Passive control of fluid-structure instabilities by means of piezo-shunts

Marco CARINI, Jean-Lou PFISTER & Olivier MARQUET
AEROFLEX project

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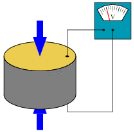
return on innovation

Piezo-electricity : a wide range of applications

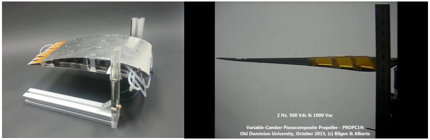
Piezo-electricity : *Electric response of certain materials to a mechanical stress due to their microscopic structure*



Sensing & Actuation from everyday life ...

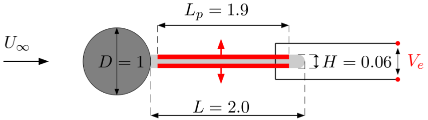


... to the **Aerospace Research**

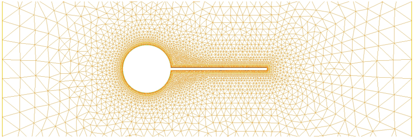


Focus on : using piezos to control fluid-structure global instabilities

2D Model problem



Fluid region



Solid region

- Rigid cylinder + elastic plate + piezo patches.
- Incompressible flow at $Re = U_\infty D/\nu = 80$.
- Solid-to-fluid density ratio $\rho = 50$ and plate bending stiffness $K = 0.3$.
- Electromechanical coupling coefficient $k_e = 0.57009$.

Modelling framework

- *Incompressible Navier-Stokes equations on the deforming domain :*

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}\cdot \left(\boldsymbol{u} \otimes \boldsymbol{u} + p\boldsymbol{I} - \frac{1}{Re}\left(\boldsymbol{\nabla}\boldsymbol{u} + \boldsymbol{\nabla}\boldsymbol{u}^T \right) \right) = \boldsymbol{0},$$

$$\boldsymbol{\nabla}\cdot \boldsymbol{u} = 0,$$

- *Linear isotropic elastic material under small deformations :*

$$\rho \frac{\partial^2 \boldsymbol{\xi}_s}{\partial t^2} - \boldsymbol{\nabla}\cdot \boldsymbol{\sigma}_s(\boldsymbol{\xi}_s) = \boldsymbol{0},$$

- *Piezo-patch modelling :*

$$\rho \frac{\partial^2 \boldsymbol{\xi}_s}{\partial t^2} - \boldsymbol{\nabla}\cdot \boldsymbol{\sigma}_s(\boldsymbol{\xi}_s, V_e) = \boldsymbol{0},$$

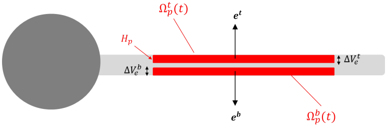
$$\boldsymbol{\nabla}\cdot \boldsymbol{d}(V_e, \boldsymbol{\xi}_s) = 0.$$

- Arbitrary Lagrangian Eulerian (**ALE**) formulation.
- **Monolithic approach** using FEM discretisation.

Electro-mechanical coupling

Discretized piezo-structure eqs.

$$
\begin{aligned}
M \frac{d^2 \boldsymbol{\xi}_s}{dt^2} + K \boldsymbol{\xi}_s - K_p \, \Delta V_e &= \mathbf{0}, \\
K_p^T \boldsymbol{\xi}_s + C_p \, \Delta V_e &= Q_e,
\end{aligned}
$$



- C_p is the equivalent piezo-patches capacitance.
- K_p is the **electromechanical coupling matrix** bewteen σ_s^{xx} and the **transverse electric field** e_y.
- **Parallel connection** $\Delta V_e^t = \Delta V_e^b = \Delta V_e$.
- **Opposite poling** y**-direction** $\Rightarrow$ **only bending modes are affected** :

