Fluid-structure stability analysis of flexible splitter plates

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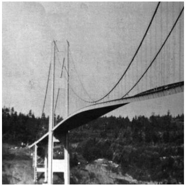
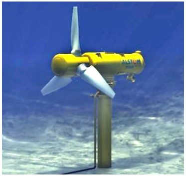
Fluid-structure vibrations are involved in many engineering fields...

Civil engineering

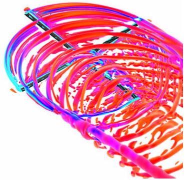


Offshore industry



Wind energy

Aerospace engineering



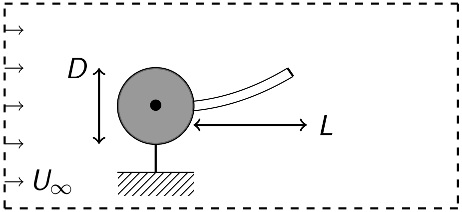
...the identification of vibration thresholds due to coupling is often crucial.

Introduction

To find vibration thresholds, a **linear approach** is sufficient :

- With reduced models (Dowell, Clark et Cox 2004 ; Eloy et al. 2008)
- Only few studies with Navier-Stokes + elasticity :
  - « transpiration » (Fernández et Le Tallec 2003)
  - « full linearization » (Pedraglio 2015)

Configuration (Lee et You 2013) :



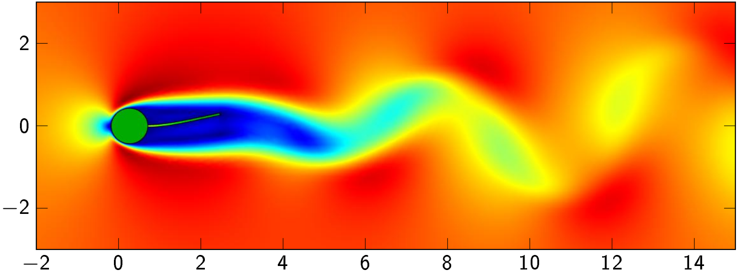
- Plate length $L/D = 2$, thickness $E/D = 0.06$
- Density ratio $\rho_{\text{solid}}/\rho_{\text{fluid}} = 84.7$
- Reynolds number $\mathcal{R}e = U_\infty D/\nu = 100$
- Varying parameter : plate **bending stiffness** (rigidity) $\mathcal{K}_{\text{B}}$

Model : incompressible flow + elasticity (Saint-Venant Kirchhoff)

Outline

An unsteady simulation

Configuration of Lee & You, $\mathcal{K}_B = 0.02$:



velocity magnitude

Unsteady simulations, effect of $\mathcal{K}_\mathrm{B}$

Results from Lee et You 2013 (DNS) : vortex shedding in the wake & symmetric structure oscillations for all cases, and

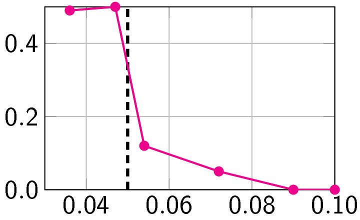


Plate max. displacement *vs* $\mathcal{K}_\mathrm{B}$

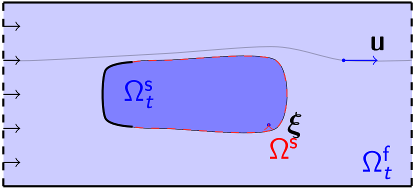
Threshold at $\mathcal{K}_\mathrm{B} \sim 0.05$.

