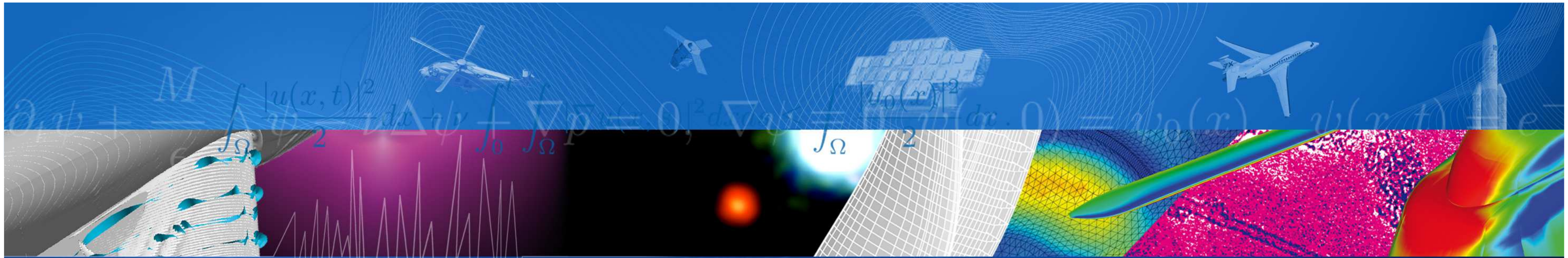

Identifying the “wavemaker” of fluid/structure instabilities

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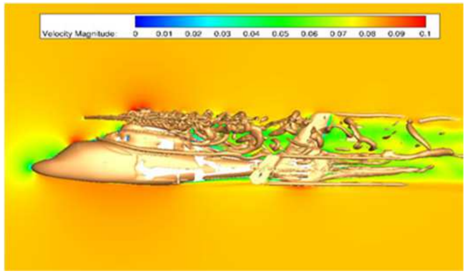
Context

Flow-induced structural vibrations

Civil engineering

Aeronautics



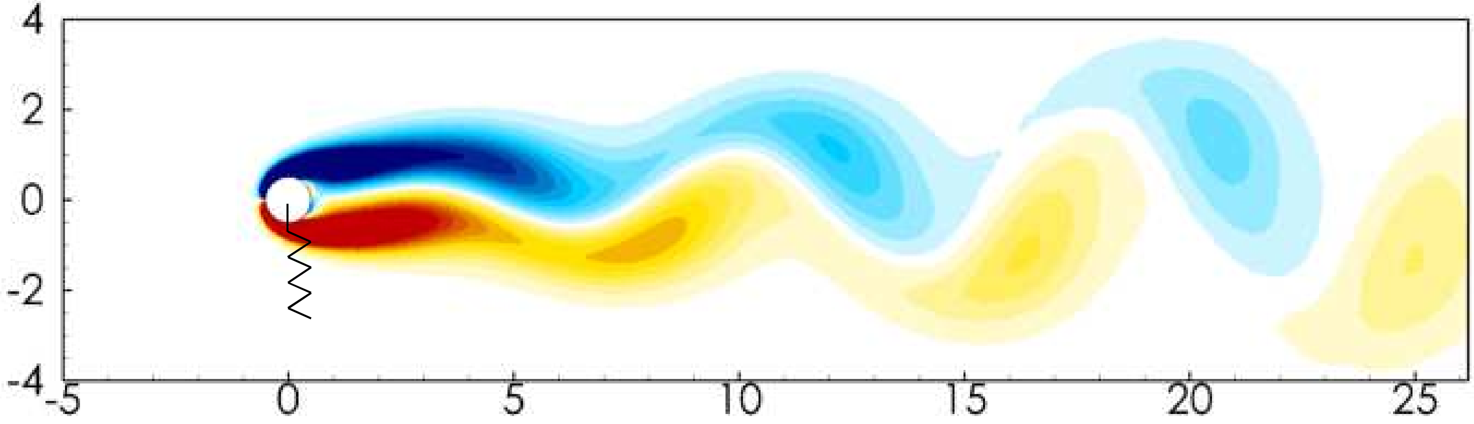
Offshore-marine industry

Stability analysis of the fluid/structure problem

A tool to predict the onset of vibrations

Model problem: spring-mounted cylinder flow

One spring - cross-stream



Structural frequency	Structural damping	Density ratio	Reynolds number
$\kappa_s = \sqrt{\kappa/m}$	γ_s	$\rho = \rho_s/\rho_f$	$Re = \frac{U_\infty D}{\nu}$
$0.4 < k_s < 1.2$ $(\omega_v = 0.73)$	$\gamma_s = 0.01$	$\rho = 10^6, 200, 10$	$Re = 40$

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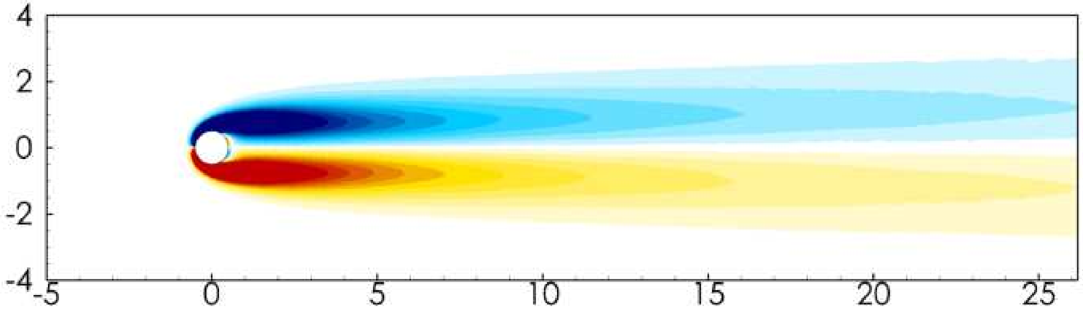
Stability analysis of the coupled fluid/solid problem

Steady solution – No solid component



$$q'(x,t) = (q_f, q_s)(x)e^{(\lambda + i\,\omega)t} + c.c.$$

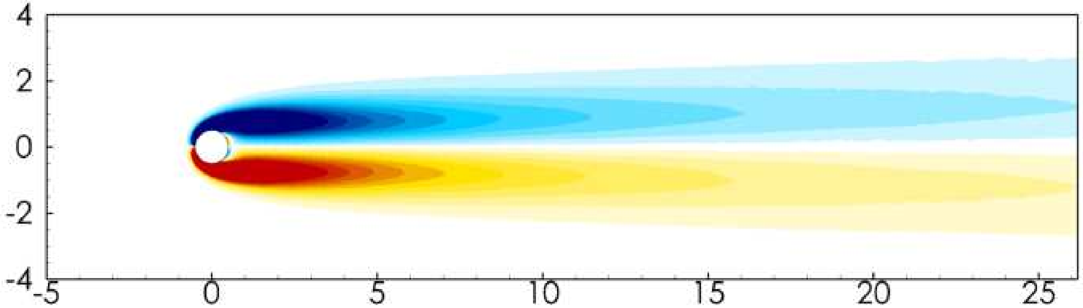
Fluid/solid components

Growth rate/frequency

$$\begin{pmatrix} L_f(Re) & C_{fs} \\ \rho^{-1}C_{sf} & L_s(k_s,\gamma_s) \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega) \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Diagonal operators: intrinsinc dynamics

