



Prediction of flutter instability in turbulent flow based on Linear Stability Analysis

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Introduction

Fluid-Structure system exhibit various types of **instability** phenomenons



Introduction

Fluid-Structure system exhibit various types of **instability** phenomenons



- Accurate fluid-structure modeling is needed

Introduction

Turbulent flows

Streamlined body - **Attached** flow
Heaving and **pitching** involved



Potential flow modelling
Analytical methods [Theodorsen 1935]

Bluff body - **Separated** flow
Only pitching motion involved



No robust modelling
Experiments / Time-marching simulations

As previously mentioned, one of the complications associated with bridge-deck aerodynamics is that the flow will typically separate from the leading edge; hence, analytical methods, such as those presented by Theodorsen (1935) for the flow over airfoils, cannot be employed to determine the aerodynamic derivatives for bridge decks. Although numerical methods (briefly discussed later in this section) are now being proposed, at present, resort must be made to experimental techniques to obtain accurate estimates of the aerodynamic derivatives.

in [Paidoussis 2011]

Introduction

Turbulent flows

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Analytical methods [Theodorsen 1935]

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Proposed method

Linear stability analysis of the coupled fluid-structure interaction problem

with a **turbulent flow** modelled with **Reynolds Averaged Navier Stokes (RANS)** equations

Part 1 – Configuration and Modelling

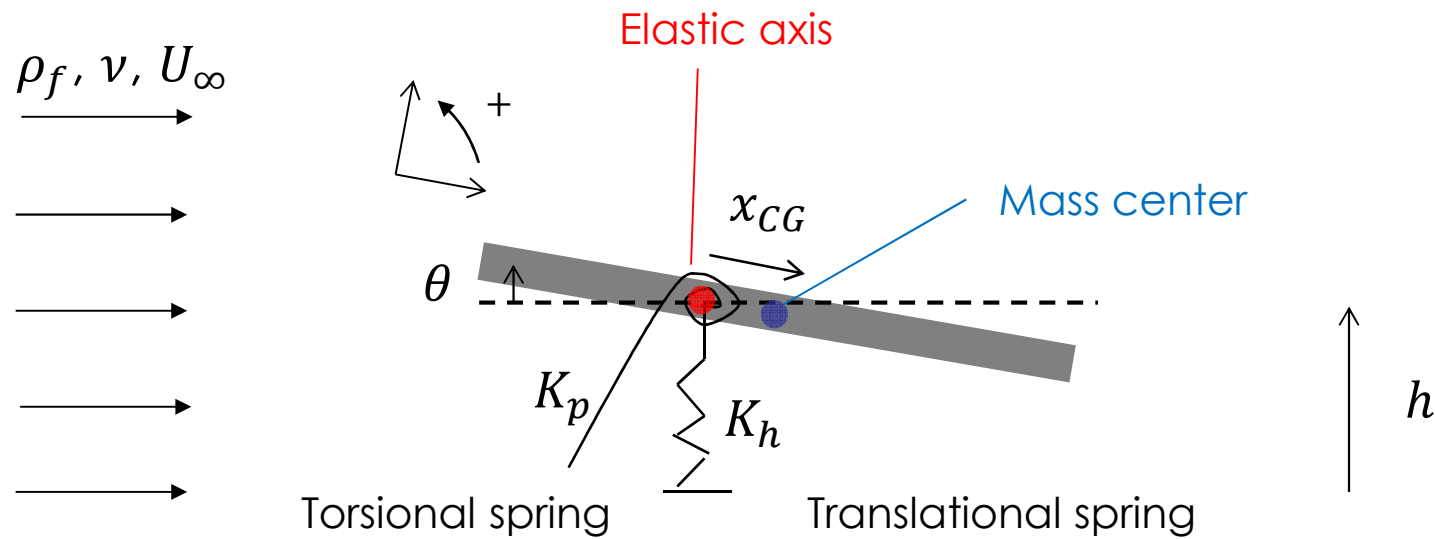
Part 2 – An **elongated plate** mounted on two springs

Part 3 – A **short plate** mounted on two springs

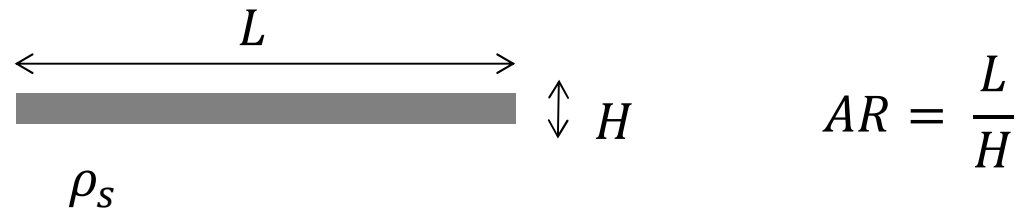
Part 1 : Configuration and Modelling

Aeroelastic configuration

Heaving and pitching rigid body in turbulent incompressible flows

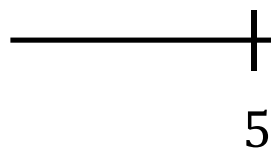


Aeroelastic configuration



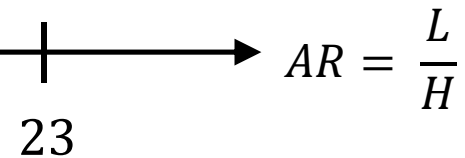
Two aspect ratios are investigated

Short plate



« Bridge type »

Elongated plate



« Airfoil type »

Modelisation

Non-dimensional numbers :

Fluid parameter : $Re = \frac{LU_\infty}{\nu}$

Solid parameter :

$$\Omega = \frac{\omega_{0h}}{\omega_{0p}}$$

ω_{0h} : non-dimensional **heaving** natural frequency

$$U^* = \frac{1}{\omega_{0p}}$$

ω_{0p} : non-dimensional **pitching** natural frequency

Coupling parameter : $\tilde{m} = \frac{\rho_s}{\rho_f}$

Shape parameters : $AR = \frac{L}{H}$ $a = \frac{x_{CG}}{L}$

Modélisation

Non-dimensional numbers :

Fluid parameter :

$$Re = 2,7 \cdot 10^4$$

Solid parameter :

$$\Omega = 0,8$$

$$U^* = \frac{1}{\omega_{0p}}$$

Coupling parameter :

$$\tilde{m} = 10^4$$

Shape parameters :

$$AR = \frac{L}{H}$$

$$a = 0,08$$

Fluid-structure modelling

Fluid variables

\mathbf{q}_f

Solid variables

\mathbf{q}_s

$$\begin{aligned} \text{Fluid equation} \quad & \frac{\partial \mathbf{q}_f}{\partial t} = \mathcal{R}_f(\mathbf{q}_f) + \mathcal{C}_{fs}(\mathbf{q}_f, \mathbf{q}_s) \\ \text{Solid equation} \quad & \frac{d\mathbf{q}_s}{dt} = \mathcal{C}_{sf}(\mathbf{q}_f) + \mathcal{R}_s(\mathbf{q}_s) \end{aligned}$$

Coupled fluid-structure equations

Fluid-structure modelling

Fluid dynamics

- RANS approach
- Spalart-Allmaras turbulent model

$$\mathbf{q}_f = (u, v, p, \tilde{v})^T$$

$$\begin{aligned}\frac{\partial \mathbf{q}_f}{\partial t} &= \mathcal{R}_f(\mathbf{q}_f) + \mathbf{C}_{fs}(\mathbf{q}_f, \mathbf{q}_s) \\ \frac{d\mathbf{q}_s}{dt} &= \mathbf{C}_{sf}(\mathbf{q}_f) + \mathcal{R}_s(\mathbf{q}_s)\end{aligned}$$

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$$\frac{d\mathbf{q}_s}{dt} = \mathbf{C}_{sf}(\mathbf{q}_f) + \mathcal{R}_s(\mathbf{q}_s)$$

$$\mathbf{q}_s = (h, \theta, \dot{h}, \dot{\theta})^T$$

Solid dynamics

- 2-DOF rigid body

$$\ddot{h} + \omega_{0h}^2 h + a \ddot{\theta} = 0$$

$$\ddot{\theta} + \omega_{0p}^2 \theta + f(a) \ddot{h} = 0$$

Fluid-structure modelling

Fluid dynamics

- RANS approach
- Spalart-Allmaras turbulent model

$$\mathbf{q}_f = (u, v, p, \tilde{v})^T$$

Solid-to-fluid coupling

- Interface conditions
- Non-inertial volumic terms [Mougin et al. 2002]

$$\frac{\partial \mathbf{q}_f}{\partial t} = \mathcal{R}_f(\mathbf{q}_f) + \mathbf{C}_{fs}(\mathbf{q}_f, \mathbf{q}_s)$$
$$\frac{d\mathbf{q}_s}{dt} = \mathbf{C}_{sf}(\mathbf{q}_f) + \mathcal{R}_s(\mathbf{q}_s)$$

$$\mathbf{q}_s = (h, \theta, \dot{h}, \dot{\theta})^T$$

Fluid-to-solid coupling

- Fluid force - Lift
- Fluid moment

Solid dynamics

- 2-DOF rigid body

$$\ddot{h} + \omega_{0h}^2 h + a \ddot{\theta} = 0$$

$$\ddot{\theta} + \omega_{0p}^2 \theta + f(a) \dot{h} = 0$$

Linear stability analysis

Two classical ingredients :

Perturbation decomposition

$$\begin{array}{l} \text{Fluid} \\ \text{Solid} \end{array} \quad \begin{array}{l} \mathbf{q}_f(\mathbf{x}, t) = \boxed{\mathbf{Q}_f(\mathbf{x})} + \boxed{\epsilon \mathbf{q}'_f(\mathbf{x}, t)} \\ \mathbf{q}_s(t) = \boxed{\mathbf{0}} + \boxed{\epsilon \mathbf{q}'_s(t)} \end{array}$$

base state perturbation

Modal decomposition

$$\begin{array}{l} \text{Fluid} \\ \text{Solid} \end{array} \quad \begin{array}{l} \mathbf{q}'_f(\mathbf{x}, t) = \hat{\mathbf{q}}_f(\mathbf{x}) e^{\sigma t} + c.c \\ \mathbf{q}'_s(t) = \hat{\mathbf{q}}_s e^{\sigma t} + c.c \end{array}$$

Linear stability analysis

Growth rate

$$\lambda = \Re[\sigma]$$

Frequency

$$\omega = \Im[\sigma]$$

Fluid-solid mode

$$(\hat{\mathbf{q}}_f, \hat{\mathbf{q}}_s)$$

Coupled eigenvalue problem

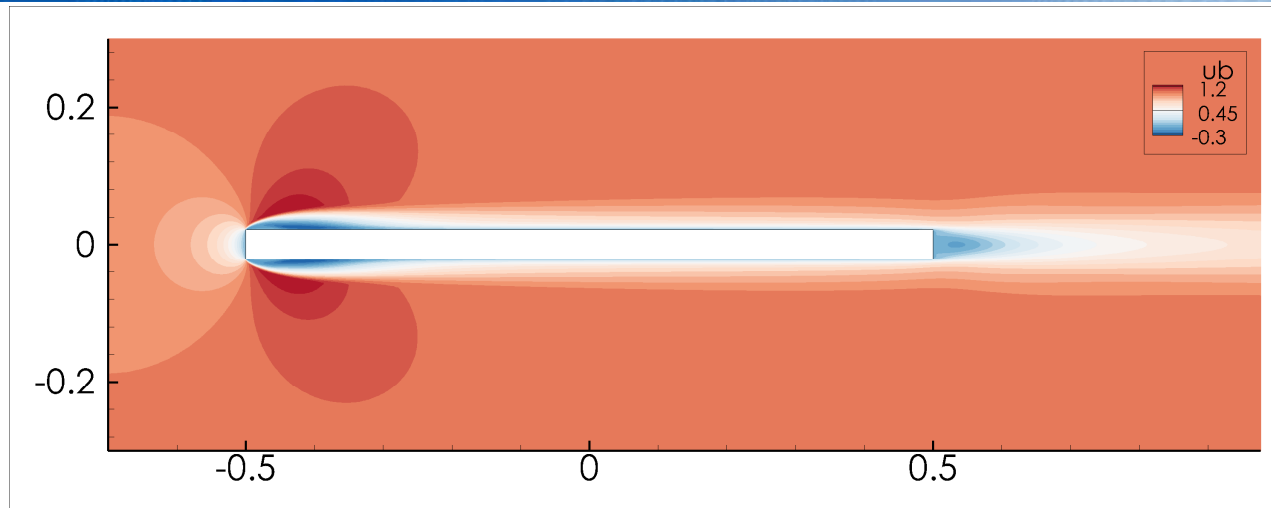
$$\sigma B \begin{pmatrix} \hat{\mathbf{q}}_f \\ \hat{\mathbf{q}}_s \end{pmatrix} = \underbrace{\begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}}_{\text{Fluid-structure Jacobian matrix}} \begin{pmatrix} \hat{\mathbf{q}}_f \\ \hat{\mathbf{q}}_s \end{pmatrix}$$

Mass matrix
(spatial discretization)