$$K_p^T \boldsymbol{\xi}_s = 0, \quad \text{for pure traction/compression}$$

Connecting a R-shunt circuit

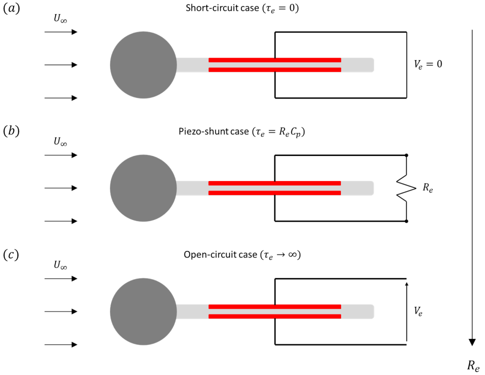
1st order electr. dynamics

$$\frac{dQ_e}{dt} + \frac{1}{\tau_e} Q_e = 0,$$

where

$$\tau_e = R_e C_p$$

is the *characteristic electric time*.



- Short-circuit ⇒ No electro-mechanical coupling ⇒ pure fluid-structure behaviour.
- Open-circuit ⇒ Maximal electro-mechanical coupling.

Fluid-Structure-Electric System

$$\mathcal{B}_f \frac{\partial \boldsymbol{q}_f}{\partial t} - \mathcal{R}_f(\boldsymbol{q}_f, \boldsymbol{q}_s) = \boldsymbol{0}, \quad \text{on } \Omega_f^r$$

Fluid Eqs.

$$\mathcal{B}_s \frac{\partial \boldsymbol{q}_s}{\partial t} - \mathcal{R}_s(\boldsymbol{q}_s, \boldsymbol{q}_s) = \boldsymbol{0}, \quad \text{on } \Omega_s^r$$

Solid Eqs.

$$\mathcal{B}_p \frac{d\boldsymbol{q}_e}{dt} - \mathcal{R}_p(\boldsymbol{q}_e, \boldsymbol{q}_s) = \boldsymbol{0},$$

Electr. Eqs.

where

$$\boldsymbol{q}_f = \begin{pmatrix} \boldsymbol{u} \\ p \end{pmatrix}, \qquad \boldsymbol{q}_s = \begin{pmatrix} \boldsymbol{\xi}_s \\ \partial \boldsymbol{\xi}_s / \partial t \end{pmatrix}, \qquad \boldsymbol{q}_e = \begin{pmatrix} Q_e \\ V_e \end{pmatrix}.$$

and in compact abstract from :

$$\mathcal{B} \frac{\partial \boldsymbol{q}}{\partial t} = \mathcal{R}(\boldsymbol{q}).$$

where $\boldsymbol{q} = \{\boldsymbol{q}_f, \boldsymbol{q}_s, \boldsymbol{q}_e\}^T$.

Global stability analysis

- Nonlinear **fixed equilibrium solution** $\boldsymbol{q}_B$:

$$\mathcal{R}(\boldsymbol{q}_B) = 0.$$

- **Linearized perturbations around** $\boldsymbol{q}_B$:

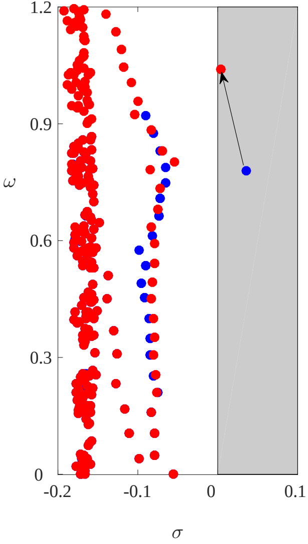
$$\boldsymbol{q}(\boldsymbol{x}, t) = \boldsymbol{q}_B(\boldsymbol{x}) + \epsilon \hat{\boldsymbol{q}}(\boldsymbol{x}) e^{\lambda t}, \quad \text{with } \epsilon \ll 1.$$

- **Generalised Eigenvalue problem** :

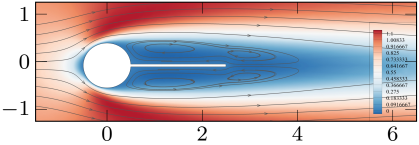
$$\lambda \mathcal{B} \hat{\boldsymbol{q}} = \mathcal{A} \hat{\boldsymbol{q}},$$

where :

$$\mathcal{A} = \left. \frac{\partial \mathcal{R}(\boldsymbol{q})}{\partial \boldsymbol{q}} \right|_{\boldsymbol{q}_B} = \begin{pmatrix} \mathcal{A}_{ff} & \mathcal{I}_{fs} & 0 \\ \mathcal{I}_{fs} & \mathcal{A}_{ss} & \mathcal{A}_{sp} \\ 0 & \mathcal{A}_{ps} & \mathcal{A}_{pp} \end{pmatrix}.$$



