Assessment of Fictitious Domain method for Linear Stability Analysis of Fluid-Structure systems

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Département Aérodynamique Fondamentale et Expérimentale

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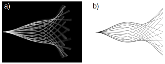
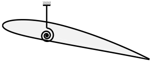
Linear Stability Analysis & FSI

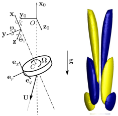
Solid modelling complexity



Fluttering flag (C.Eloy, *JFM* 2008)
Potential flow + 1D elastic solid



Spring-Mounted airfoil
Quasi-Steady theory + damped oscillator



Zig-Zag mode (J.Tchoufag, *JFM* 2014)
Navier-Stokes + rigid-solid

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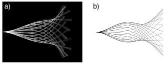

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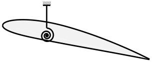

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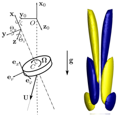

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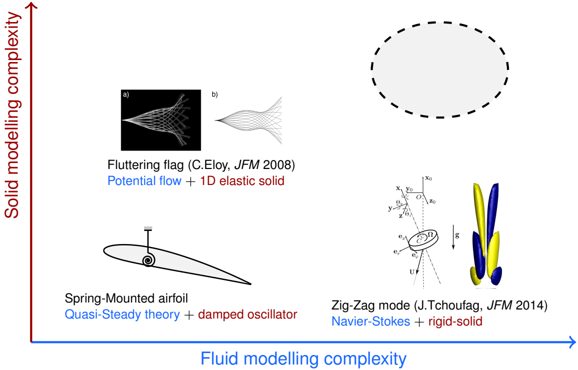


Objective

Assess the use of **Fictitious Domain** approach for **Stability Analysis** of **FSI** systems

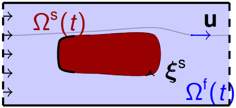
Plan

1. Introduction
2. Conforming vs. Non-Conforming
3. App.1 : VIV on rigid cylinder
4. App. 2 : cylinder with flexible appendice
5. Conclusion

Numerical frameworks for FSI

Description of the separate problems

Fluid : Eulerian description
in domain $\Omega^{f}(t)$
Solid : Lagrangian description
in domain Ω_0^{s}



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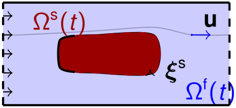
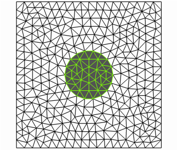


Description of coupled FSI problems

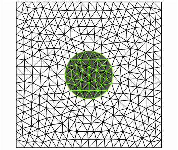
Conforming methods



Arbitrary Lagrangian-Eulerian method (ALE)
(J.Donea, *Enc. Comp. Mech.* 2004)

- Artificial unknowns : mesh displacement in the **fluid region**
- Added equation : **fluid mesh movement** equation

Non-Conforming methods



Fictitious Domain method (FD)
(R.Glowinsky, *Int. J. Numer. Meth. Fluid* 1999)

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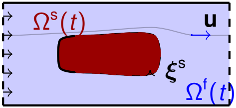

Description of coupled FSI problems

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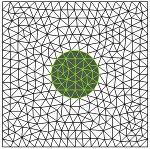

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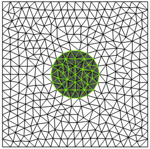

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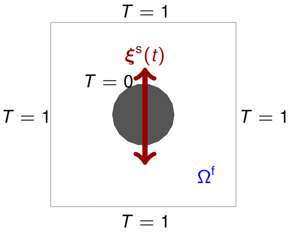
An illustrative example

Heat equation in a moving domain $\Omega^{\mathrm{f}}(\xi^{\mathrm{s}}(t))$:

$$\Delta T = 0 \text{ in } \Omega^{\mathrm{f}}(\xi^{\mathrm{s}}(t))$$



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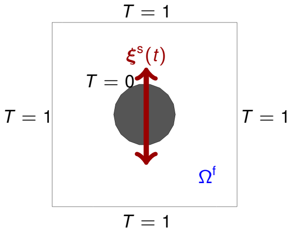
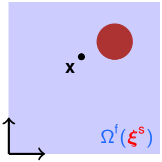


- How is the fluid-structure coupling handled in ALE vs. Fictitious Domain frameworks ?

