Extended mean-flow analysis of periodic flows

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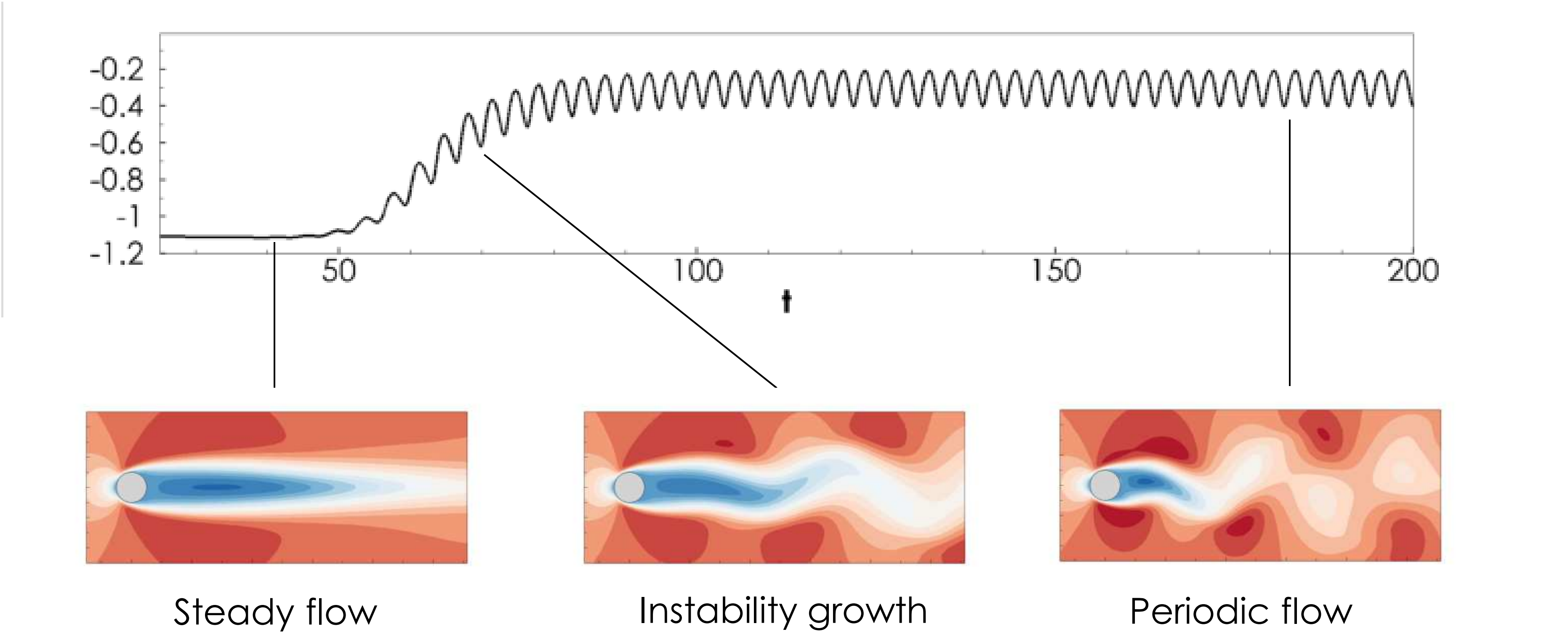
Introduction

Periodic flow resulting from
the nonlinear saturation of a linear instability



Periodic flow resulting from
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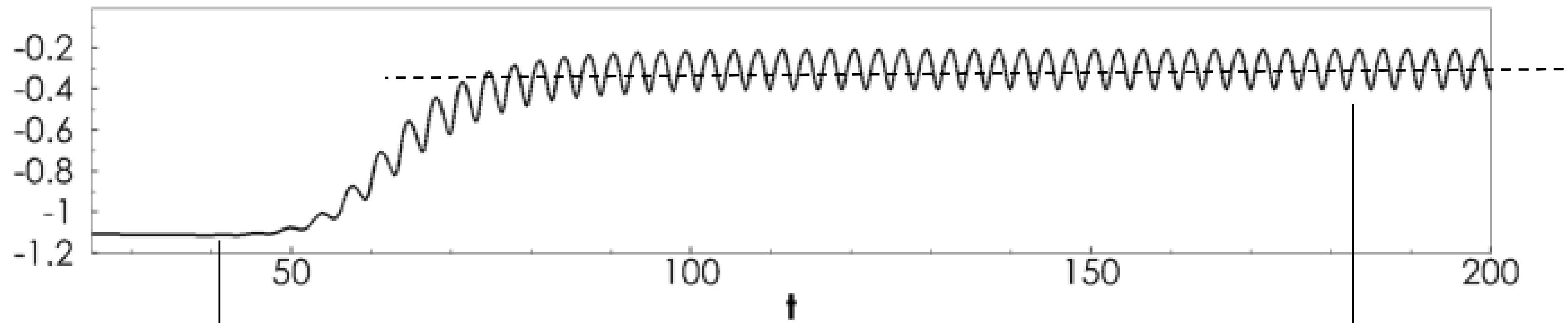
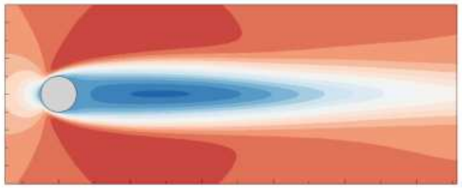
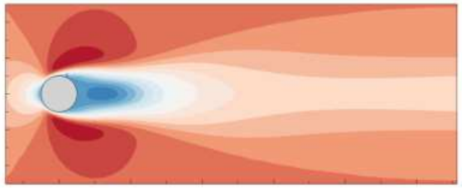




Mean flow distortion

Zielinska et al (1997)



Base flow

Mean flow

Introduction

The nonlinear saturation is due to two mechanisms

1 – Mean flow distortion (circular cylinder flow)

- Mean flow analysis - *Barkley (2002)*

  Eigenvalue analysis of a mean flow (computed from DNS)

  Real **Z**ero **I**maginary **F**requency property – Turton et al. (2015)

- Self-consistent model - *Mantic-Lugo et al (2015)*

  Reconstruction of the mean flow assuming the RZIF property.

The nonlinear saturation is due to two mechanisms

2 – Interaction of higher-harmonics (open-cavity flow)

- Weakly nonlinear analysis - *Sipp & Lebedev (2007)*
- Second-order self-consistent model - *Meliga (2017)*

Extended mean-flow analysis

An eigenvalue analysis that accounts for both effects.

Outlines

1 – Extended mean-flow analysis

2 – Results for laminar flows

- Circular-cylinder flow
- Open-cavity flow

3 – Conclusion/Perspectives

Periodic flow and Fourier decomposition

$$\frac{\partial \boldsymbol{q}}{\partial t} = L(\boldsymbol{q}) + N(\boldsymbol{q}, \boldsymbol{q})$$

Linear operator — Nonlinear operator (quadratic)

Periodic solutions

$$\boldsymbol{q}(\boldsymbol{x}, t+T) = \boldsymbol{q}(\boldsymbol{x}, t) \qquad T = 2\,\pi/\omega$$

Fourier decomposition

$$\boldsymbol{q}(\boldsymbol{x}, t) = \boldsymbol{q_0}(\boldsymbol{x}) + \left(A\,\boldsymbol{q_1}(\boldsymbol{x})\,e^{i\,\omega\,t} + c.c.\right) + \left(A^2\boldsymbol{q_2}(\boldsymbol{x})\,e^{i\,2\,\omega\,t} + c.c.\right) + \cdots$$

Mean flow — First harmonic — Second harmonic

Harmonic balanced equations

A set of time-independent coupled nonlinear equations

Mean flow equation

$$-L(\boldsymbol{q_0}) - N(\boldsymbol{q_0}, \boldsymbol{q_0}) = A^2 \left(N(\boldsymbol{q_1}, \boldsymbol{q_1^*}) + N(\boldsymbol{q_1^*}, \boldsymbol{q_1}) \right) + A^4 \left(N(\boldsymbol{q_2}, \boldsymbol{q_2^*}) + N(\boldsymbol{q_2^*}, \boldsymbol{q_2}) \right)$$

First-harmonic equation

$$i\,\omega\,\boldsymbol{q_1} - L(\boldsymbol{q_1}) - N(\boldsymbol{q_0}, \boldsymbol{q_1}) - N(\boldsymbol{q_1}, \boldsymbol{q_0}) = A^2 \left(N(\boldsymbol{q_2}, \boldsymbol{q_1^*}) + N(\boldsymbol{q_1^*}, \boldsymbol{q_2}) \right)$$

Second-harmonic equation

$$2\,i\,\omega\,\boldsymbol{q_2} - L(\boldsymbol{q_2}) - N(\boldsymbol{q_0}, \boldsymbol{q_2}) - N(\boldsymbol{q_2}, \boldsymbol{q_0}) = N(\boldsymbol{q_1}, \boldsymbol{q_1})$$

Harmonic balanced equations

A set of time-independent coupled nonlinear equations

Base flow equation

$$-L(\boldsymbol{q_0}) - N(\boldsymbol{q_0}, \boldsymbol{q_0}) = 0$$

First-harmonic equation

$$i\,\omega\,\boldsymbol{q_1} - L(\boldsymbol{q_1}) - N(\boldsymbol{q_0}, \boldsymbol{q_1}) - N(\boldsymbol{q_1}, \boldsymbol{q_0}) = A^2\,(\,N(\boldsymbol{q_2}, \boldsymbol{q_1^*}) + N(\boldsymbol{q_1^*}, \boldsymbol{q_2})\,)$$

Second-harmonic equation

$$2\,i\,\omega\,\boldsymbol{q_2} - L(\boldsymbol{q_2}) - N(\boldsymbol{q_0}, \boldsymbol{q_2}) - N(\boldsymbol{q_2}, \boldsymbol{q_0}) = N(\boldsymbol{q_1}, \boldsymbol{q_1})$$

Mean-flow analysis

First-harmonic equation

$$i\,\omega\,\boldsymbol{q_1} - L_0(\boldsymbol{q_1}) = A^2\,L_2(\boldsymbol{q_1^*}) + \cdots$$

Mean-flow operator

Second-harmonic operator

$$L_0(\boldsymbol{q_1}) = L(\boldsymbol{q_1}) + N(\boldsymbol{q_0},\boldsymbol{q_1}) + N(\boldsymbol{q_1},\boldsymbol{q_0})$$

$$L_2(\boldsymbol{q_1^*}) = N(\boldsymbol{q_2},\boldsymbol{q_1^*}) + N(\boldsymbol{q_1^*},\boldsymbol{q_2})$$

