Fluid-solid-electric stability analysis for the control of a flexible splitter plate

M. Carini, J.-L. Pfister and O. Marquet

ONERA-DAAA, Meudon

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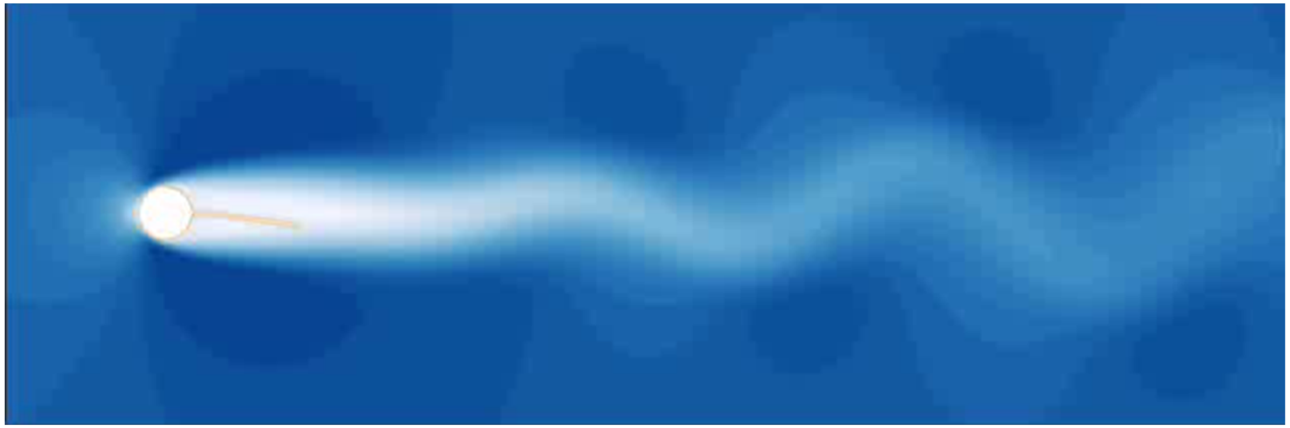
Introduction

Temporal simulation of fluid-solid interaction



1. Investigate the dynamics based on fluid-solid stability analysis
2. Control by stabilisation with piezo-electric patches

Outlines

1. Configuration, physical and numerical models

2. Fluid-solid stability analysis

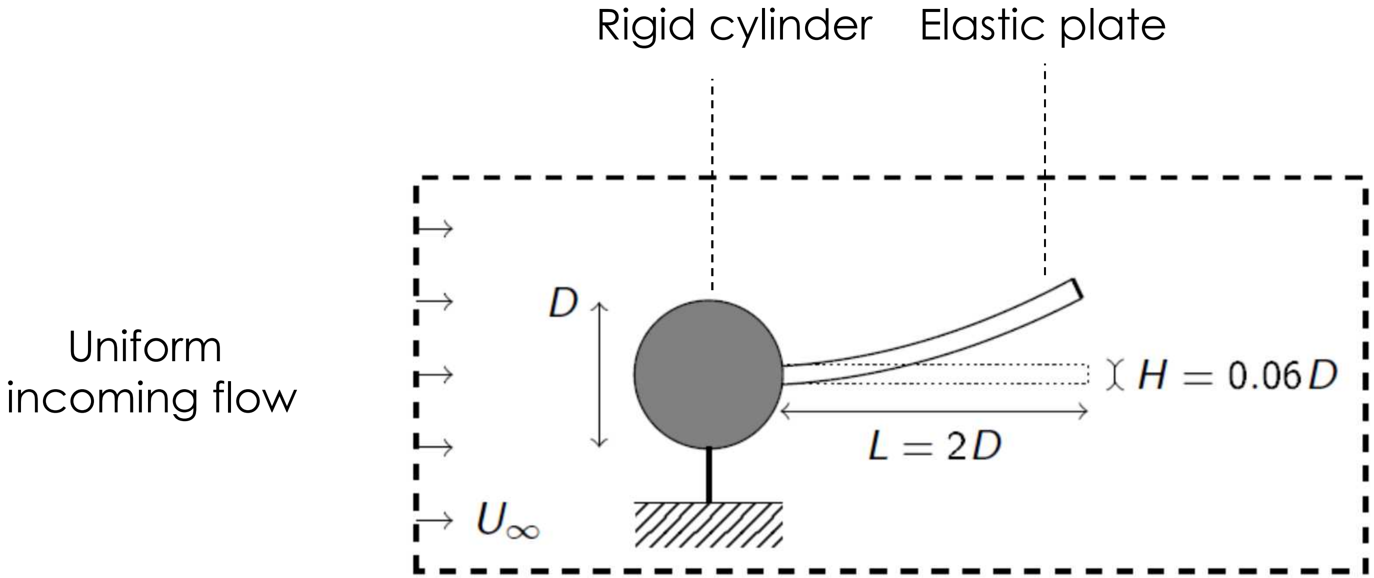
3. Stabilization using piezoelectric patches

3.1 Fluid-solid-electric stability analysis

3.2 Results for short- and open-circuit configurations

3.3 Results for resistive

Flow configuration and nondimensional parameters



- Incompressible laminar flow - Reynolds number $Re = \dfrac{U_\infty D}{\nu} = 80$
- Elastic plate
  - Bending stiffness $K_B = 0.3$
  - Poisson coefficient $\nu_s = 0.4$
- Solid-to-fluid density ratio $\rho_s/\rho_f = 50$

Physical modelling

- **Fluid model** – Eulerien :
  Incompressible Navier-Stokes equations

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}\cdot\left(\boldsymbol{u}\otimes\boldsymbol{u} + p\boldsymbol{I} - \frac{1}{Re}\left(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^T\right)\right) = \boldsymbol{0},$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0,$$

  Time-dependent domain $\Omega_f(t)$

- **Solid model** - Lagrangian:
  Linear isotropic material under small deformations

$$\rho\frac{\partial^2 \boldsymbol{\xi}_s}{\partial t^2} - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma}_s(\boldsymbol{\xi}_s) = \boldsymbol{0},$$