Steady solution – No solid component



$$q'(x,t) = (q_f, q_s)(x)e^{(\lambda+i\,\omega)t} + c.c.$$

Fluid/solid components

Growth rate/frequency

Fluid operator

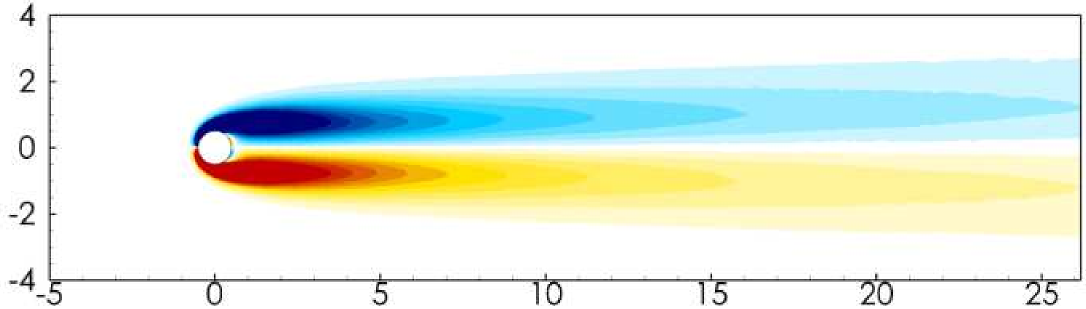
$$\begin{pmatrix} L_f(Re) & C_{fs} \\ \rho^{-1}C_{sf} & L_s(k_s,\gamma_s) \end{pmatrix}\begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega)\begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Solid operator
(damped harmonic oscillator)

Cossu & Morino (JFS, 2000)

Off-diagonal operators: coupling terms

Steady solution – No solid component



$$q'(x,t) = (q_f, q_s)(x)e^{(\lambda + i\,\omega)t} + c.c.$$

Fluid/solid components

Growth rate/frequency

Coupling operator
(boundary condition + non-inertial terms)

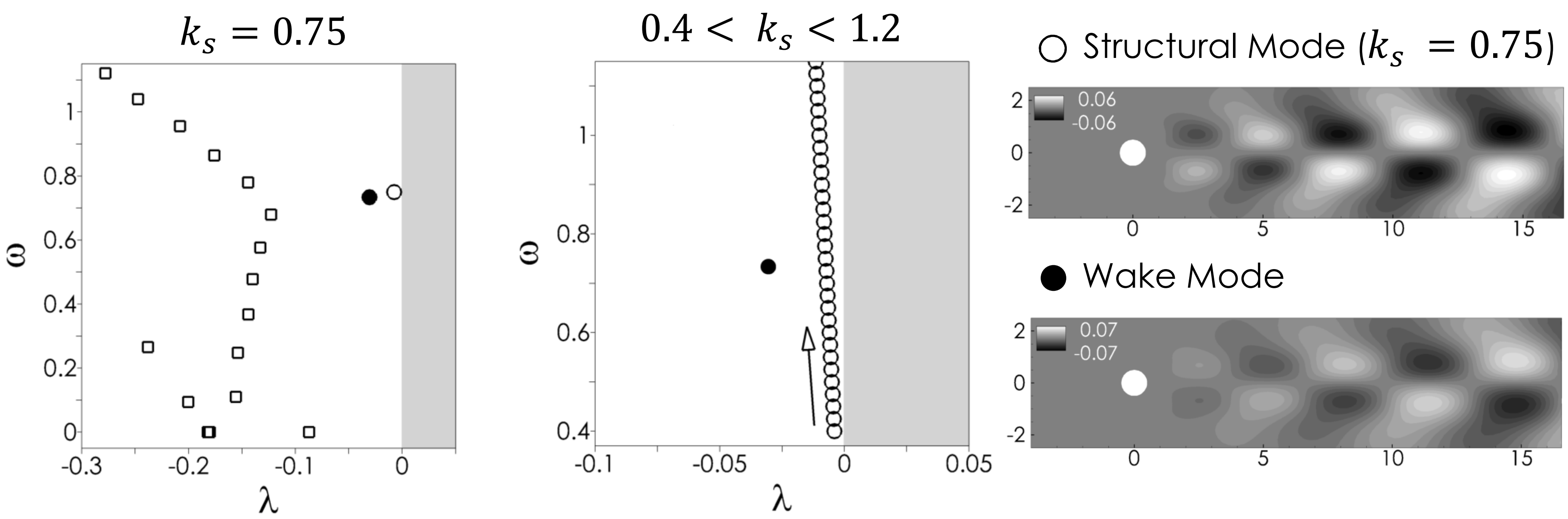
$$\begin{pmatrix} L_f(Re) & C_{fs} \\ \rho^{-1}C_{sf} & L_s(k_s,\gamma_s) \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = (\lambda + i\omega) \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Coupling operator
Weighted fluid force

Mougin & Magaudet (IJMF, 2002), Jenny et al. (JFM 2004)

Stability analysis at large density ratio

$$\rho = 10^6$$



Methods to identify structural and wake modes

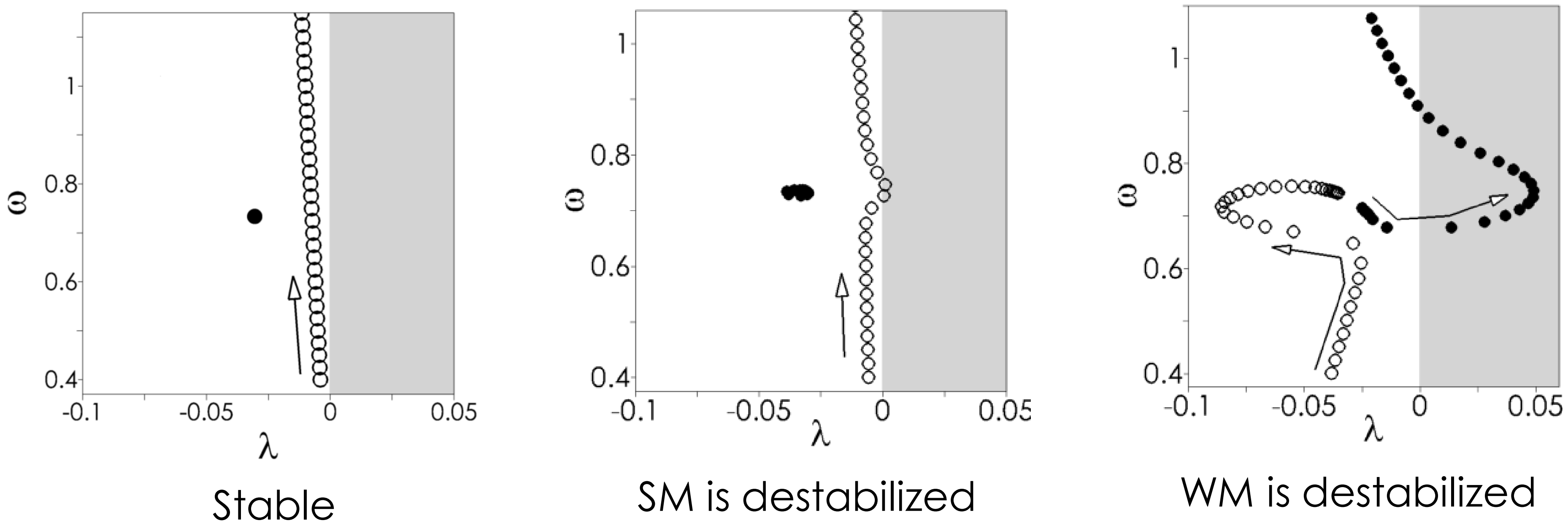
- Vary the structural frequency

- Look at the vertical displacement/velocity (not the fluid component)

Stability analysis at smaller density ratio

○ SM　　Effect of decreasing the density ratio　　● WM

$\rho = 10^6 \longrightarrow \rho = 200 \longrightarrow \rho = 10$



- Stronger interaction between the two branches
- The two branches exchange their « nature » for small ρ
- Coalescence of modes (not seen here)

 Zhang et al (JFM 2015), Meliga & Chomaz (JFM 2011)

Objective and outlines

Objective:

- Identify the « wavemaker » of a fluid/solid eigenmode

- Quantify the respective contributions of fluid and solid dynamics to the eigenvalue of a coupled eigenmode

Outlines:

1 - Presentation of the operator/eigenvalue decomposition

2 - The infinite mass ratio limit ($\rho = 10^6$)

3 - Finite mass ratio ($\rho = 200$ and $\rho = 10$)

State of the art for the method

- Energetic approach of eigenmodes (Mittal et al, JFM 2016)

  - Transfer of energy from the fluid to the solid / Growth rate
  - Does not identify the « wavemaker » region in the fluid

- Wavemaker analysis (Giannetti & Luchini, JFM 2008, …)

  - Structural sensitivity analysis of the eigenvalue problem.

