Identifying the frequency selection of fluid/structure instabilities when the interaction is large

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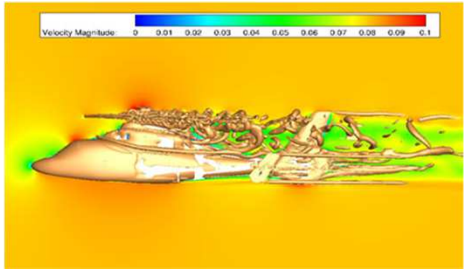
Context

Flow-induced structural vibrations

Civil engineering

Aeronautics



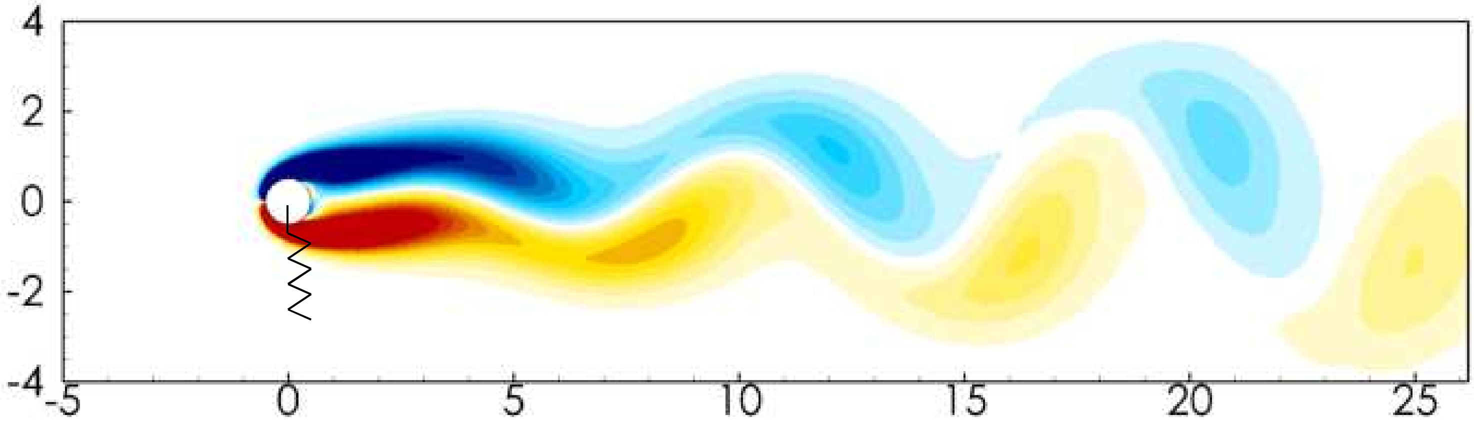
Offshore-marine industry

Predict the onset of vibrations (based on stability analysis)
and control them

The origin of fluid/structure instabilities ?

A model problem spring-mounted cylinder flow

One spring in the cross-stream direction



Structural frequency

$\kappa_s = \sqrt{^{\kappa}/_{m}}$

Structural damping

γ_s

Density ratio

$\rho = {^{\rho_s}}/{_{\rho_f}}$

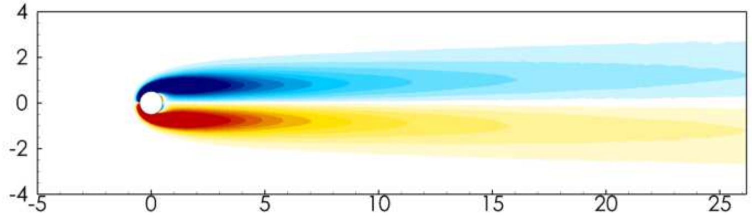
Reynolds number

$Re = \frac{U_\infty D}{\nu} = 40$

Self-sustained oscillations

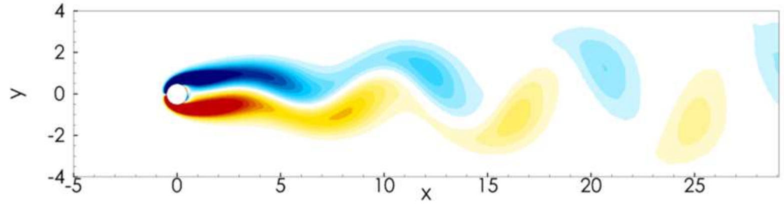
$$Re = 40\,, \rho = 10\,, \gamma_s = 0$$

$k_s = 0.6$



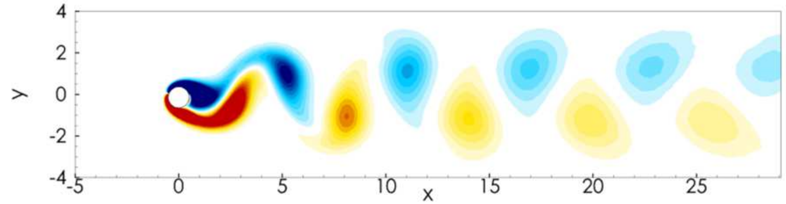
No oscillation

$k_s = 0.7$



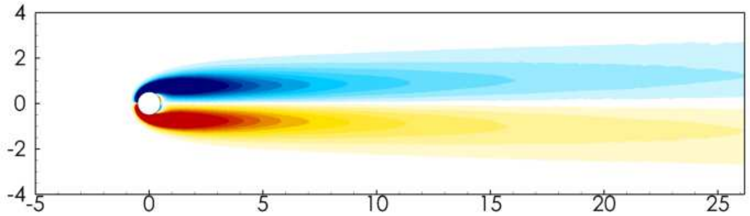
Weak oscillation

$k_s = 0.9$



Strong oscillation

$k_s = 1.1$



No oscillation

Outlines

1 – Stability analysis of the fluid/structure problem

a – Operator definition and formalism

b – Results for weak and strong interaction

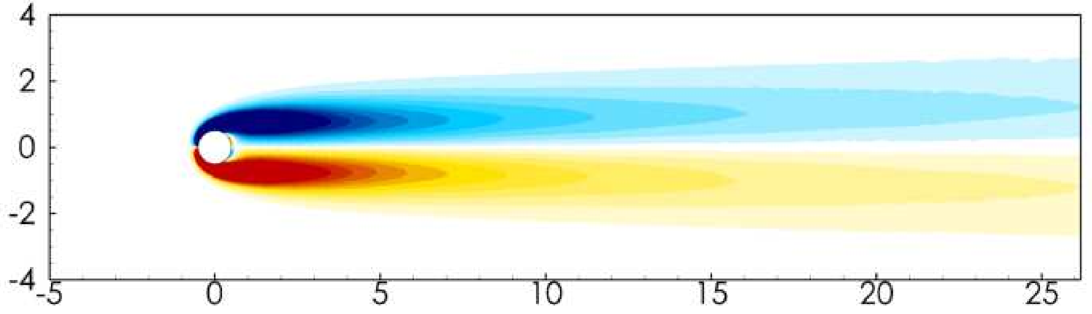
2 – Identification the driving dynamics of coupled modes

a – Operator decomposition approach

b – Results for strong interaction

Stability analysis of the coupled fluid/solid problem

Stability of the steady solution (fixed cylinder)



$$q'(x,t) = (q_f, q_s)(x) e^{(\lambda + i\,\omega)t} + c.c.$$

Fluid/solid components

Growth rate/frequency

Linearized fluid equations

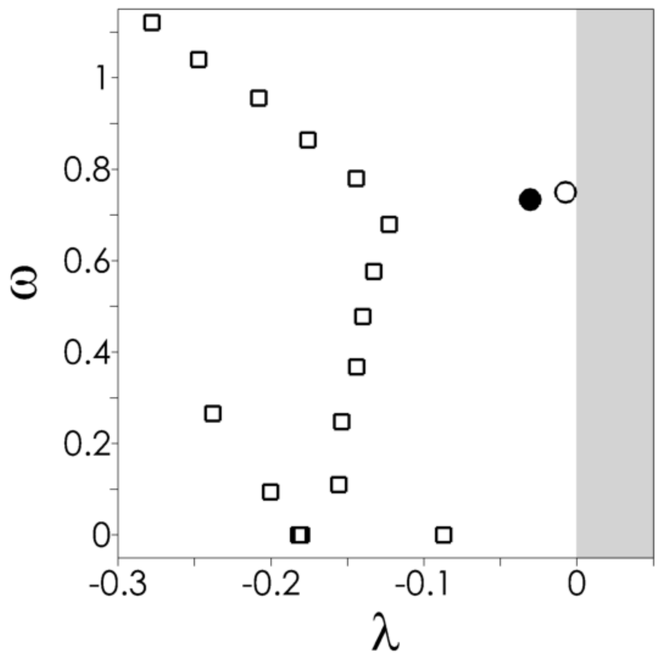
$$\begin{pmatrix} L_f(Re) & C_f \\ \rho^{-1} C_s & L_s(k_s, \gamma_s) \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega) \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Damped harmonic oscillator

Results – Eigenvalue spectrum

$\rho = 10^6$ $k_s = 0.75$

Eigenvalue spectrum

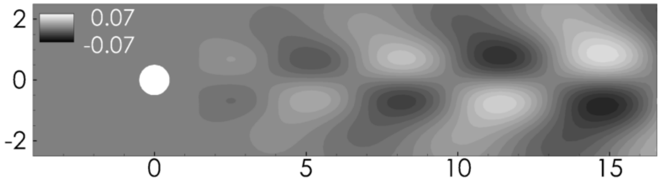


Are the coupled modes driven by the fluid or the solid dynamics ?