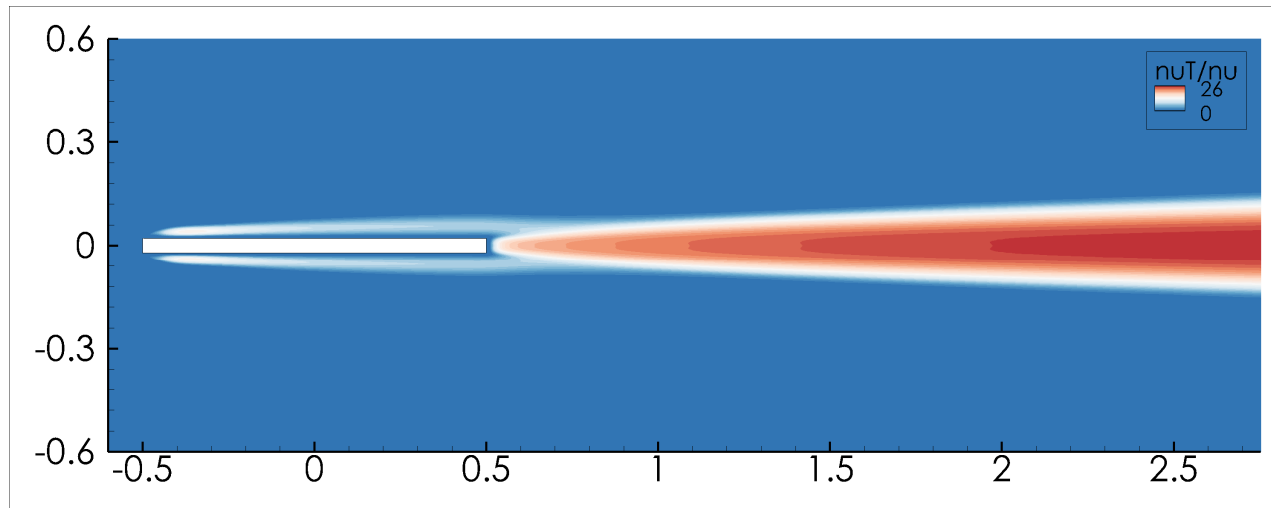
Fluid-structure
Jacobian matrix

Part 2 : An **elongated plate** mounted on two springs ($AR=23$)

Steady base flow

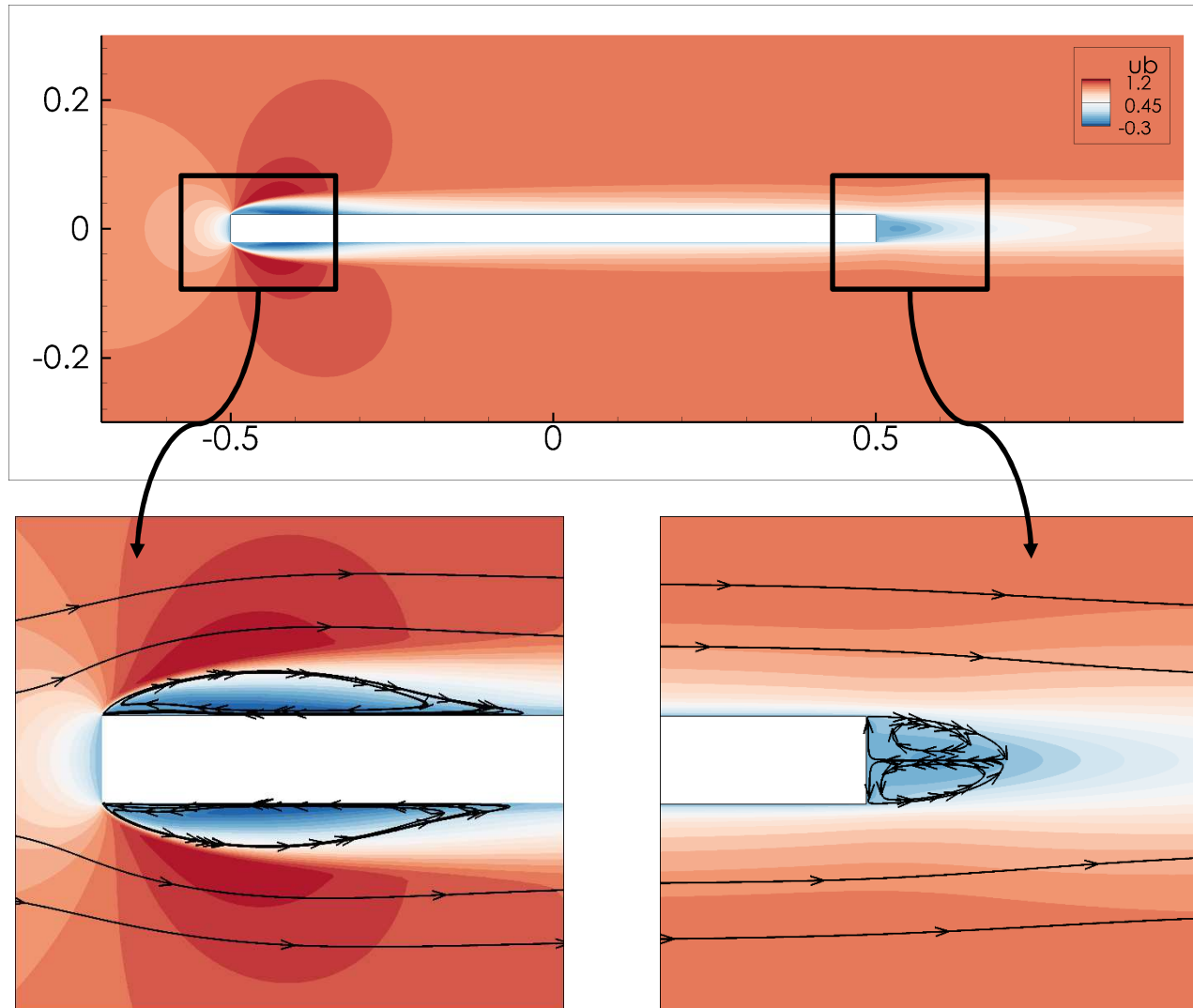


Axial velocity ($Re = 27500$)



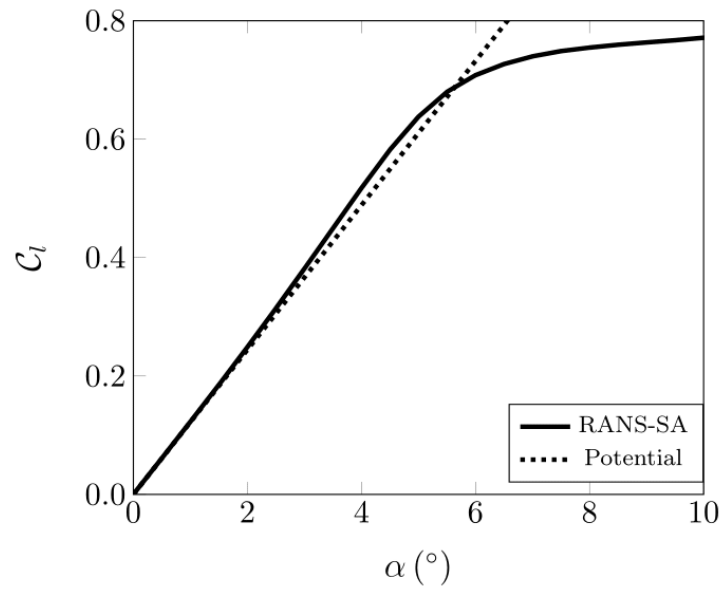
Turbulent to kinematic viscosity ratio

Steady base flow

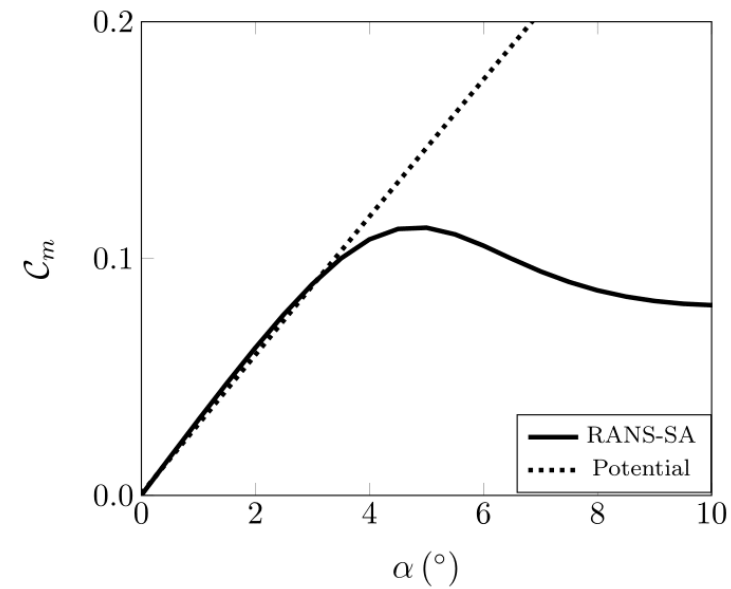


Steady base flow

Lift coefficient



Moment coefficient



Present study

Potential flows

$$\left. \frac{\partial C_l}{\partial \alpha} \right|_{\alpha=0}$$

7,0

7,0

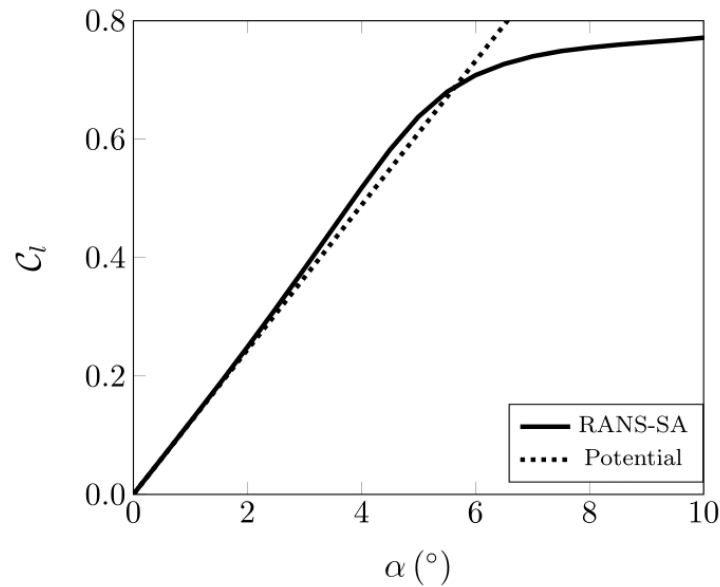
$$\left. \frac{\partial C_m}{\partial \alpha} \right|_{\alpha=0}$$

1,8

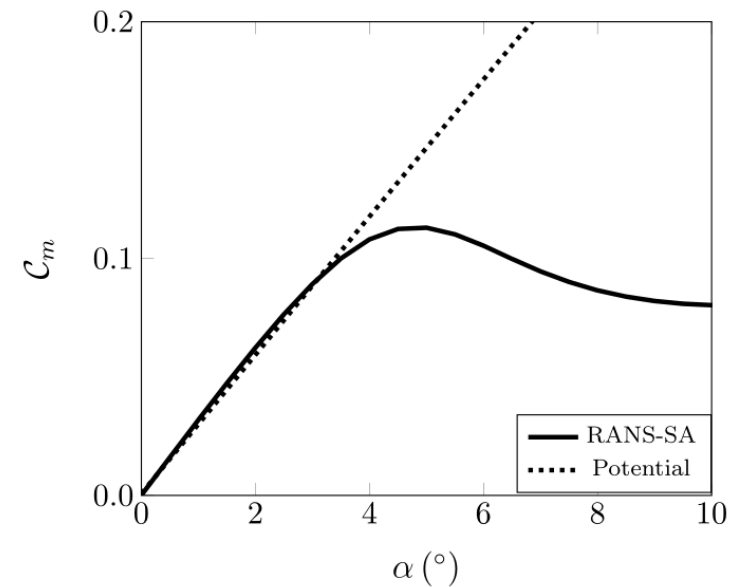
1,7

Steady base flow

Lift coefficient



Moment coefficient



Present study

Potential flows

$$\left. \frac{\partial C_l}{\partial \alpha} \right|_{\alpha=0}$$

7,0

7,0

$$\left. \frac{\partial C_m}{\partial \alpha} \right|_{\alpha=0}$$

1,8

1,7



Small influence of detached areas on slopes at $\alpha = 0^\circ$

Linear Stability Analysis

Coupled fluid-structure system

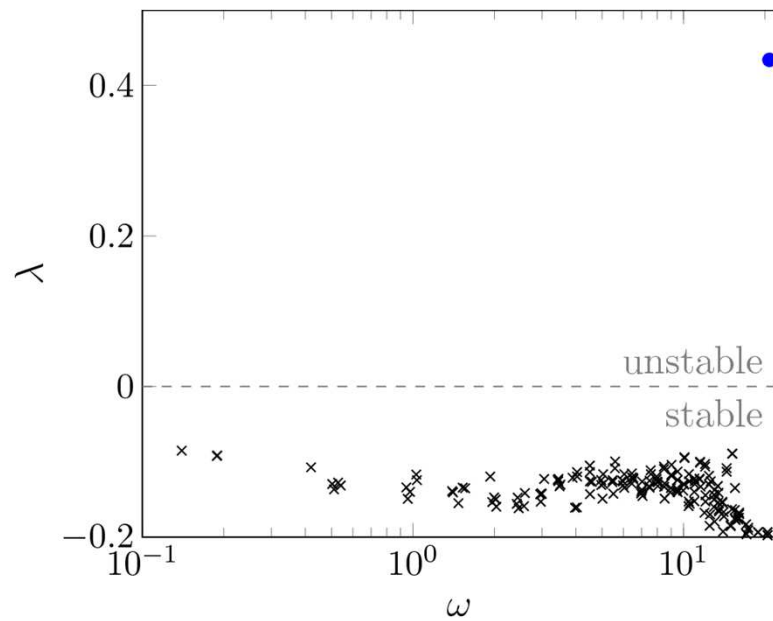
$$\sigma B \hat{q} = A \hat{q}$$

$$A = \begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}$$

Linear Stability Analysis

Uncoupled fluid system

$$\sigma_f \mathbf{B}_{ff} \hat{\mathbf{q}}_f = \mathbf{A}_{ff} \hat{\mathbf{q}}_f$$