  - Largest eigenvalue variation induced by any perturbation of the operator ?

  - Output of this analysis is an inequality. We would like an identify !

- **Operator/Eigenvalue decomposition**

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From operator to eigenvalue decomposition

Operator decomposition

$$L\,q = \;(L_a + L_b)q = \sigma\,q$$

In general, q is not an eigenmode of L_a or L_b , so

$$L_a\,q = \sigma_a\,q + r_a \qquad\qquad L_b\,q = \sigma_b\,q + r_b$$

with residuals $r_a \neq 0, r_b \neq 0$ but $r_a = -r_b = r$

Eigenvalue decomposition

$$\sigma_a + \sigma_b = \sigma$$

How to compute the eigenvalue contributions σ_a/σ_b ?

Computing eigenvalue contributions

Expansion of the residual on the set of other eigenmodes q_k

$$r = \sum_k r_k q_k \longrightarrow L_a\, q = \sigma_a\, q + \sum_k r_k q_k$$

Orthogonal projection on the mode q
using the adjoint mode q^+

$$q^{+H}(L_a\, q) = \sigma_a \underbrace{(q^{+H} q)}_{=1} + \sum_k r_k \underbrace{(q^{+H} q_k)}_{=0}$$

Normalisation $= 1$ $= 0$ Bi-orthogonality

Adjoint mode-based decomposition

$$\sigma = \sigma_a + \sigma_b$$

$$\sigma_a = q^{+H}(L_a\, q) \qquad \sigma_b = q^{+H}(L_b\, q)$$

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Why this particular eigenvalue decomposition?

For an identical decomposition of the operator,
other eigenvalue decompositions are possible

Non-orthogonal projection on the mode q

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_a + \hat{\sigma}_b$$

$$\hat{\sigma}_a = q^H (L_a\, q) \qquad \hat{\sigma}_b = q^H (L_b\, q)$$

But it includes contributions from other eigenmodes

$$L_a\, q = \sigma_a\, q + \sum_k r_k q_k \qquad L_b\, q = \sigma_b\, q - \sum_k r_k q_k$$

$$\hat{\sigma}_{a/b} = \sigma_{a/b} \pm \sum_k r_k \;(q^H q_k) \quad \neq 0$$

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Application to the spring-mounted cylinder flow

$$\begin{pmatrix} L_f & C_{fs} \\ \rho^{-1} C_{sf} & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} \left(L_f\, q_f + C_{fs}\, q_s \right)$$

Solid contribution

$$\sigma_s = q_s^{+H} \left(L_s\, q_s + \rho^{-1} C_{sf}\, q_f \right)$$

$$\sigma_f = \int_D q_f^{+*}(x) \cdot \left(L_f\, q_f + C_{fs}\, q_s \right)(x)\ dx$$

Local contributions

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Infinite mass ratio - Fluid Modes

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ \mathbf{0} & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} \left(L_f\, q_f + C_{fs}\, q_s\right)$$

Solid contribution

$$\sigma_s = q_s^{+H} \left(L_s\, q_s + \rho^{-1} C_{sf}\, q_f\right)$$

Fluid Modes

$$\boldsymbol{q_s} = \mathbf{0}$$

$$\boxed{\sigma_f = q_f^{+H} L_f\, q_f = \sigma}$$

OK

$$\rho^{-1} = 0$$

$$\boxed{\sigma_s = 0}$$

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Infinite mass ratio - Structural Mode

$\rho^{-1} = 0$

$$\begin{pmatrix} L_f & C_{fs} \\ \mathbf{0} & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Adjoint mode-based decomposition

$$\sigma = \sigma_f + \sigma_s$$

Fluid contribution

$$\sigma_f = q_f^{+H} \left(L_f \, q_f + C_{fs} \, q_s \right)$$

Solid contribution

$$\sigma_s = q_s^{+H} \left(L_s \, q_s + \rho^{-1} C_{sf} \, q_f \right)$$

$\boldsymbol{q_f^+ = 0}$

Structural Mode

$\rho^{-1} = 0$

$$\sigma_f = 0$$

OK

$$\sigma_s = q_s^{+H} L_s \, q_s = \sigma$$

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Infinite mass ratio – Direct mode-based decomposition

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ 0 & L_s \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_f + \hat{\sigma}_s$$

Fluid contribution

$$\hat{\sigma}_f = \boldsymbol{q_f^H} \left(L_f \, q_f + C_{fs} \, q_s \right)$$

Solid contribution

$$\hat{\sigma}_s = \boldsymbol{q_s^H} \left(L_s \, q_s + \rho^{-1} C_{sf} \, q_f \right)$$

$$L_f q_f + C_{fs} q_s = \sigma q_f$$

Structural Mode

$$\rho^{-1} = 0$$

$$\boxed{\hat{\sigma}_f = \sigma \left(q_f^H \, q_f \right)}$$

NOT OK

$$\boxed{\hat{\sigma}_s = \sigma \left(q_s^H \, q_s \right)}$$

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Infinite mass ratio – Direct mode-based decomposition

$$\rho^{-1} = 0 \qquad \begin{pmatrix} L_f & C_{fs} \\ 0 & L_s \end{pmatrix}\begin{pmatrix} q_f \\ q_s \end{pmatrix} = \sigma \begin{pmatrix} M_f & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} q_f \\ q_s \end{pmatrix}$$

Direct mode-based decomposition

$$\sigma = \hat{\sigma}_f + \hat{\sigma}_s$$

Fluid contribution

$$\hat{\sigma}_f = \boldsymbol{q_f^H} \, (L_f \, q_f + C_{fs} \, q_s)$$

Solid contribution

$$\hat{\sigma}_s = \boldsymbol{q_s^H} (L_s \, q_s + \rho^{-1} C_{sf} \, q_f)$$

$$(\sigma I - L_f) q_f = C_{fs} q_s$$

Structural Mode

$$\rho^{-1} = 0$$

$$\boxed{\hat{\sigma}_f = \sigma \, (q_f^H \, q_f)}$$

NOT OK
(large fluid response)