- **Base flow**

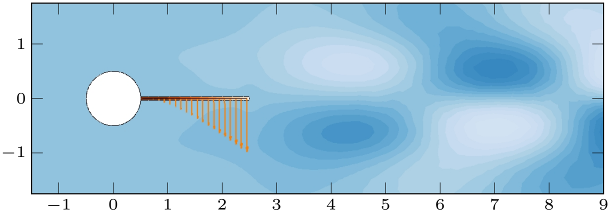


- **Global spectrum** for **Short** and **Open circuit** case.

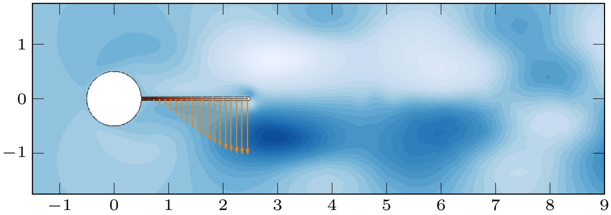
- Investigate the mode transition :
  - based on the **added electrical stiffness** effect.
  - by varying τ_e.

Leading fluid-structure-electric modes

Short-circuit



Open-circuit

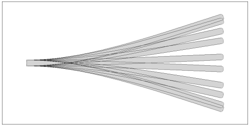


When using the same normalization, the maximum y-displacement is increased by 3 **orders** of magnitude in the open-circuit case.

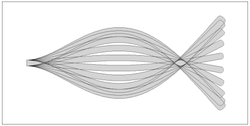
Structure mode kinematic analysis

Projection of q_s on the structure free-vibration modes

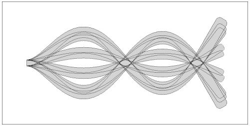
$\omega_1 = 0.84$



$\omega_2 = 5.25$



$\omega_3 = 14.59$



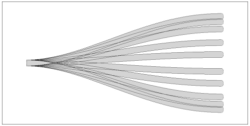
Projection coefficient amplitude

	Mode 1	Mode 2	Mode 3	Mode 4
Short-circuit mode	1.00	2.89×10^{-3}	1.99×10^{-4}	5.29×10^{-5}
Open-circuit mode	0.99	1.53×10^{-1}	2.61×10^{-2}	1.14×10^{-2}

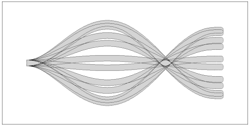
Structure mode kinematic analysis (continued)

Projection of q_s on the piezo-structure free-vibration modes

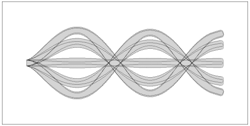
$\omega_1 = 1.33$



$\omega_2 = 7.64$



$\omega_3 = 17.66$



Projection coefficient amplitude

	Mode 1	Mode 2	Mode 3	Mode 4
Open-circuit mode	1.00	1.70×10^{-3}	4.42×10^{-4}	3.39×10^{-5}

Structure mode dynamic analysis

For the free-vibrating piezo-elastic plate in open-circuit we have :

$$-\omega^2 M\hat{\boldsymbol{\xi}}_s + (K + K_p C_p^{-1} K_p^T)\hat{\boldsymbol{\xi}}_s = 0$$

By projection onto the base of free-vibrating modes of the elastic plate only, X_s, we obtain