  Time-independent (reference) domain - Ω_s^r

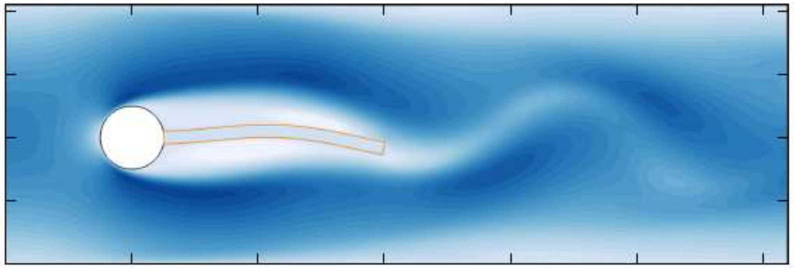
- **Fluid-solid interface**
  Stress and velocity continuity

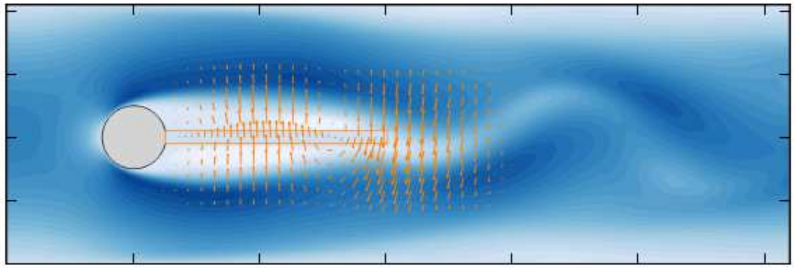
Fluid/structure coupling and numerical treatment

Arbitrary **L**agrangian **E**ulerian mapping

Time-dependent domains

Time-independent reference domains





Solid domain

The displacement variable $\boldsymbol{\xi}_s(t)$ maps

Deformed domain $\longleftarrow$ Reference domain

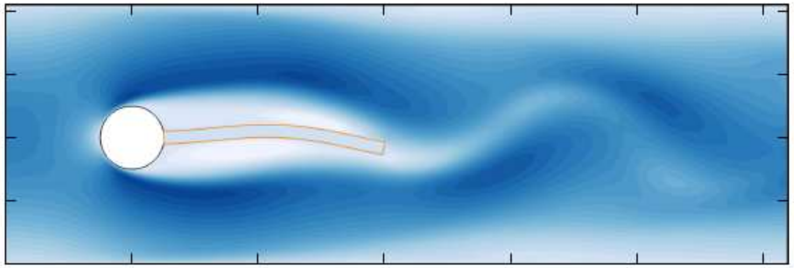
$\Omega_s(t)$ $\qquad$ Ω_s^r

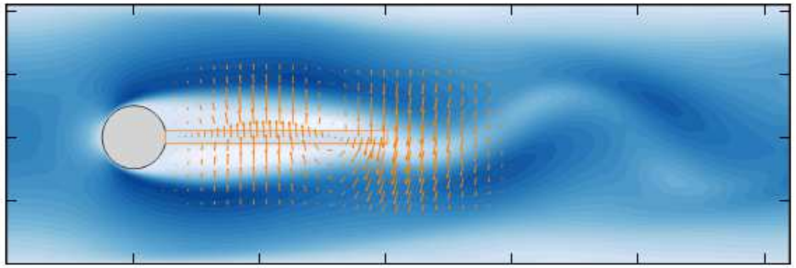
Fluid/structure coupling and numerical treatment

Arbitrary **L**agrangian **E**ulerian mapping

Time-dependent domains

Time-independent reference domains





Fluid domain

An artificial displacement variable $\xi_f(t)$ is introduced

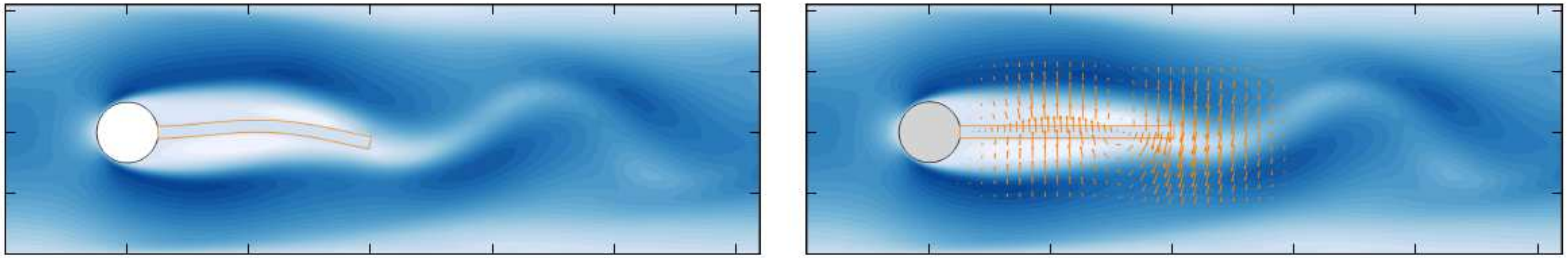
Deformed domain ⟵ Reference domain

$\Omega_f(t)$ Ω_f^r

Fluid/structure coupling and numerical treatment

Arbitrary **L**agrangian **E**ulerian mapping

Time-dependent domains | **Time-independent** reference domains



Fluid domain

An artificial displacement variable $\xi_f(t)$ is introduced which satisfies an arbitrary extension equation

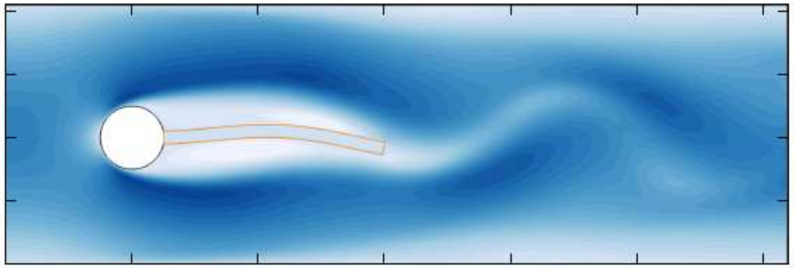
$$E\left(\xi_f(t)\right) = 0 \qquad \xi_f(t) = \xi_s(t)$$

in Ω_f^r on Γ_{fs}^r

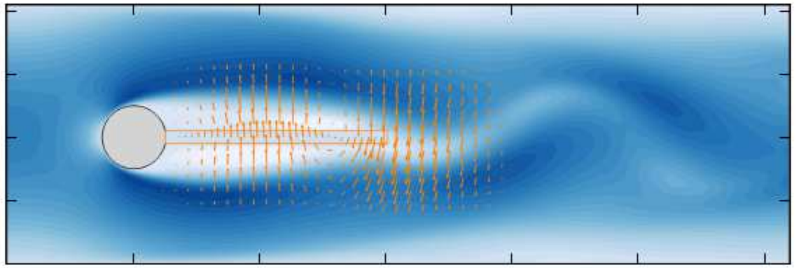
Fluid/structure coupling and numerical treatment