$$\boxed{\hat{\sigma}_s = \sigma \, (q_s^H \, q_s)}$$

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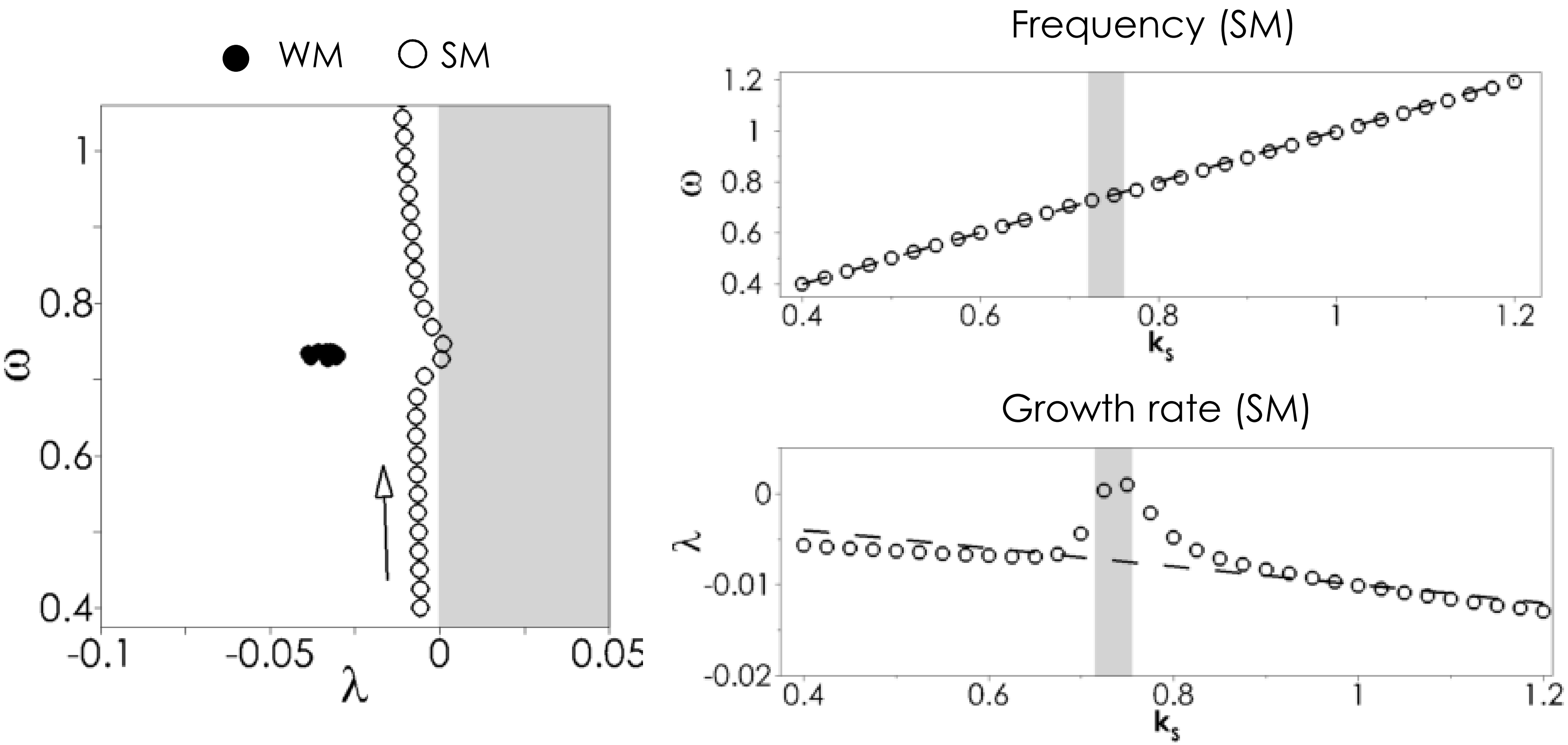
Mass ratio $\rho = 200$: Structural Mode



- The frequency is quasi-equal to k_s
- The growth rate gets positive for $k_s$close ω_v

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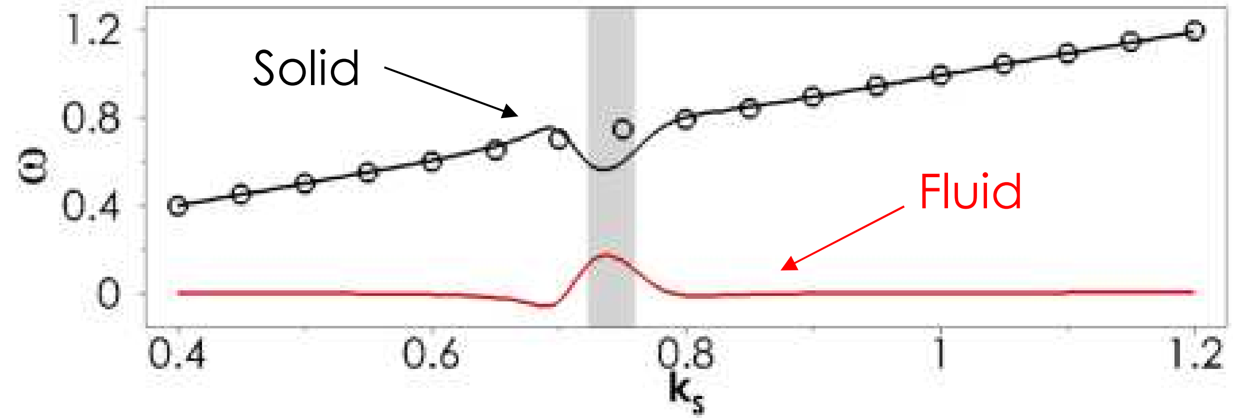
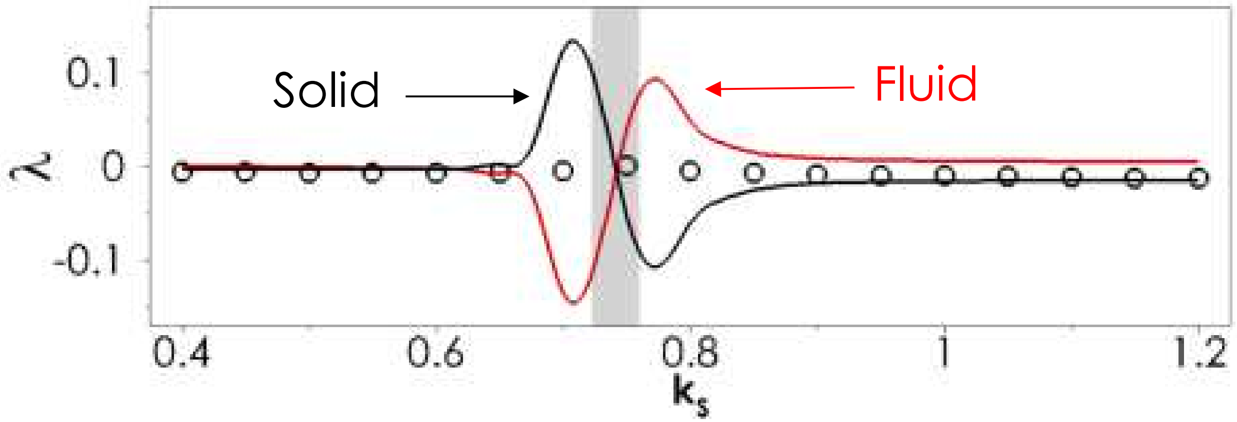
Structural Mode: frequency/growth rate decomposition

Frequency



Growth rate



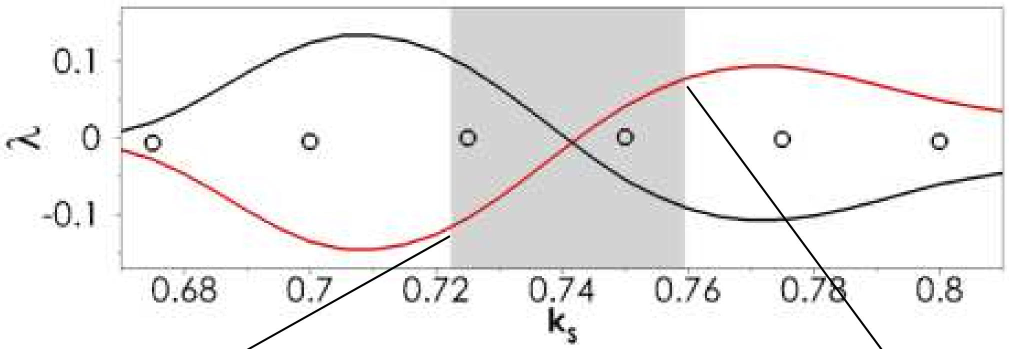
Very large (resonance) and opposite contributions