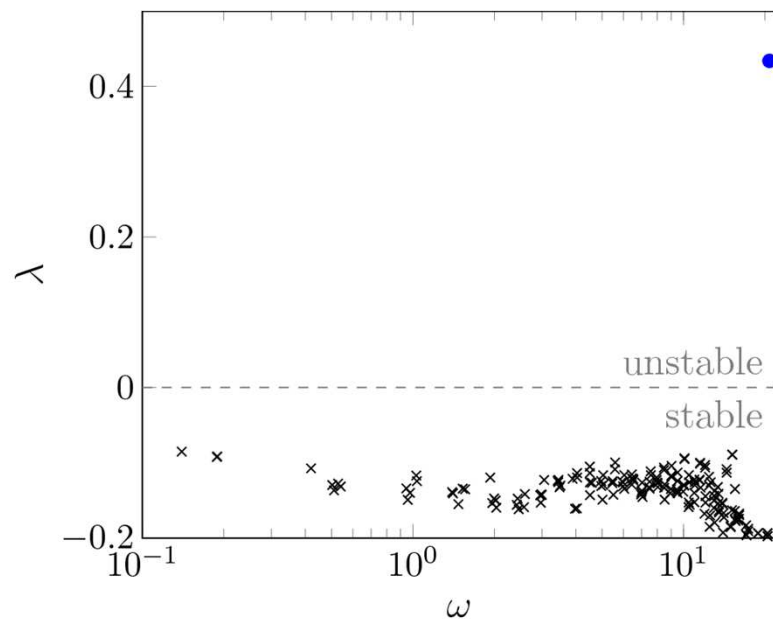


Fluid system spectrum

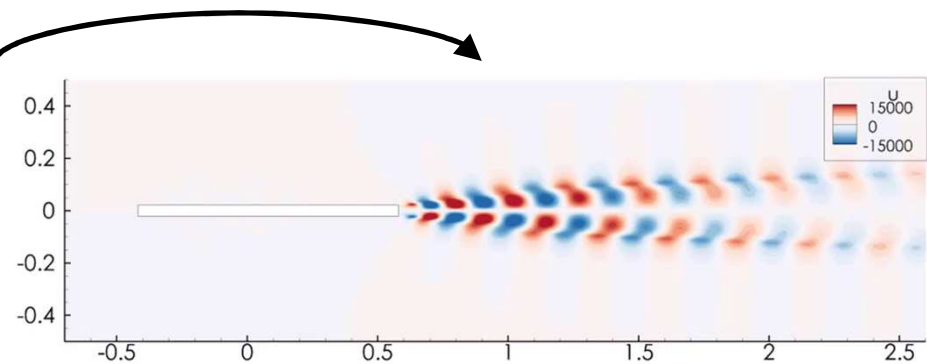
Linear Stability Analysis

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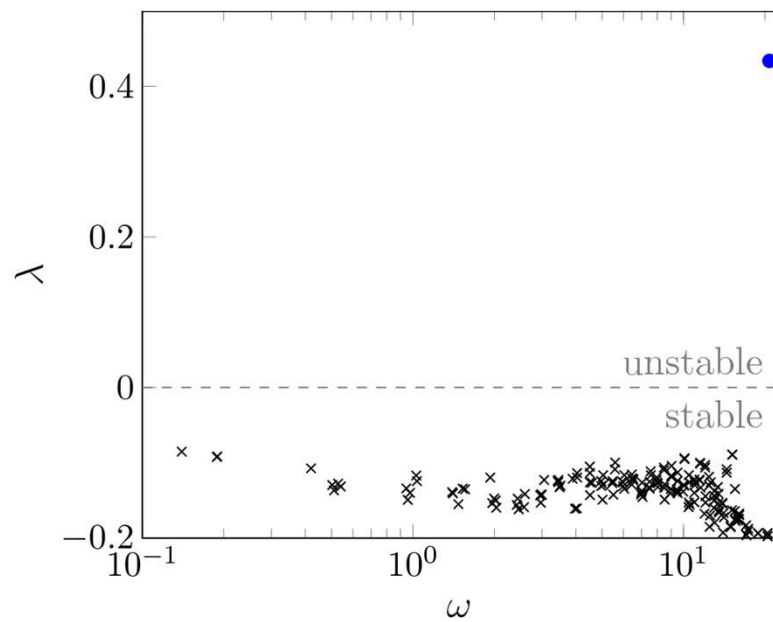
Fluid system spectrum



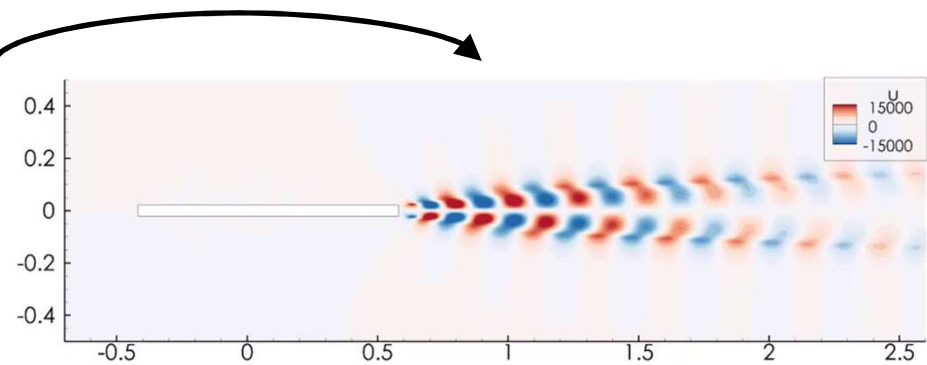
Linear Stability Analysis

Uncoupled fluid system

$$\sigma_f \mathbf{B}_{ff} \hat{\mathbf{q}}_f = \mathbf{A}_{ff} \hat{\mathbf{q}}_f$$



Fluid system spectrum

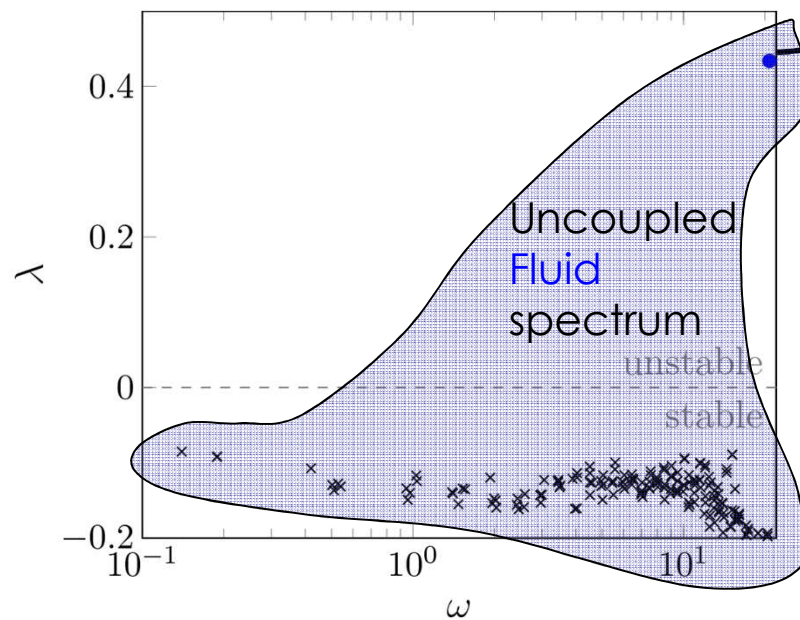


$$\omega = 20,7 \quad (St_H \approx 0,14)$$

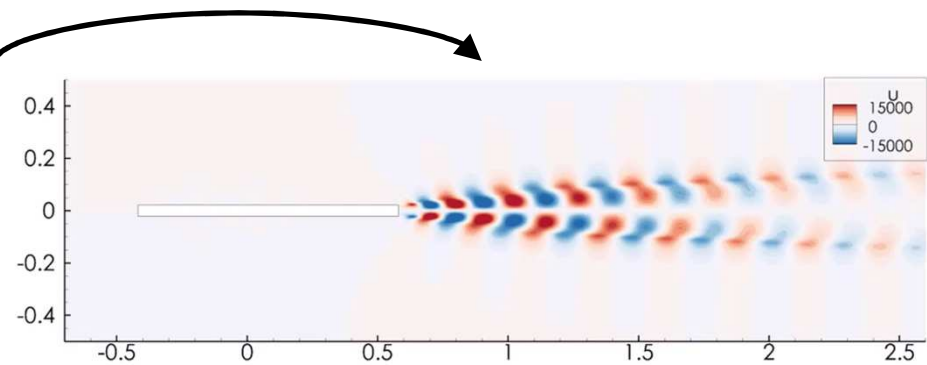
Linear Stability Analysis

Uncoupled fluid system

$$\sigma_f \mathbf{B}_{ff} \hat{\mathbf{q}}_f = \mathbf{A}_{ff} \hat{\mathbf{q}}_f$$



Fluid system spectrum

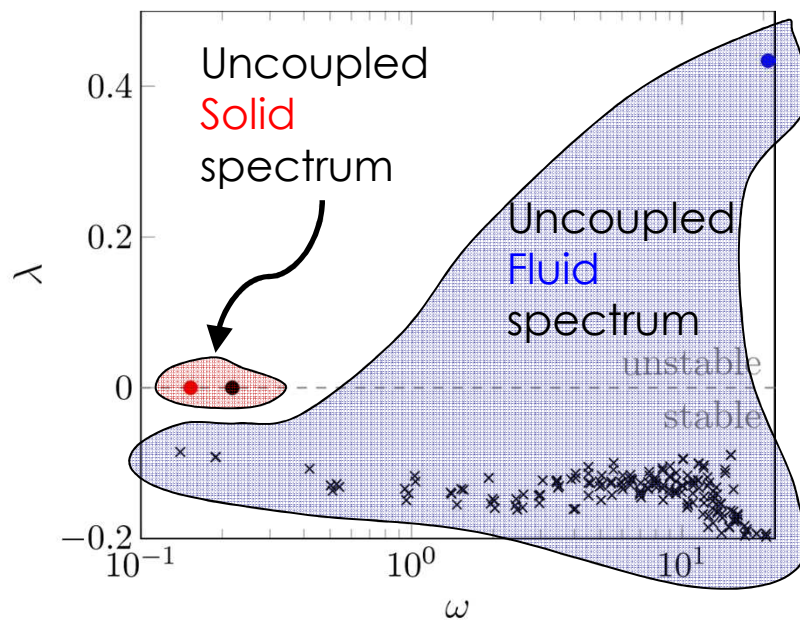


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Linear Stability Analysis

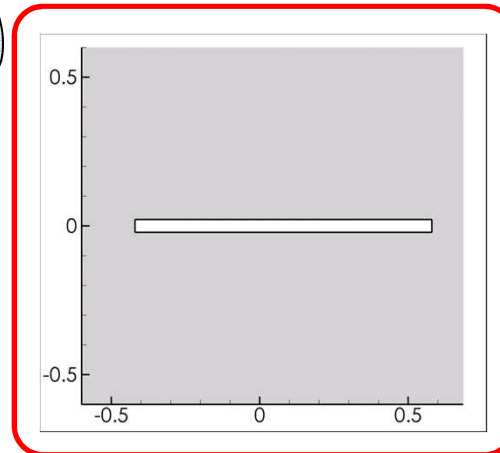
Uncoupled solid system

$$\sigma_s \mathbf{B}_{ss} \hat{\mathbf{q}}_s = \mathbf{A}_{ss} \hat{\mathbf{q}}_s$$