Neglect the second-harmonic

$$i\,\omega\,\boldsymbol{q_1} - L_0(\boldsymbol{q_1}) = 0$$

Eigenvalue analysis of the mean flow operator

$$(\sigma_m + i\,\omega_m)\,\boldsymbol{\hat{q}_m} = L_0(\boldsymbol{\hat{q}_m})$$

$$\boxed{\sigma_m \sim 0\,;\;\; \omega_m \sim \omega\,;\; \boldsymbol{\hat{q}_m} \sim \boldsymbol{q_1}}$$

Extended mean-flow analysis

First-harmonic equation and its complex conjugate

$$i\,\omega\,\boldsymbol{q_1} - L_0(\boldsymbol{q_1}) - A^2 L_2(\boldsymbol{q_1^*}) = \boldsymbol{0} + \cdots$$

$$i\,\omega\,\boldsymbol{q_1^*} + L_0(\boldsymbol{q_1^*}) + A^2 L_2^*(\boldsymbol{q_1}) = \boldsymbol{0} + \cdots$$

Extended first-harmonic equation

$$i\,\omega \begin{pmatrix} \boldsymbol{q_1} \\ \boldsymbol{q_1^*} \end{pmatrix} - \begin{pmatrix} L_0 & A^2 L_2 \\ -A^2 L_2^* & -L_0 \end{pmatrix} \begin{pmatrix} \boldsymbol{q_1} \\ \boldsymbol{q_1^*} \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{pmatrix} + \cdots$$

Extended mean-flow analysis

$$(\sigma_e + i\,\omega_e) \begin{pmatrix} \boldsymbol{\phi_e} \\ \boldsymbol{\psi_e} \end{pmatrix} = \begin{pmatrix} L_0 & A^2 L_2 \\ -A^2 L_2^* & -L_0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\phi_e} \\ \boldsymbol{\psi_e} \end{pmatrix}$$

$$\sigma_e \sim 0\,; \quad \omega_e \sim \omega\,; \boldsymbol{\phi_e} \sim \boldsymbol{q_1}; \boldsymbol{\psi_e} \sim \boldsymbol{q_1^*}$$

Outlines

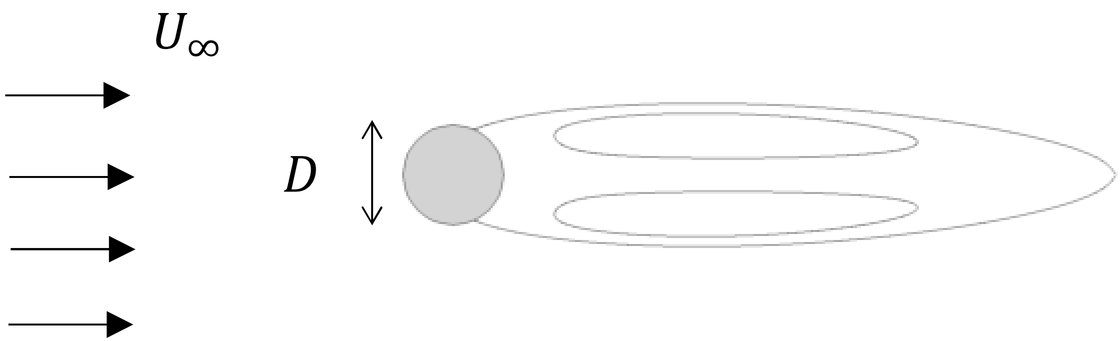
1 – Extended mean flow analysis of periodic flows

2 – Results for laminar flows

- Circular-cylinder flow
- Open-cavity flow (with rounded corners)

3 – Conclusion/perspectives

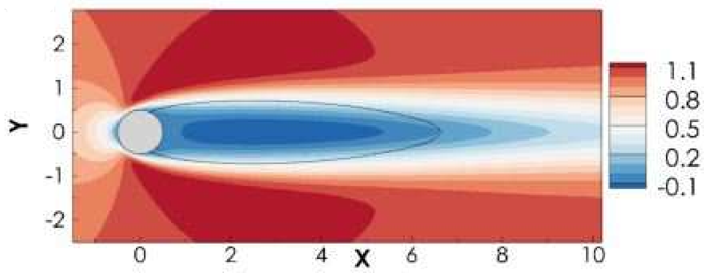
Circular cylinder flow configuration



$$Re = \frac{U_\infty D}{\nu} = 100$$

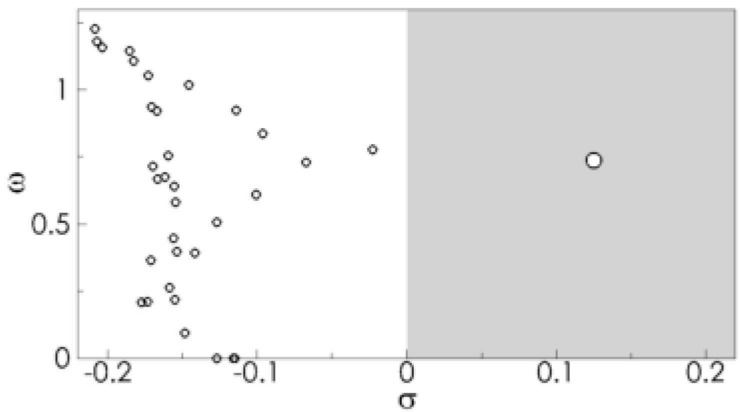
Circular cylinder flow – Base flow analysis

Base flow o



$\sigma_b = 0.125$

$\omega_b = 0.739$

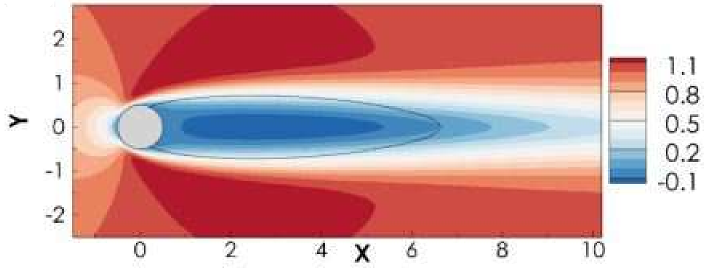


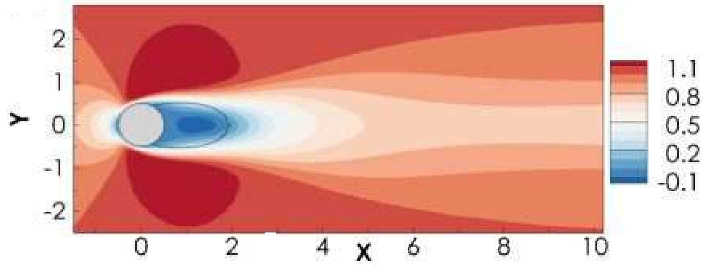
Circular cylinder flow – Mean-flow analysis

Base flow ○

$\omega = 1.044$

Mean flow ●





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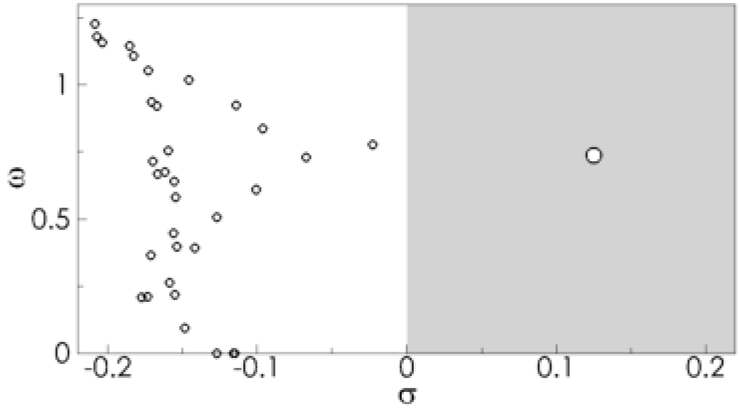
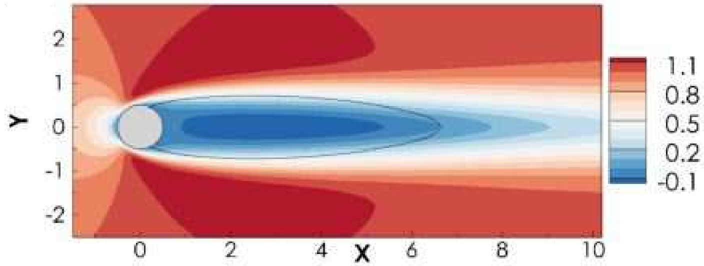
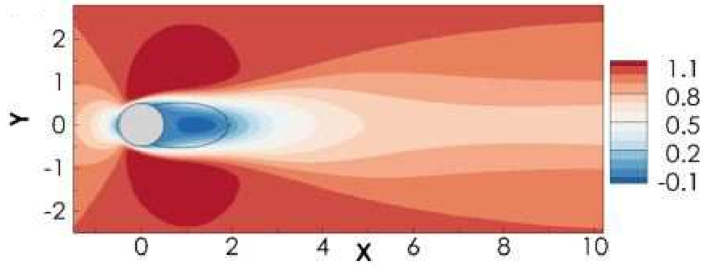