Spatial distribution of the fluid component

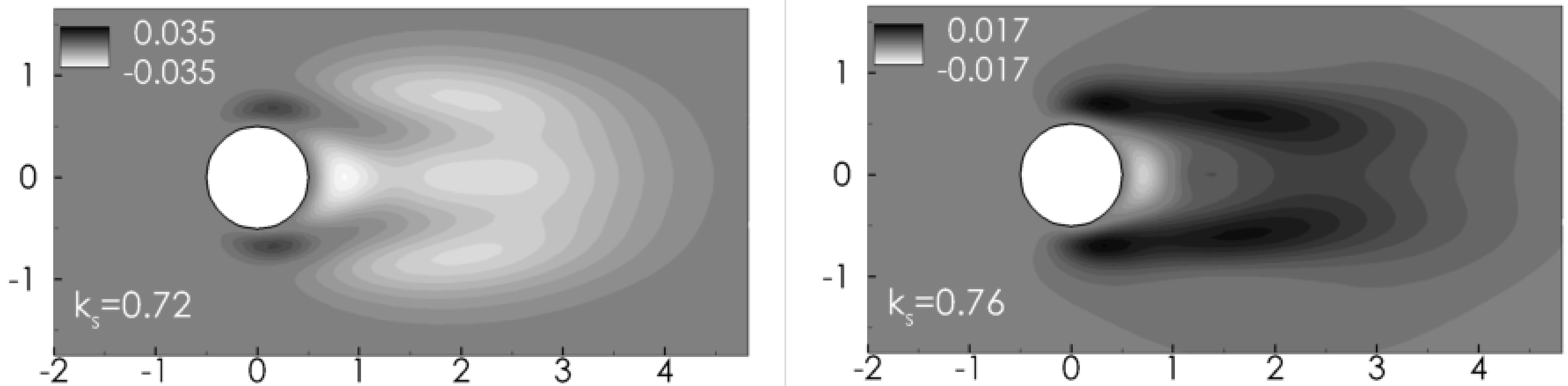
The solid contribution induces destabilisation

Growth rate

The fluid contribution induces destabilisation



Local fluid contributions



Phase change between q_f^+ and $L_f\ q_f$

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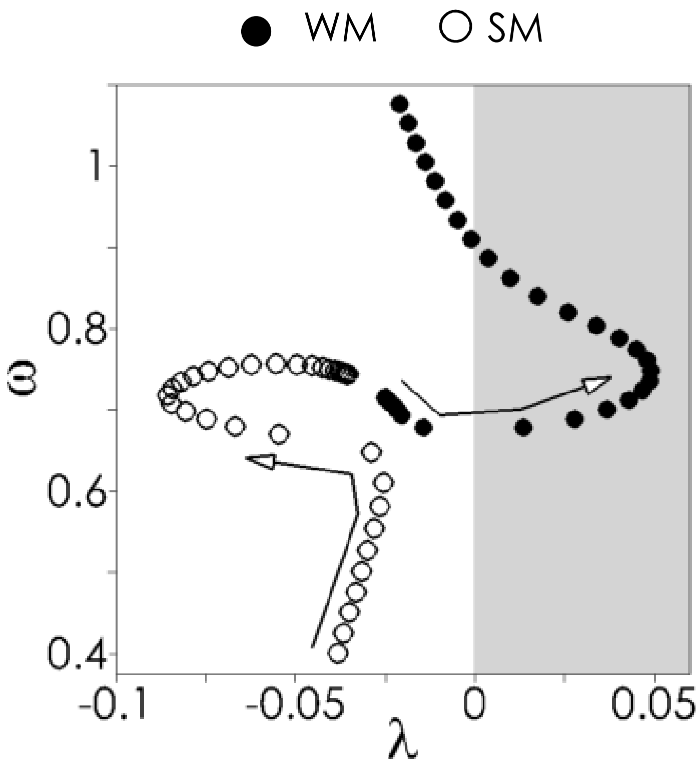
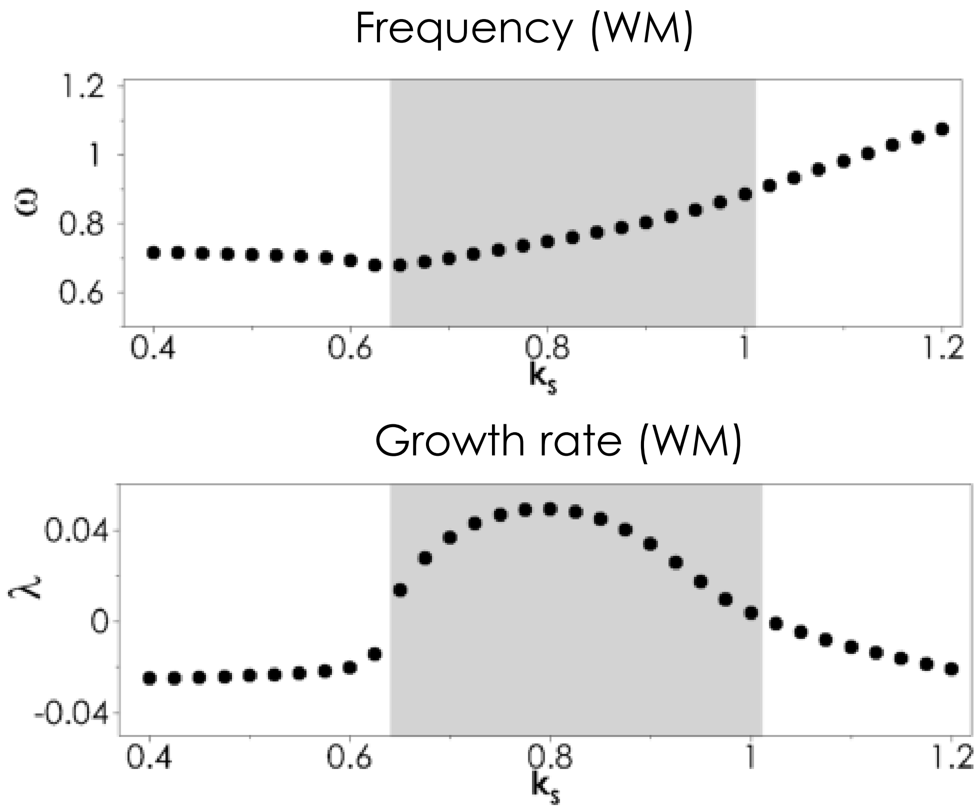
Mass ratio $\rho = 10$: Wake Mode