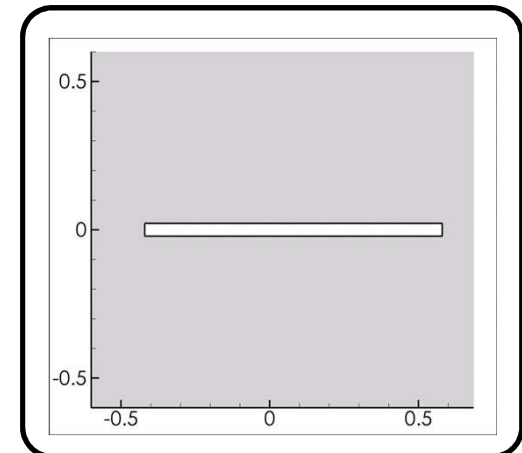
$$U^* = 5$$

Low-frequency mode



$$\frac{\omega}{\omega_{0p}} = 0,76$$

High-frequency mode

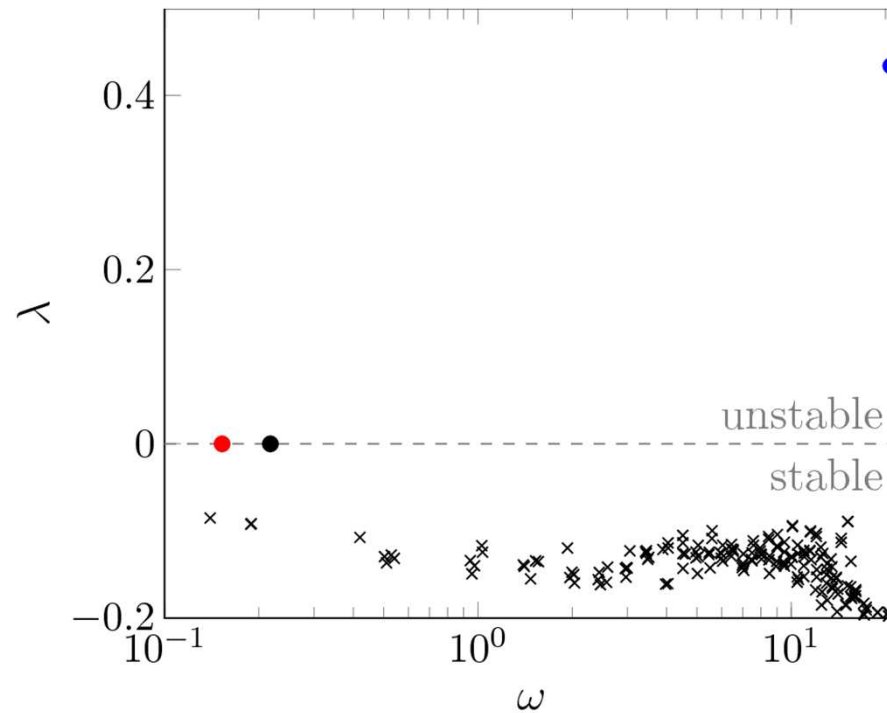


$$\frac{\omega}{\omega_{0p}} = 1,08$$

➤ **Inertial coupling** between heaving and pitching

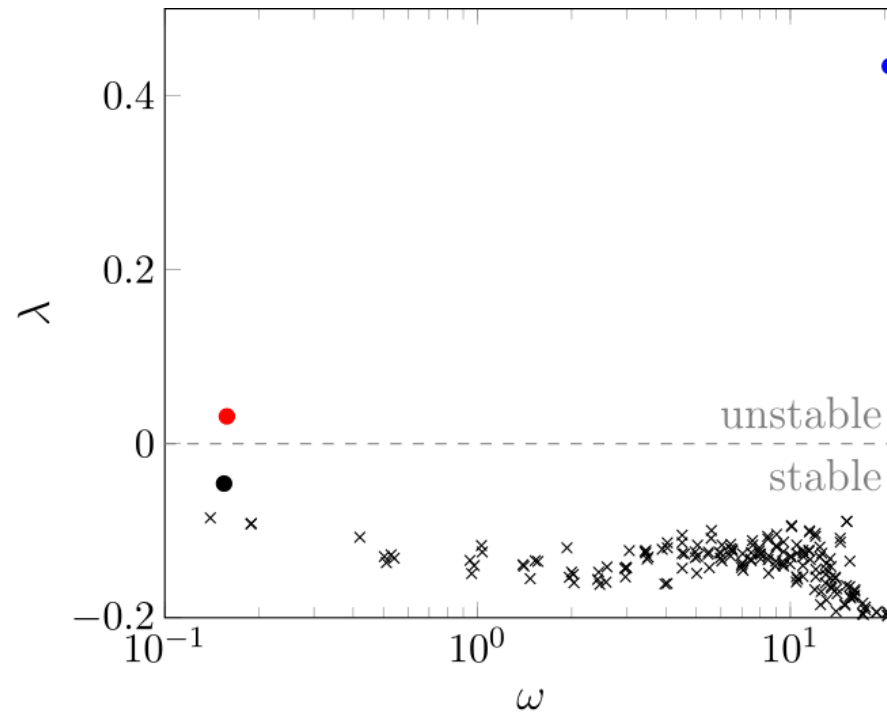
Linear Stability Analysis

Uncoupled fluid and solid systems



Linear Stability Analysis

Coupled fluid-structure system

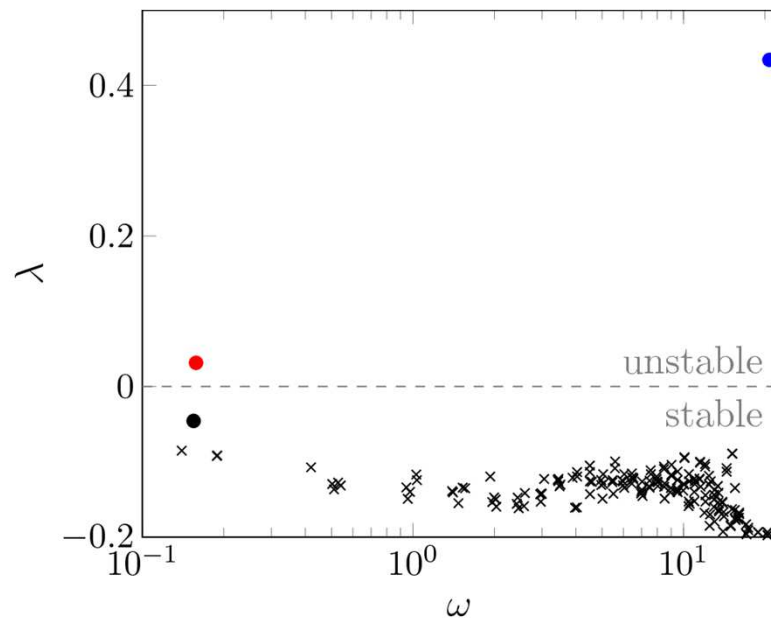


Linear Stability Analysis

Coupled fluid-structure system

$$\sigma B \hat{q} = A \hat{q}$$

$$A = \begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}$$



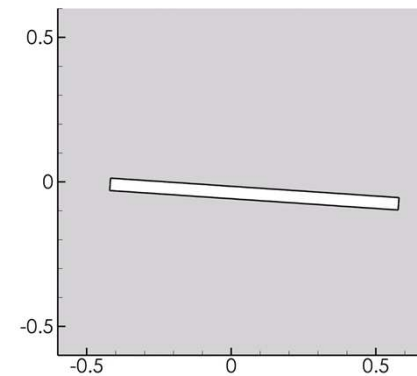
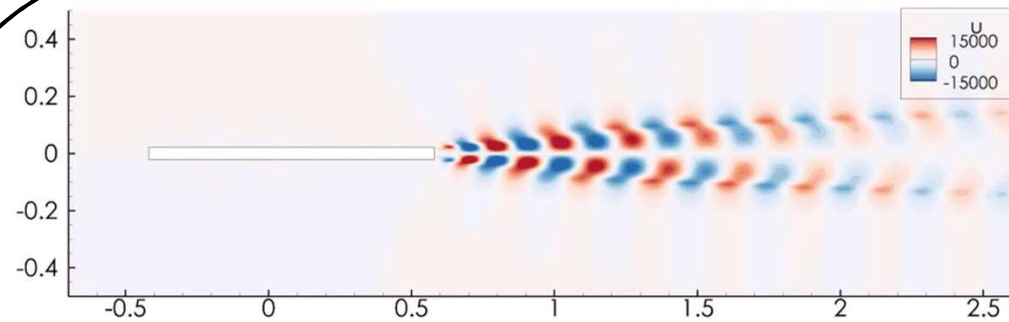
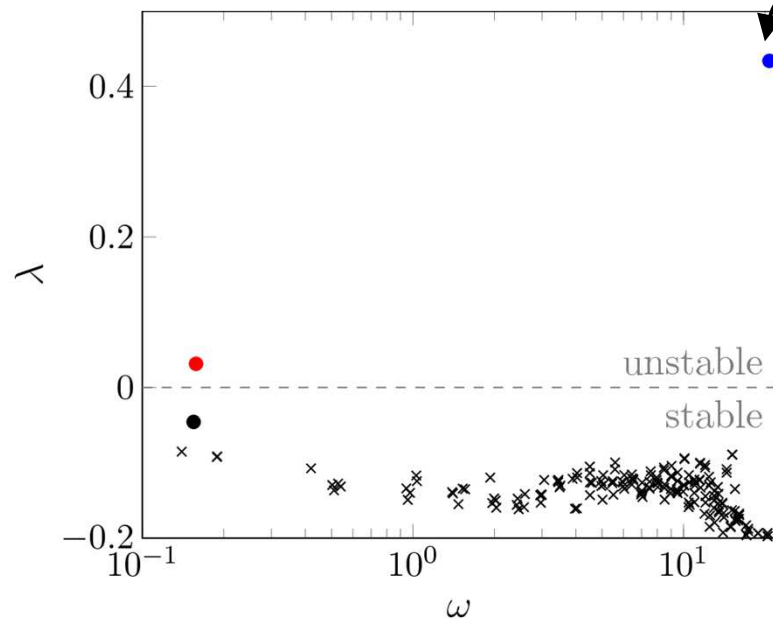
$$U^* = 5 \quad \tilde{m} = 10^4$$

Linear Stability Analysis

Coupled fluid-structure system

$$\sigma \hat{q} = A \hat{q}$$

$$A = \begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}$$



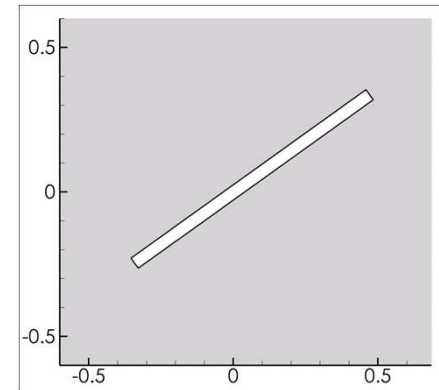
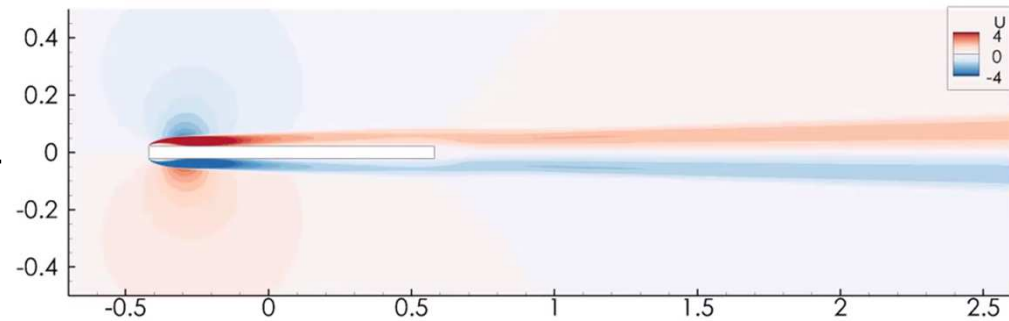
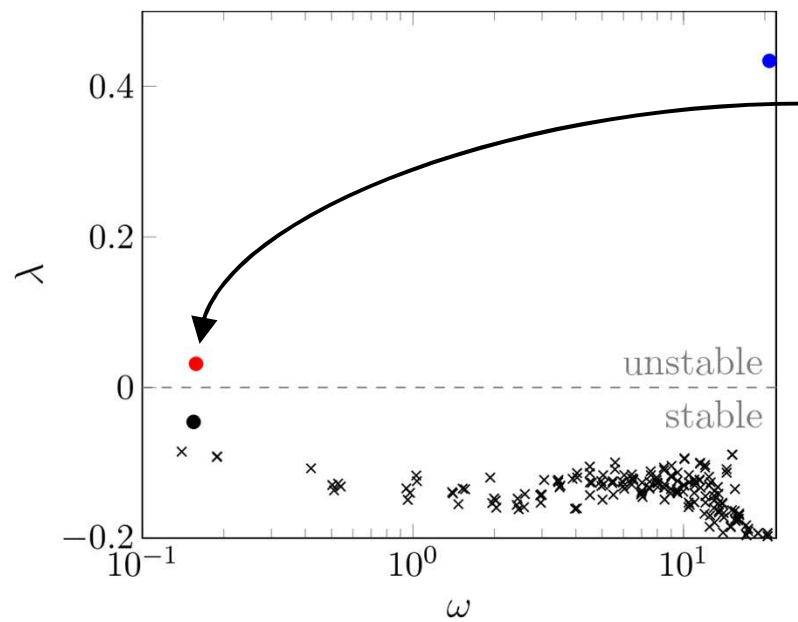
Mode kinetic energy : $E_s = 1$ $E_f = 10^8$

Linear Stability Analysis

Coupled fluid-structure system

$$\sigma \hat{q} = A \hat{q}$$

$$A = \begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}$$



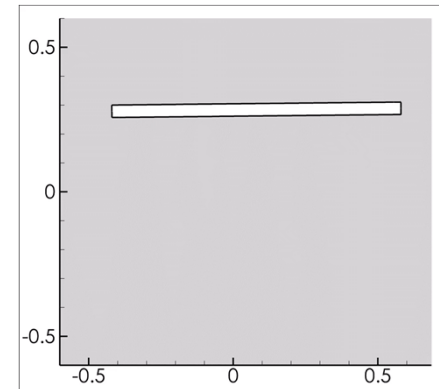
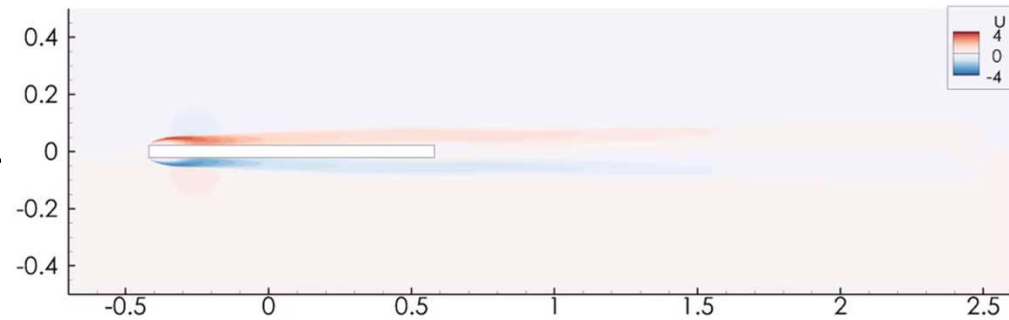
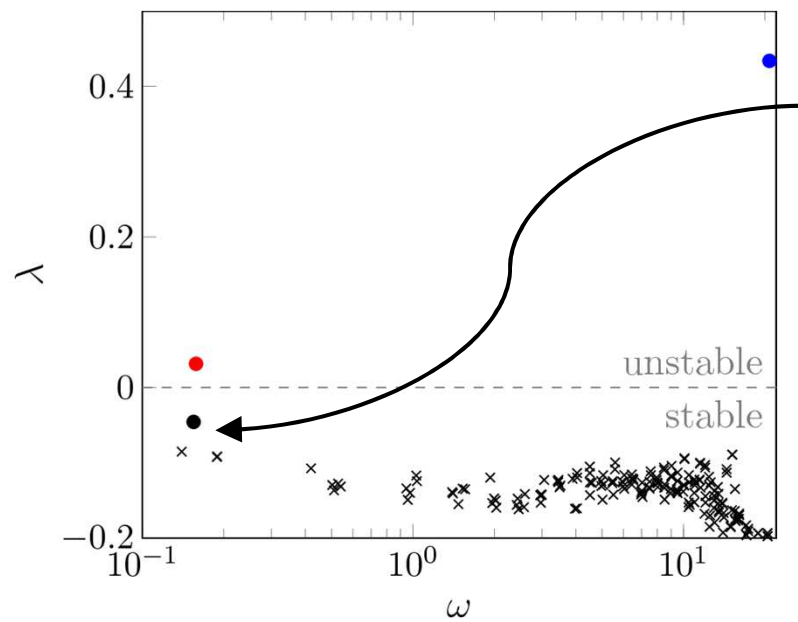
Mode kinetic energy : $E_s = 1$ $E_f = 4,1$