Results – Components of eigenvector

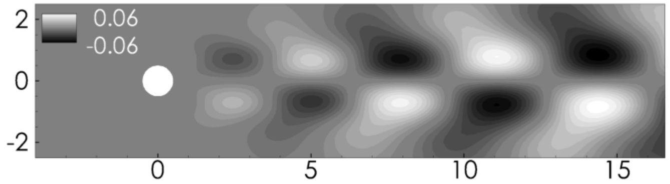
Infinite mass ratio - weak interaction

●



Fluid component

○



$y_s = v_s = 0$

Solid component

$y_s = v_s \neq 0$

Fluid (wake) mode

Solid mode

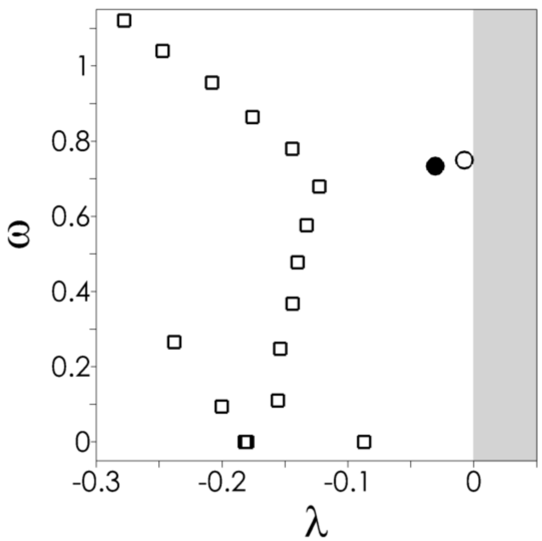
For small mass ratio - strong interaction ?

$$y_s = v_s \neq 0$$

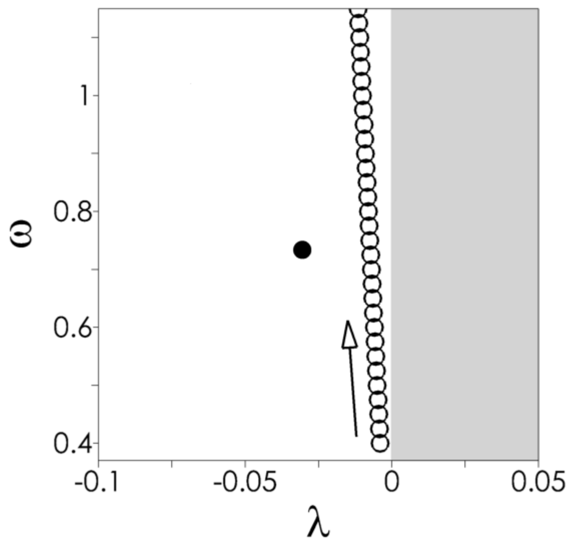
Results – Variation of stiffness

Infinite mass ratio – weak interaction

$k_s = 0.75$



$0.4 < k_s < 1.2$



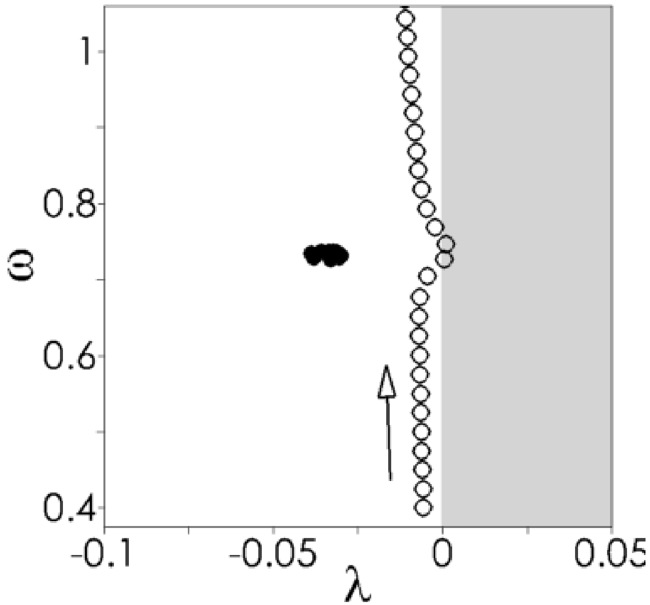
Solid mode : $\omega \sim k_s$ (= structural frequency)

Wake modes : $\omega \sim \omega_v$ (= vortex-shedding frequency)

Results – Variation of stiffness

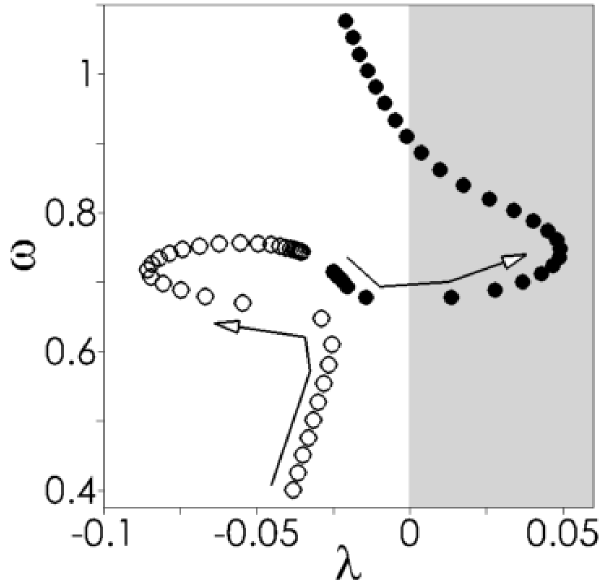
Finite mass ratio – strong interaction

$\rho = 200$



Destabilization of **solid** branch

$\rho = 10$



Destabilization of **fluid** branch?

The two branches exchange their « nature » for small mass ratio

Zhang et al (JFM 2015), Meliga & Chomaz (JFM 2011)

Outlines

1 – Stability analysis of the fluid/structure problem

a – Operator definition and formalism

b – Results (various mass ratio / structural frequency)

2 – Identification of the driving dynamics

a – Operator decomposition approach

b – Results

The eigenvalue problem

Eigenvalue problem - Coupled operator

$$\begin{pmatrix} L_f(Re) & C_f \\ \rho^{-1} C_s & L_s(k_s, \gamma_s) \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega) \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Infinite mass ratio

$$\begin{pmatrix} L_f(Re) & C_f \\ 0 & L_s(k_s, \gamma_s) \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega) \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Fluid mode = eigenvalue/vector of L_f

Solid mode = eigenvalue/vector of L_s

From operator to eigenvalue decomposition

Operator decomposition

$$L\,q = \;(L_a + L_b)q = \sigma\,q$$

In general, q is not an eigenmode of L_a or L_b , so

$$L_a\,q = \sigma_a\,q + r_a \qquad\qquad L_b\,q = \sigma_b\,q + r_b$$

with residuals $r_a \neq 0, r_b \neq 0$ but $r_a = -r_b = r$

Eigenvalue decomposition

$$\sigma_a + \sigma_b = \sigma$$

How to compute the eigenvalue contributions σ_a/σ_b ?