ALE formalism : an illustrative example

Current configuration



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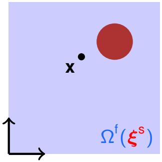

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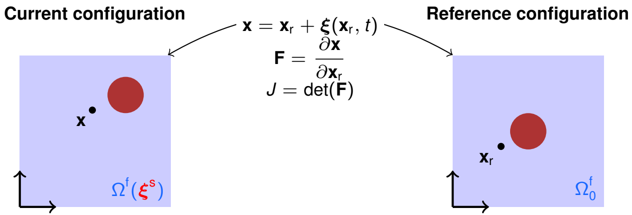

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$$\mathbf{x} = \mathbf{x}_r + \boldsymbol{\xi}(\mathbf{x}_r, t)$$
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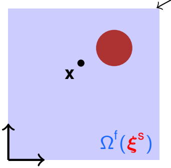

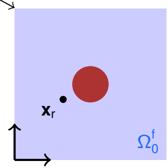

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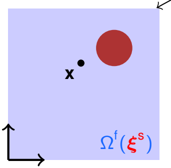

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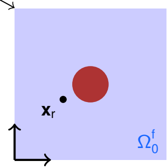
Reference configuration



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- The **geometrical non-linearity** can be put either : in the fluid integration domain or in the fluid integrand

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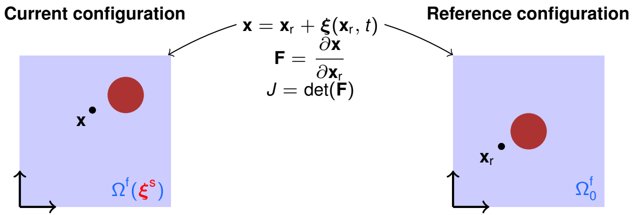

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- The **geometrical non-linearity** can be put either : in the fluid integration domain or in the fluid integrand
- Imagine what will happen with full Navier-Stokes ...

Fictitious Domain formalism : an illustrative example

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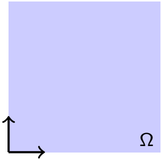

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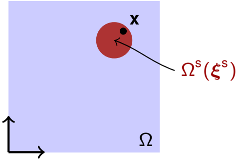

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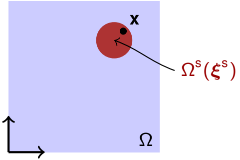

$$\underbrace{\int_{\Omega} \nabla T \cdot \nabla v \, d\mathbf{x}}_{\text{physical equation on } \textbf{total} \text{ domain}}$$

$$+ \quad \underbrace{\int_{\Omega^s(\boldsymbol{\xi}^s)} (T - 0)\, \mu, \, d\mathbf{x}}_{\text{constraint}}$$

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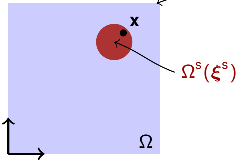

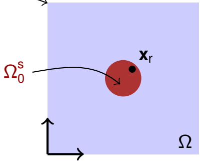

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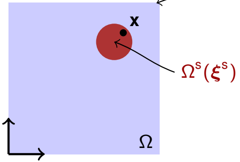

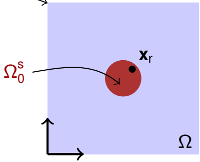

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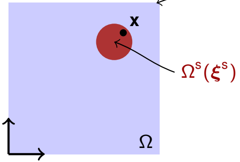

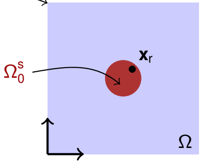

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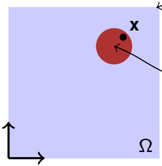

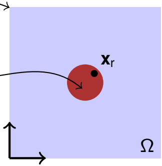

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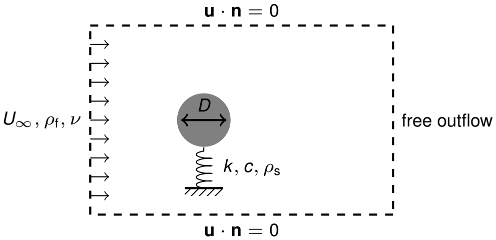
- **Nothing different** with the full Navier-Stokes ...