Circular cylinder flow – Mean-flow analysis

Base flow ○ $\omega = 1.044$ Mean flow ●





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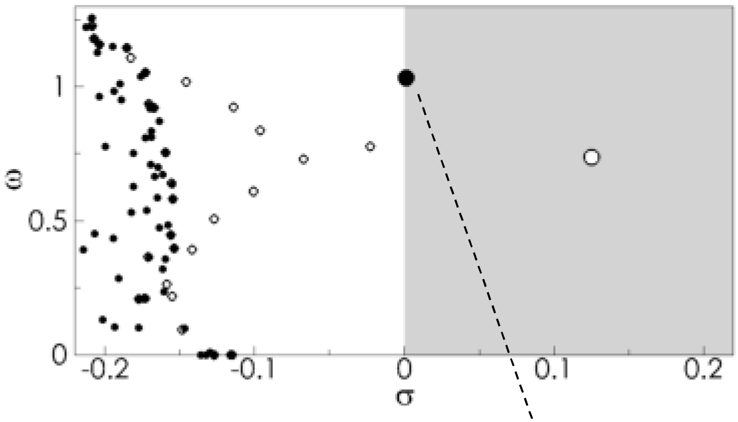
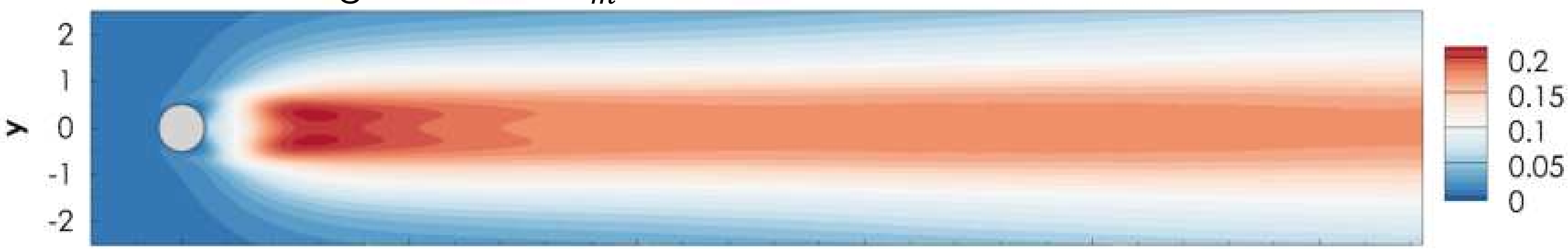


$\boldsymbol{\sigma_m = 0.002}$

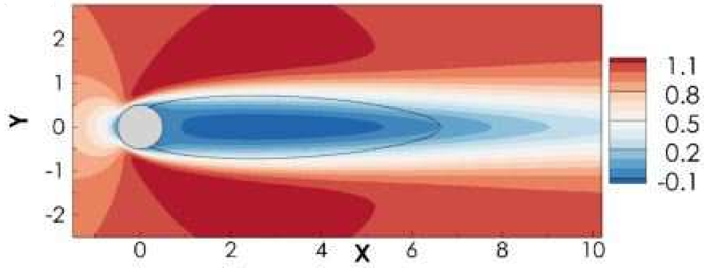
$\boldsymbol{\omega_m = 1.032}$

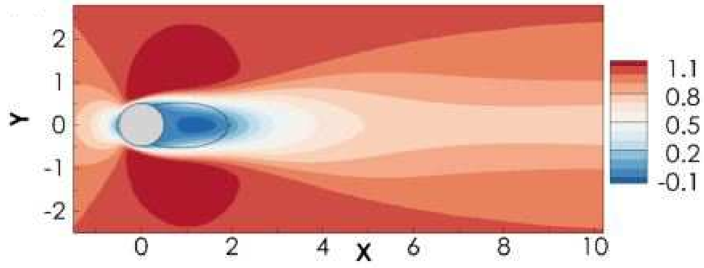
Mean flow eigenmode - ω_m



Circular cylinder flow – Mean-flow analysis

Base flow ○ $\omega = 1.044$ Mean flow ●





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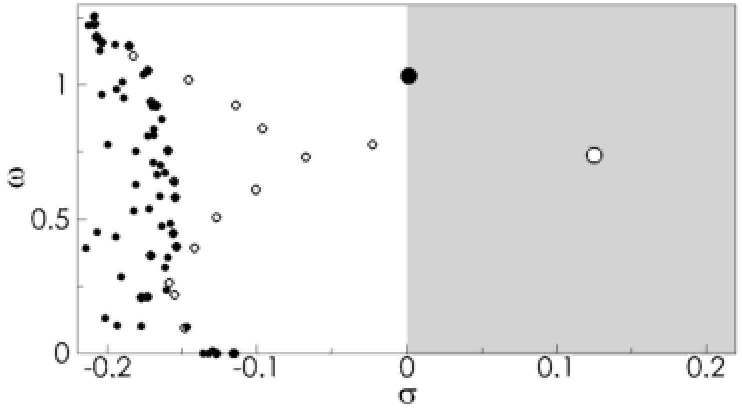

$\boldsymbol{\sigma_m = 0.002}$

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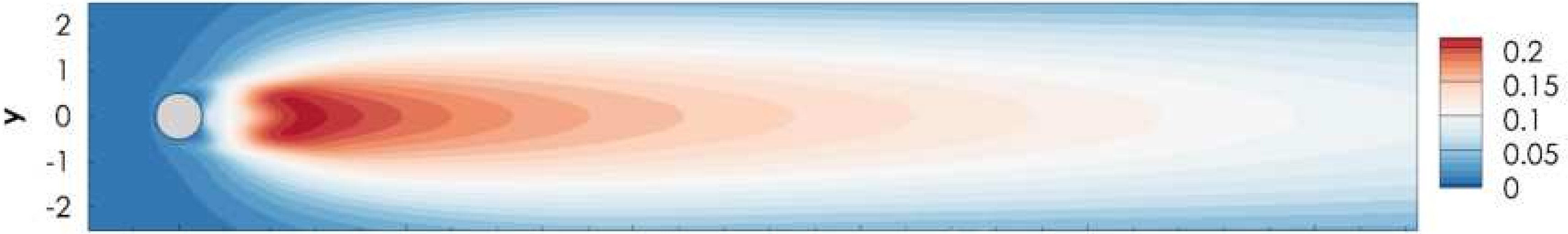
First Fourier mode - ω



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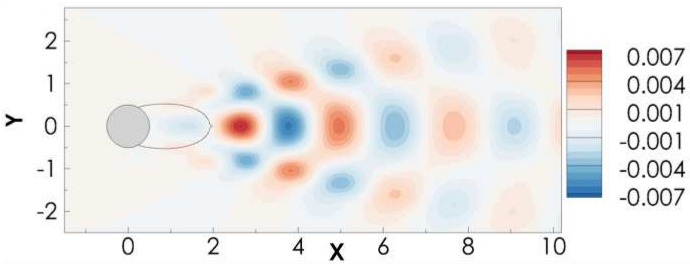
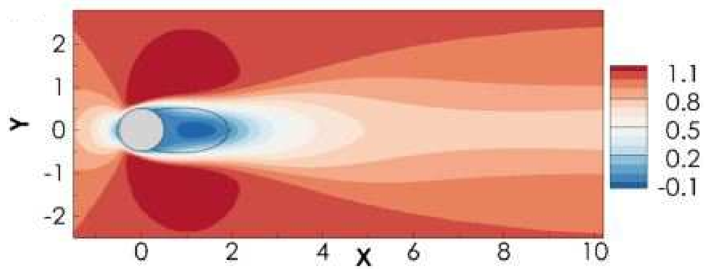
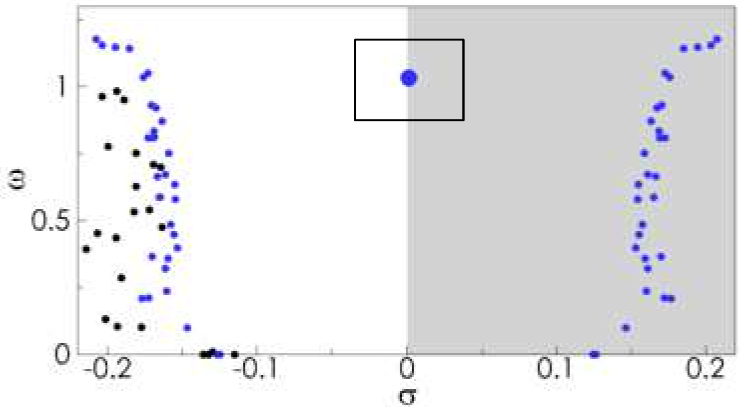
Circular cylinder flow – Extended mean-flow analysis

Second-Harmonic ● $\omega = 1.044$ Mean flow ●







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Circular cylinder flow – Extended mean-flow analysis

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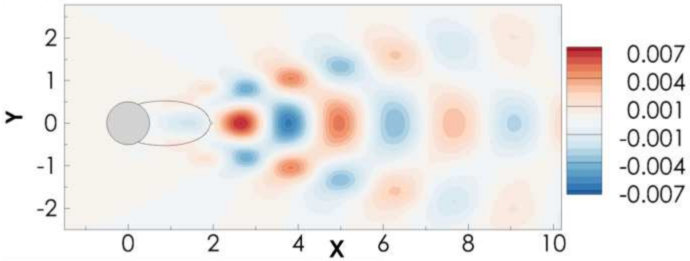
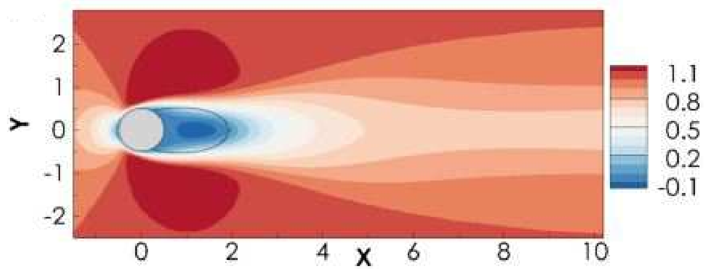
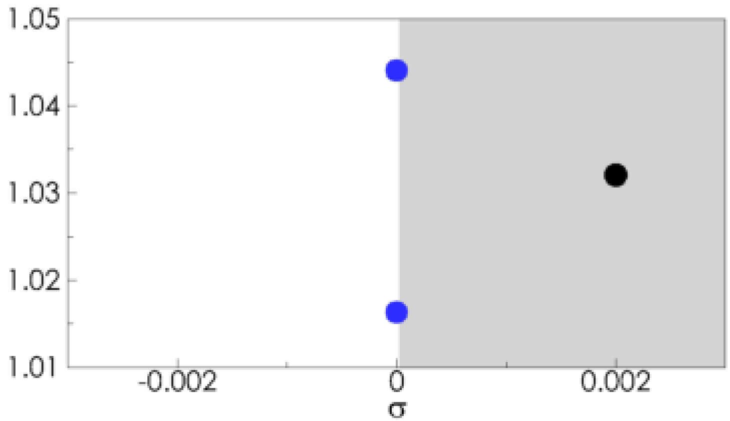