Computing eigenvalue contributions

Expansion of the residual on the set of **other eigenmodes** q_k

$$r = \sum_k r_k q_k \longrightarrow L_a\, q = \sigma_a\, q + \sum_k r_k q_k$$

Orthogonal projection on the mode q
using the adjoint mode q^+

$$q^{+H}(L_a\, q) = \sigma_a \underbrace{(q^{+H} q)}_{=1} + \sum_k r_k \underbrace{(q^{+H} q_k)}_{=0}$$

Normalisation $= 1$ ... $= 0$ Bi-orthogonality

Adjoint mode-based decomposition

$$\sigma = \sigma_a + \sigma_b$$

$$\sigma_a = q^{+H}(L_a\, q) \qquad \sigma_b = q^{+H}(L_b\, q)$$

Why this particular eigenvalue decomposition?

For an identical decomposition of the operator,
other eigenvalue decompositions are possible

Non-orthogonal projection on the mode q

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_a + \hat{\sigma}_b$$

$$\hat{\sigma}_a = q^H (L_a\, q) \qquad \hat{\sigma}_b = q^H (L_b\, q)$$

But it includes contributions from other eigenmodes

$$L_a\, q = \sigma_a\, q + \sum_k r_k q_k \qquad L_b\, q = \sigma_b\, q - \sum_k r_k q_k$$

$$\hat{\sigma}_{a/b} = \sigma_{a/b} \pm \sum_k r_k \; (q^H q_k) \quad \neq 0$$

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Application to the spring-mounted cylinder flow

$$\begin{pmatrix} L_f & C_f \\ \rho^{-1} C_s & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} \left(L_f\, q_f + C_f\, q_s\right)$$

Adjoint fluid component

Solid contribution

$$\sigma_s = q_s^{+H} \left(L_s\, q_s + \rho^{-1} C_s\, q_f\right)$$

Adjoint solid component

Application to the spring-mounted cylinder flow

$$\begin{pmatrix} L_f & C_f \\ \rho^{-1} C_s & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

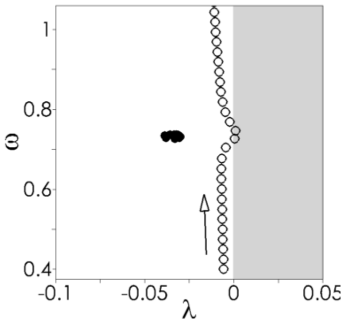
$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution	**Solid contribution**
$\sigma_f = \sigma\,(q_f^{+H}\, q_f)$	$\sigma_s = \sigma(q_s^{+H}\, q_s)$

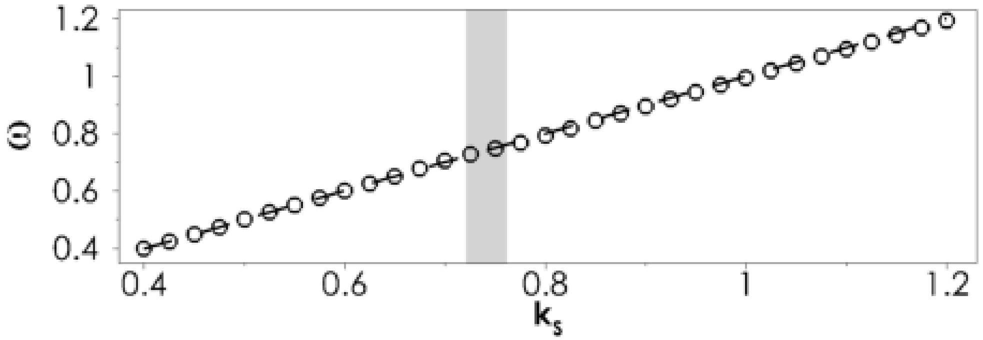
Direct and adjoint fluid components

Direct and adjoint solid components

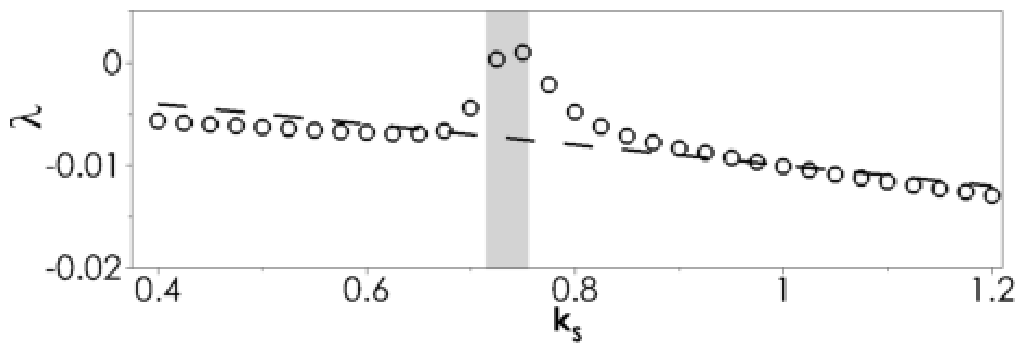
Stability results for $\rho = 200$ – Solid branch



