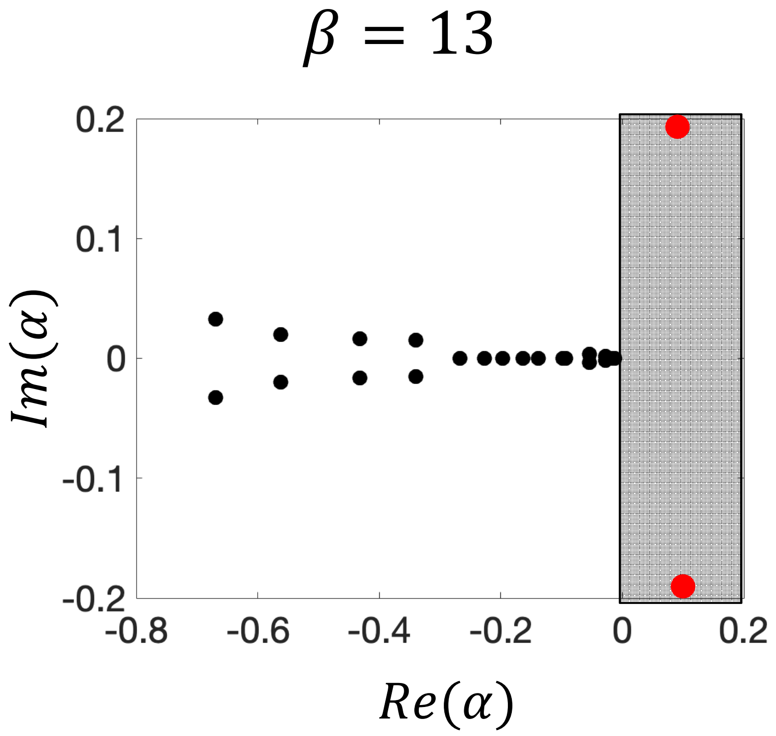


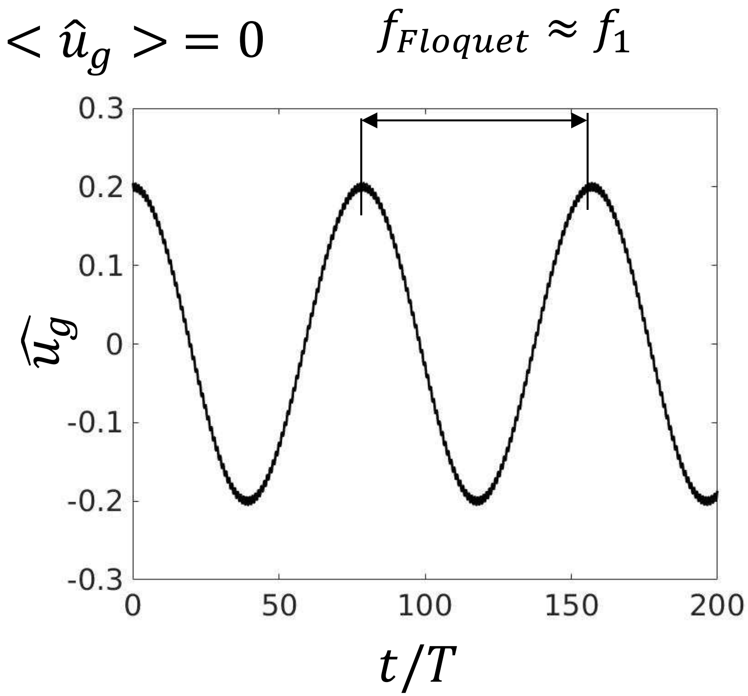


$< \hat{u}_g > \neq 0$

- Mode breaks spatial symmetry
- Synchronous $\mathrm{Im}(\boldsymbol{\alpha}) = 0$
- Net average speed