Linear Stability Analysis

Coupled fluid-structure system

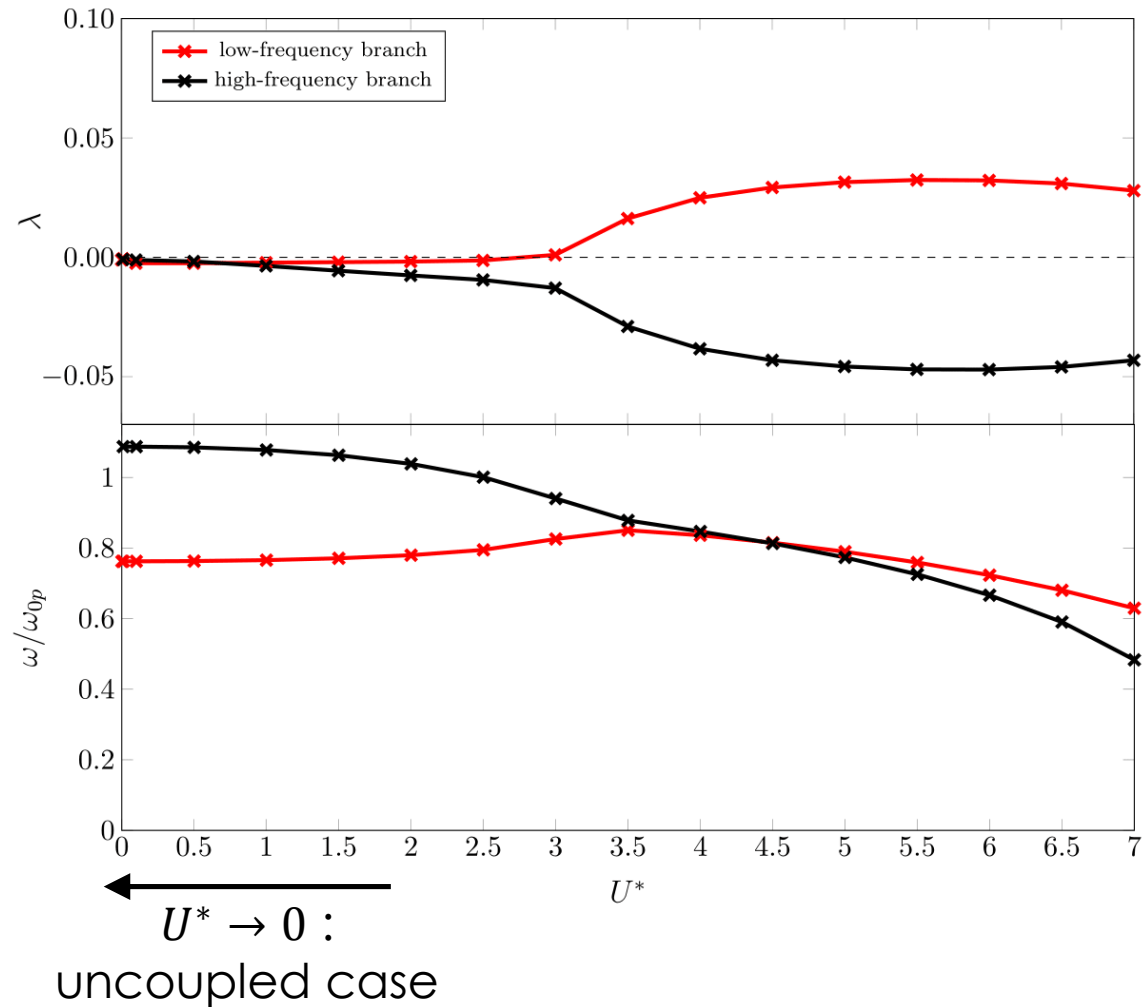
$$\sigma \hat{q} = A \hat{q}$$

$$A = \begin{pmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{pmatrix}$$



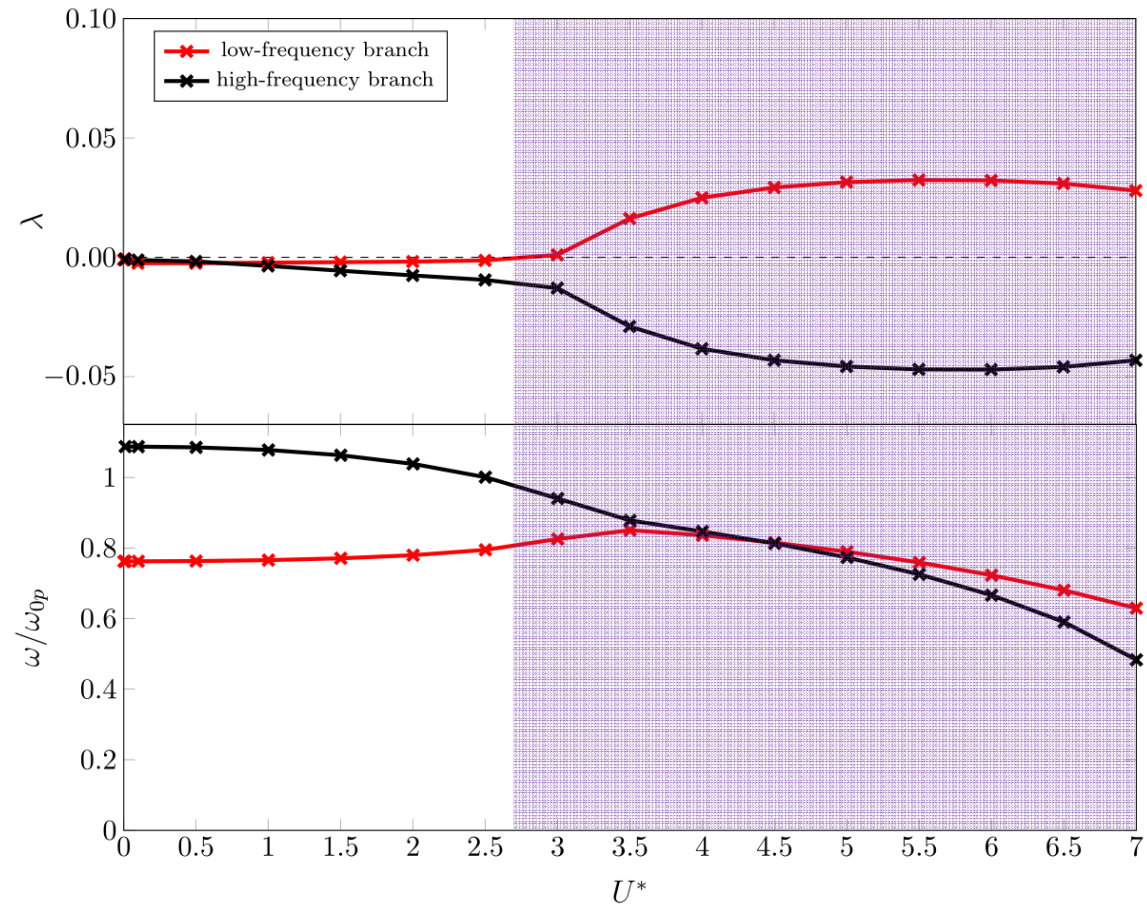
Mode kinetic energy : $E_s = 1$ $E_f = 1,1$

Aeroelastic instabilities of a 2-DOF elongated plate



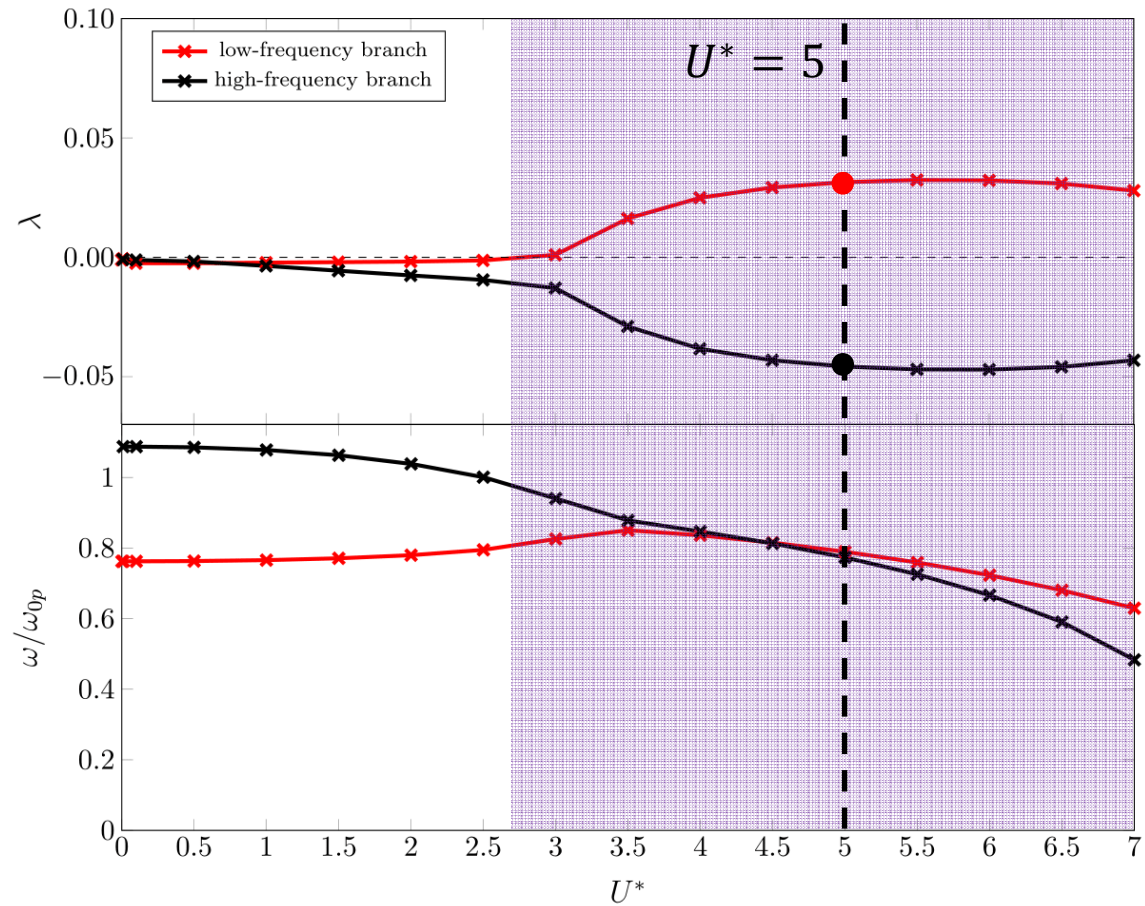
Aeroelastic instabilities of a 2-DOF elongated plate

Coupled mode flutter



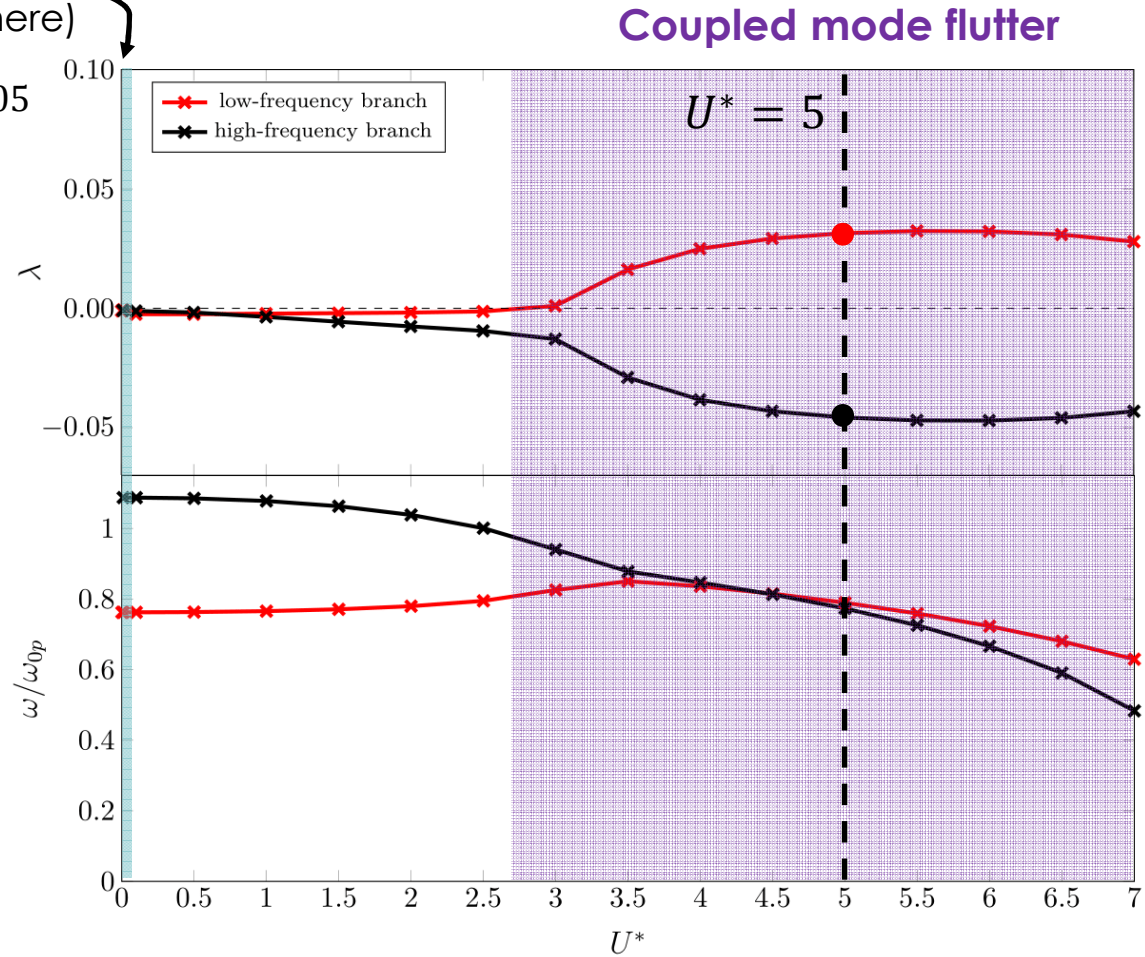
Aeroelastic instabilities of a 2-DOF elongated plate