Frequency



Growth rate



Solid branch: Frequency decomposition

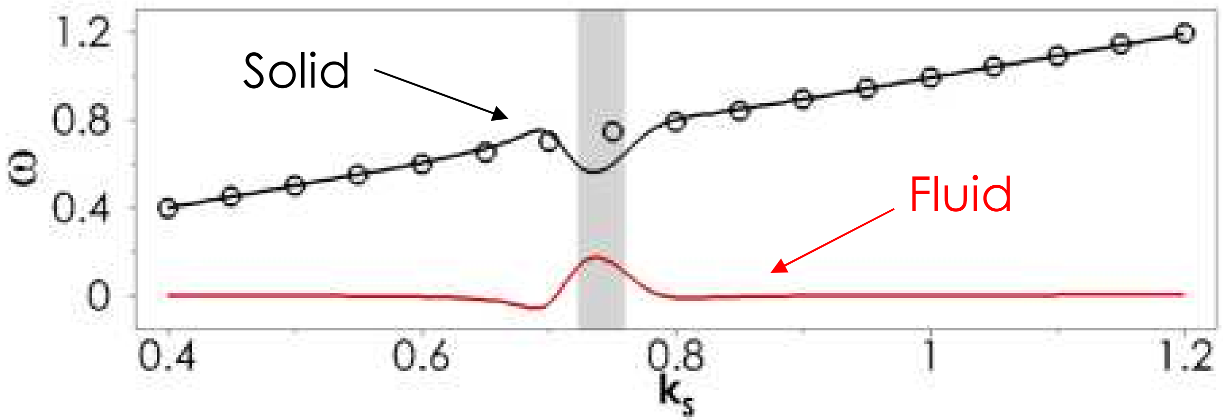
$$\omega = \omega_f + \omega_s$$

Fluid contribution

$$\omega_f = \Im(\sigma_f)$$

Solid contribution

$$\omega_s = \Im(\sigma_s)$$



The frequency is selected by the solid dynamics

Solid branch: Growth rate decomposition

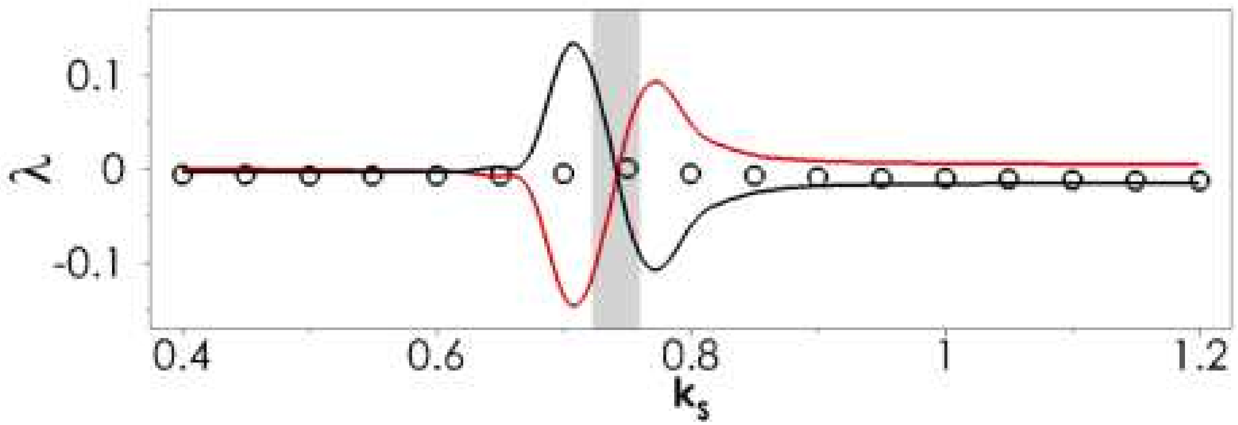
$$\lambda = \lambda_f + \lambda_s$$

Fluid contribution

$$\lambda_f = \Re(\sigma_f)$$

Solid contribution

$$\lambda_s = \Re(\sigma_s)$$



Large and opposite contributions in the unstable region

Solid branch: Growth rate decomposition

$$\lambda = \lambda_f + \lambda_s$$

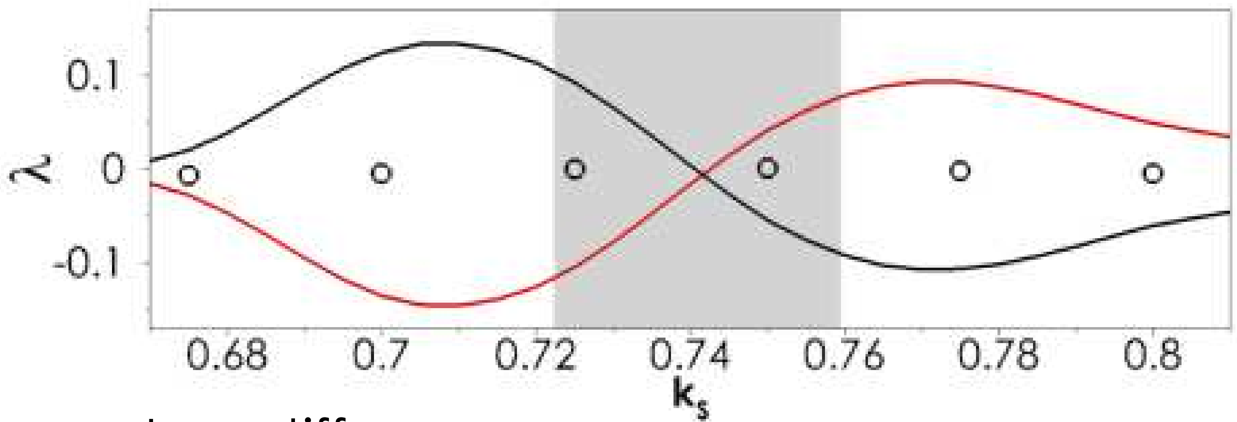
Fluid contribution

$$\lambda_f = \Re(\sigma_f)$$

Solid contribution

$$\lambda_s = \Re(\sigma_s)$$

$$\lambda > 0$$



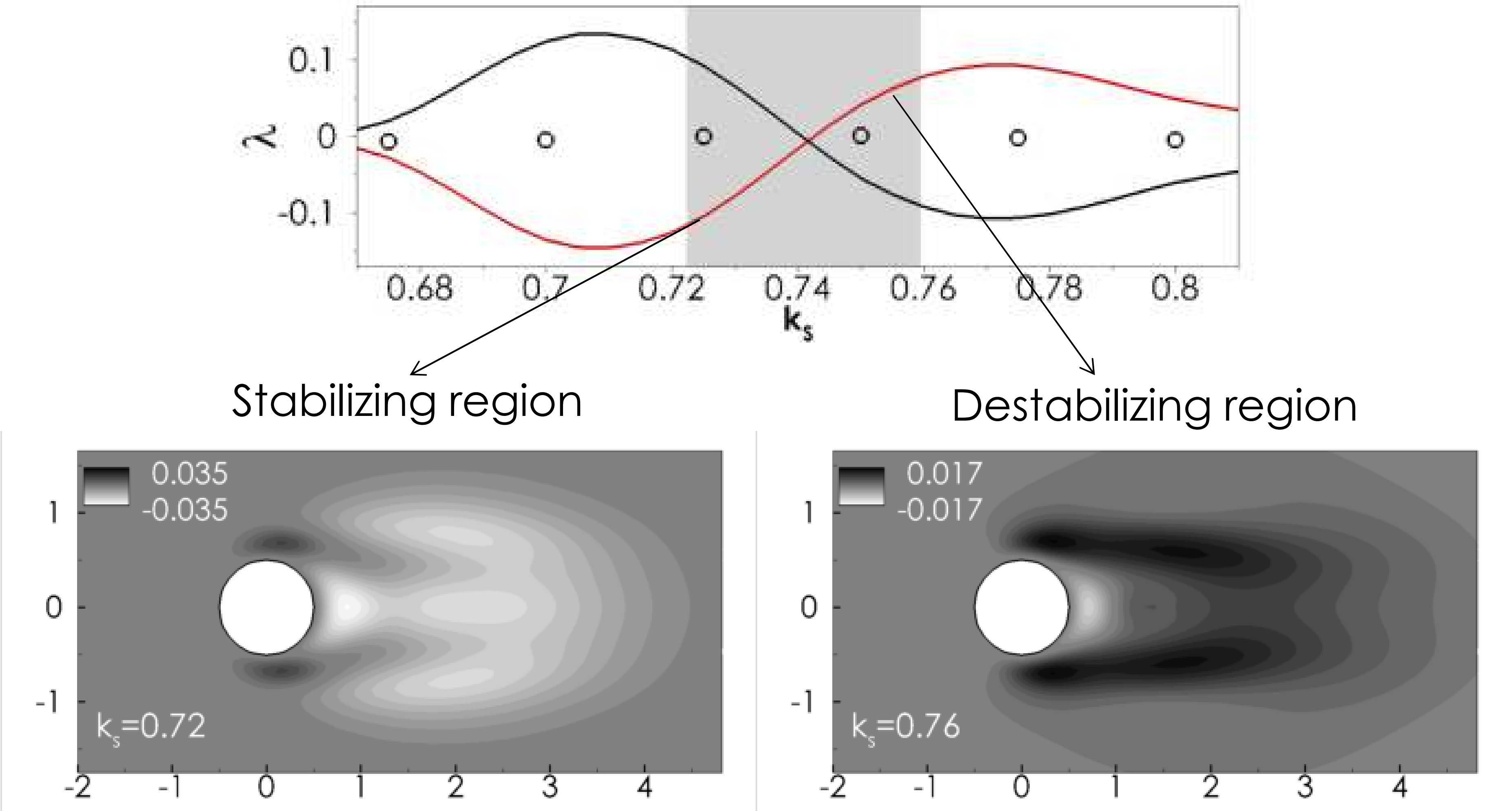
Low stiffness

$|\lambda_s| > |\lambda_f|$
destabilization by the solid contribution

High stiffness

$|\lambda_f| > |\lambda_s|$
destabilization by the fluid contribution

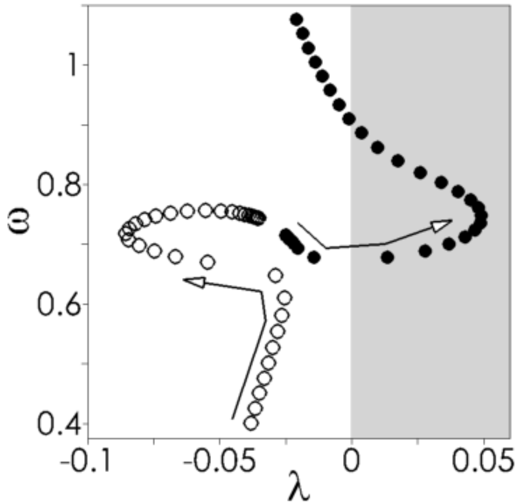
Local (fluid) contribution to the growth