$\boldsymbol{\sigma_e = 10^{-12}}$

$\boldsymbol{\omega_e^1 = 1.044}$

$\omega_e^2 = 1.016$



$\boldsymbol{\sigma_m = 0.002}$

$\boldsymbol{\omega_m = 1.032}$

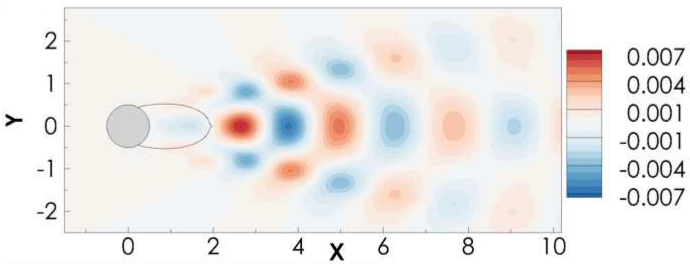
Two eigenvalues with zero growth-rate

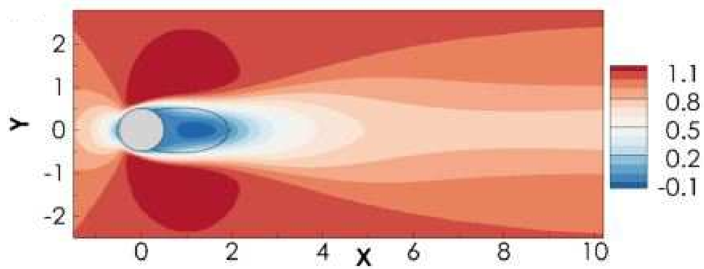
The frequency of one eigenvalue is the nonlinear frequency

$$\boldsymbol{\omega_e^1 = \omega}$$

Circular cylinder flow – Extended mean-flow analysis

Second-Harmonic ● $\omega = 1.044$ Mean flow ●





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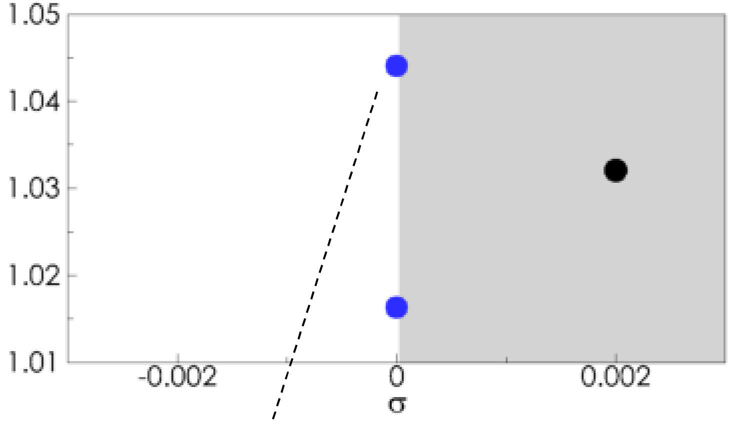

$\boldsymbol{\sigma_m = 0.002}$

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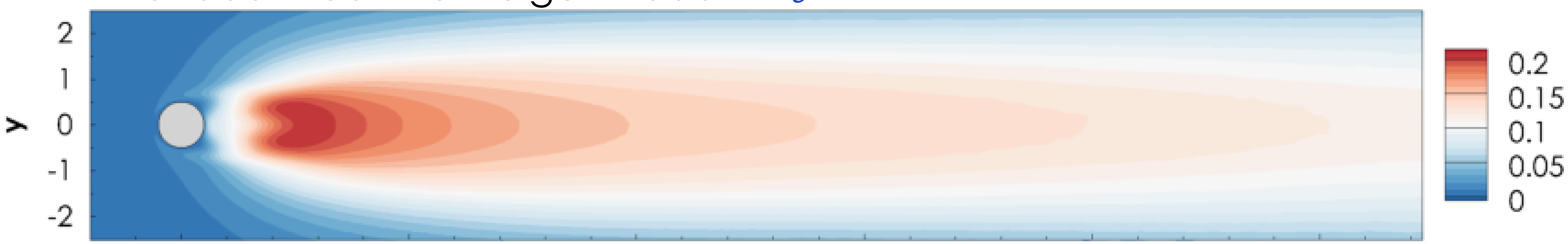
Extended mean-flow eigenmode - ω_e^1



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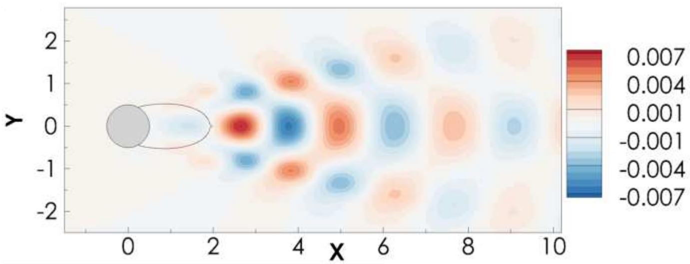
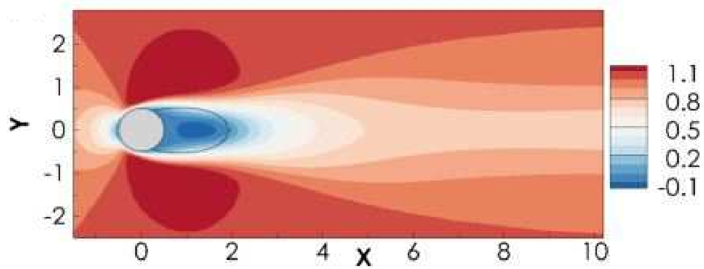
Circular cylinder flow – Extended mean-flow analysis

Second-Harmonic ● $\omega = 1.044$ Mean flow ●

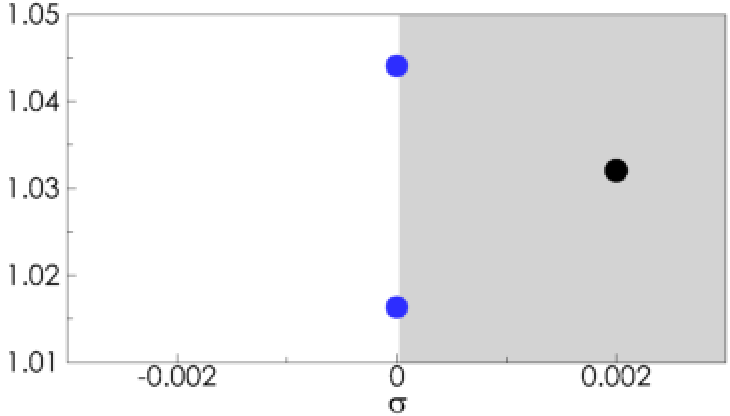




$\boldsymbol{\sigma_e = 10^{-12}}$

$\boldsymbol{\omega_e^1 = 1.044}$

$\omega_e^2 = 1.016$



$\boldsymbol{\sigma_m = 0.002}$

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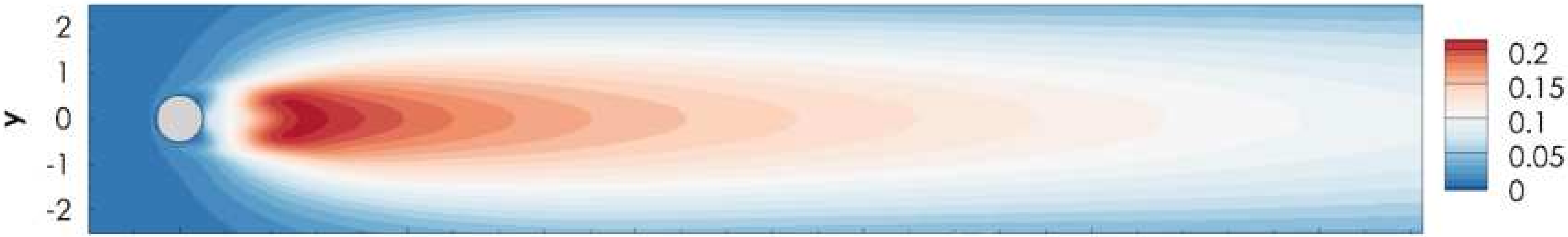
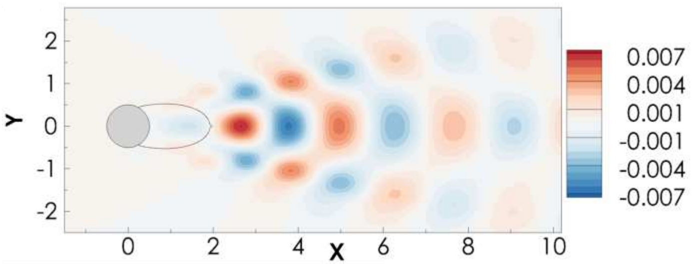
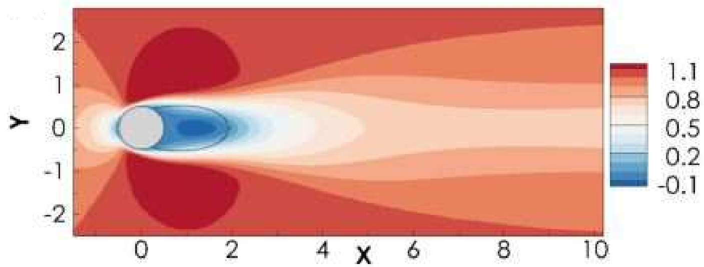
First Fourier mode - ω