Coupled mode flutter

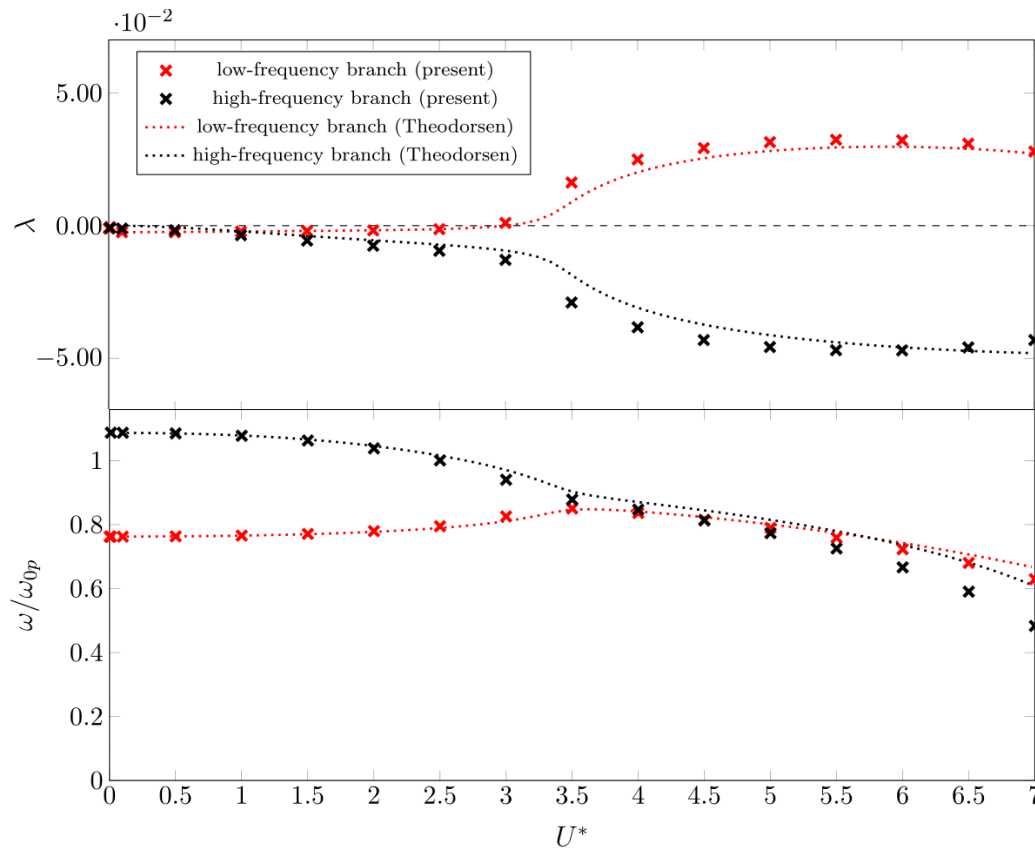


Aeroelastic instabilities of a 2-DOF elongated plate

VIV
(not investigated here)

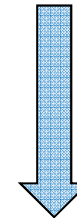
$$U_{VIV}^* \approx \frac{1}{2\pi St_H AR} \approx 0,05$$


Validation



	U_{crit}^*	$\frac{\omega}{\omega_{0p}}$
Present study	2,9	0,81
Theodorsen	3,0	0,81

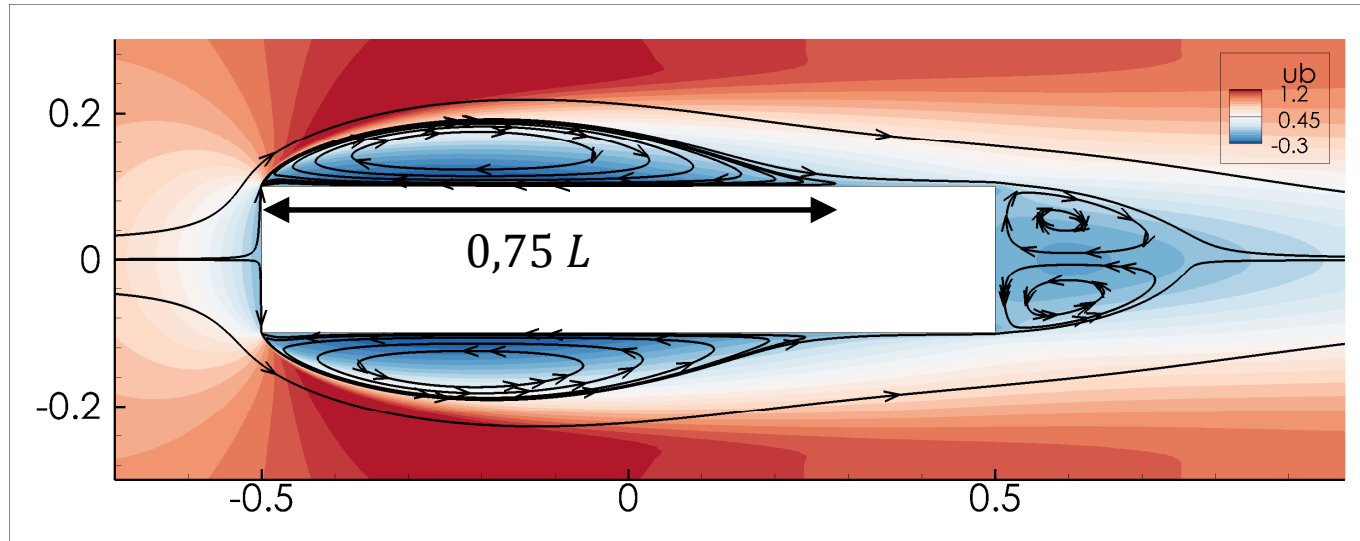
Comparison to Theodorsen model



- Modelisation **validated against** classical **Theodorsen** flutter theory
- **Maginal role of detached areas** on instability thresholds

Part 3 : An **short plate** mounted on two springs ($AR=5$)

Steady Base Flow

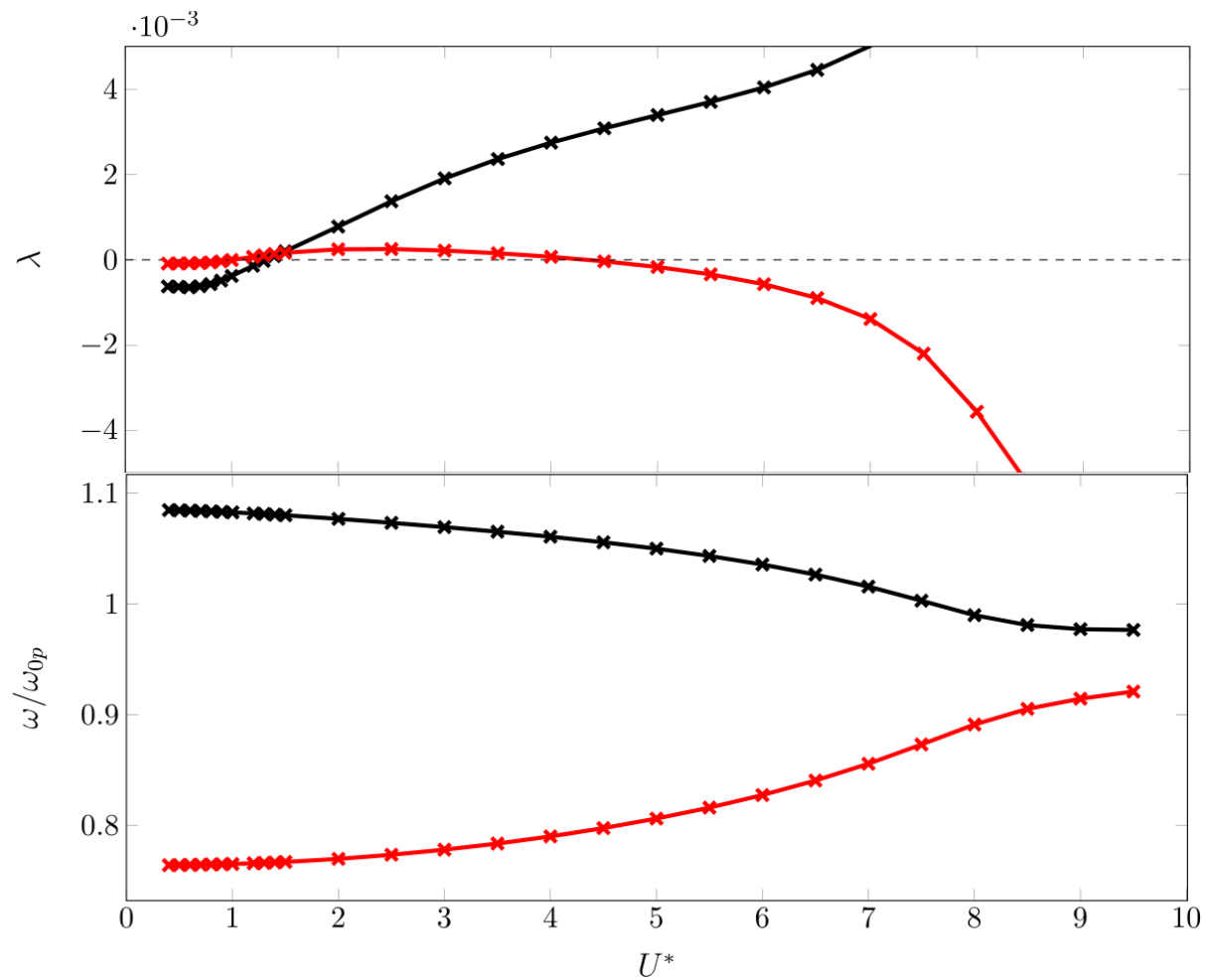


	$\frac{\partial C_l}{\partial \alpha} \Big _{\alpha=0}$	$\frac{\partial C_m}{\partial \alpha} \Big _{\alpha=0}$
Present study	9,15	0,95
Potential flows	8,4	1,7

Large leading-edge **detached areas**

- Non-negligible effect on steady aerodynamic coefficient

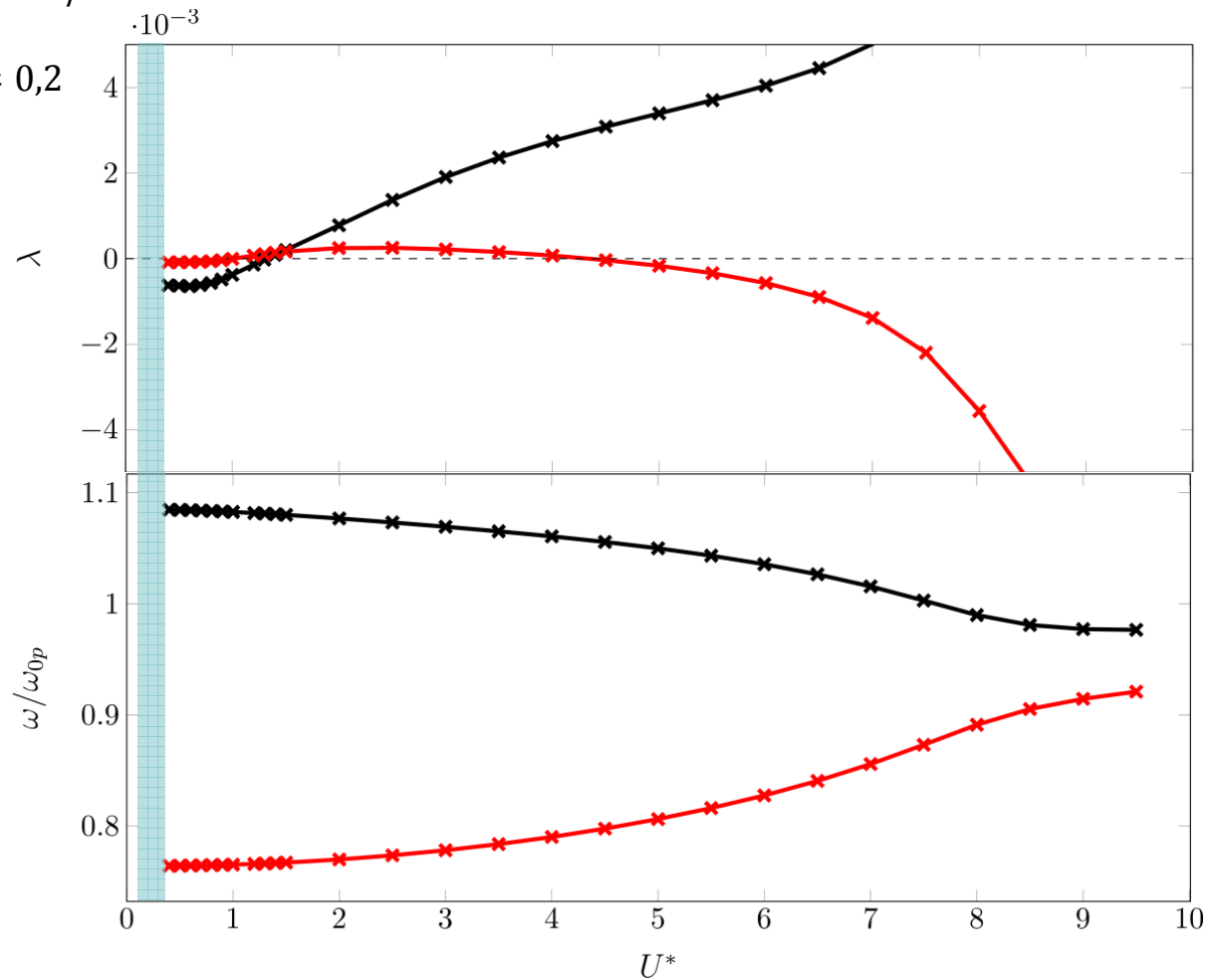
Aeroelastic instabilities of a 2-DOF short plate