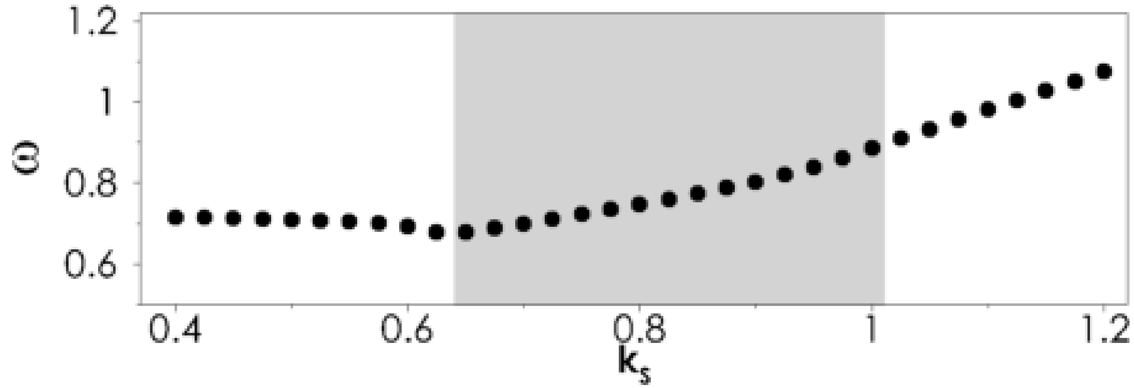
Phase difference between q_f^+ and q_f

Stability results for $\rho = 10$

○ SM Eigenvalue spectrum ● WM



Frequency of the Wake Mode branch



« So called » fluid branch: Frequency decomposition

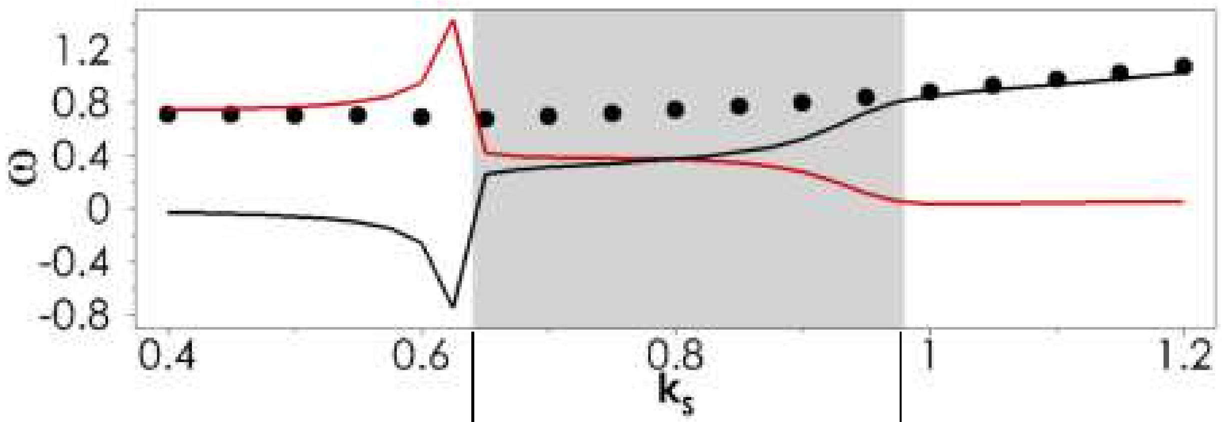
$$\omega = \omega_f + \omega_s$$

Fluid contribution

$$\omega_f = \Im(\sigma_f)$$

Solid contribution

$$\omega_s = \Im(\sigma_s)$$



Low stiffness

$\omega \sim \omega_f$

Frequency selection by the fluid dynamics

$\omega_f \sim \omega_s$

High stiffness

$\omega \sim \omega_s$

Frequency selection by the solid dynamics

Conclusion

- Operator decomposition approach applied to coupled fluid/solid modes
- No need to vary the parameters (mass ratio or stiffness), need to determine the adjoint modes.
- Results similar to « wavemaker » analysis (structural sensitivity)
- **Not a variation** of eigenvalues but **a decomposition** of eigenvalues
- Extension to more complex solid dynamics (Jean-Lou Pfister - PhD)

Thank you

erc European Research Council

Pure modes (infinite mass ratio)

	Direct	Adjoint
	$\sigma \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \begin{pmatrix} F & C_{fs} \\ 0 & S \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$	$\sigma^* \begin{pmatrix} a_f \\ a_s \end{pmatrix} = \begin{pmatrix} F^H & 0 \\ {C_{fs}}^H & S^H \end{pmatrix} \begin{pmatrix} a_f \\ a_s \end{pmatrix}$
Fluid modes	$\sigma\, q_f = F\, q_f$ $q_s = 0$	$\sigma_f^* a_f^p = F^H a_f^p$ $(\sigma_f^* I - S^H) a_s^p = C_{fs}^H a_f^p$
Solid modes	$\sigma\, q_s = S\, q_s$ $(\sigma I - F)\, q_f = C_{fs} q_s$	$\sigma_s^* a_s^p = S^H a_s^p$ $a_f^p = 0$

Projection of coupled problem on pure fluid modes

$$(\sigma\, I - F)\, q_f = C_{fs}\, q_s$$

$$(\sigma I \;\; - S) q_s = \rho^{-1} C_{sf}\, q_f$$

$$a_f^{p^H}(\sigma\, I \; - F) q_f + a_s^{p^H}(\sigma\, I - S) q_s - a_f^{p^H} C_{fs}\; q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\left(\sigma^* a_f^p - F^H a_f^p\right)^H q_f + \left(\sigma^* a_s^p - S^H a_s^p - C_{fs}^H a_f^p\right)^H q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$(\sigma - \sigma_f)(a_f^{p^H} q_f) + (\sigma - \sigma_f)(a_s^{p^H} q_s) = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\boxed{(\sigma - \sigma_f) = \frac{\rho^{-1} a_s^{p^H} C_{sf}\, q_f}{(a_f^{p^H} q_f + a_s^{p^H} q_s)}}$$

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Projection of coupled problem on pure solid modes

$$(\sigma\, I - F)\, q_f = C_{fs}\, q_s$$

$$(\sigma I \;\; - S) q_s = \rho^{-1} C_{sf}\, q_f$$

$$a_f^{p^H} (\sigma\, I \;\; - F) q_f + a_s^{p^H} (\sigma\, I - S) q_s - a_f^{p^H} C_{fs}\;\, q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\left(\sigma^* a_f^p - F^H a_f^p\right)^H q_f + \left(\sigma^* a_s^p - S^H a_s^p - C_{fs}^H a_f^p\right)^H q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

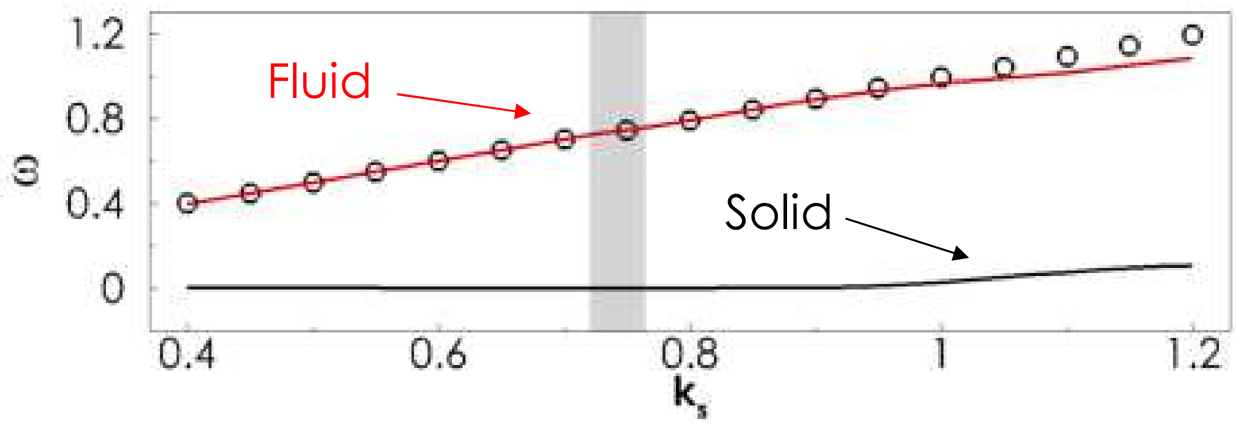
$$(\sigma - \sigma_s)(a_s^{p^H} q_s) = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\boxed{(\sigma - \sigma_s) = \frac{\rho^{-1} a_s^{p^H} C_{sf}\, q_f}{a_s^{p^H} q_s}}$$