Arbitrary **L**agrangian **E**ulerian mapping

Time-dependent domains



Time-independent reference domains



New variable $\xi_f(t)$, the fluid mesh displacement

Additional equation to spread the displacement at the interface

$$E\big(\xi_f(t)\big) = 0 \qquad \xi_f(t) = \xi_s(t)$$

$$\Omega_f^r \qquad \Gamma_{fs}^r$$

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Steady solutions of the fluid/solid equations

The nonlinear coupled fluid/structure problem is

$$B\frac{\partial \boldsymbol{q}}{\partial t} = R(\boldsymbol{q}) \qquad \boldsymbol{q}(\boldsymbol{x},t) = \left(\boldsymbol{u}, p, \boldsymbol{\xi_f}, \boldsymbol{\xi_s}, \dot{\boldsymbol{\xi}}_{\boldsymbol{s}}\right)^T$$

written in a time-independent reference domain $\Omega^r = \Omega_f^r \cup \Omega_s^r$

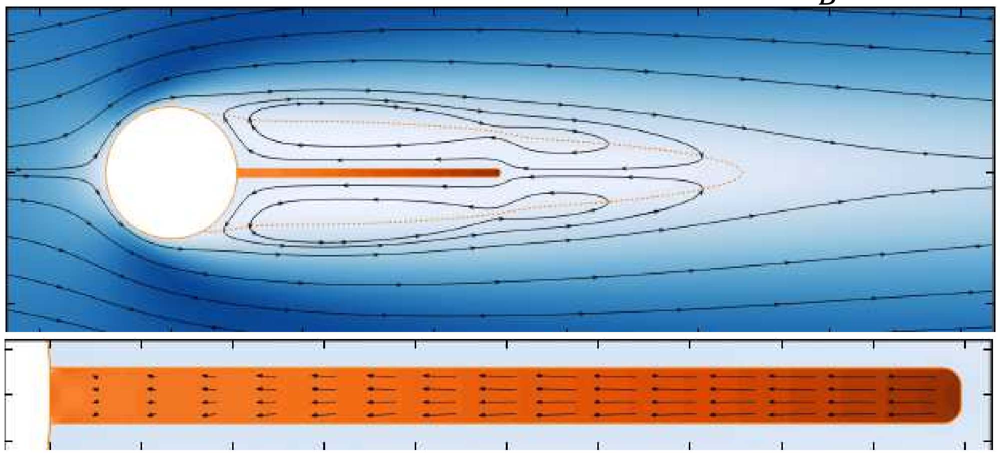
Steady solutions

$$R(\boldsymbol{q_b}) = 0$$

$Re = 80$ $\qquad$ $K_B = 0.015$

Fluid velocity

Solid displacement



Flow recirculation

Small compression

Linear stability of the fluid/solid equations

Linear stability analysis

$$\boldsymbol{q}(x,t) = \boldsymbol{q_b}(\boldsymbol{x}) + \epsilon \left(\widehat{\boldsymbol{q}}(x)\, e^{(\sigma+i\,\omega)t} + \widehat{\boldsymbol{q}}^*(x)\, e^{(\sigma-i\,\omega)t} \right)$$

σ: growth rate ω: frequency

Eigenvalue problem

$$(\,\sigma + i\,\omega)\boldsymbol{B}\,\widehat{\boldsymbol{q}} + \boldsymbol{A}(\boldsymbol{q_b})\,\widehat{\boldsymbol{q}} = \boldsymbol{0}$$

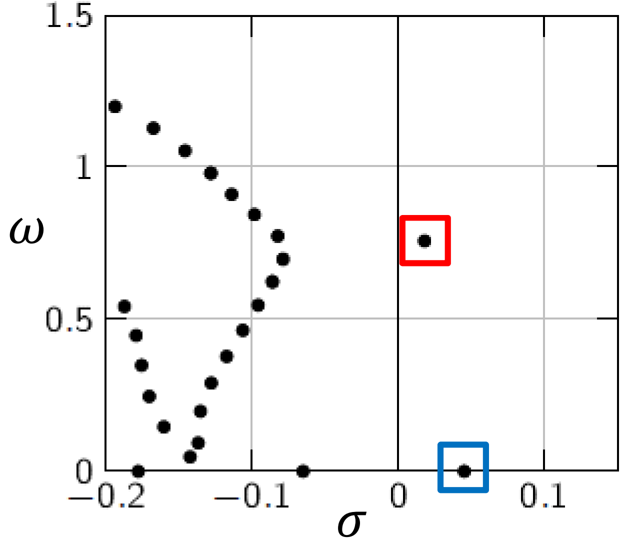
$$\boldsymbol{A}(\boldsymbol{q_b})\,\widehat{\boldsymbol{q}} = \begin{pmatrix} \boldsymbol{A_{ff}} & \boldsymbol{A_{fs}} \\ \boldsymbol{A_{sf}} & \boldsymbol{A_{ss}} \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{q}}_{\boldsymbol{f}} \\ \widehat{\boldsymbol{q}}_{\boldsymbol{s}} \end{pmatrix}$$

$$\widehat{\boldsymbol{q}}_{\boldsymbol{f}} = (\widehat{\boldsymbol{u}}, \hat{p}, \widehat{\boldsymbol{\xi}}_{\boldsymbol{f}}) \qquad \widehat{\boldsymbol{q}}_{\boldsymbol{s}} = (\widehat{\boldsymbol{\xi}}_{\boldsymbol{s}}, \widehat{\dot{\boldsymbol{\xi}}}_{\boldsymbol{s}})$$