- $\mathcal{K}_\mathrm{B} < 0.05$: high structure deformation regime
- $\mathcal{K}_\mathrm{B} > 0.05$: small structure deformation regime

Outline

Lagrangian/Eulerian

Velocity $\mathbf{u}$, displacement ξ, reference domain Ω, moving domain $\Omega(t)$



The governing equations do not belong to the same space :

$$\mathcal{B}_F \frac{\partial \mathbf{u}}{\partial t} - \mathcal{R}_F(\mathbf{u}, p) = 0 \qquad \mathbf{x} \in \Omega^f(t) \qquad \textbf{Fluid - Eulerian}$$

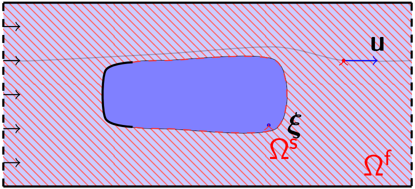
$$\frac{\partial^2 \xi}{\partial t^2} - \mathcal{R}_S(\xi) = 0 \qquad \mathbf{X} \in \Omega^s \qquad \textbf{Solid - Lagrangian}$$

$$\mathcal{R}_I(\mathbf{u}, p)(\mathbf{x}) + \mathcal{R}_I(\xi)(\mathbf{X}) = 0 \qquad \text{Interface - Both !}$$

How to cope with moving domains ?

Arbitrary Lagrangian Eulerian formulation (Donea et al. 2004)

Velocity $\mathbf{u}$, displacement ξ, reference domain Ω, moving domain Ω_t



Idea : introduce an arbitrary extension map E in the fluid region $\rightarrow$ dependant variables functions of fixed reference coordinates $\mathbf{X}$

$$\mathbf{x} = \mathbf{X} + \xi(\mathbf{X}, t) \qquad \text{In the solid (elasticity)}$$

$$\mathbf{x} = E(\mathbf{X}, t) \qquad \text{In the fluid (extension)}$$

Extension (fluid mesh) equation : $-\mathcal{R}_{\mathrm{E}}(\xi|_{\Omega^{\mathrm{f}}}, \xi|_{\Omega^{\mathrm{s}}})(\mathbf{X}) = 0$ in Ω^{f}

Coupled problem in reference domain

System written in the fixed reference configuration :

$$\tilde{\mathcal{B}}_{\mathsf{F}} \frac{\partial \mathbf{u}}{\partial t} - \tilde{\mathcal{R}}_{\mathsf{F}}(\mathbf{u}, p, \boldsymbol{\xi}) = 0 \qquad \Omega^{\mathsf{f}} \qquad (\textbf{F}\text{luid})$$

$$-\mathcal{R}_{\mathsf{E}}(\boldsymbol{\xi}) = 0 \qquad \Omega^{\mathsf{f}} \qquad (\textbf{E}\text{xtension})$$

$$\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} - \mathcal{R}_{\mathsf{S}}(\boldsymbol{\xi}) = 0 \qquad \Omega^{\mathsf{s}} \qquad (\textbf{S}\text{olid})$$

$$-\tilde{\mathcal{R}}_{\mathsf{I}}(\mathbf{u}, p, \boldsymbol{\xi}) = 0 \qquad \Omega^{\mathsf{s}} \cap \Omega^{\mathsf{f}} \qquad (\textbf{I}\text{nterface})$$

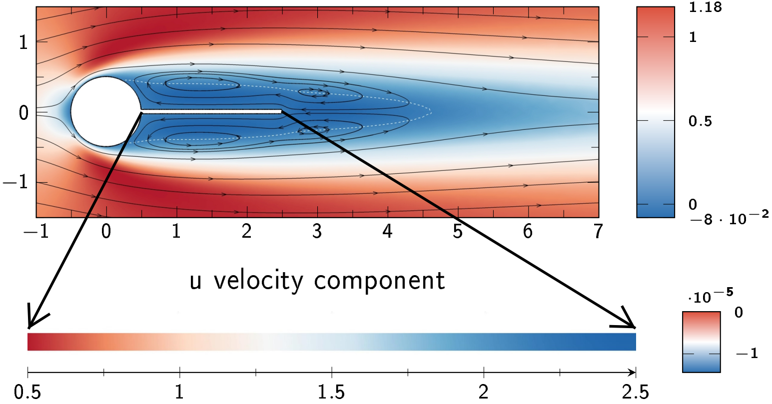
More compact way with $\mathbf{q}(\mathbf{x}, t) = [\mathbf{u}, p, \boldsymbol{\xi}|_{\Omega^{\mathsf{f}}}, \boldsymbol{\xi}|_{\Omega^{\mathsf{s}}}, \partial_t \boldsymbol{\xi}|_{\Omega^{\mathsf{s}}}]$

$$\mathcal{B}\frac{\partial \mathbf{q}}{\partial t} - \mathcal{R}(\mathbf{q}) = 0 \qquad \Omega = \Omega^{\mathsf{f}} \cup \Omega^{\mathsf{s}}$$