Problem set-up



Modelisation

- **Fluid model** : Navier-Stokes
- **Solid model** : damped linear oscillator

Non-dimensional numbers :

$$\mathcal{R}e = \frac{\rho_f U_\infty D}{\nu} = 40$$

$$\omega_0 = \sqrt{\frac{k}{m}} \frac{D}{U_\infty} = 0.8,$$

$$\zeta = \frac{c}{2\sqrt{km}} = 0.01$$

$$\tilde{m} = \frac{\rho_s}{\rho_f} = 10$$

Numerics :

- Finite elements modelisation in `Freefem++`(Taylor-Hood elements)
- Newton's method for baseflows calculations
- Shift and invert method for eigenvalue computations with `ARPACK`

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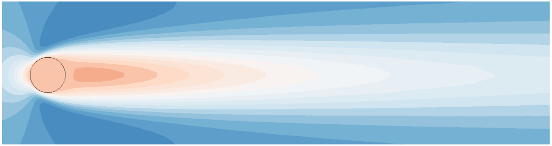
Comparison of stationary fields

Fictitious Domain



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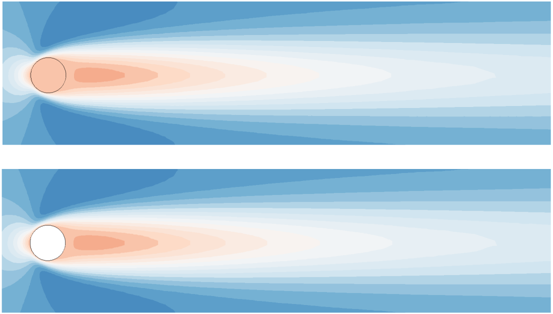

FIGURE – Baseflows : x-velocity

Comparison of stationary fields

Fictitious Domain

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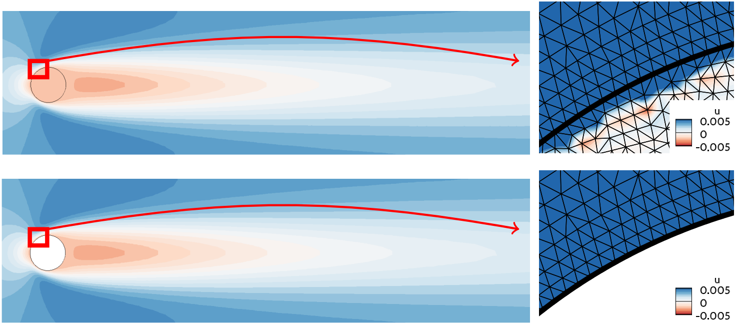

FIGURE – Baseflows : x-velocity

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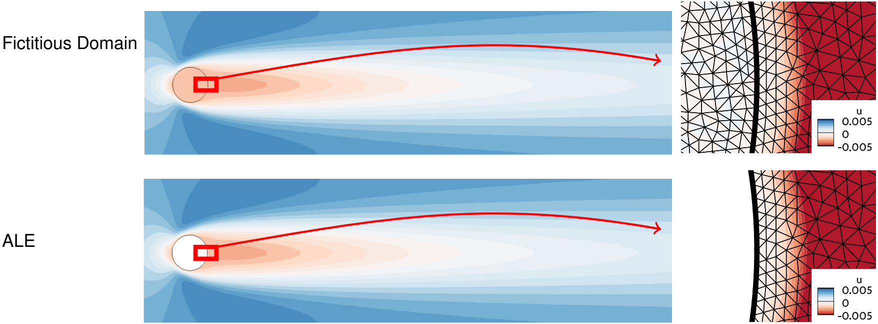