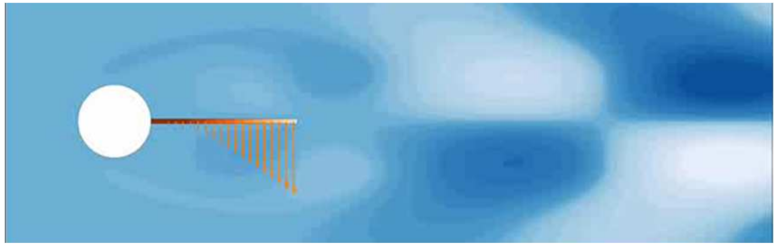
Results – Eigenvalues and eigenmodes

$Re = 80$ $\qquad (\sigma + i\,\omega)\boldsymbol{B}\,\widehat{\boldsymbol{q}} + \boldsymbol{A}(\boldsymbol{q_b})\,\widehat{\boldsymbol{q}} = \boldsymbol{0}$ $\qquad K_B = 0.015$

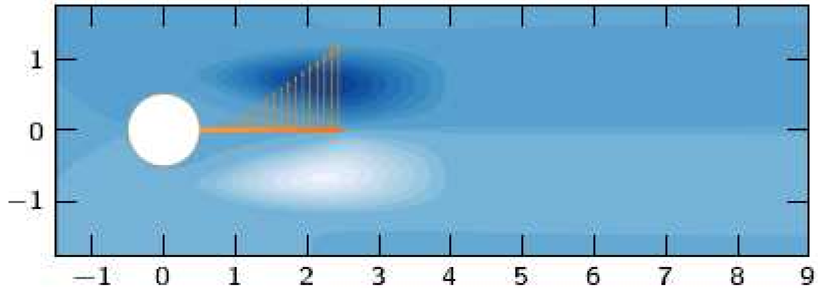
Eigenvalue spectrum



Two unstable modes



Unsteady mode



Steady mode

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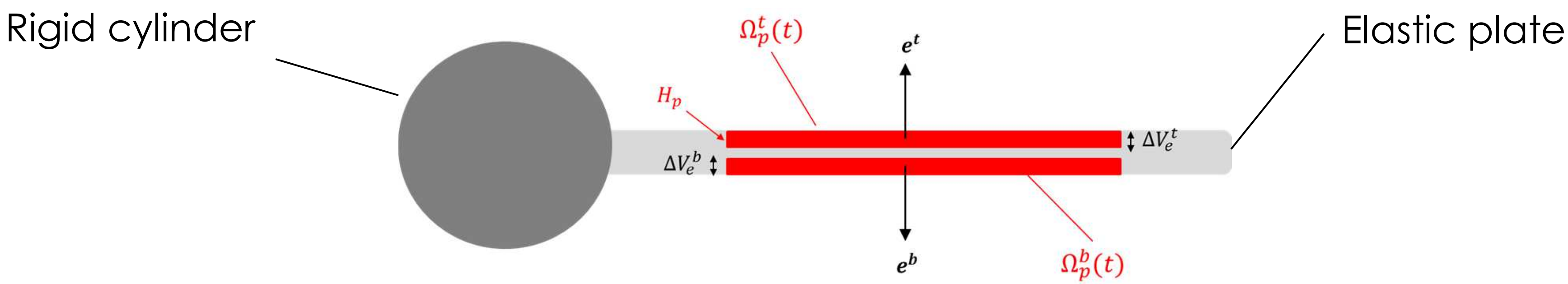
Piezoelectric patches

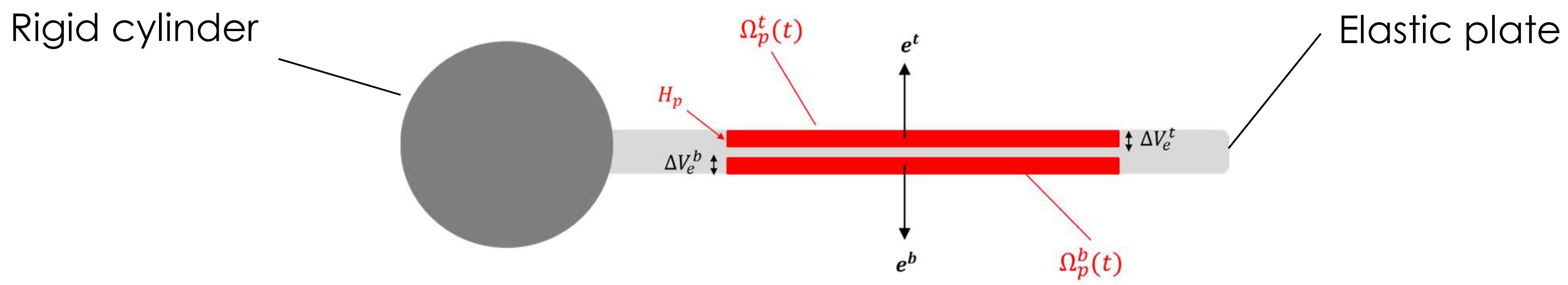


- Two piezoelectric patches
- Continuous modelling of one piezo-patch

$$\rho \frac{\partial^2 \boldsymbol{\xi}_s}{\partial t^2} + \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}_s(\boldsymbol{\xi}_s, V_e) = 0 \qquad \boldsymbol{\nabla} \cdot \boldsymbol{d}(V_e, \boldsymbol{\xi}_s) = 0$$

The Cauchy stress tensor is modified to take into account the electro-mechanical coupling effects

Piezoelectric patches



Discrete modelling of two piezo-patches (Thomas et al. 2009) which are

- connected in parallel $\left(\Delta V_e^t = \Delta V_e^b = V_e\right)$
- polarized in the transverse direction $(e_x = 0\ )$
- with opposive direction $\left(e_y^t = -e_y^b\ \right)$