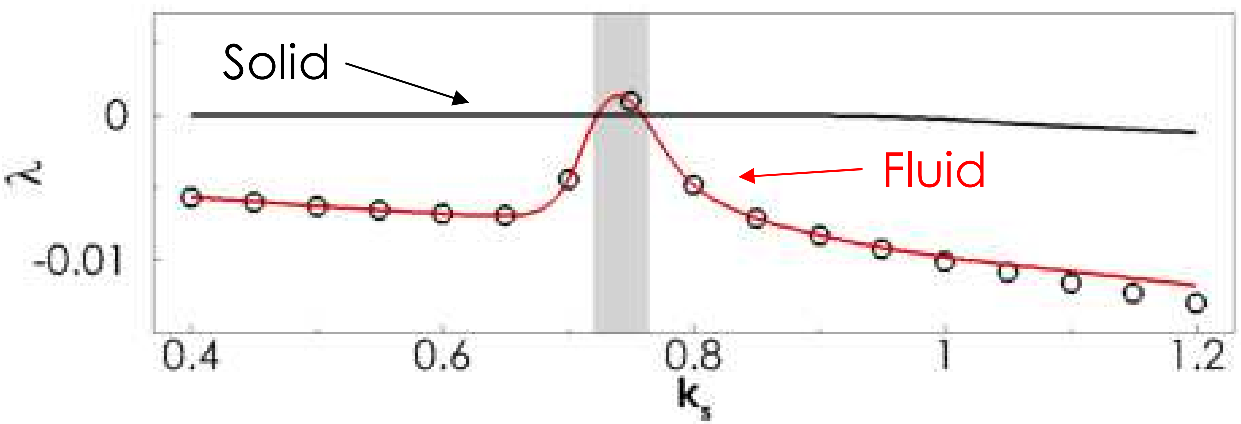
Direct-based decomposition of the unstable mode

$\rho = 200$

Frequency



Growth rate



Infinite mass ratio - Fluid Modes

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ \mathbf{0} & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} \left(L_f \, q_f + C_{fs} \, q_s \right)$$

Solid contribution

$$\sigma_s = q_s^{+H} \left(L_s \, q_s + \rho^{-1} C_{sf} \, q_f \right)$$

$$\boldsymbol{q_s} = \mathbf{0}$$

Fluid Modes

$$\rho^{-1} = 0$$

$$\boxed{\sigma_f = q_f^{+H} L_f \, q_f = \sigma}$$

OK

$$\boxed{\sigma_s = 0}$$

Infinite mass ratio - Structural Mode

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ \mathbf{0} & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} (L_f \, q_f + C_{fs} \, q_s)$$

Solid contribution

$$\sigma_s = q_s^{+H} (L_s \, q_s + \rho^{-1} C_{sf} \, q_f)$$

$$\boldsymbol{q_f^+} = \mathbf{0}$$

Structural Mode

$$\rho^{-1} = 0$$

$$\sigma_f = 0$$

OK

$$\sigma_s = q_s^{+H} L_s \, q_s = \sigma$$

Infinite mass ratio – Direct mode-based decomposition

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ 0 & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_f + \hat{\sigma}_s$$

Fluid contribution

$$\hat{\sigma}_f = \boldsymbol{q_f^H} \left(L_f \, q_f + C_{fs} \, q_s \right)$$

Solid contribution

$$\hat{\sigma}_s = \boldsymbol{q_s^H} \left(L_s \, q_s + \rho^{-1} C_{sf} \, q_f \right)$$

$$L_f q_f + C_{fs} q_s = \sigma q_f$$

Structural Mode

$$\rho^{-1} = 0$$

$$\hat{\sigma}_f = \sigma \left(q_f^H \, q_f \right)$$

NOT OK

$$\hat{\sigma}_s = \sigma \left(q_s^H \, q_s \right)$$

Infinite mass ratio – Direct mode-based decomposition

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ 0 & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_f + \hat{\sigma}_s$$

Fluid contribution

$$\hat{\sigma}_f = \boldsymbol{q_f^H} \left(L_f \, q_f + C_{fs} \, q_s \right)$$

Solid contribution

$$\hat{\sigma}_s = \boldsymbol{q_s^H} \left(L_s \, q_s + \rho^{-1} C_{sf} \, q_f \right)$$

$$(\sigma I - L_f) q_f = C_{fs} q_s$$

Structural Mode

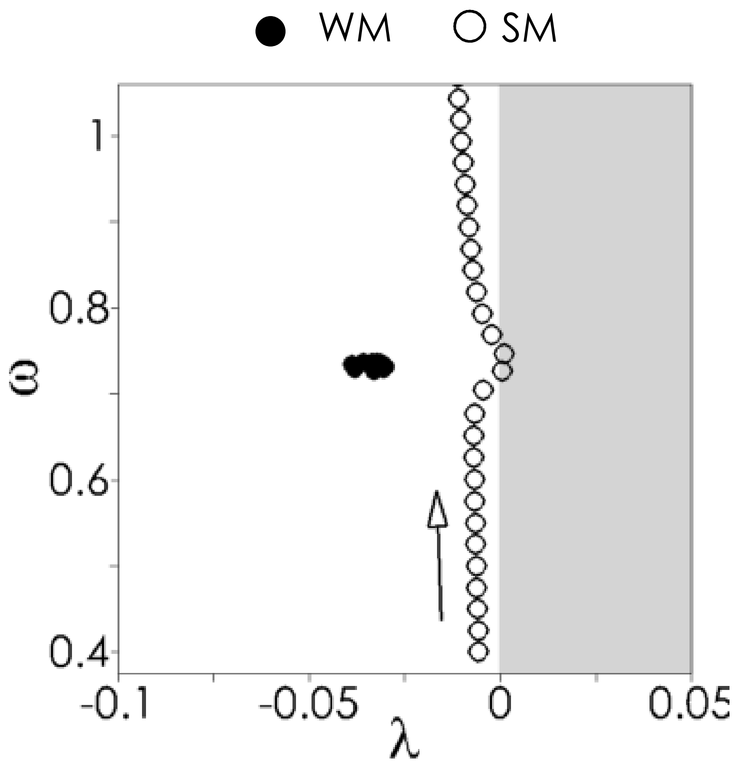
$$\rho^{-1} = 0$$

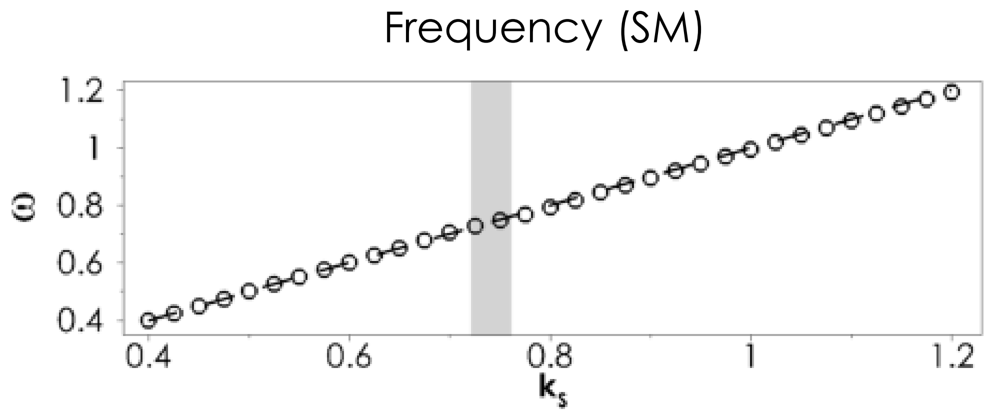
$$\boxed{\hat{\sigma}_f = \sigma \left(q_f^H \, q_f \right)}$$