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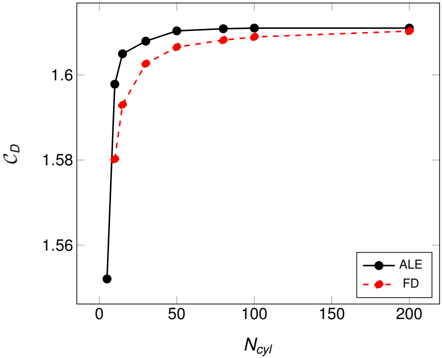


- FD converges towards ALE value when refining on the solid walls
- Excellent comparison of converged $\mathcal{C}_D$ values

Eigenvalue problem : comparison of spectrums

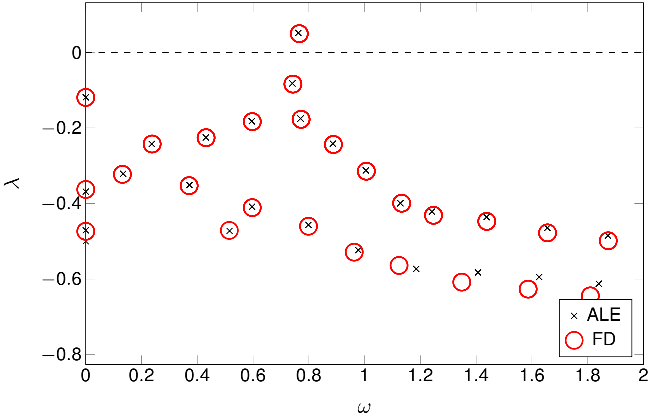
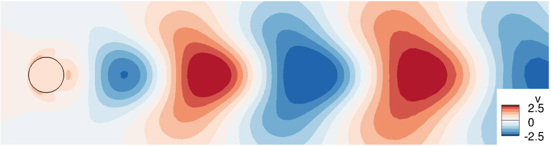