Here Ω (ref. configuration) is the rigid solid + corresponding fluid domain

Base state ($\mathcal{K}_B = 0.044$)

Base state $\mathbf{q}_b(\mathbf{x})$: $\mathcal{R}(\mathbf{q}_b) = 0$ + symmetry/x-axis.



solid axial displacement : small compression

Stability problem

In the reference configuration $\rightarrow$ domain motion expressed in operators $\rightarrow$ **straightforward linearization**. Looking for **normal modes**

$$\hat{\mathbf{q}}(\mathbf{x}) \exp(\lambda + \mathbf{i}\omega)t$$

in the linearized problem :

$$\left.\frac{\partial \mathcal{R}}{\partial \mathbf{q}}\right|_{\mathbf{q}_b} \hat{\mathbf{q}} = (\lambda + \mathbf{i}\omega)\mathcal{B}\hat{\mathbf{q}}$$

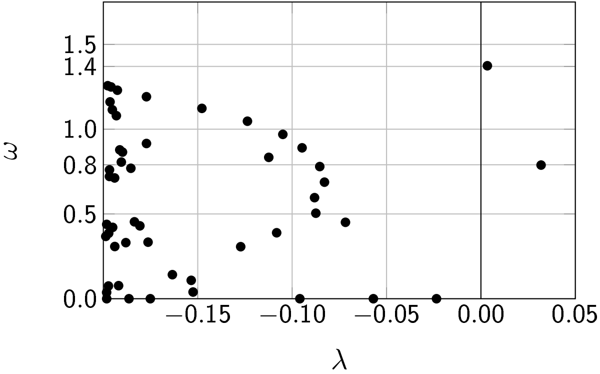
Eigenvalue problem implemented in `FreeFem++` with an exact Jacobian.

Outline

1. Non-linear simulations
2. Linear stability approach for fluid-structure problem
3. Stability results
   - Case $\mathcal{K}_{\mathrm{B}} = 0.044$
   - Effect of the rigidity
   - Case $\mathcal{K}_{\mathrm{B}} = 0.02$
4. Conclusion

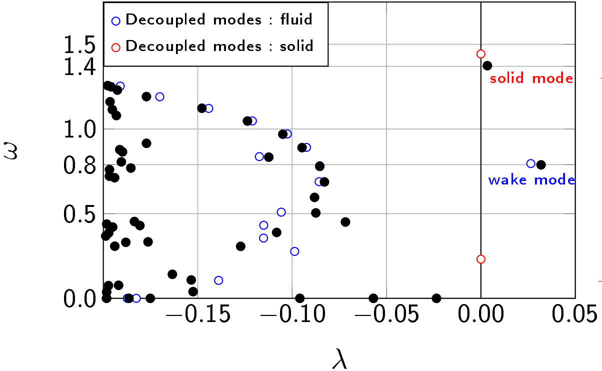
$\mathcal{K}_\mathrm{B} = 0.044$ – spectra

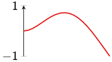
Case of Lee et You 2013 ($\mathcal{K}_\mathrm{B} = 0.044$) :

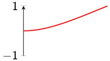


$\mathcal{K}_\mathrm{B} = 0.044$ – spectra

Case of Lee et You 2013 ($\mathcal{K}_\mathrm{B} = 0.044$) :