Circular cylinder flow – Extended mean-flow analysis

Second-Harmonic ● $\omega = 1.044$ Mean flow ●





$\boldsymbol{\sigma_e = 10^{-12}}$

$\boldsymbol{\omega_e^1 = 1.044}$

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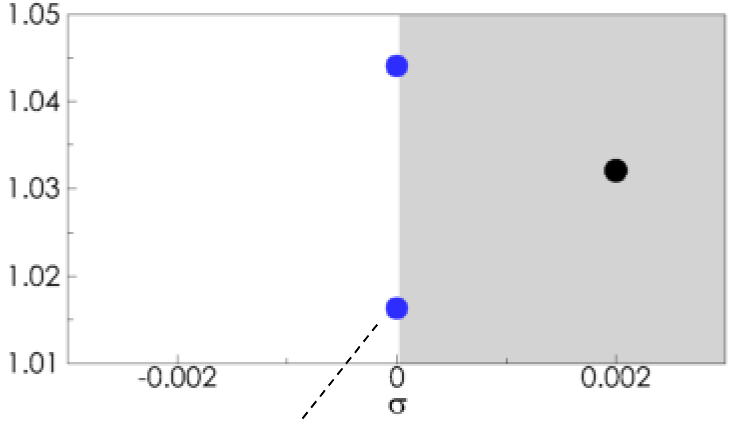
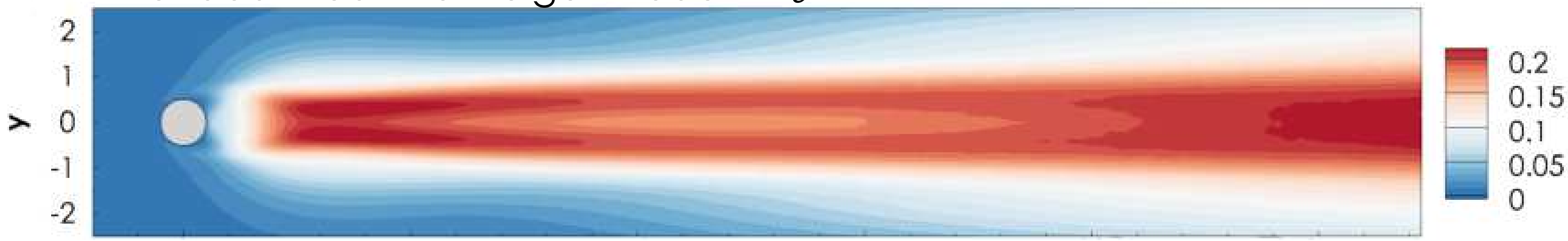


$\boldsymbol{\sigma_m = 0.002}$

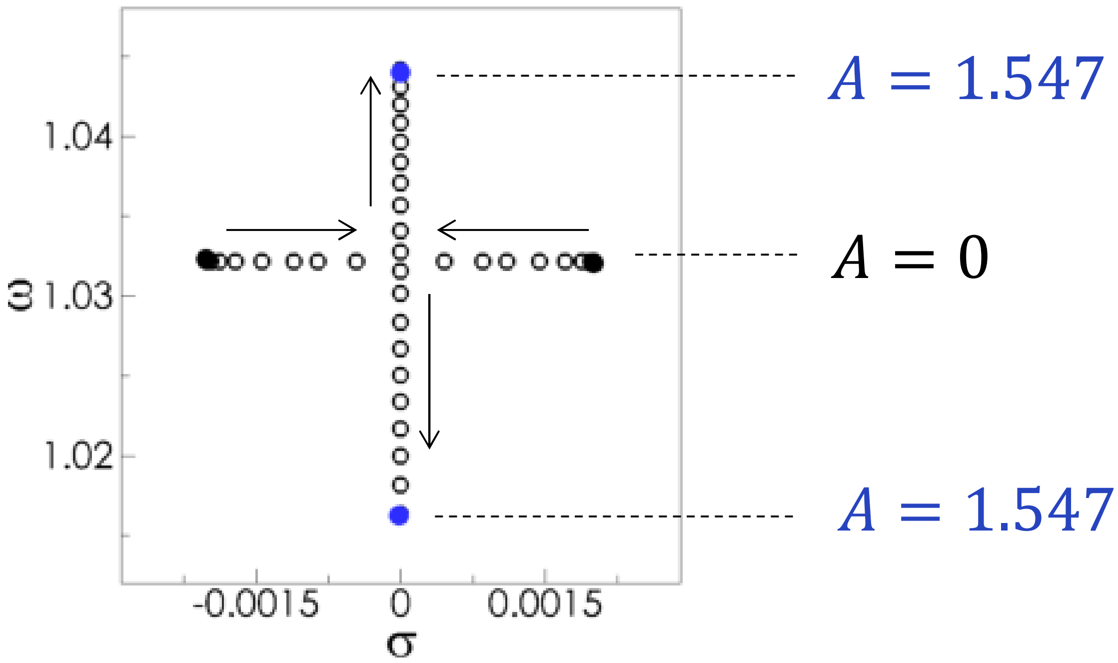
$\boldsymbol{\omega_m = 1.032}$

Extended mean-flow eigenmode - ω_e^2



Circular cylinder flow – Extended mean-flow analysis

$$L_e = \begin{pmatrix} L_0 & \boldsymbol{A}^2 L_2 \\ -\boldsymbol{A}^2 L_2^* & -L_0 \end{pmatrix}$$



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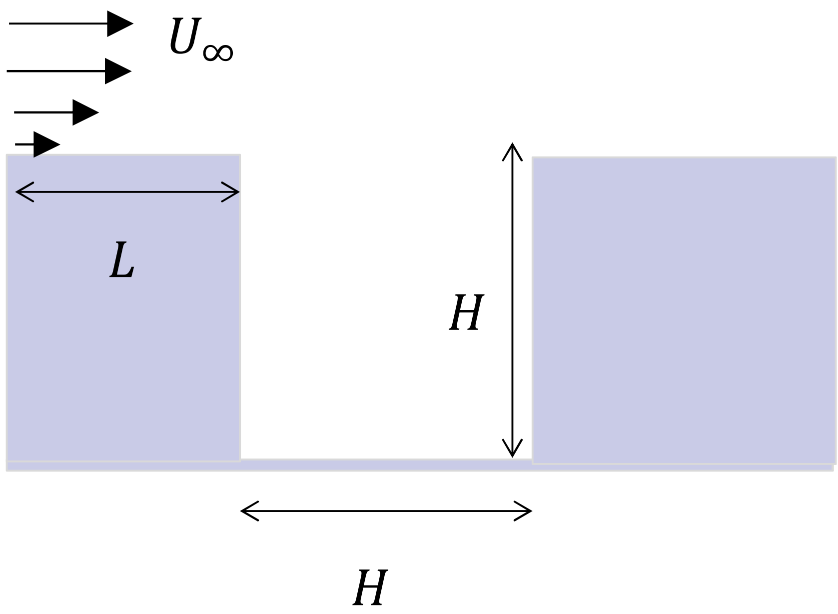
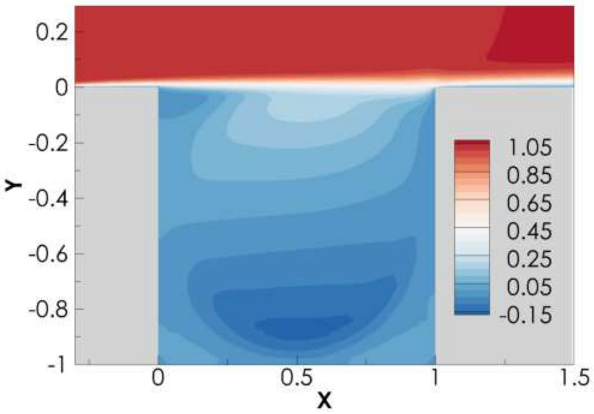
Open cavity flow configuration



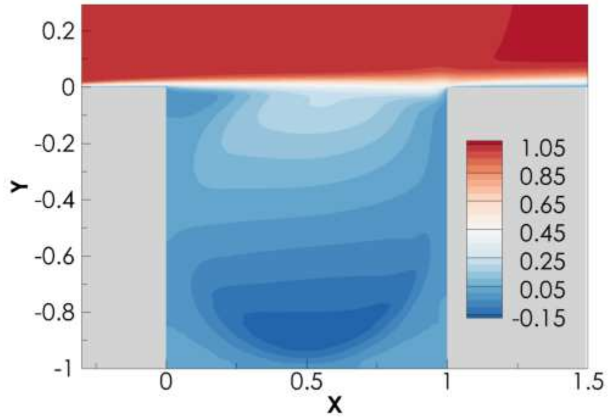
$$4400 < Re = \frac{U_\infty H}{\nu} < 4600$$

Open-cavity flow – Mean-flow analysis

Base flow ○

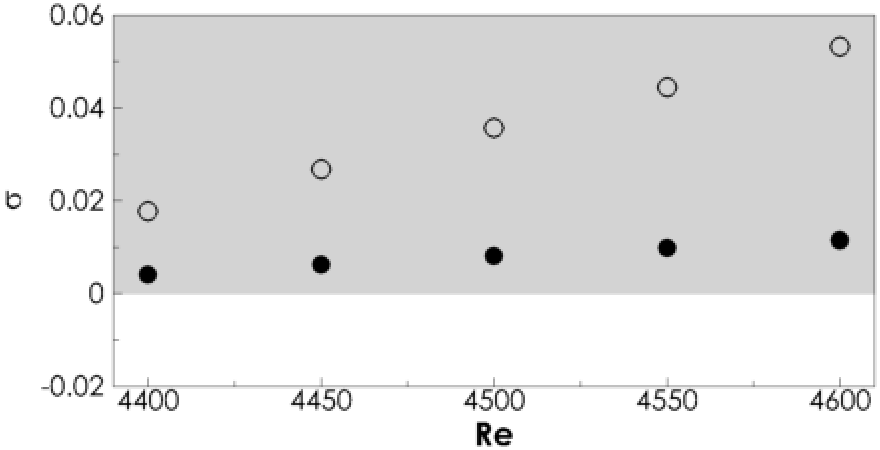


Mean flow ●



Weaker mean-flow distortion

Growth rate



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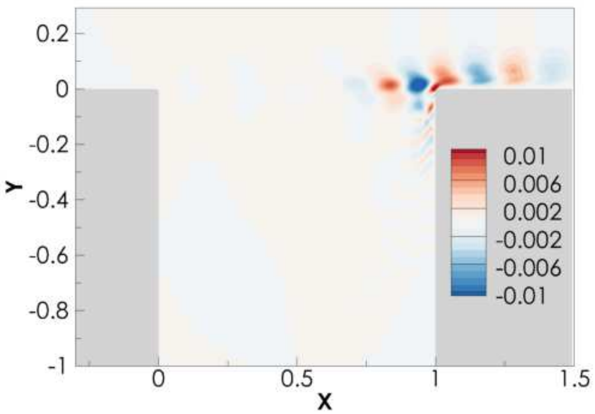
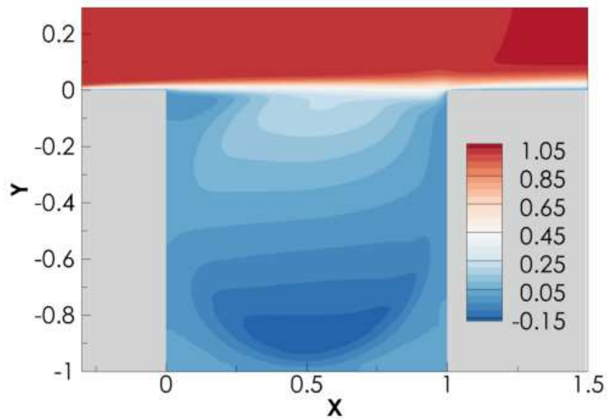
Open-cavity flow – Extended mean-flow analysis