Aeroelastic instabilities of a 2-DOF short plate

VIV
(not investigated here)

$$U_{VIV}^* \approx \frac{1}{2\pi St_H AR} \approx 0,2$$

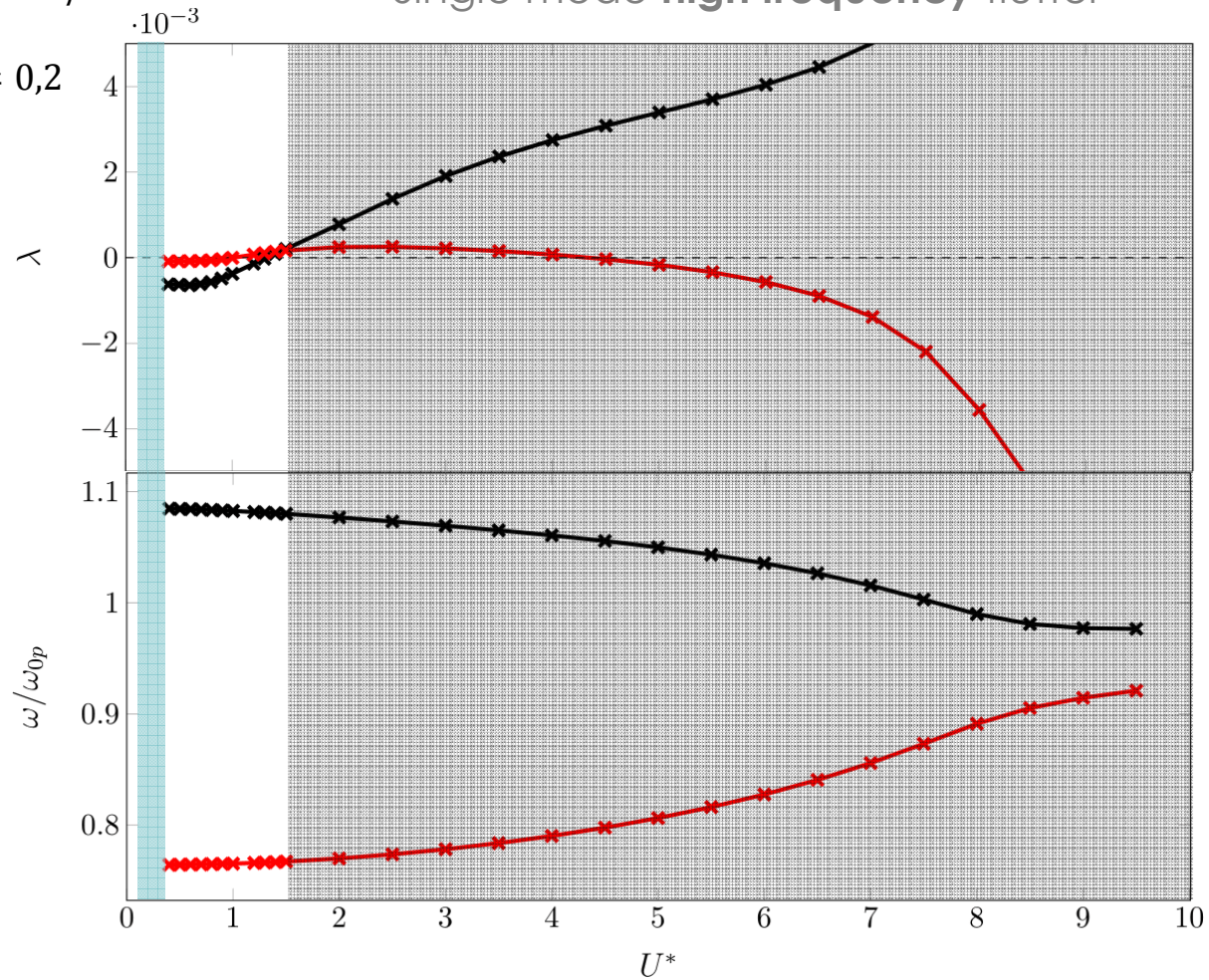


Aeroelastic instabilities of a 2-DOF short plate

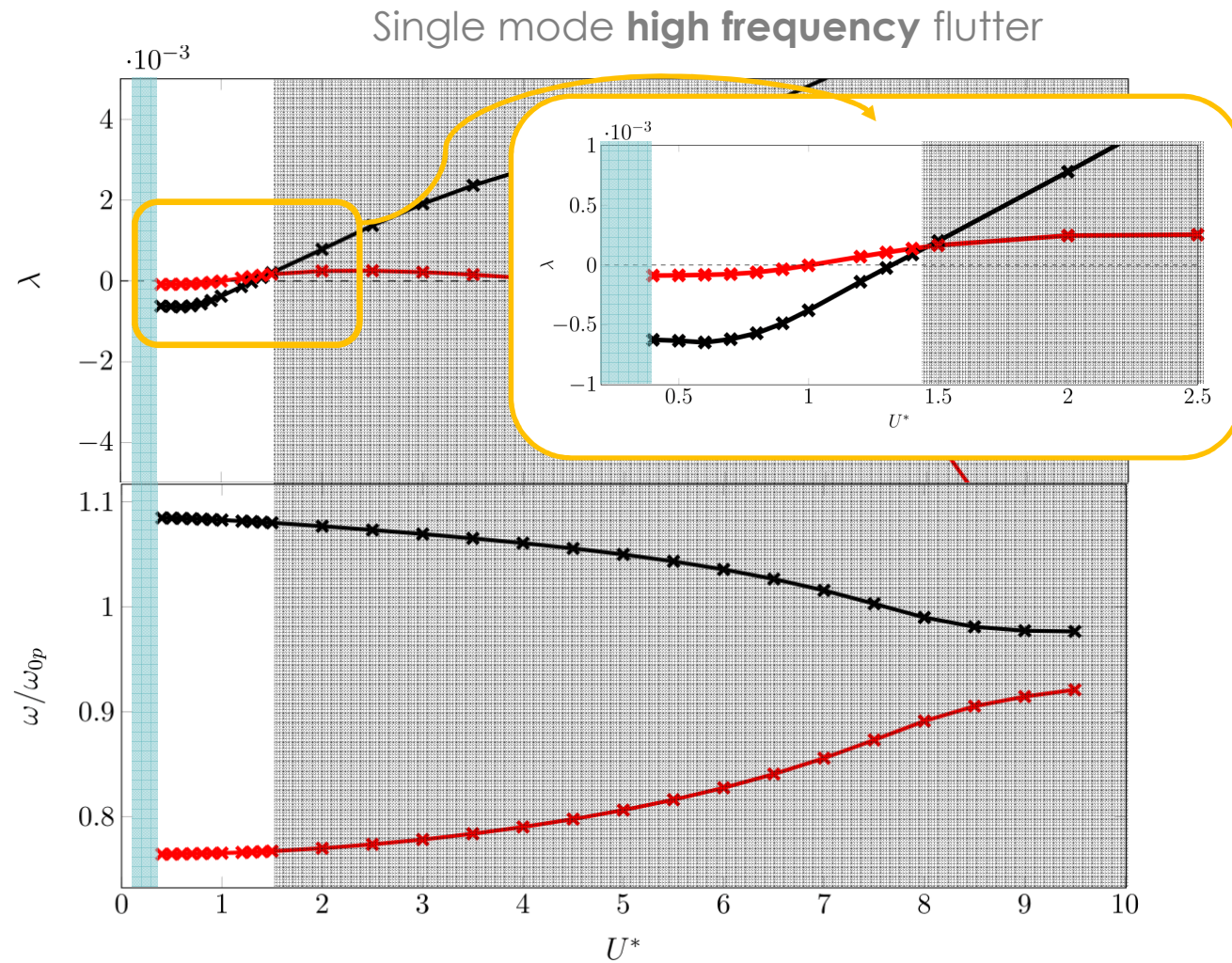
VIV
(not investigated here)

$$U_{VIV}^* \approx \frac{1}{2\pi St_H AR} \approx 0,2$$

Single mode **high frequency** flutter



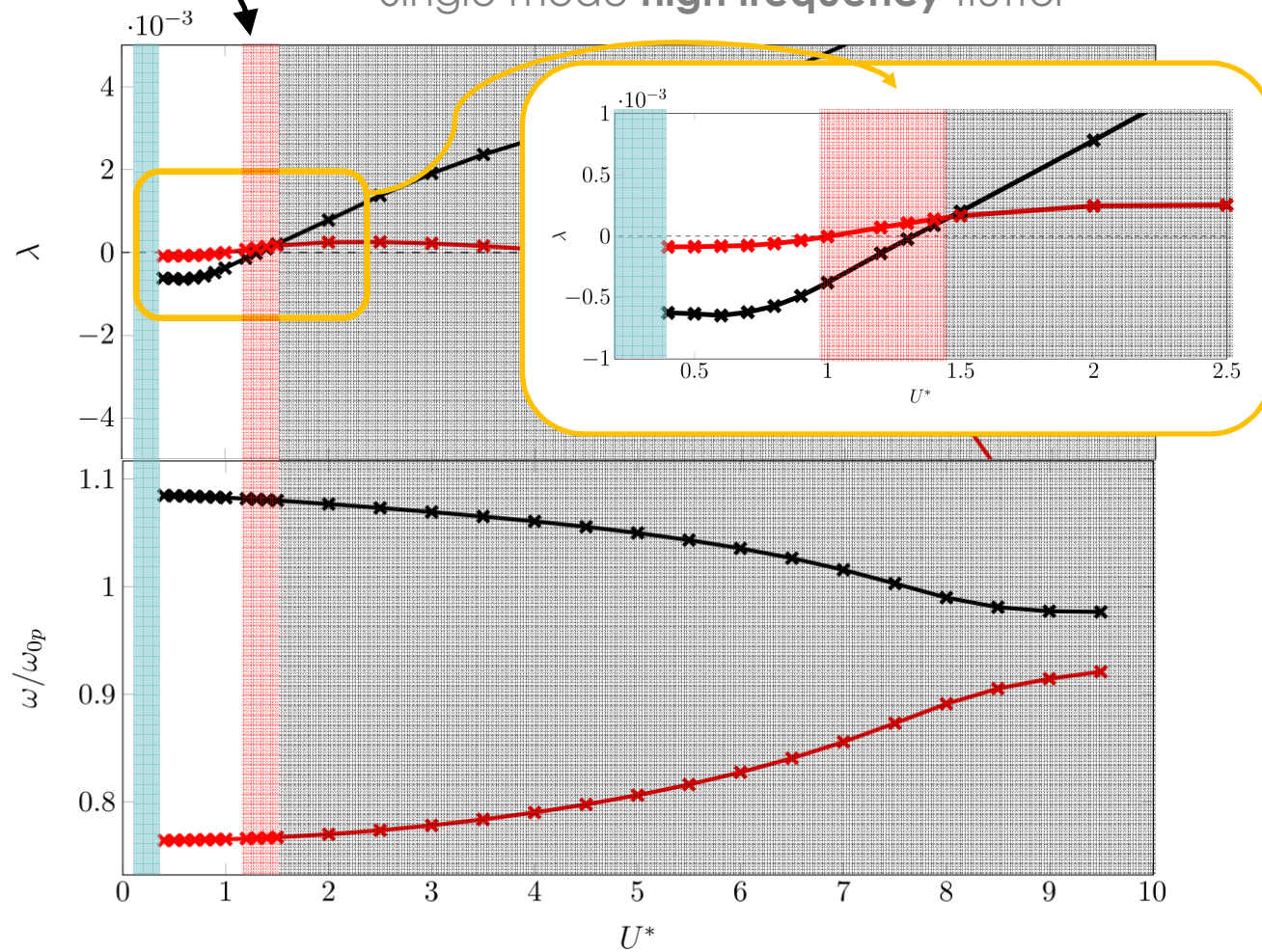
Aeroelastic instabilities of a 2-DOF short plate