FIGURE – Spectrums

- FD and ALE spectrums are similar

Comparison of eigenmodes

Fictitious Domain



Comparison of eigenmodes

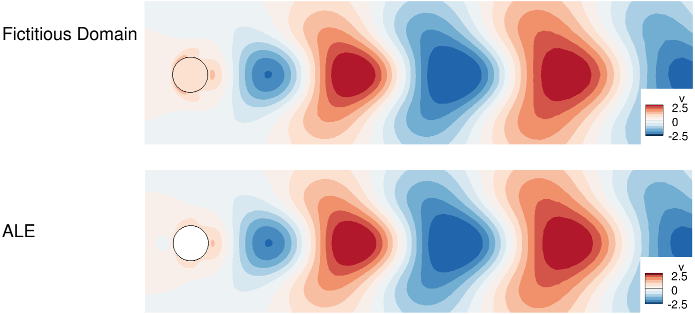


FIGURE – Eigenmodes : x-velocity

Comparison of eigenmodes

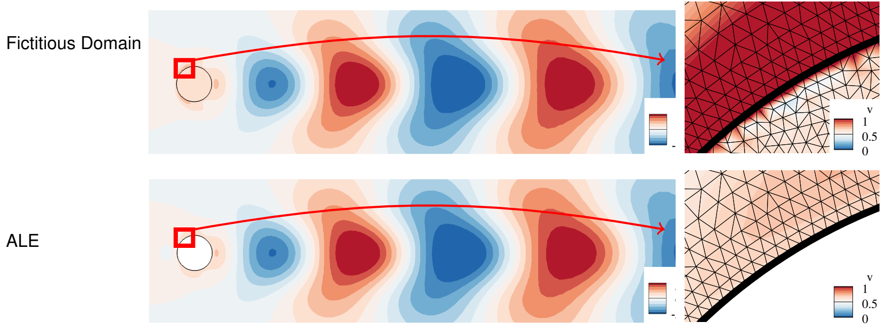


FIGURE – Eigenmodes : x-velocity

Comparison of eigenmodes

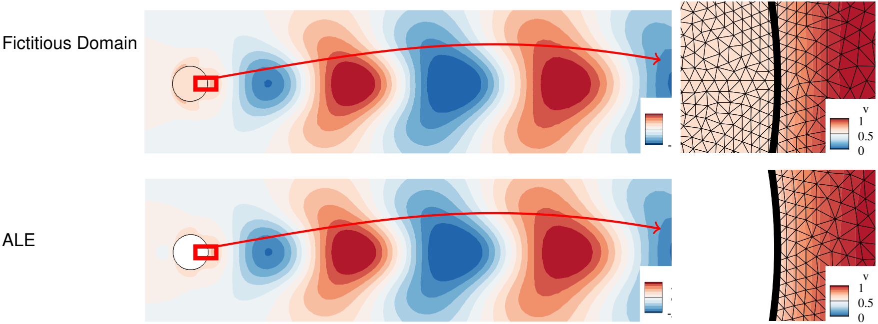
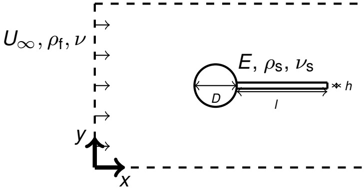


FIGURE – Eigenmodes : y-velocity

Problem set-up

Geometry



Modelisation

- **Fluid model** : Navier-Stokes
- **Solid model** : linear elasticity

Non-dimensional numbers :

$$\mathcal{R}e = \frac{\rho_f U_\infty D}{\nu_f} = 100$$

$$\mathcal{K}_b = \frac{Eh^3}{12\rho_f U_\infty^2 l^3} = 0.04, \ \nu_s = 0.35,$$

$$\frac{h}{l} = 0.03$$

$$\tilde{m} = \frac{\rho_s}{\rho_f} = 85$$

Numerics :

- Finite elements modelisation in `Freefem++`(Taylor-Hood elements)
- Newton's method for baseflows calculations
- Shift and invert method for eigenvalue computations with `ARPACK`

Comparison of stationary fields

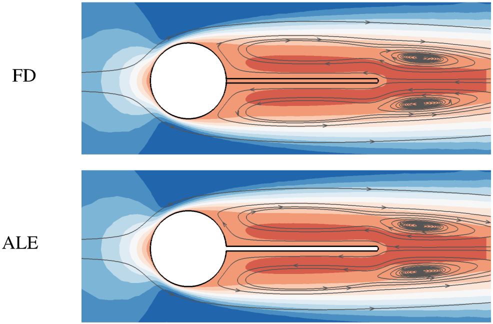


FIGURE – Stationary solution : ALE vs FD

Comparison of stationary fields

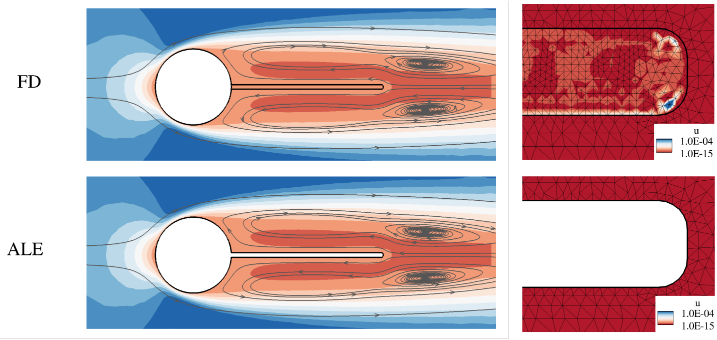
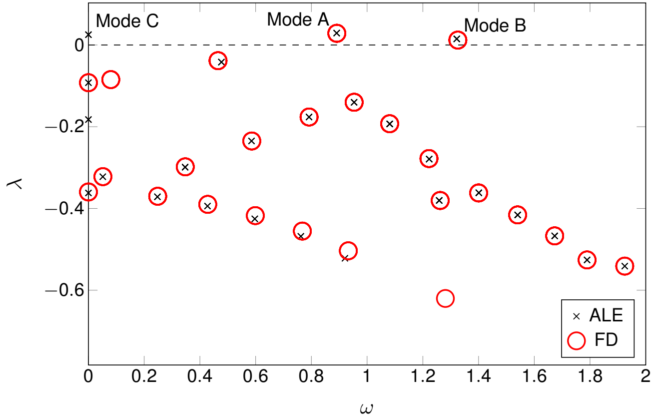