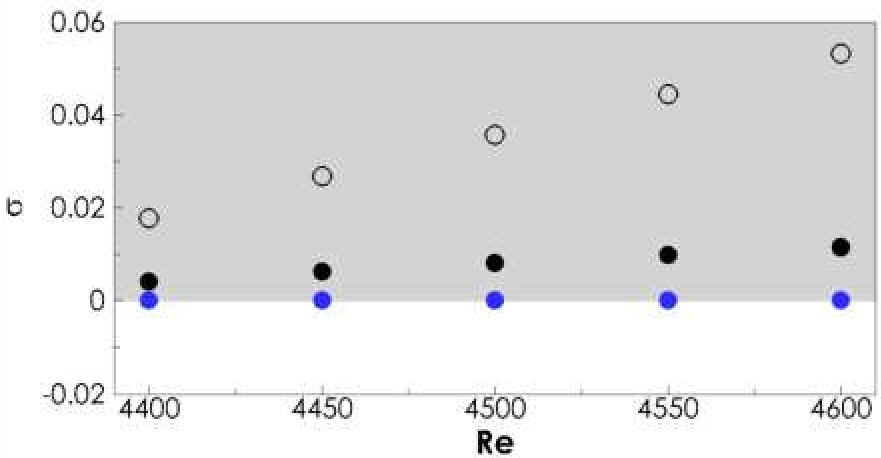
Second-harmonic ●



Mean flow ●



Extended mean-flow ●



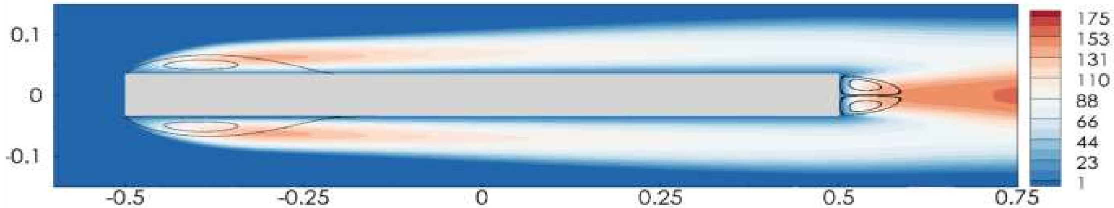
Conclusion and perspectives

Conclusion

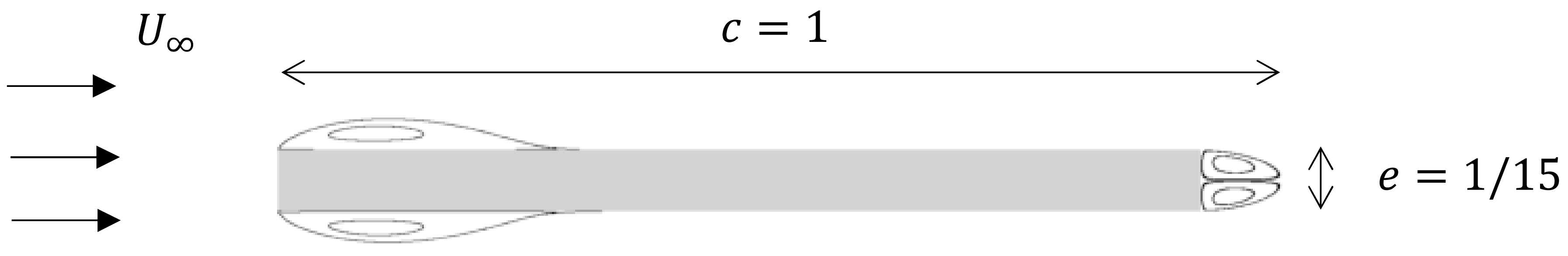
- A new eigenvalue analysis of periodic flow accounting for the two mechanisms of nonlinear saturation
- This analysis gives a **R**eal **Z**ero **I**maginay **F**requency Mode

Perspectives

- Develop a model where the second-harmonic is reconstructed
- Extension to fluid/structure problems and turbulent flows modelled with a RANS approach



Flow configuration and turbulent flow model



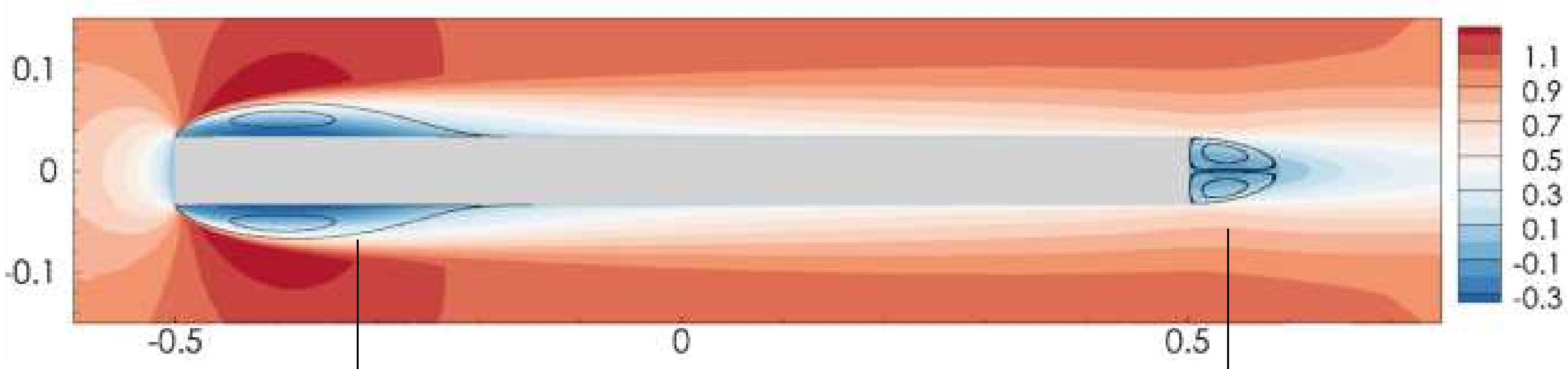
$$Re = \frac{U_\infty c}{\nu} = 1.5\ 10^5$$

- Turbulent flow modelled with **R**eynolds **A**veraged **N**avier **S**tokes equations
- Spalart-Almarras model for the turbulent eddy viscosity ν_t
- Frozen-viscosity approach:

  - Steady equations solved with the Spalart-Almarras model
  - Unsteady equations solved with frozen turbulent eddy viscosity ν_t

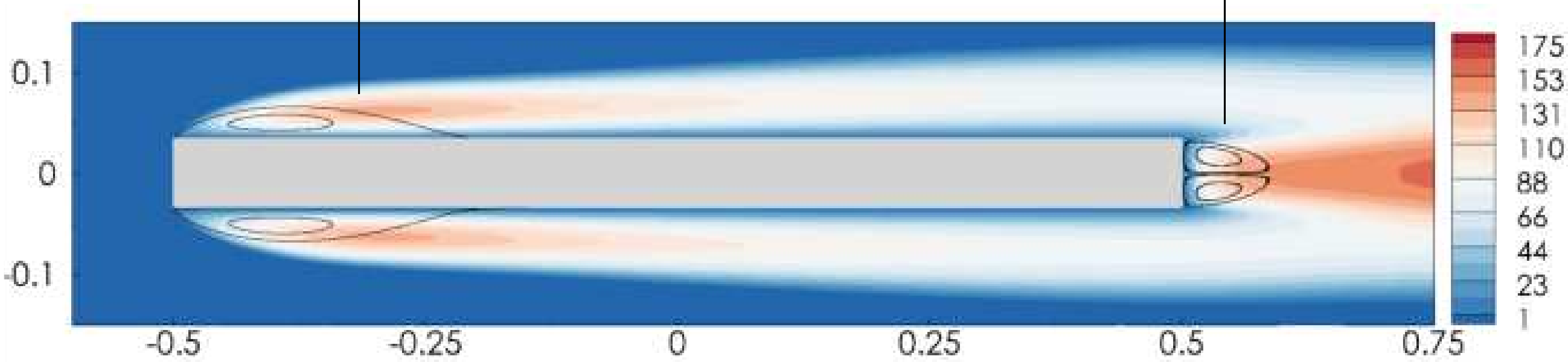
Base flow - Steady solution of RANS equations

Streamwise velocity u



Leading-edge recirculation regions

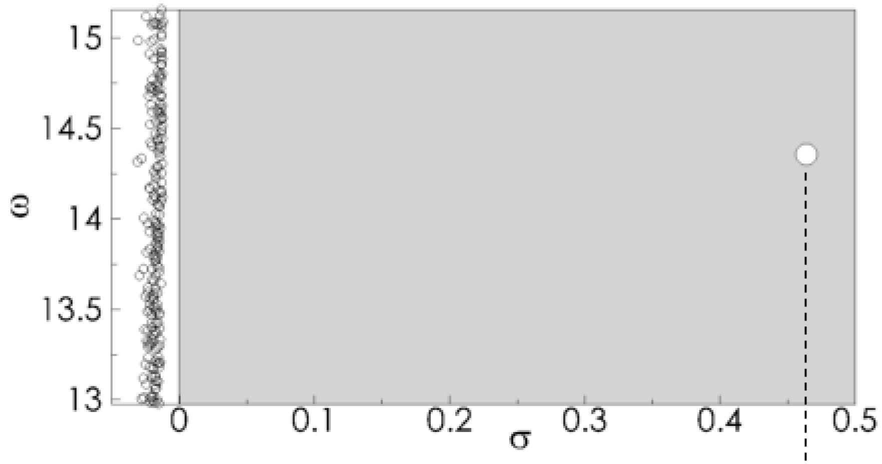
Trailing-edge recirculation regions



Turbulent eddy viscosity ν_t/ν

Stability analysis with frozen eddy-viscosity

Eigenvalue spectrum



Unstable eigenmode

Streamwise velocity u

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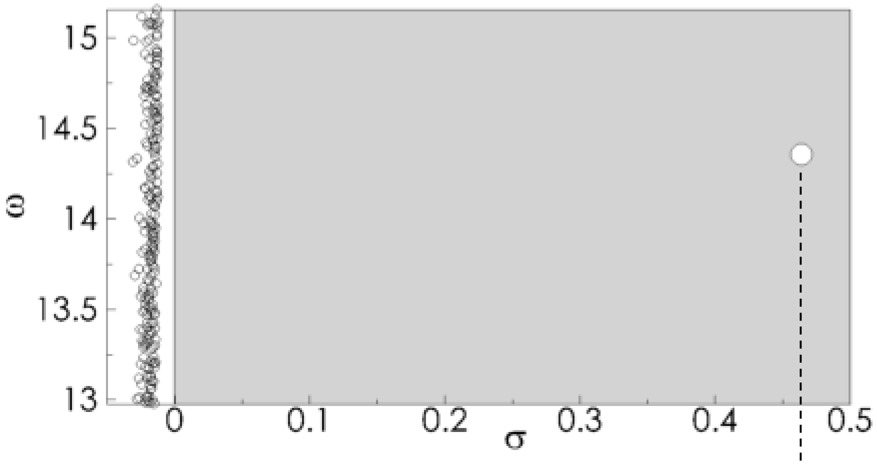
Stability analysis with frozen eddy-viscosity