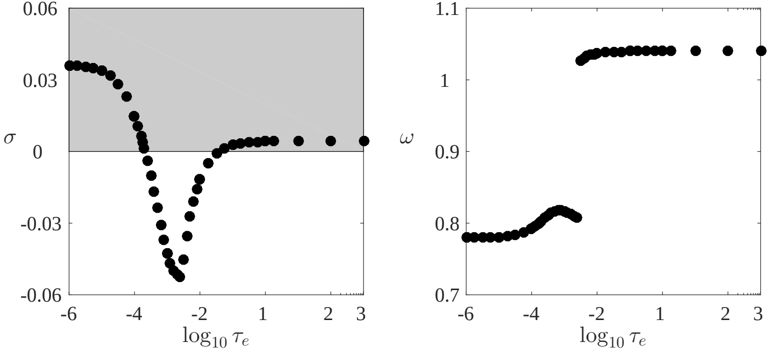
$$\omega^2 \hat{\boldsymbol{\zeta}} = \begin{bmatrix} \omega_{0,1}^2 & & & \\ & \omega_{0,2}^2 & & \\ & & \ddots & \\ & & & \omega_{0,n}^2 \end{bmatrix} \hat{\boldsymbol{\zeta}} + \Delta\Omega_p \hat{\boldsymbol{\zeta}}, \quad \text{with } \Delta\Omega_p = X_s^H K_p C_p^{-1} K_p^T X_s$$

Truncation at the first 6 modes yields $\omega_1 = 1.4$ (compared to $\omega_{1,\text{exact}} = 1.33$) and

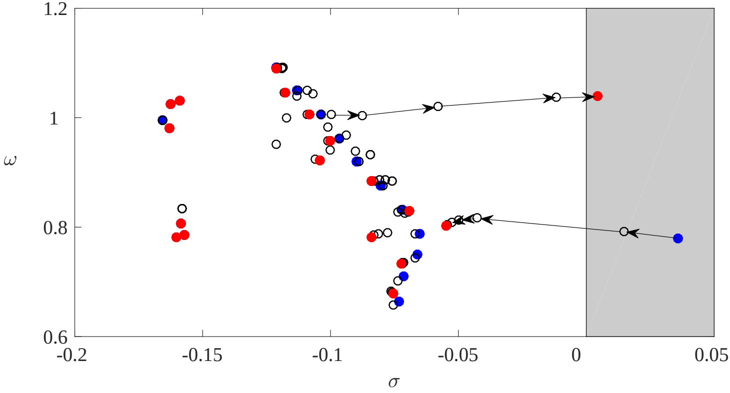
$$abs(\hat{\boldsymbol{\zeta}}_1) = \left(0.9834, 0.1782, 0.0312, 0.0134, 3.33 \times 10^{-10}, 0.0050\right)$$

⇒ the first free-vibrating piezo mode essentially results from the **combination of the first two free vibration modes** of the elastic plate.

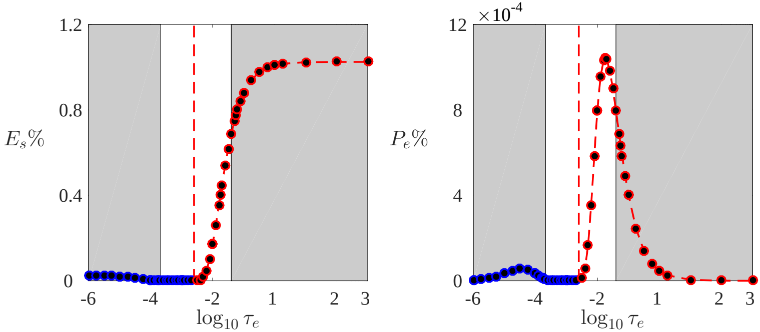
Leading eigenvalue



Increasing τ_e



Leading mode



- The white area denotes the stabilization range.
- The red dashed line corresponds to the maximal damping of the leading mode.

Conclusions

- **Structure-added electrical stiffness** effect as expected.
- **Fluid-structure mode** (*mainly driven by the unstable fluid dynamics*) **controlled by** exploiting the piezo **electro-mechanical coupling**.
- Open/close circuit **mode selection** explained by continuosly varying R_e :
  - the original fluid-structure mode is increasingly damped as R_e is increased.
  - for large enough values of R_e a second mode is destabilized (*water-bed effect*).
  - effective **stabilization** by passive control within a **finite range** of R_e values.

Ongoing developments

- Varying elastic parameters to address other types of fluid-structure modes.
- Introducing a second-order electric dynamics.
- Active feedback control.

Thanks for your attention.

(Any Question ?)