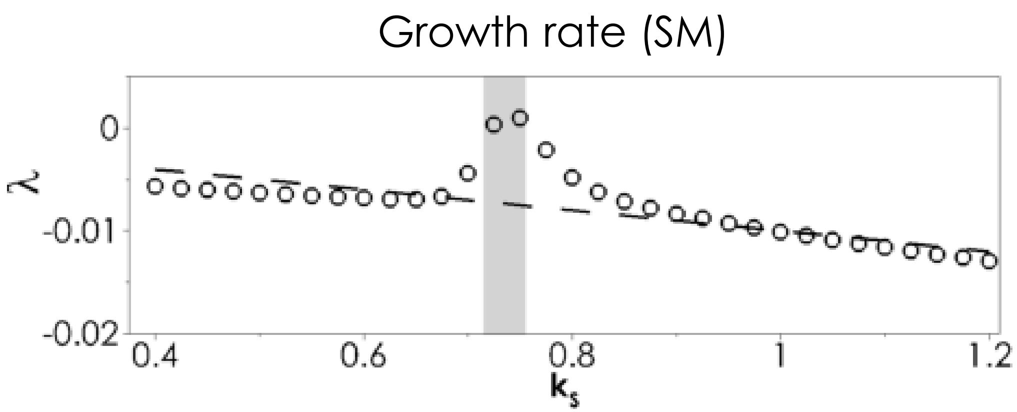
NOT OK
(large fluid response)

$$\boxed{\hat{\sigma}_s = \sigma \left(q_s^H \, q_s \right)}$$

Results for $\rho = 200$ – Solid Mode



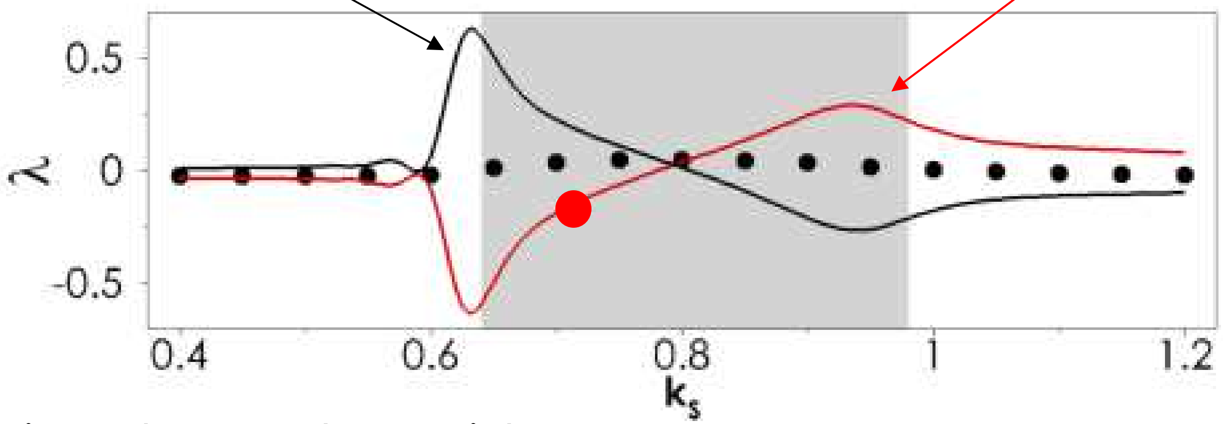




- The frequency is quasi-equal to k_s
- The growth rate gets positive for $k_s$close ω_v

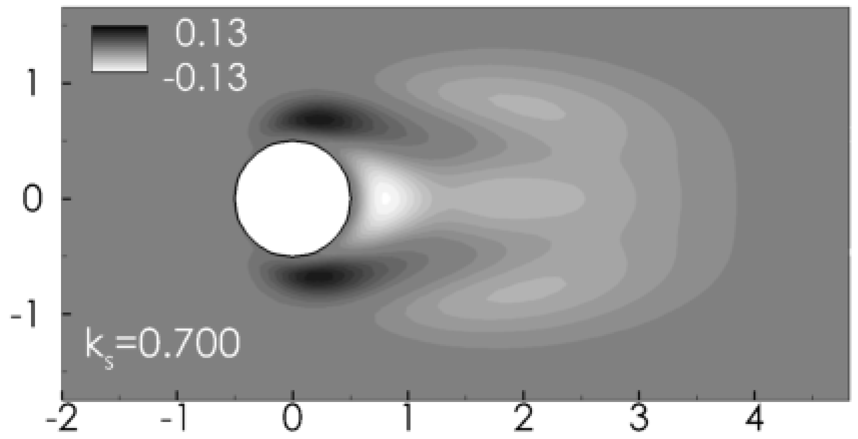
Wake Mode: growth rate decomposition

$\rho = 10$



Small k_s
destabilization due to the solid

Large k_s
destabilization due to the fluid



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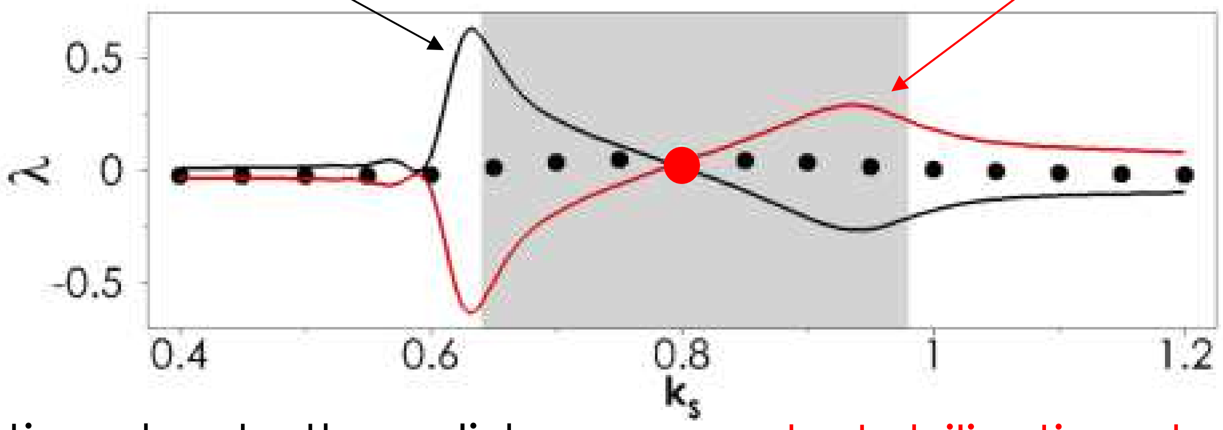
Wake Mode: growth rate decomposition

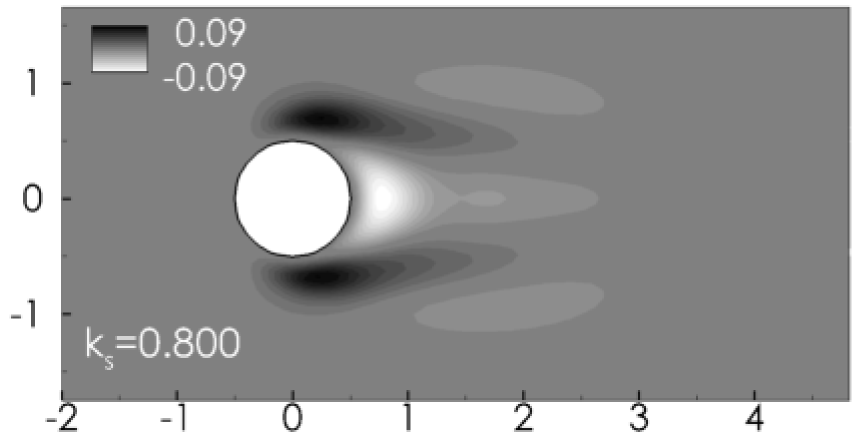
$\rho = 10$

Solid — Growth rate — Fluid



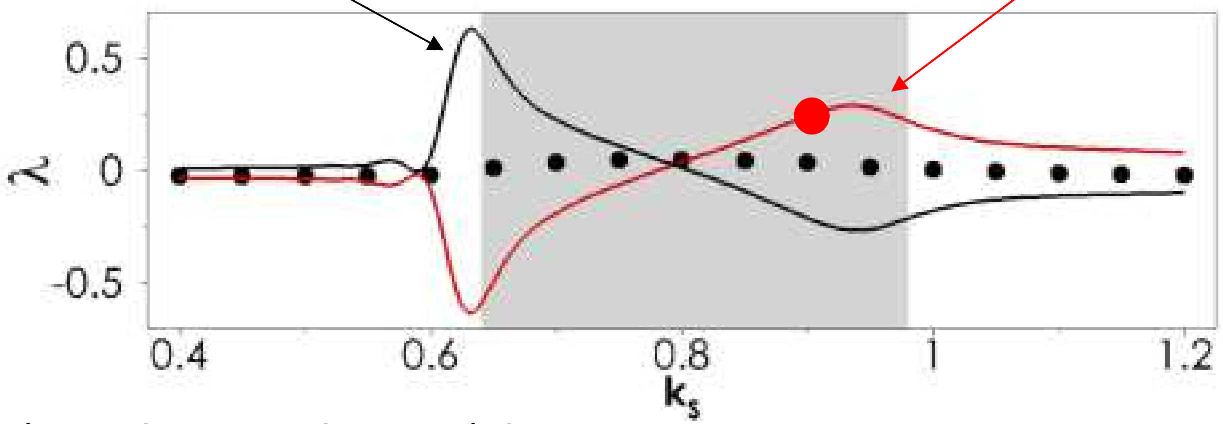
Small k_s
destabilization due to the solid