Aeroelastic instabilities of a 2-DOF short plate

Single mode **low frequency flutter**

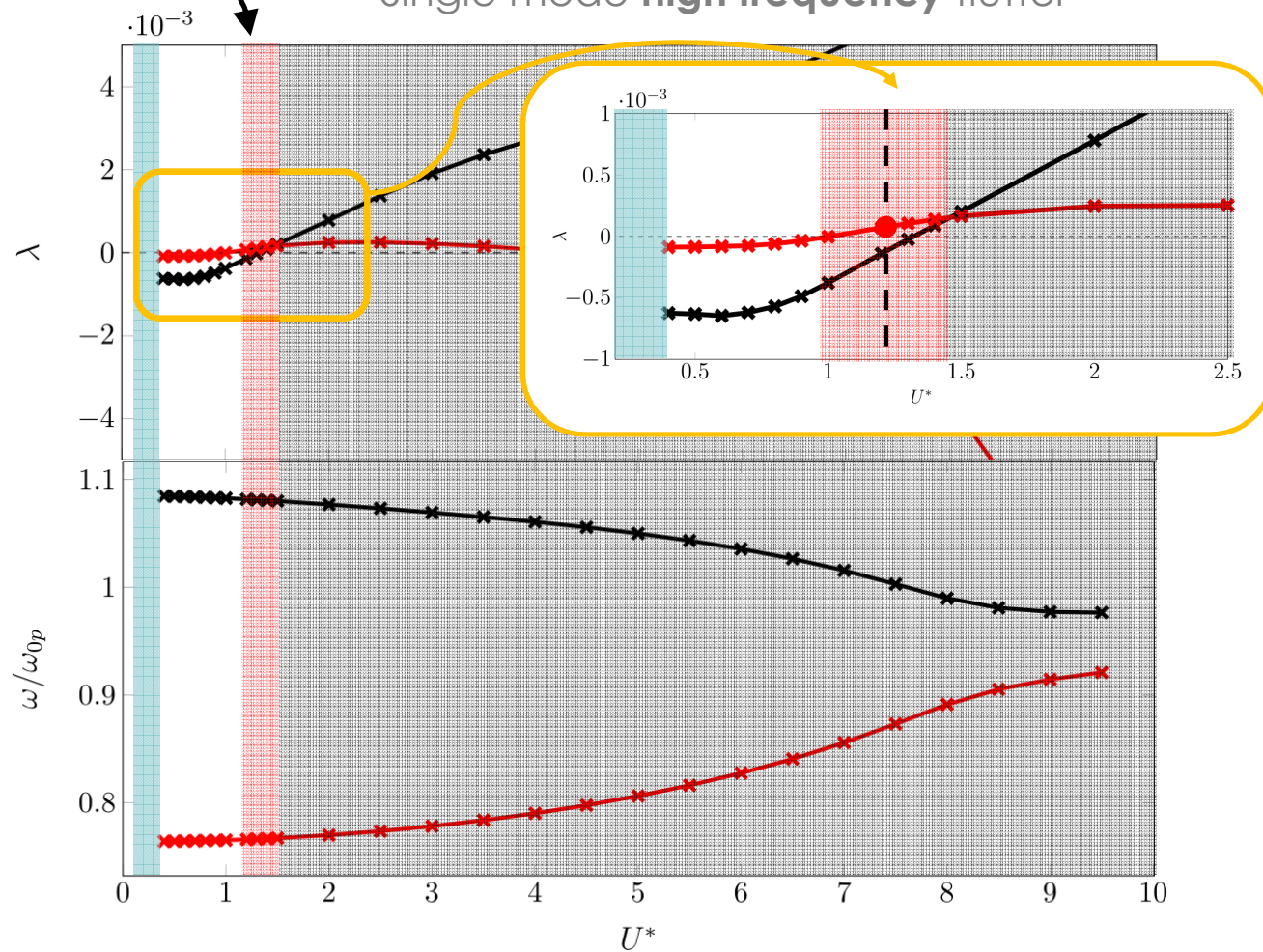
Single mode **high frequency flutter**



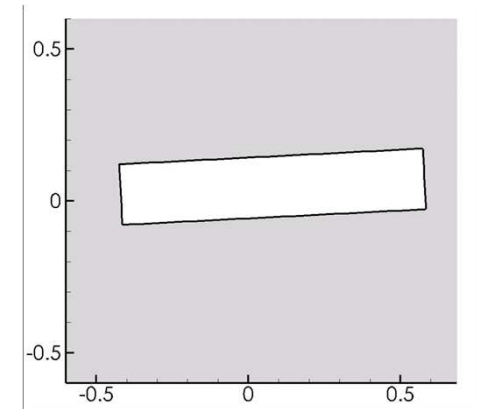
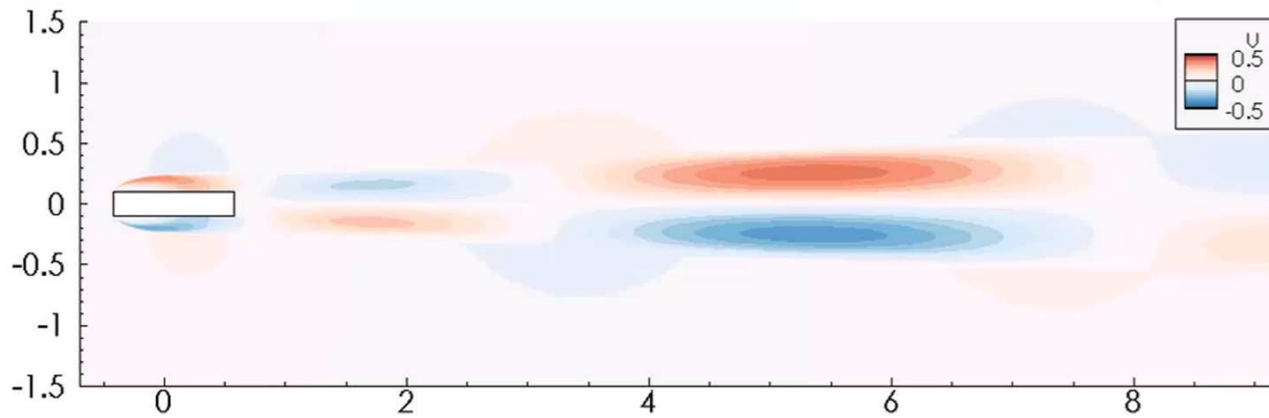
Aeroelastic instabilities of a 2-DOF short plate

Single mode **low frequency flutter**

Single mode **high frequency flutter**



Aeroelastic instabilities of a 2-DOF short plate

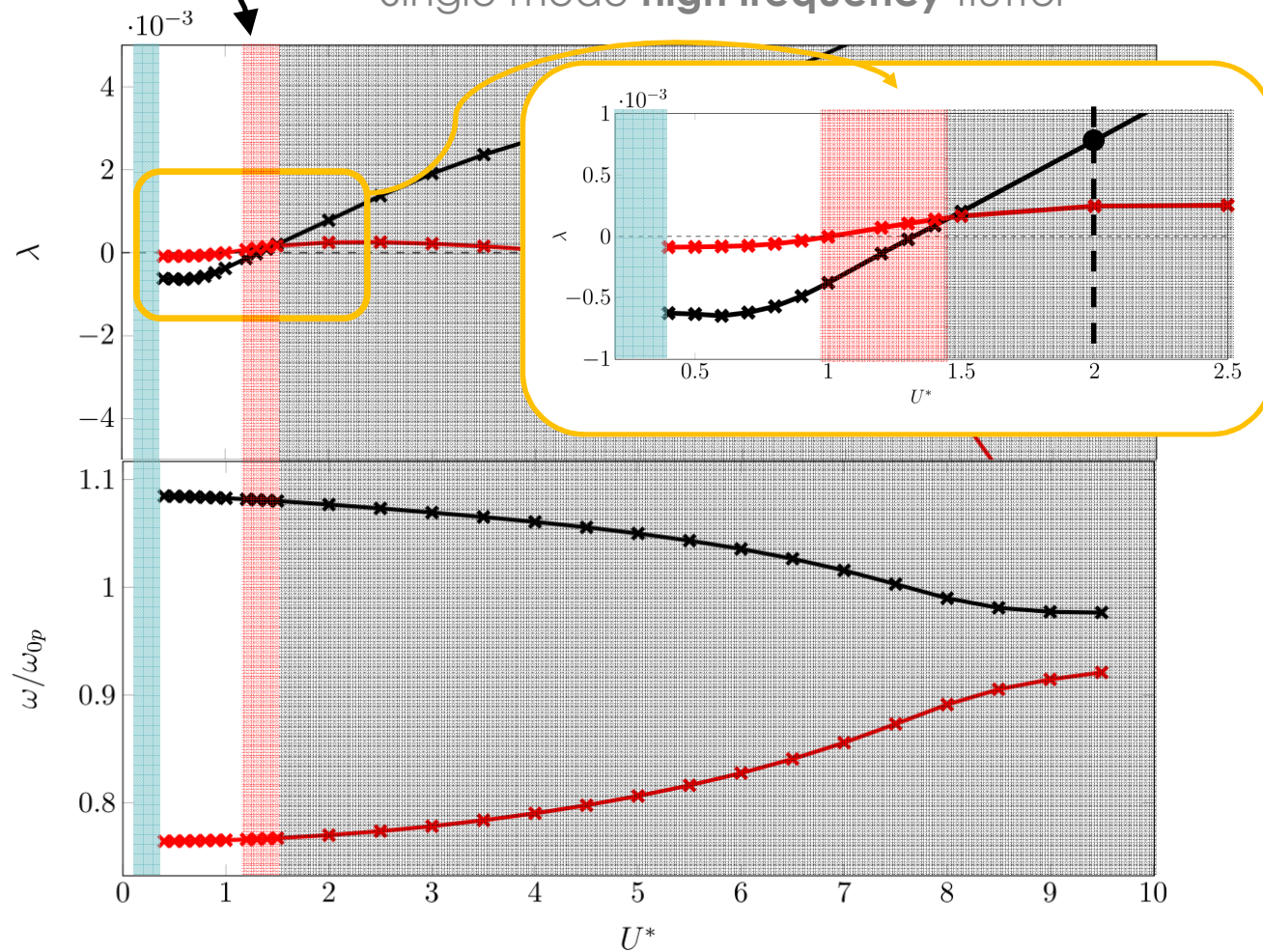


Low frequency flutter mode

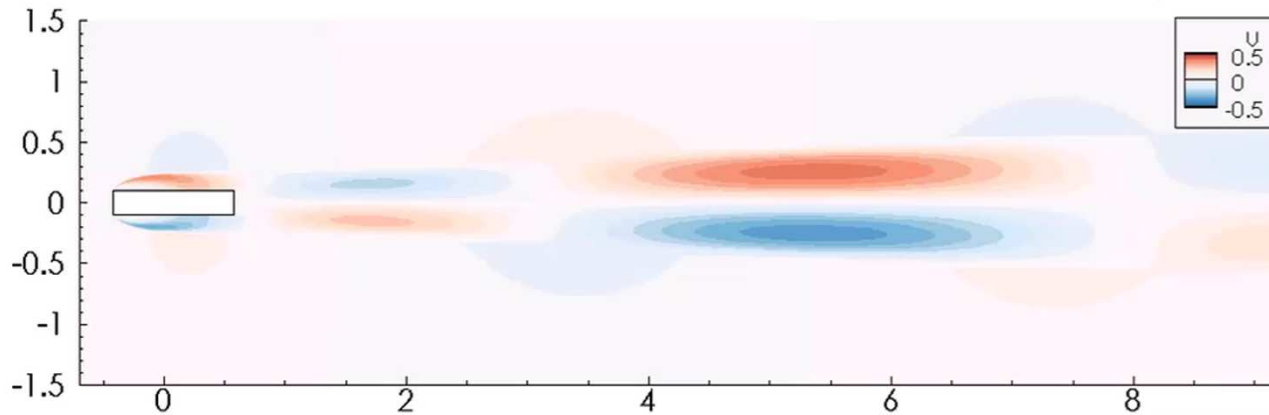
Aeroelastic instabilities of a 2-DOF short plate

Single mode **low frequency flutter**

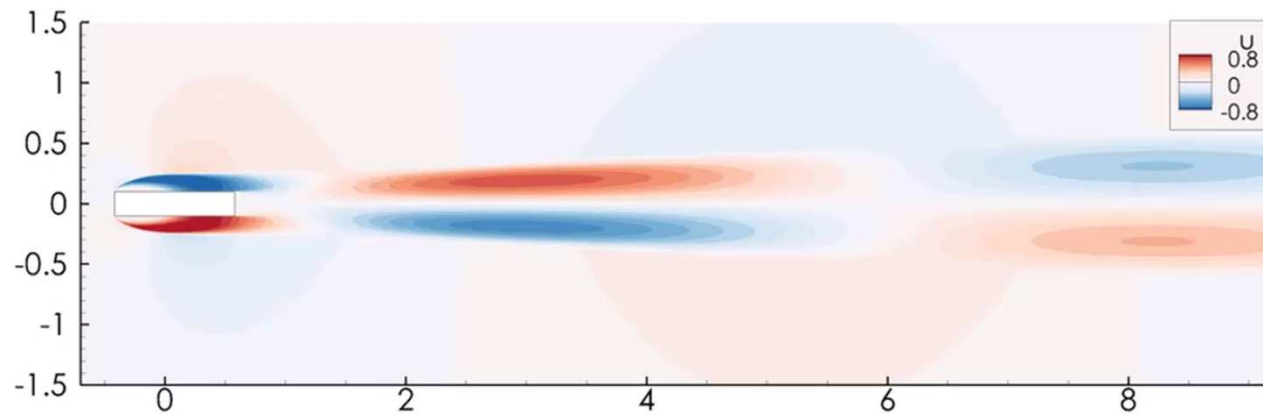
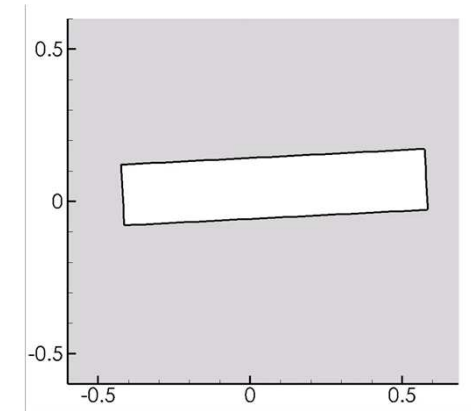
Single mode **high frequency flutter**



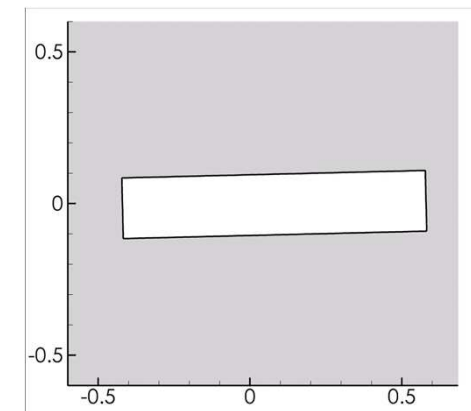
Aeroelastic instabilities of a 2-DOF short plate



Low frequency flutter mode



High frequency flutter mode



Comparison to Theodorsen ...

	U_{crit}^*	ω/ω_{0p}
Present study (low frequency branch)	0,98	0,77
Present study (high frequency branch)	~1,4	1,08
Theodorsen	4,3	1,03

- Clear difference bewteen RANS and Theodorsen
- Well known phenomena : **single-mode flutter** is **difficult to model**
 - Neither Theodorsen nor other simple method work
- ✓ **Interest of using a full RANS modelling**

Conclusion & Perspectives

Conclusions :

- **Single-mode and coupled-mode flutter in turbulent flow** have been investigated through **global linear stability analysis ...**
 - Streamlined body : coupled-mode flutter
 - Bluff body : single-mode flutter
- A **full RANS modelling** of the fluid has been used
- The role of recirculation areas in stability has been discussed
 - Theodorsen's model can be extended to the case of limited detached areas

Conclusion & Perspectives

Perspectives :

- Experimental results on the case $AR = 5$ for validation
- Consider flexible structures
- 3D configurations
- Towards stabilization strategies of those unstable modes