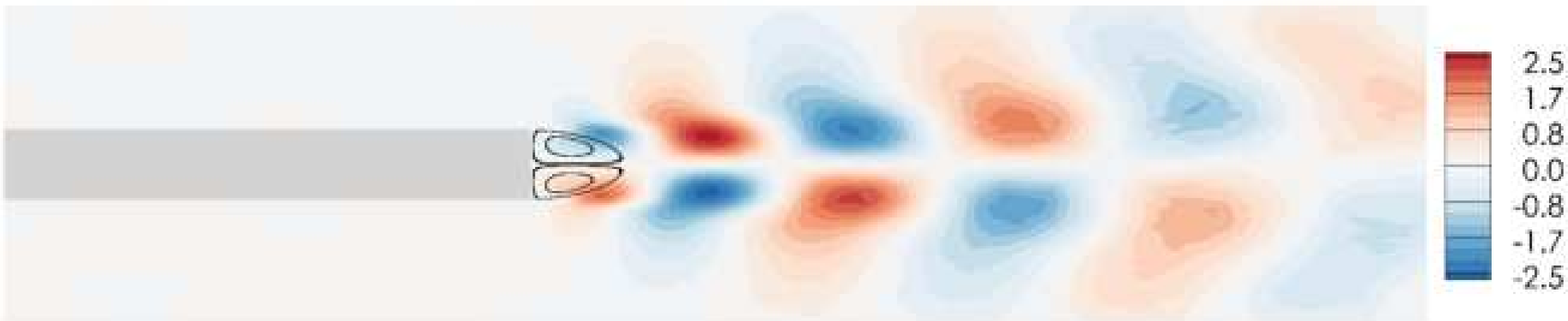
Eigenvalue spectrum



$\sigma_b = 0.463$

$\omega_b = 14.351$

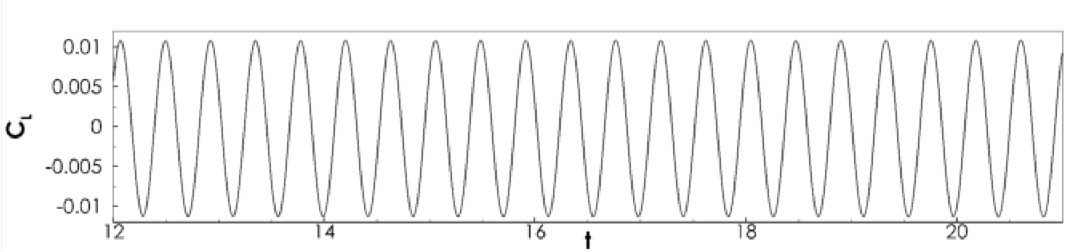
Unstable eigenmode – Zoom on trailing edge



Streamwise velocity u

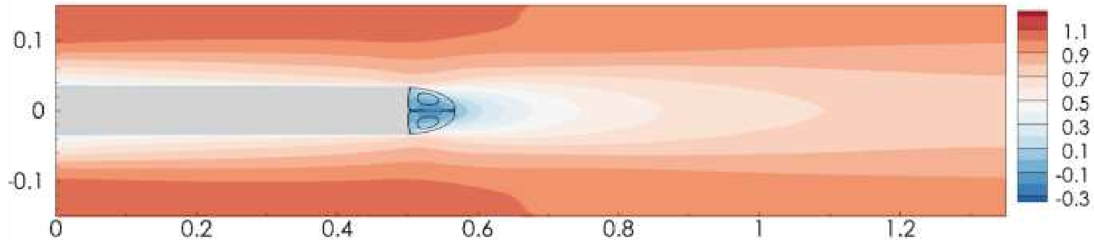
Unsteady solution with frozen-eddy viscosity

Instantaneous lift



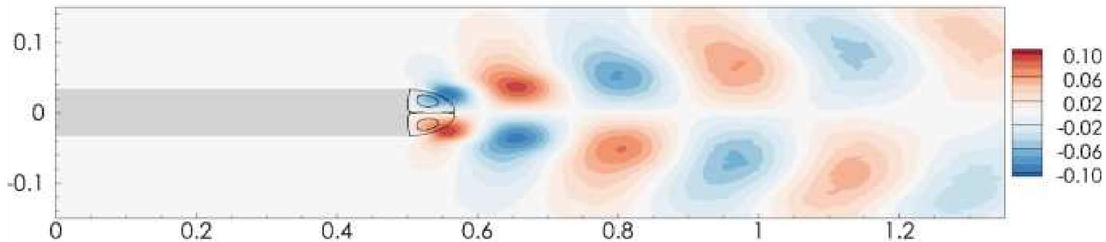
$T = 0.427$

Mean flow



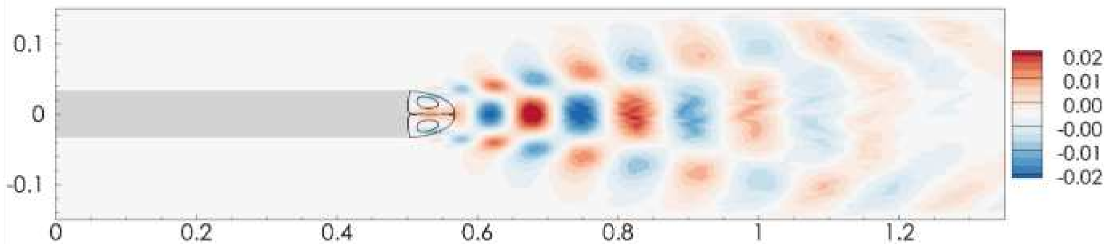
0

First harmonic



$\omega = 14.714$

Second harmonic



$2\ \omega$

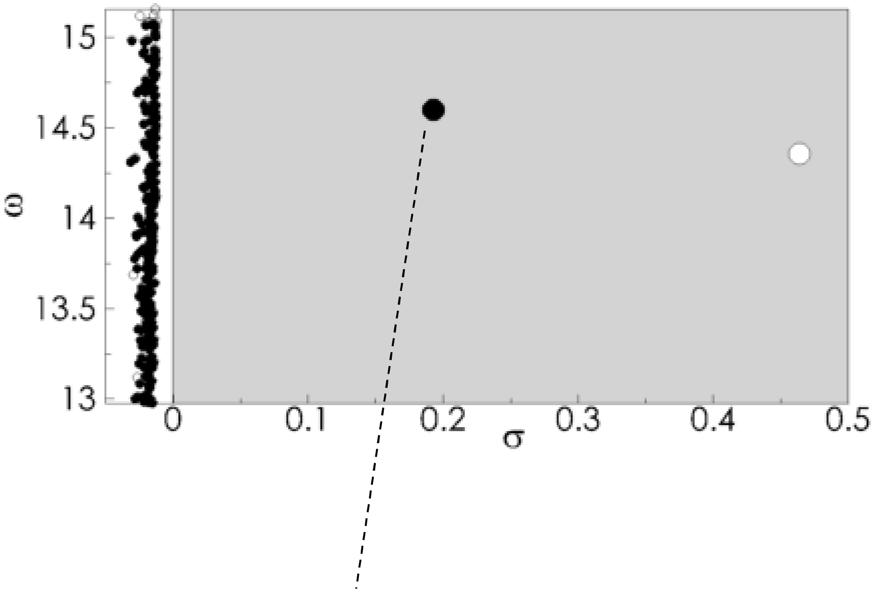
Mean-flow analysis with frozen eddy-viscosity

Mean-flow eigenvalue spectrum

$\boldsymbol{\sigma_m = 0.192}$

$\boldsymbol{\omega_m = 14.604}$

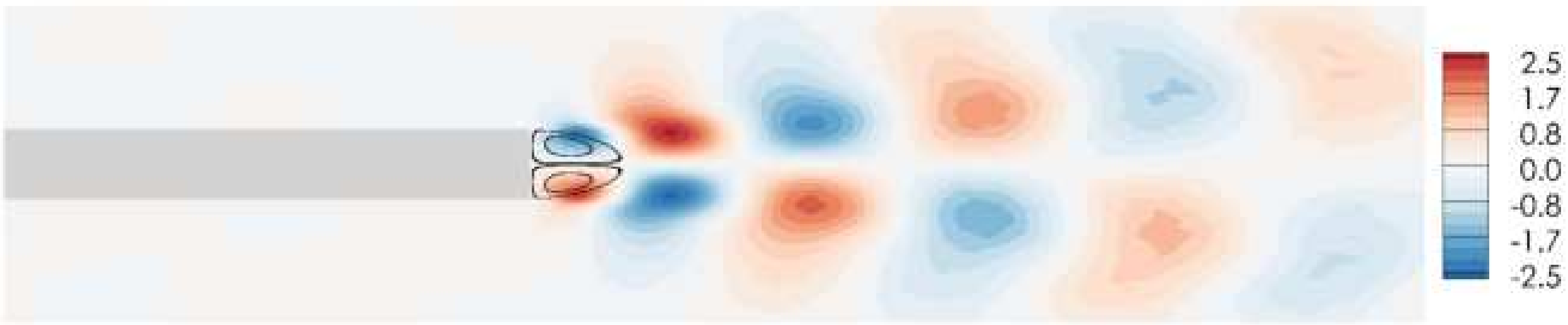
$(\omega = 14.714)$



$\sigma_b = 0.463$

$\omega_b = 14.351$

Mean-flow eigenmode



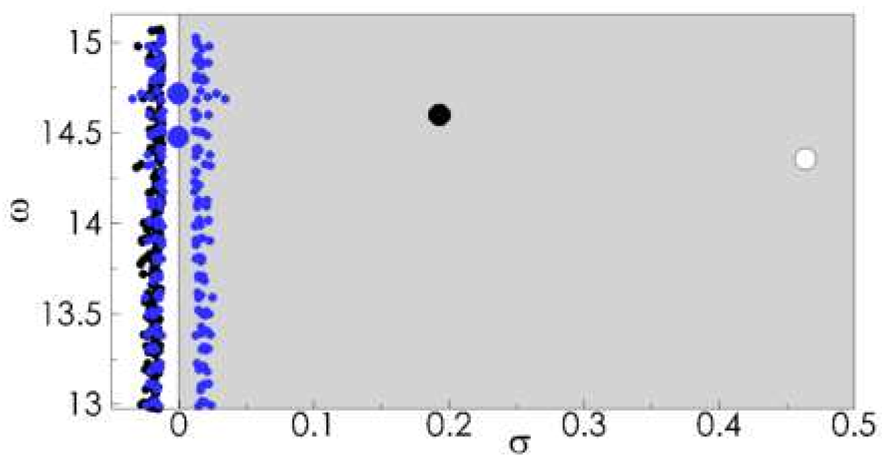
Extended mean-flow analysis with frozen eddy-viscosity