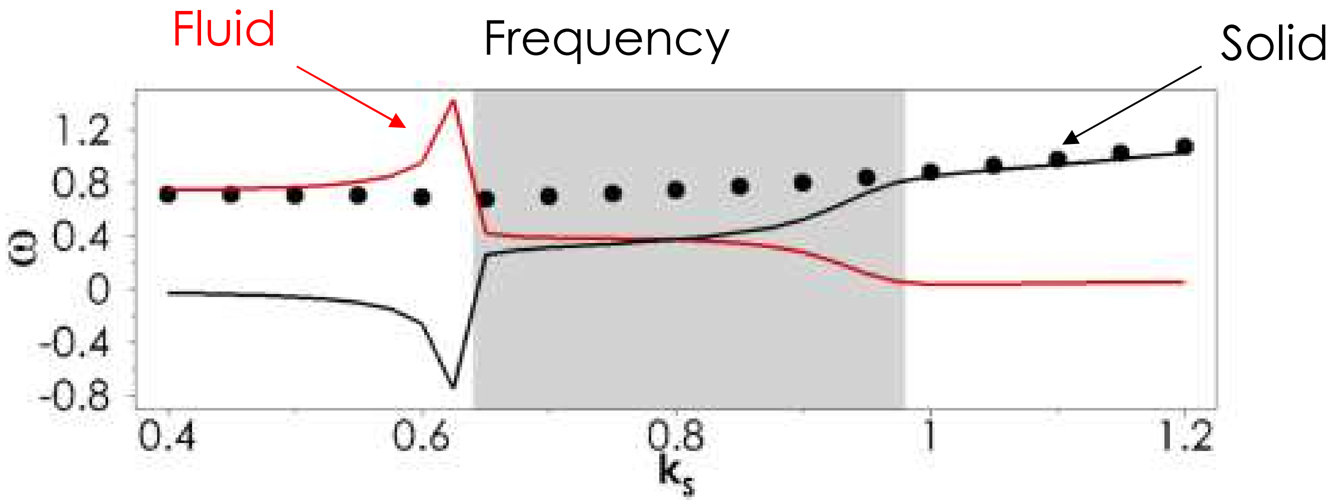




For small k_s , $\omega \sim \omega_v$ - For large k_s , $\omega \sim k_s$

Wake Mode: frequency decomposition

$\rho = 10$



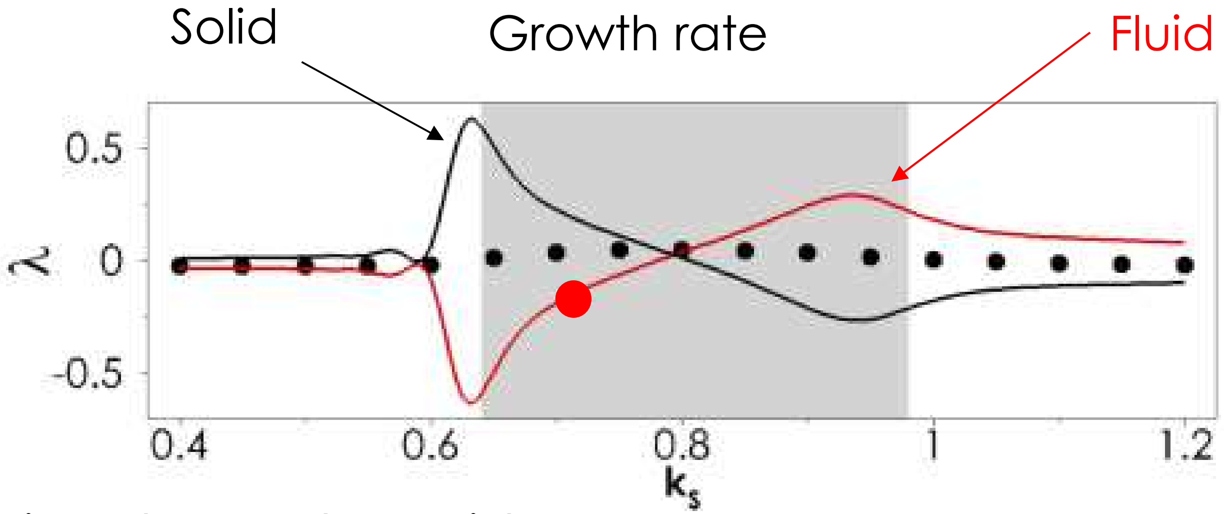
For small k_s: $\omega_f \sim \omega$ = frequency selection by the fluid

Unstable range : $\omega_f \sim \omega_s$

For large k_s: $\omega_s \sim \omega$ = frequency selection by the solid

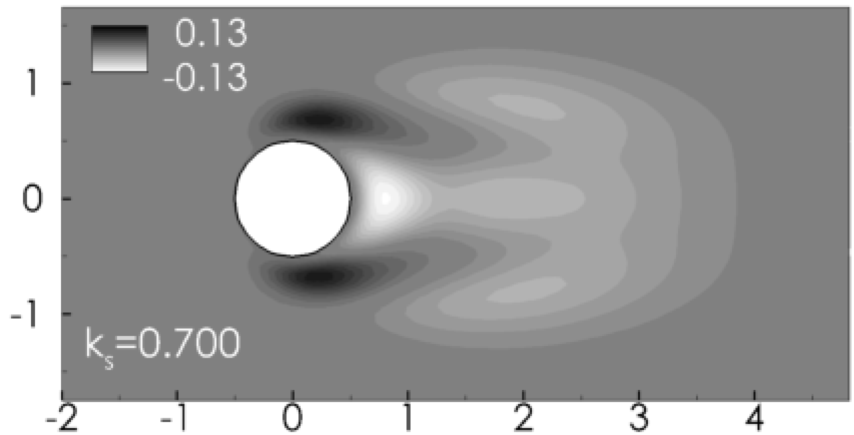
Wake Mode: growth rate decomposition

$\rho = 10$



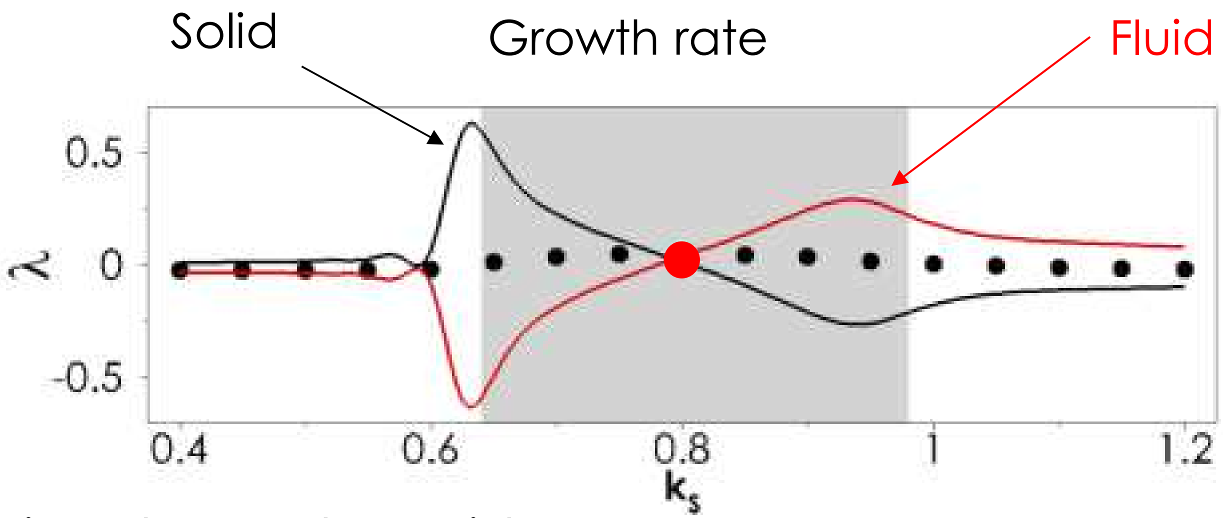
Small k_s
destabilization due to the solid