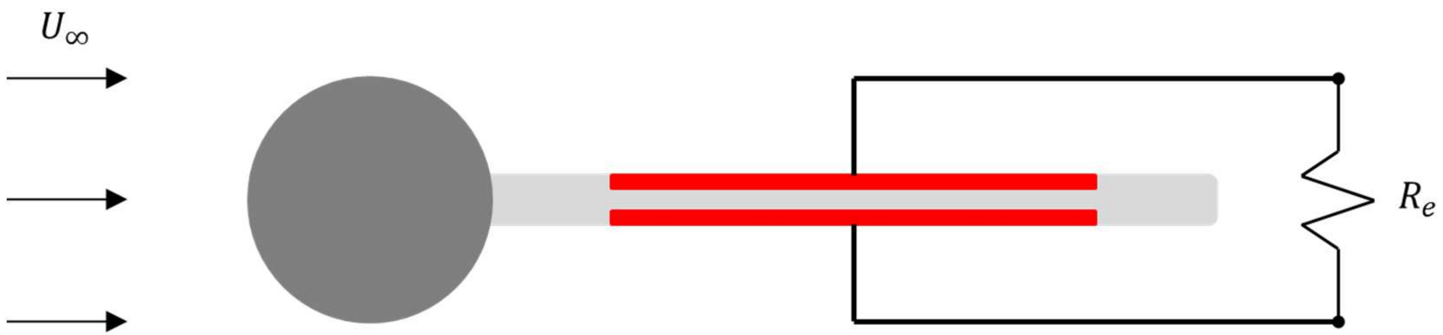
$$M\,\frac{\partial^2 \boldsymbol{\xi}_s}{\partial t^2} + K\,\boldsymbol{\xi}_s - K_p\,V_e = 0$$

$$K_p^T \boldsymbol{\xi}_s + C_p\,V_e = Q_e$$

C_p: equivalent piezo-patches capacitance

K_p: **electro-mechanical coupling** matrix (here only between σ_s^{xx} and e_x)

Piezo-shunt configuration – Resistive circuit



$$\frac{dQ_e}{dt} + \frac{1}{\tau_e} Q_e = 0$$

Characteristic electric time $\tau_e = R_e C_p$

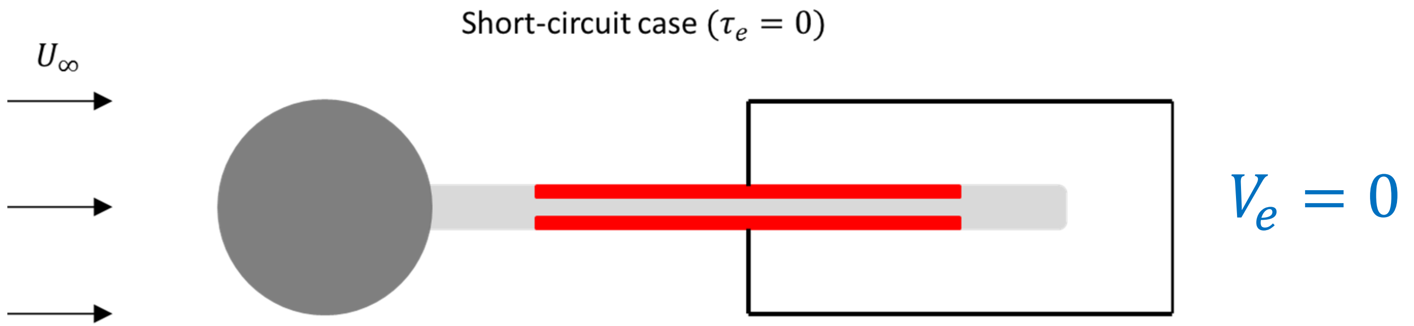
Short-circuit case

$R_e = 0$ $V_e = 0$

Open-circuit case

$R_e \to \infty$ $Q_e = 0$

Short-circuit configuration



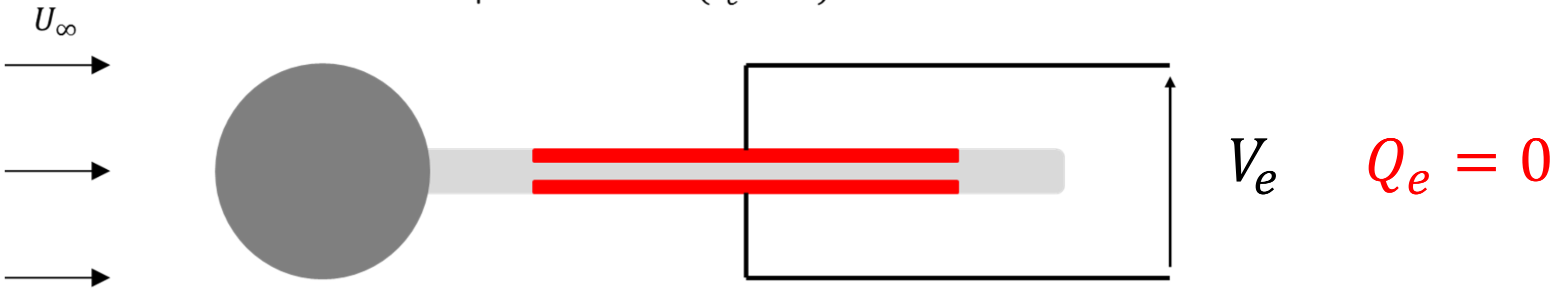
$$M \frac{\partial^2 \xi_s}{\partial t^2} + K\,\xi_s - K_p\,V_e = 0 \longrightarrow M \frac{\partial^2 \xi_s}{\partial t^2} + K\,\xi_s = 0$$

No electro-mechanical coupling

Short-circuit configuration = Fluid-solid configuration

Open-circuit configurations

Open-circuit case ($\tau_e \to \infty$)



$$K_p^T \xi_s + C_p V_e = Q_e \longrightarrow V_e = -C_p^{-1} K_p^T \xi_s$$

$$M \frac{\partial^2 \xi_s}{\partial t^2} + (K + K_p C_p^{-1} K_p^T)\, \xi_s = 0$$

Piezo-patches have an added-stiffness effect

Fluid-solid-electric eigenvalue problem

$$(\sigma + i\,\omega)\boldsymbol{B}\,\widehat{\boldsymbol{q}} + \boldsymbol{A}(\boldsymbol{q_b})\,\widehat{\boldsymbol{q}} = \boldsymbol{0}$$