$\sigma_m = 0.192$

$\omega_m = 14.604$

$(\omega = 14.714)$

Extended mean-flow eigenvalue spectrum



- **Two eigenvalues characterized by zero growth rate**

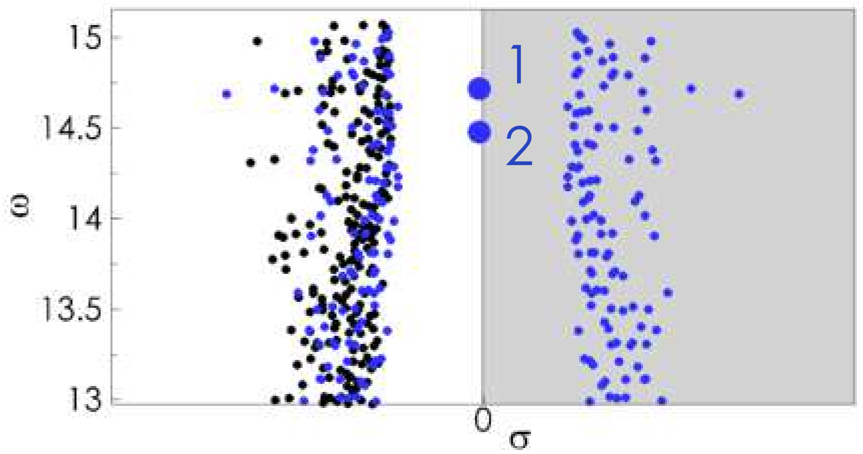
Extended mean-flow analysis with frozen eddy-viscosity

Extended mean-flow eigenvalue spectrum

$\sigma_m = 0.192$

$\omega_m = 14.604$

$(\boldsymbol{\omega} = \mathbf{14.714})$



$\boldsymbol{\sigma_e = 10^{-12}}$

$\boldsymbol{\omega_e^1 = 14.724}$

$\omega_e^2 = 14.479$

- Two eigenvalues characterized by zero growth rate
- **The frequency of one eigenmode is in excellent agreement with the non-linear frequency ω**

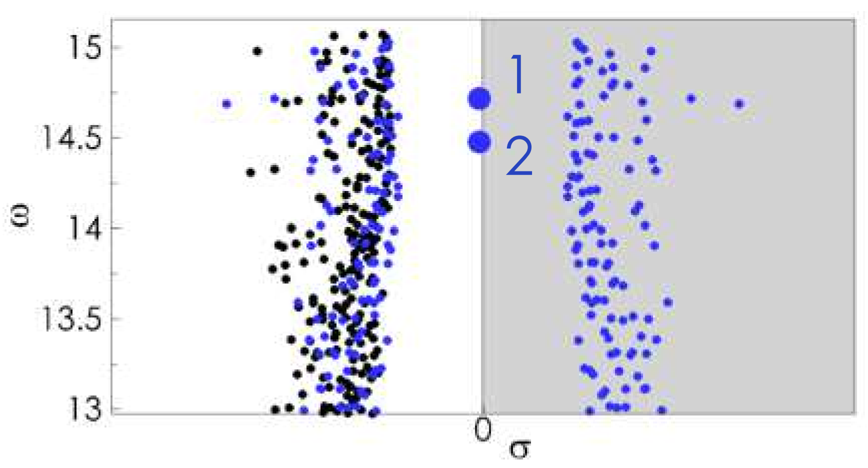
Extended mean-flow analysis with frozen eddy-viscosity

Extended mean-flow eigenvalue spectrum

$\sigma_m = 0.192$

$\omega_m = 14.604$

$(\boldsymbol{\omega} = \mathbf{14.714})$

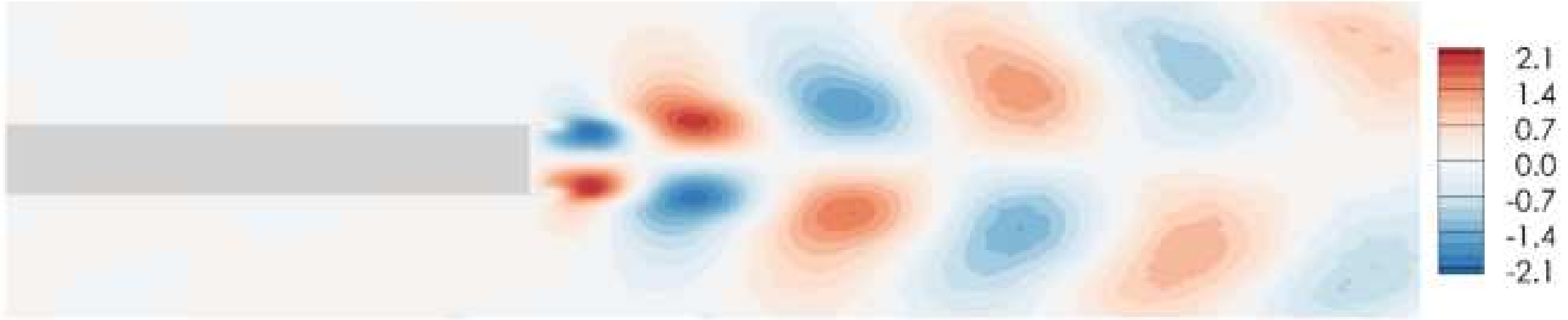


$\boldsymbol{\sigma_e = 10^{-12}}$

$\boldsymbol{\omega_e^1 = 14.724}$

$\omega_e^2 = 14.479$

Extended mean-flow eigenmode



Extended mean-flow analysis with frozen eddy-viscosity

Extended mean-flow eigenvalue spectrum

$\sigma_m = 0.192$

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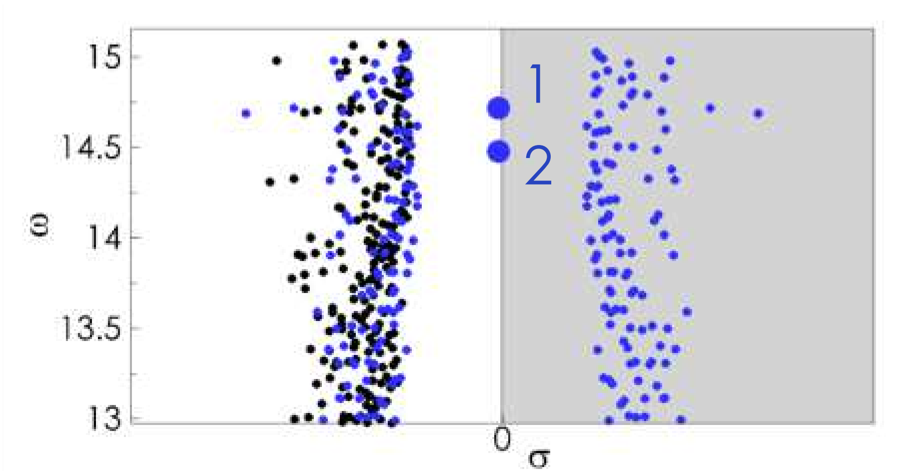
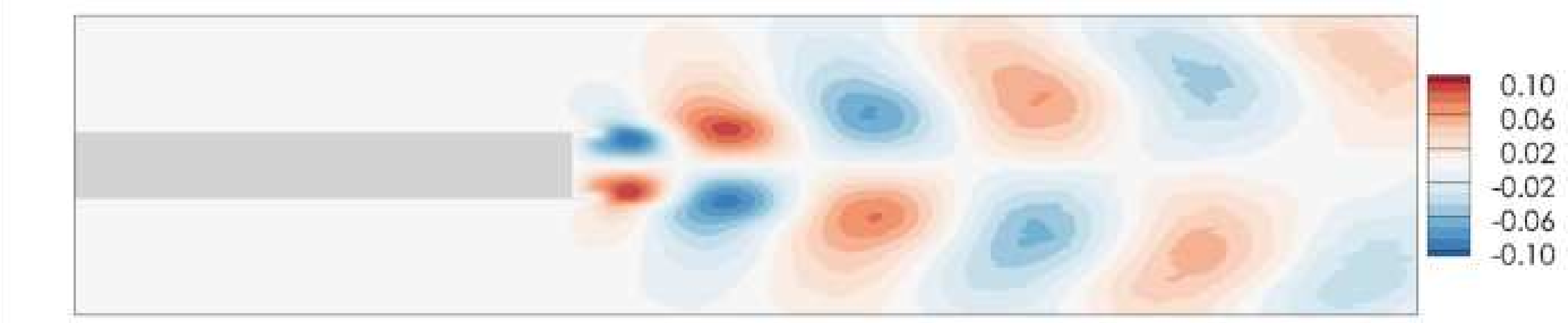


$\boldsymbol{\sigma_e} = \mathbf{10^{-12}}$

$\boldsymbol{\omega_e^1} = \mathbf{14.724}$

$\omega_e^2 = 14.479$

First harmonic



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