Large k_s
destabilization due to the fluid



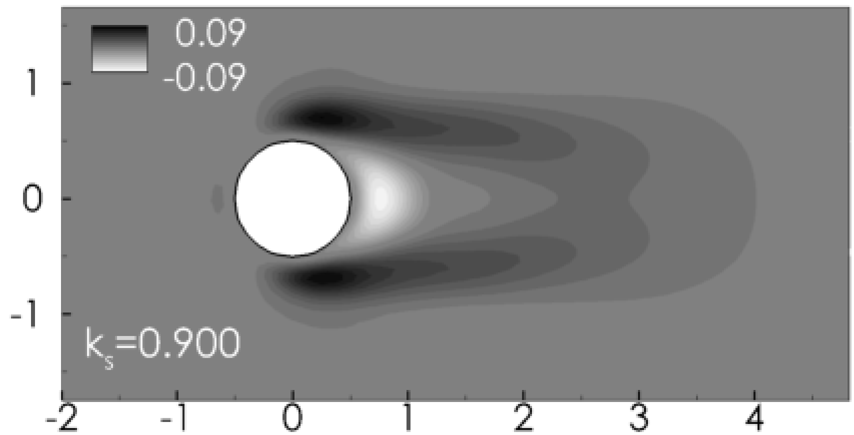
Wake Mode: growth rate decomposition

$\rho = 10$



Small k_s
destabilization due to the solid

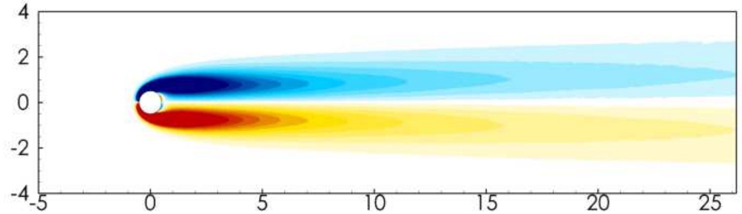
Large k_s
destabilization due to the fluid



Free oscillation

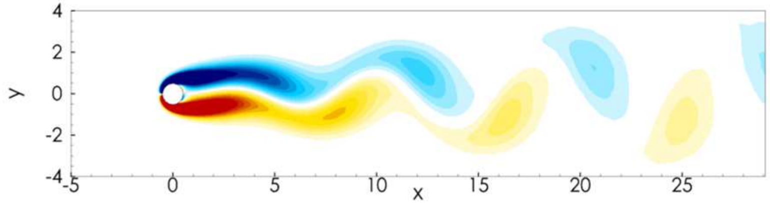
$Re = 40\ ;\ \rho = 50$

$\omega_s = 0.60$



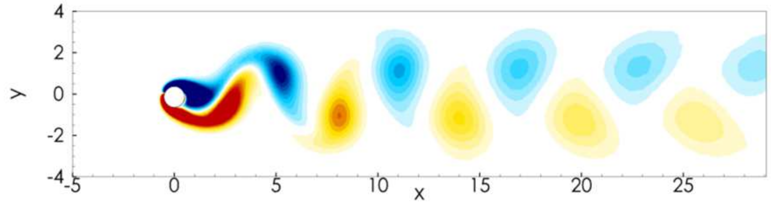
No oscillation

$\omega_s = 0.66$



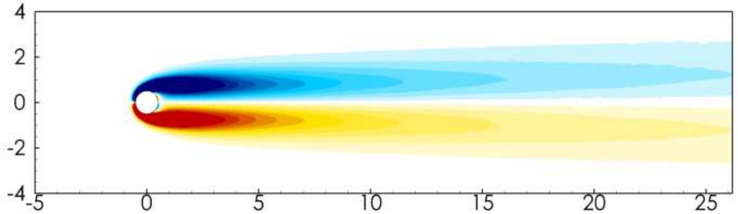
Weak oscillation

$\omega_s = 0.90$



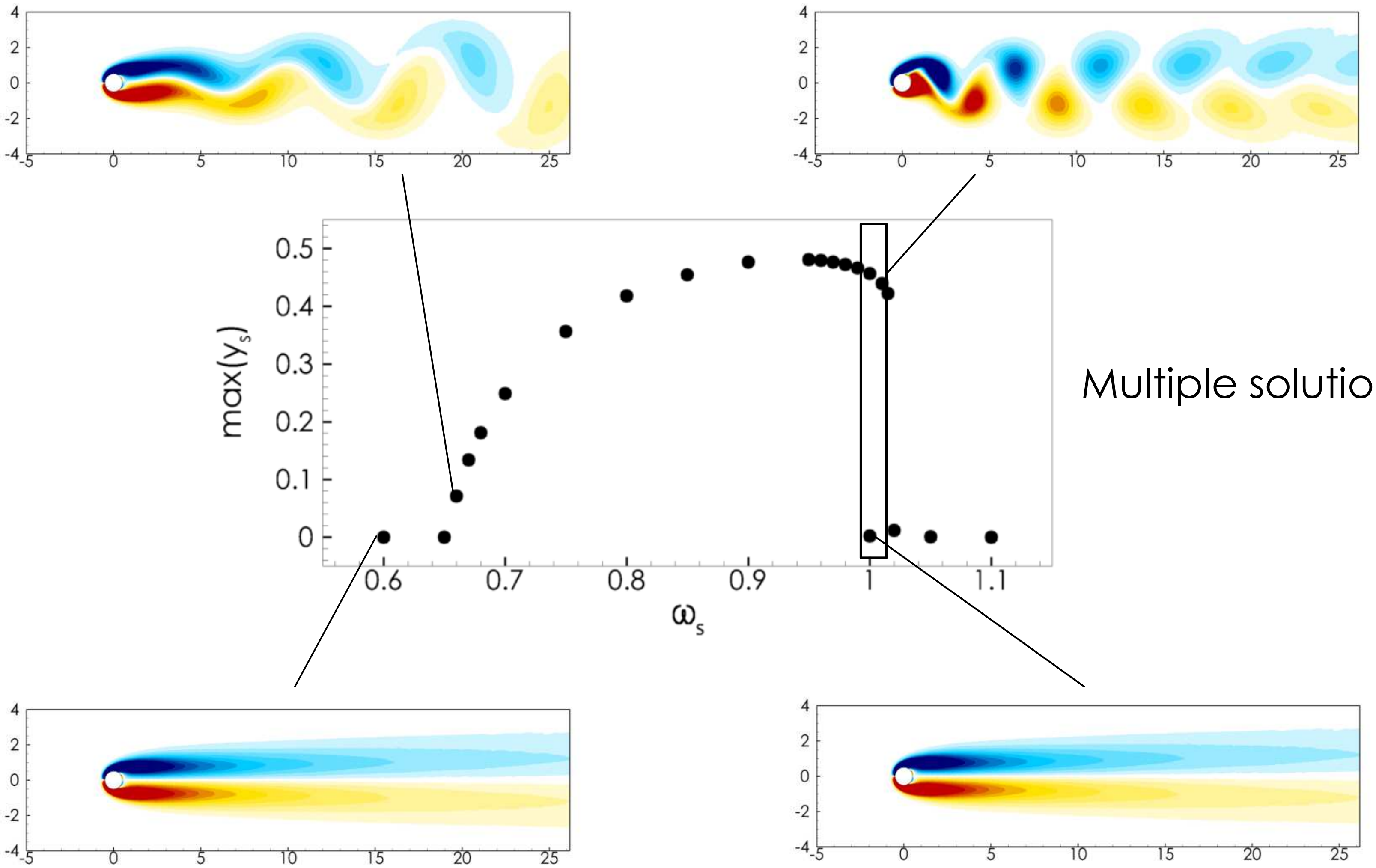
Strong oscillation

$\omega_s = 1.10$



No oscillation

Solid displacement – Fluid fields



Multiple solutions

Methods for identifying the dynamics of coupled modes

- Energetic approach (Mittal et al, JFM 2016)

  Transfer of energy from the fluid to the solid component

- Classical wavemaker analysis (Giannetti & Luchini, JFM 2008, …)

  Structural sensitivity analysis of the eigenvalue problem.

  Largest eigenvalue variation induced by operator perturbation

- **Operator/Eigenvalue decomposition**