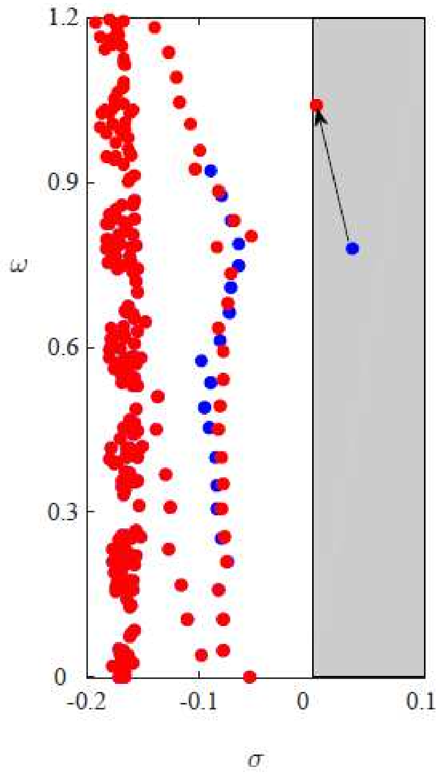
$$\boldsymbol{A}(\boldsymbol{q_b})\,\widehat{\boldsymbol{q}} = \begin{pmatrix} \boldsymbol{A_{ff}} & \boldsymbol{A_{fs}} & \boldsymbol{0} \\ \boldsymbol{A_{sf}} & \boldsymbol{A_{ss}} & \boldsymbol{A_{se}} \\ \boldsymbol{0} & \boldsymbol{A_{es}} & \boldsymbol{A_{ee}} \end{pmatrix} \begin{pmatrix} \widehat{\boldsymbol{q}}_{\boldsymbol{f}} \\ \widehat{\boldsymbol{q}}_{\boldsymbol{s}} \\ \widehat{\boldsymbol{q}}_{\boldsymbol{e}} \end{pmatrix}$$

$\widehat{\boldsymbol{q}}_{\boldsymbol{f}} = (\widehat{\boldsymbol{u}}, \hat{p}, \hat{\boldsymbol{\xi}}_{\boldsymbol{f}})$ $\widehat{\boldsymbol{q}}_{\boldsymbol{s}} = (\hat{\boldsymbol{\xi}}_{\boldsymbol{s}}, \hat{\dot{\boldsymbol{\xi}}}_{\boldsymbol{s}})$ $\widehat{\boldsymbol{q}}_{\boldsymbol{e}} = (\hat{V}_e, \hat{Q}_e)$

Results – Eigenvalue and eigenmodes

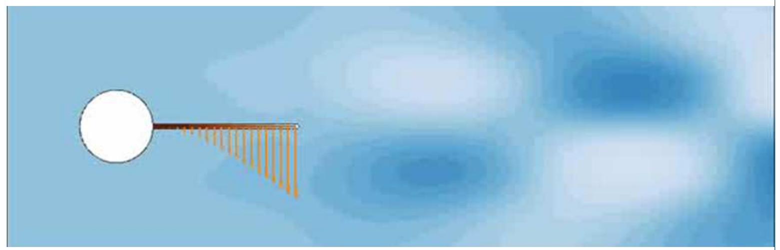
$Re = 80$ $K_B = 0.3$ $\rho = 50$ $k_e = 0.57$

Eigenvalue spectrum



Unstables eigenmodes

$\omega = 0.85$?



Fluid-solid (short-circuit)

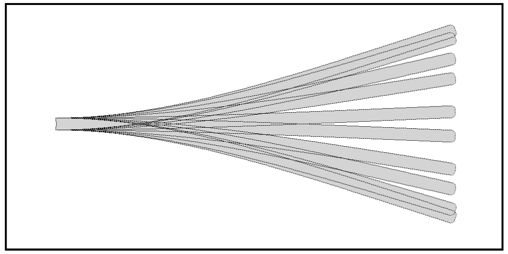
$\omega = 1.33$?

Fluid-solid-electric (open-circuit)

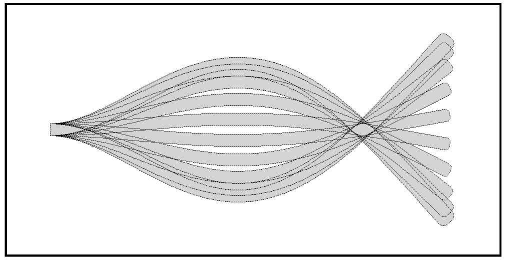
Maximal displacement increased by 3 orders of magnitude !

Free-vibration modes

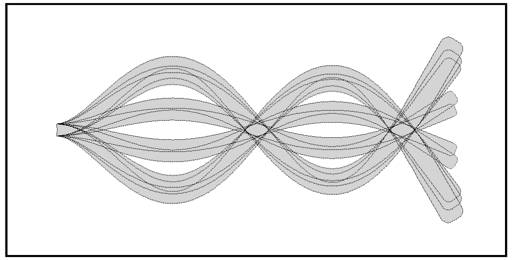
Free-vibration modes of the elastic plate



$\boldsymbol{\omega_1 = 0.85}$

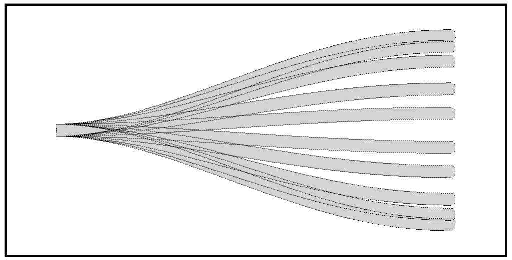


$\omega_2 = 5.25$

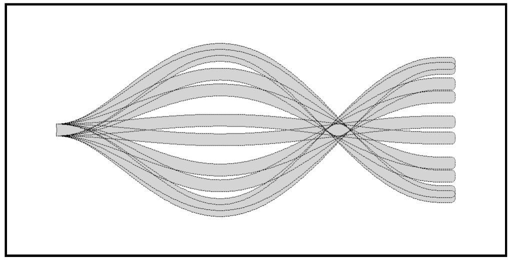


$\omega_3 = 14.59$

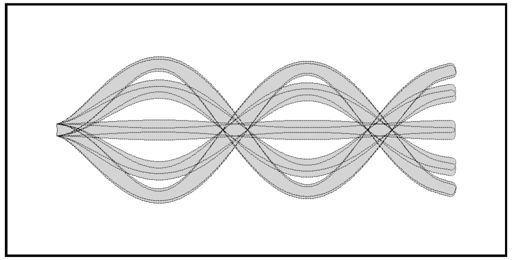
Free-vibration modes of the elastic plate with piezo-patches