Back & Forth regime

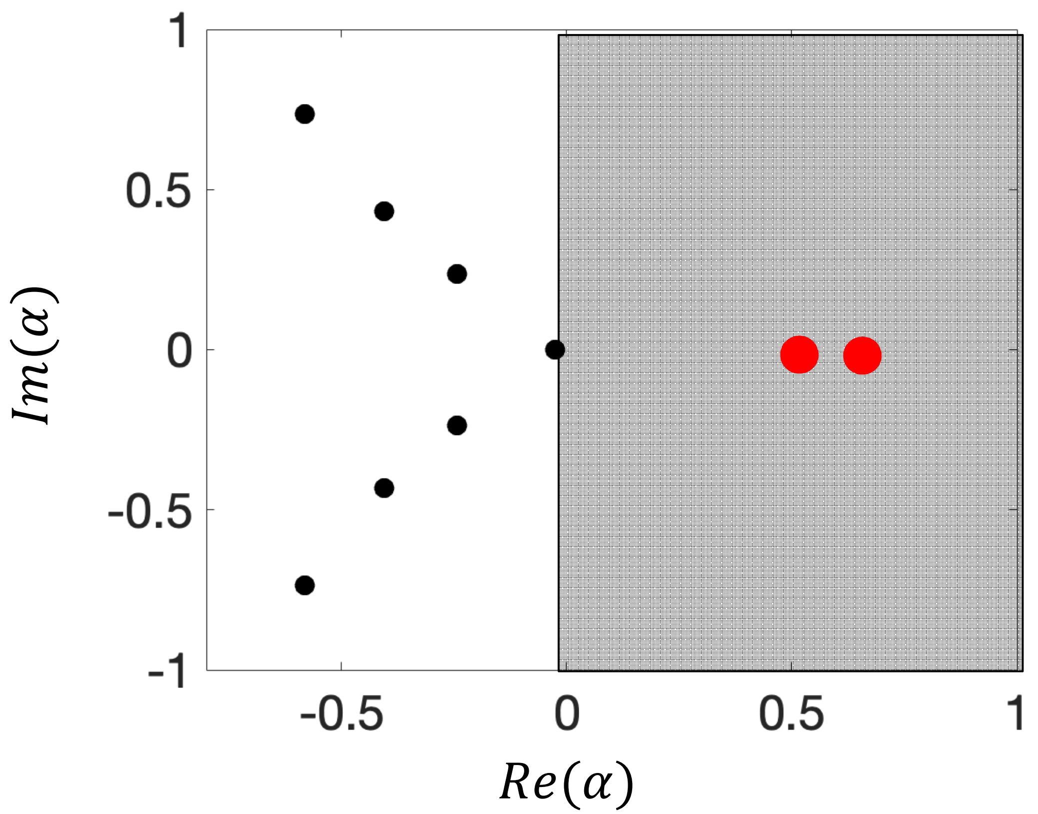




- Mode breaks spatial symmetry
- Asynchronous – Changes base-flow frequency
- Zero average speed $< \hat{u}_g > = 0$

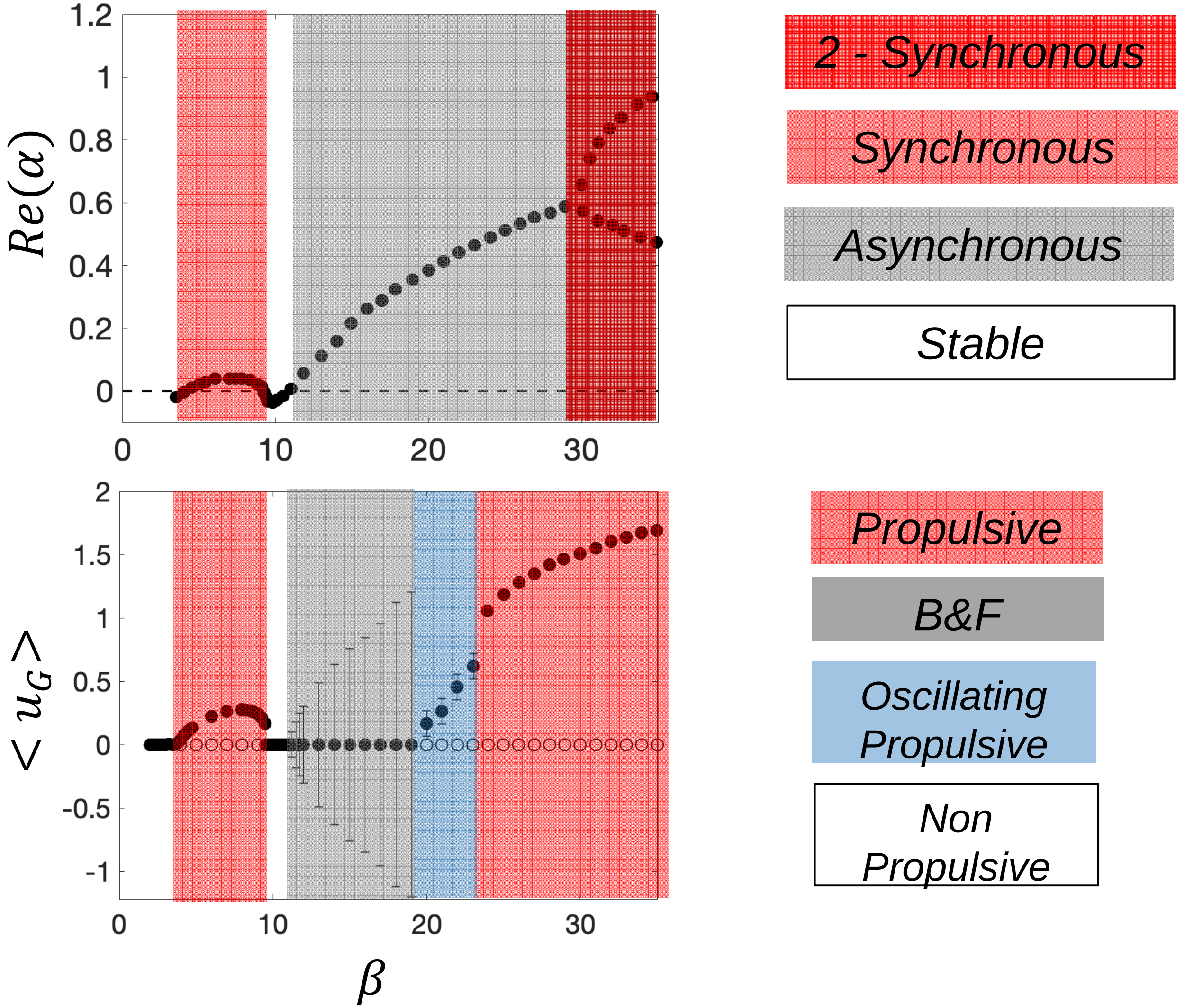
Propulsive regime again

$\beta = 29$



- Two Synchronous Modes
- Net average speed

Leading unstable mode x Non linear regimes



➢ Conclusions:

- Regimes observed through unsteady non-linear simulations for a vertically flapping foil;
- **Unidirectional propulsion** can be explained by a **synchronous unstable Floquet mode**;
- **Back & Forth regimes** can be explained by **asynchronous Floquet modes** with zero average speed;
- Propulsive Oscillating – Nonlinearities?
- A simple Floquet analysis with $\boldsymbol{\rho} \rightarrow \infty$ fails to predict these thresholds

➢ Prospects:

- Extend the presented analysis for other amplitudes and mass ratios;
- Adjoint modes;
- Interaction between flapping wings.

Questions?