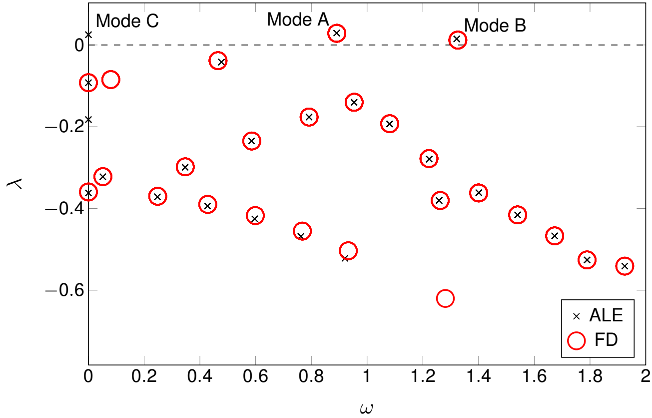
FIGURE – Stationary solution : ALE vs FD

Eigenvalue problem : comparison of spectrums



- FD and ALE spectrums are similar ...

Eigenvalue problem : comparison of spectrums



- FD and ALE spectrums are similar ...
- ... **Except** from the unstable steady mode C

Comparison of eigenmodes

Unstable unsteady modes

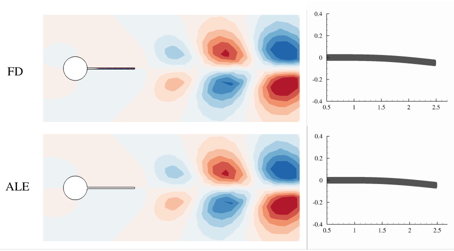


FIGURE – Mode A : pressure distribution

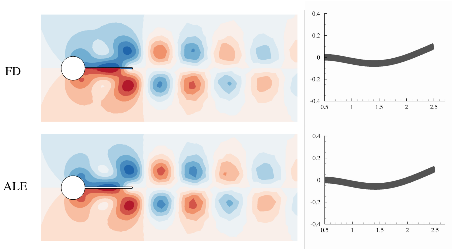


FIGURE – Mode B : pressure distribution

➥ VIV instability

➥ Mix flutter-VIV instability

What about the steady mode ?

FIGURE – Mode C : pressure distribution

What about the steady mode ?

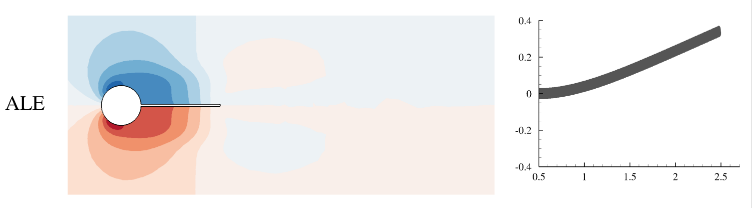


FIGURE – Mode C : pressure distribution

➥ Aeroelastic divergence instability

What about the steady mode ?

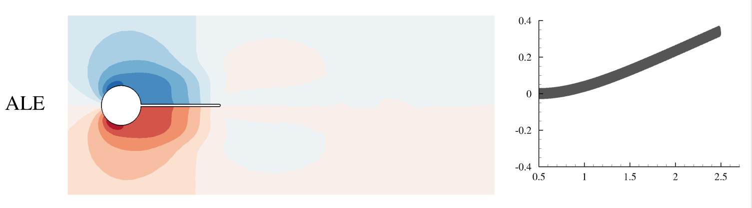


FIGURE – Mode C : pressure distribution

- Aeroelastic divergence instability
  - added stiffness due to the fluid forces

Conclusion

Present work

- We disscussed the use of a **Fictitious Domain** framework for **linear stability analysis of FSI** problems
- The **Fictitious Domain** predicts **accurate baseflows and eigenvalue spectrums** in most of our test cases at **lower computationnal cost** (7-10 times faster)
- Its precision is still in question for some types of instabilities where the interface stresses need particularly accurate evaluation

Perspectives

- **Other types of FSI instabilities** should be tested (wing flutter, galloping)
- The use of **more recent non-conformal methods** might be considered