$\boldsymbol{\omega_1 = 1.33}$



$\omega_2 = 7.64$

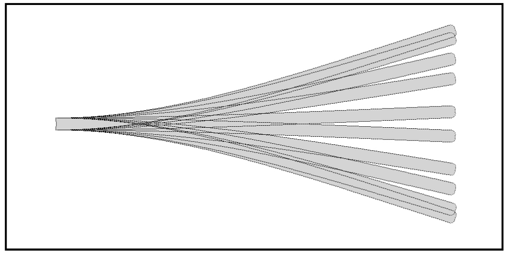


$\omega_3 = 17.66$

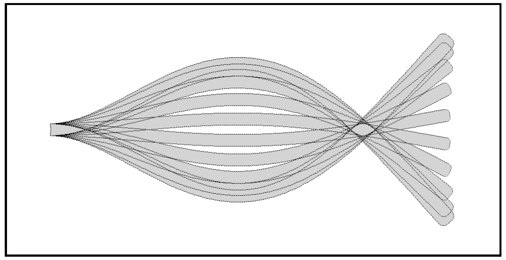
Free-vibration modes – Frequency comparison

Fluid-solid (short-circuit) $\boldsymbol{\omega = 0.85}$

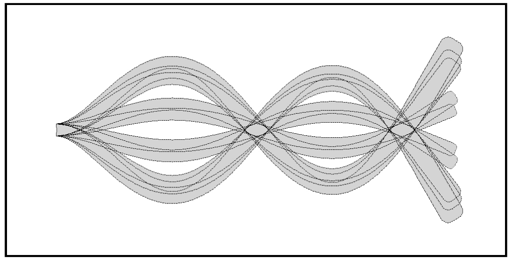
Free-vibration modes of the elastic plate



$\boldsymbol{\omega_1 = 0.85}$



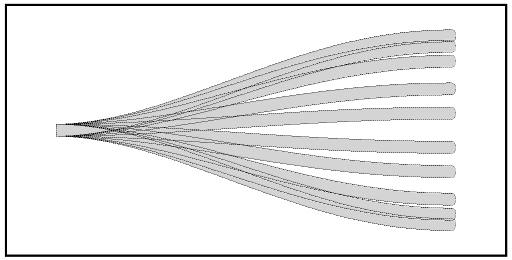
$\omega_2 = 5.25$



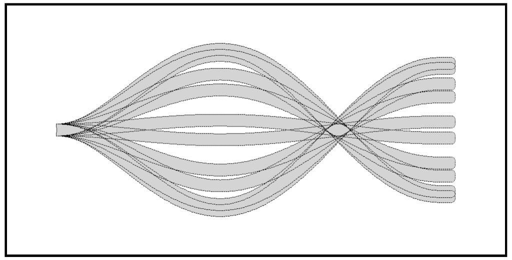
$\omega_3 = 14.59$

Fluid-solid-electric (open-circuit) $\boldsymbol{\omega = 1.33}$

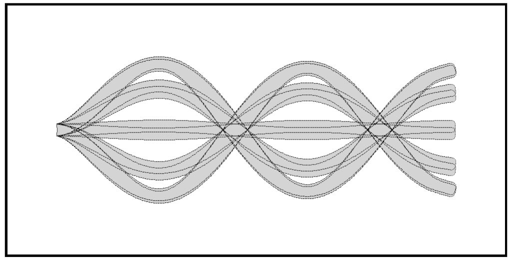
Free-vibration modes of the elastic plate with piezo-patches



$\boldsymbol{\omega_1 = 1.33}$



$\omega_2 = 7.64$

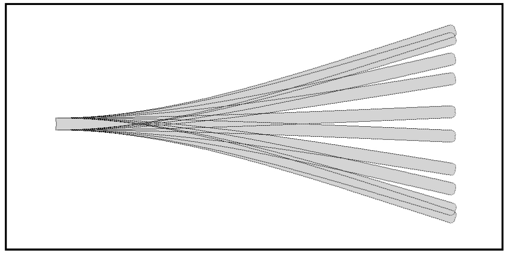


$\omega_3 = 17.66$

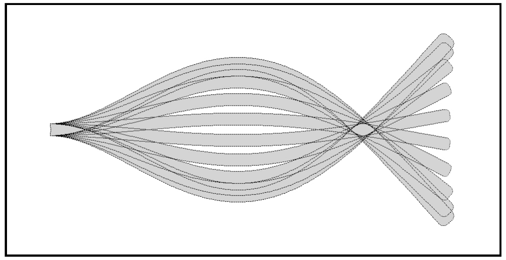
Free-vibration modes – Amplitude comparison

Fluid-solid (short-circuit)

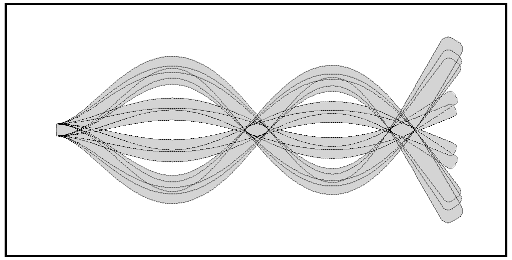
Free-vibration modes of the elastic plate



$\boldsymbol{\alpha_1 = 0.9969}$



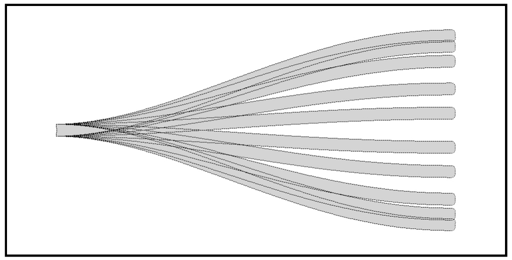
$\alpha_2 = 0.0029$



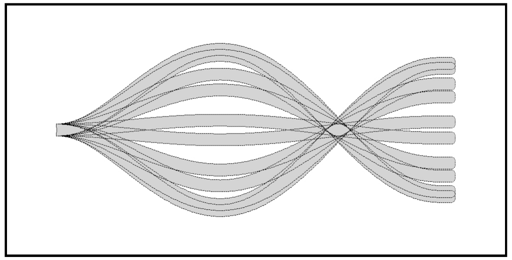
$\alpha_3 = 0.0002$

Fluid-solid-electric (open-circuit)