Large k_s
destabilization due to the fluid



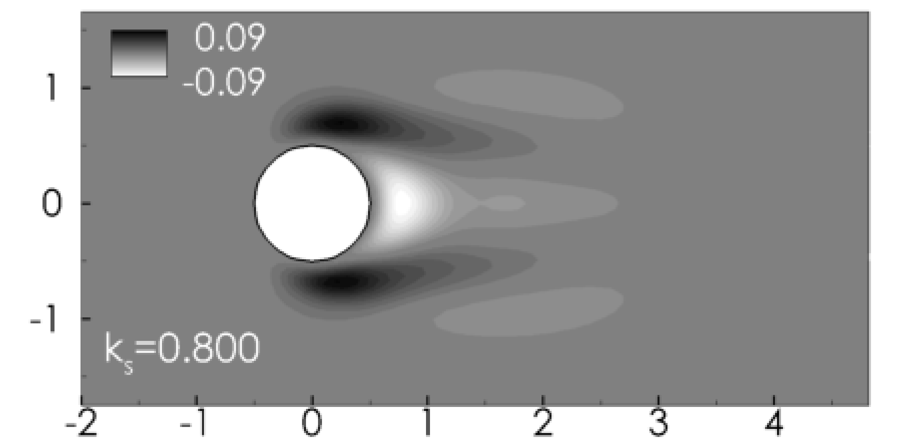
Wake Mode: growth rate decomposition

$\rho = 10$



Small k_s
destabilization due to the solid

Large k_s
destabilization due to the fluid



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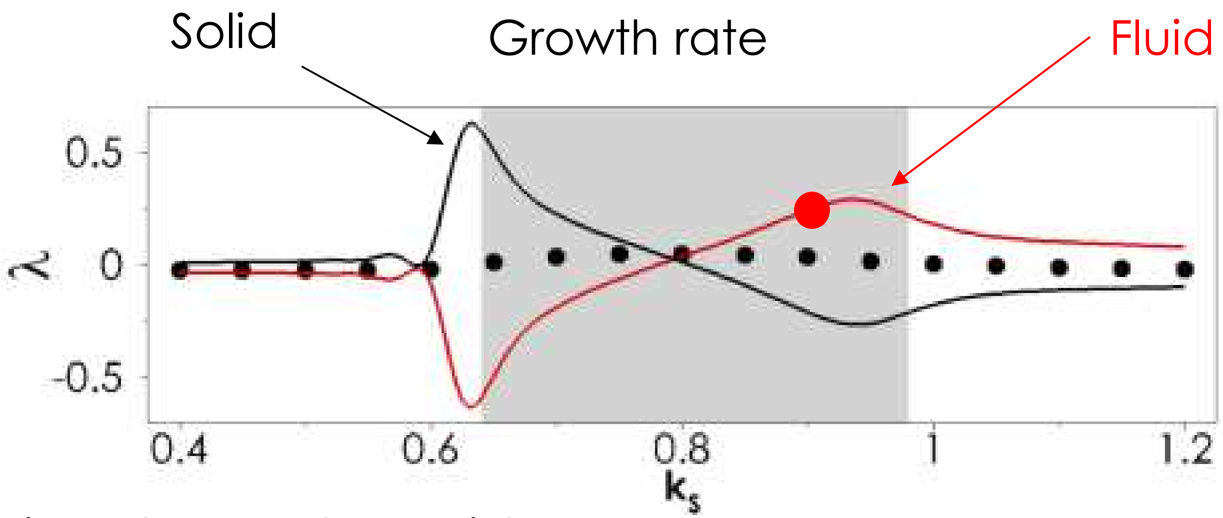
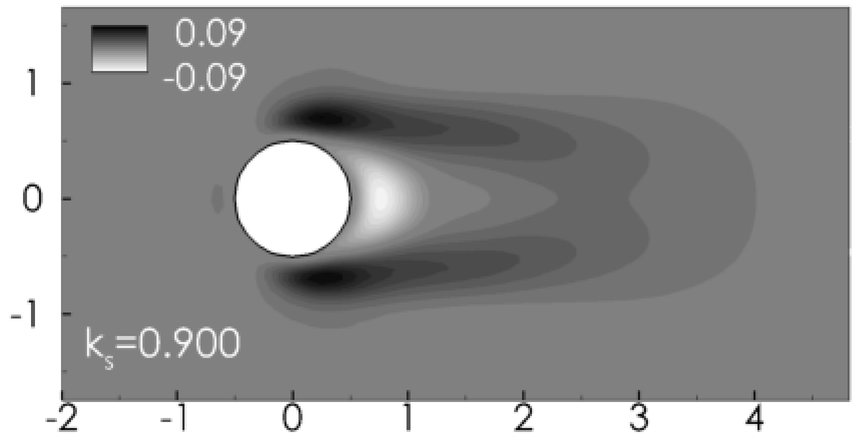
Wake Mode: growth rate decomposition

$\rho = 10$



Small k_s
destabilization due to the solid

Large k_s
destabilization due to the fluid



Conclusion & perspectives

- The adjoint-based eigenvalue decomposition enables to discuss

  - the frequency selection/ the destabilization of coupled modes
  - The localization of this process in the fluid

- Comparison with structural sensitivity (not shown here)

  Identification of the same spatial regions

- Use this decomposition in more complex fluid/structure problem

  Cylinder with a flexible splitter plate (Jean-Lou Pfister)

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Pure modes (infinite mass ratio)

	Direct	Adjoint
	$\sigma \begin{pmatrix} q_f \\ q_s \end{pmatrix} = \begin{pmatrix} F & C_{fs} \\ 0 & S \end{pmatrix} \begin{pmatrix} q_f \\ q_s \end{pmatrix}$	$\sigma^* \begin{pmatrix} a_f \\ a_s \end{pmatrix} = \begin{pmatrix} F^H & 0 \\ {C_{fs}}^H & S^H \end{pmatrix} \begin{pmatrix} a_f \\ a_s \end{pmatrix}$
Fluid modes	$\sigma\, q_f = F\, q_f$ $q_s = 0$	$\sigma_f^* a_f^p = F^H a_f^p$ $(\sigma_f^* I - S^H) a_s^p = C_{fs}^H a_f^p$
Solid modes	$\sigma\, q_s = S\, q_s$ $(\sigma I - F)\, q_f = C_{fs} q_s$	$\sigma_s^* a_s^p = S^H a_s^p$ $a_f^p = 0$

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Projection of coupled problem on pure fluid modes

$$(\sigma I - F)\, q_f = C_{fs}\, q_s$$

$$(\sigma I \;\; - S) q_s = \rho^{-1} C_{sf}\, q_f$$

$$a_f^{p^H}(\sigma I \;\; - F) q_f + a_s^{p^H}(\sigma I - S) q_s - a_f^{p^H} C_{fs}\;\, q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\left(\sigma^* a_f^p - F^H a_f^p\right)^H q_f + \left(\sigma^* a_s^p - S^H a_s^p - C_{fs}^H a_f^p\right)^H q_s = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$(\sigma - \sigma_f)(a_f^{p^H} q_f) + (\sigma - \sigma_f)(a_s^{p^H} q_s) = \rho^{-1} a_s^{p^H} C_{sf}\, q_f$$

$$\boxed{(\sigma - \sigma_f) = \frac{\rho^{-1} a_s^{p^H} C_{sf}\, q_f}{(a_f^{p^H} q_f + a_s^{p^H} q_s)}}$$

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Projection of coupled problem on pure solid modes

$$(\sigma\, I - F)\, q_f = C_{fs}\, q_s$$

$$(\sigma I \;\; - S) q_s = \rho^{-1} C_{sf}\, q_f$$

$$a_f^{p\,H} (\sigma\, I \;\; - F) q_f + a_s^{p\,H} (\sigma\, I - S) q_s - a_f^{p\,H} C_{fs}\;\, q_s = \rho^{-1} a_s^{p\,H} C_{sf}\, q_f$$

$$\left(\sigma^* a_f^p - F^H a_f^p\right)^H q_f + \left(\sigma^* a_s^p - S^H a_s^p - C_{fs}^H a_f^p\right)^H q_s = \rho^{-1} a_s^{p\,H} C_{sf}\, q_f$$

$$(\sigma - \sigma_s)(a_s^{p\,H} q_s) = \rho^{-1} a_s^{p\,H} C_{sf}\, q_f$$

$$\boxed{(\sigma - \sigma_s) = \frac{\rho^{-1} a_s^{p\,H} C_{sf}\, q_f}{a_s^{p\,H} q_s}}$$

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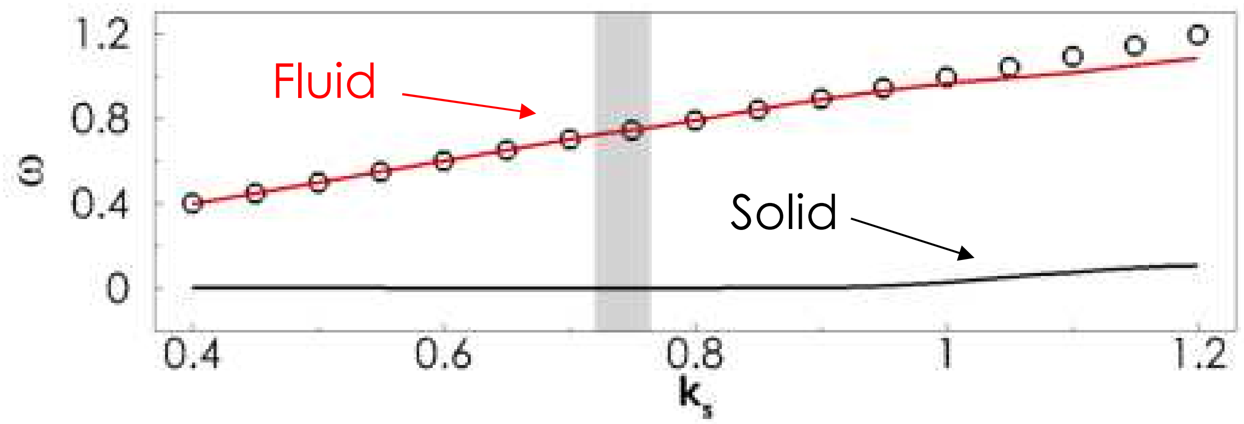
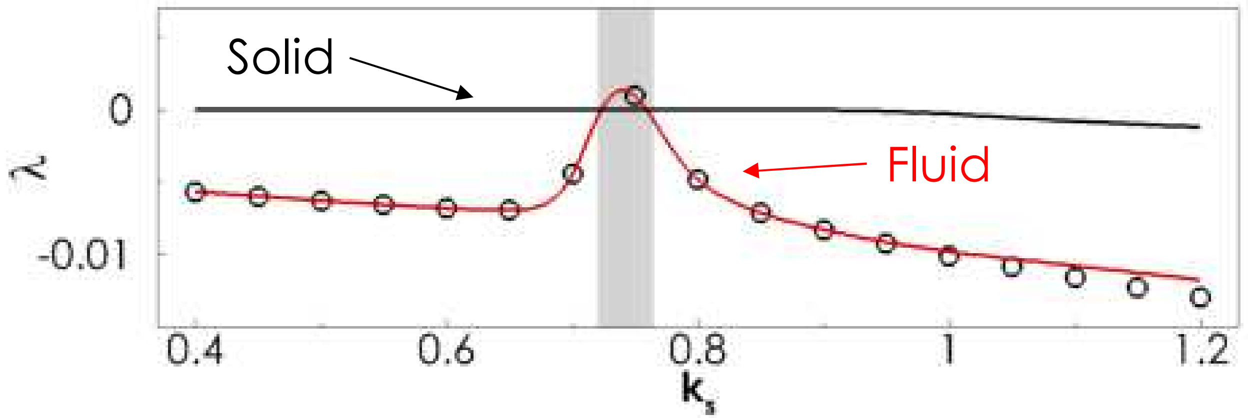
Direct-based decomposition of the unstable mode

$\rho = 200$

Frequency



Growth rate



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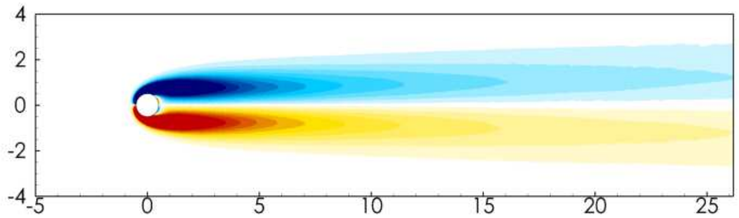
Free oscillation

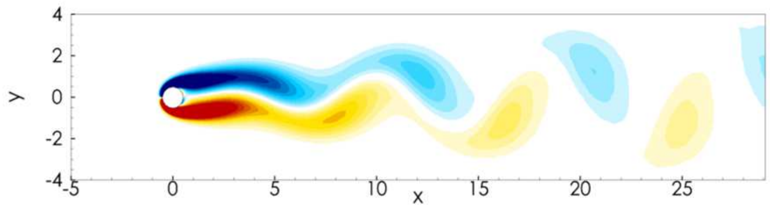
$$Re = 40\ ;\ \rho = 50$$

$\omega_s = 0.60$



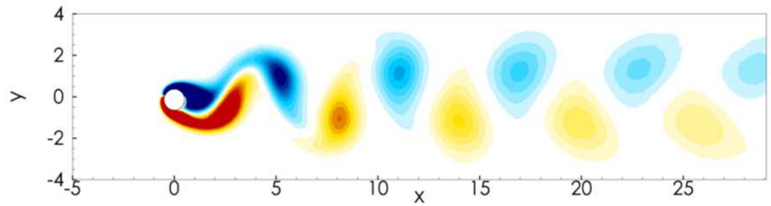
No oscillation

$\omega_s = 0.66$



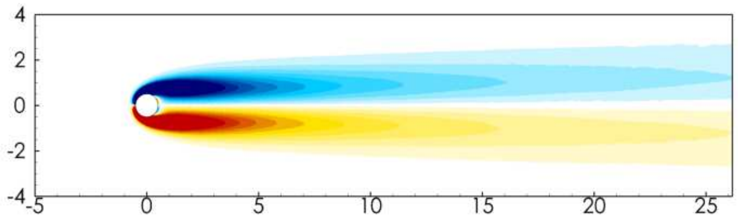
Weak oscillation

$\omega_s = 0.90$



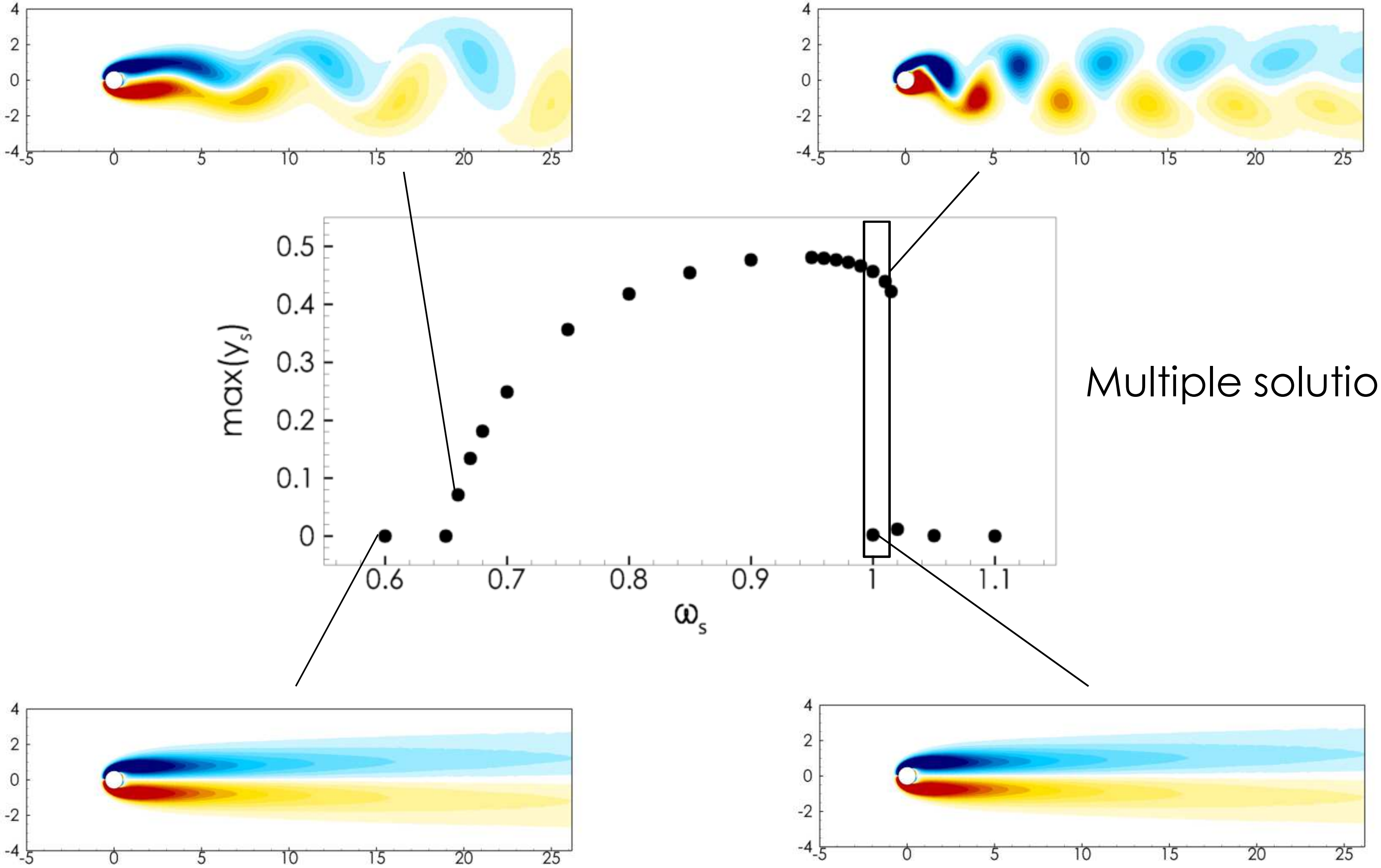
Strong oscillation

$\omega_s = 1.10$



No oscillation

Solid displacement – Fluid fields



Multiple solutions