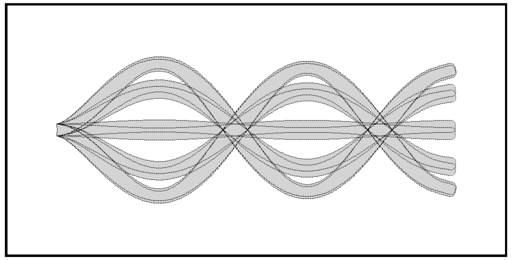
Free-vibration modes of the elastic plate with piezo-patches



$\boldsymbol{\alpha_1 = 0.9979}$



$\alpha_2 = 0.0017$

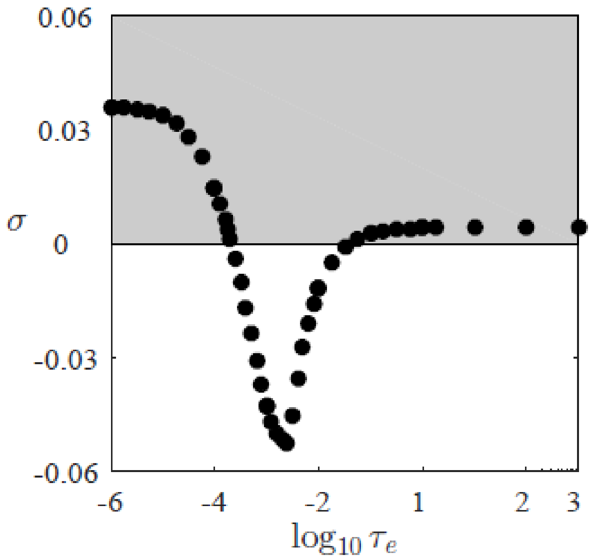


$\alpha_3 = 0.0004$

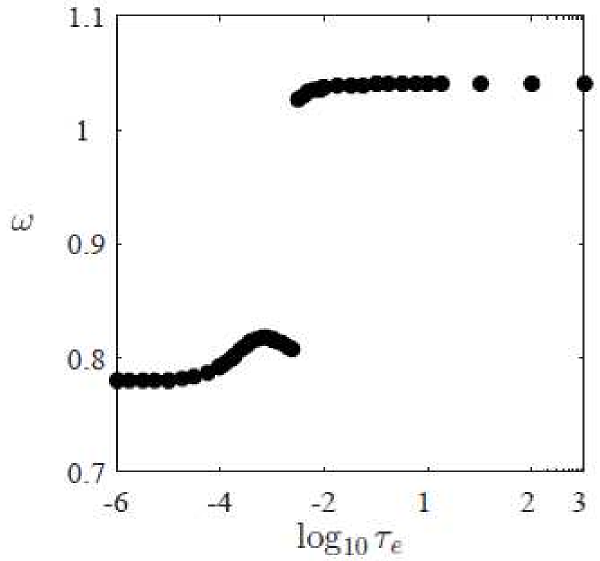
Results for the piezo R-shunt configuration

Behaviour of the leading eigenvalue when varying the resistance

Growth rate



Frequency



Stabilization in a range of electric resistance

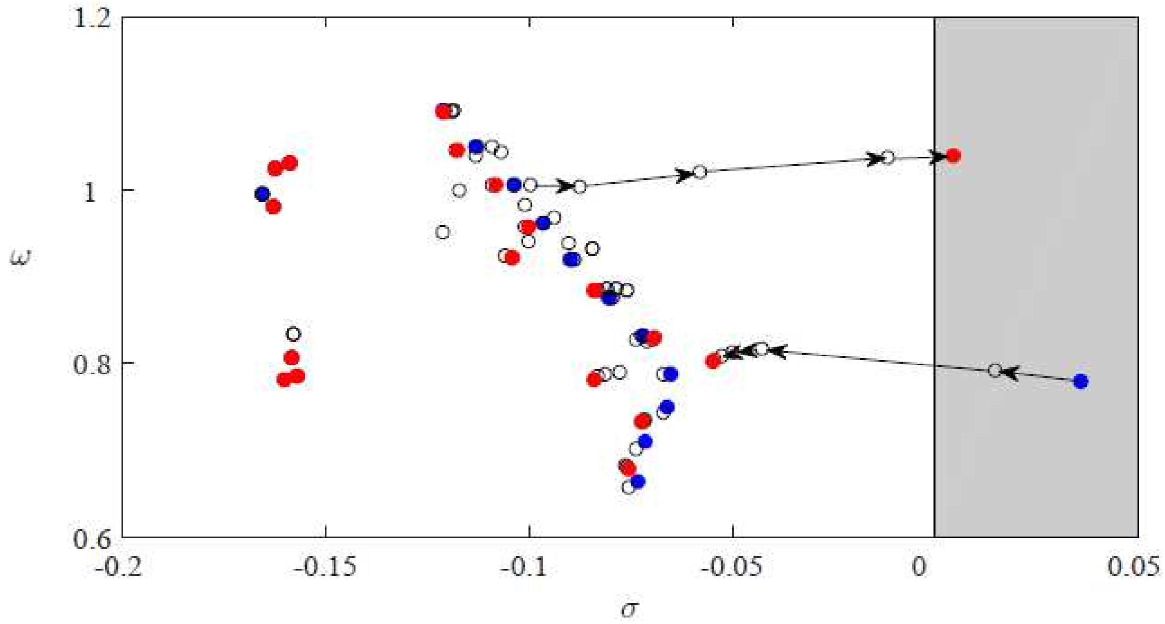
Jump of the frequency

Results - Piezo-shunt configuration

Behaviour of the leading eigenvalue when varying the resistance

Eigenvalue spectrum

Short-circuit
$R_e = 0$



Open-circuit
$R_e \to \infty$

Small $\boldsymbol{R_e}$: **stabilization** of the fluid-solid eigenmode

Large $\boldsymbol{R_e}$: **destabilization** of the fluid-solid-electric eigenmode

Perspectives

- Passive control

Investigate the effect of piezo-patches in other configurations

- Case with the nstable steady mode

- Case where the piezo-patches have other material properties

- Introduce a second-order electric dynamics (add inductance to the resistive circuit)

- Active feeback control