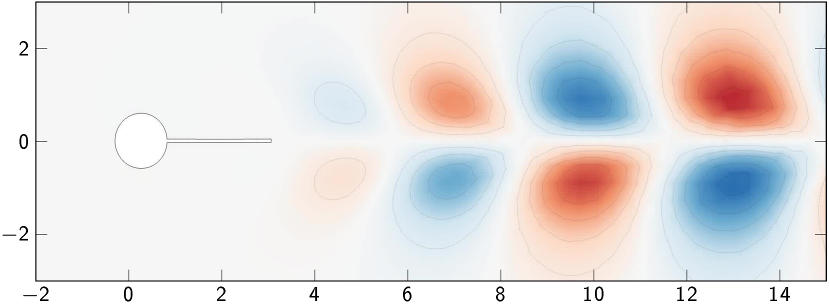




Two unstable modes ($\lambda > 0$) here :

- « wake » mode : located close to the pure hydro. wake mode
- « solid » mode : located close to the solid free vibration mode

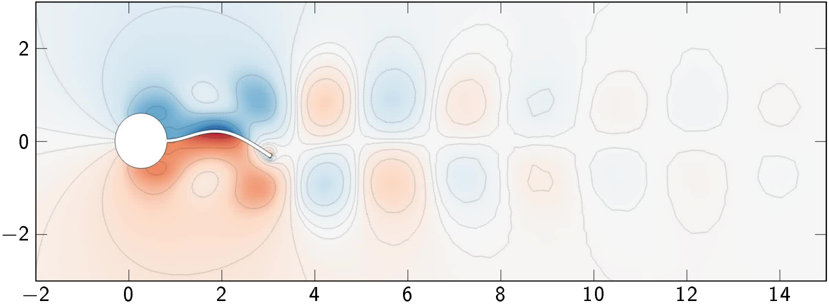
$\mathcal{K}_\mathrm{B} = 0.044$ – wake mode



wake mode pressure component, $\omega = 0.80$

- structures located mostly in the wake region
- same shape as the $\mathcal{K}_\mathrm{B} \to \infty$ (rigid plate) vortex shedding mode

$\mathcal{K}_\mathrm{B} = 0.044$ – solid mode



Solid mode, pressure component, $\omega = 1.37$

- structures in the wake region and near the solid
- shape of a second plate bending mode (two vibration nodes)

$\mathcal{K}_B = 0.044$ – Comparison with DNS

Mode shape : vortex shedding in the wake (DNS), and

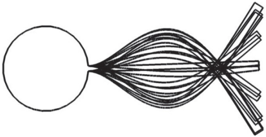


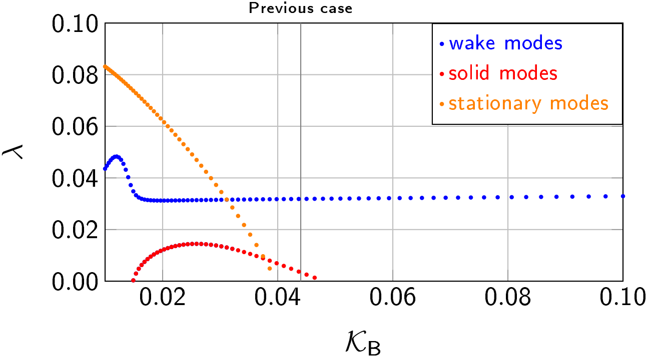
Figure : snapshots of plate deformation, from Lee et You 2013 – second plate bending mode

Frequency :

	ω
DNS	1.13
Solid mode	1.37
Wake mode	0.79

$\Rightarrow$ Non-linearly, the two modes are present

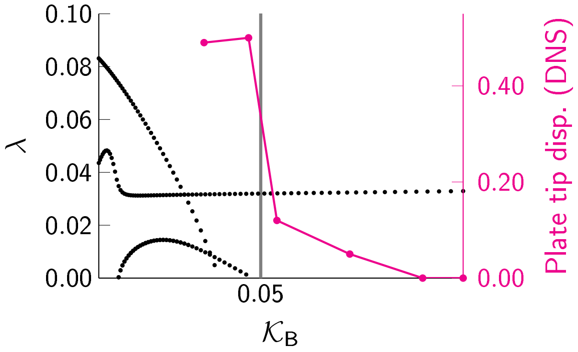
Effect of $\mathcal{K}_B$ – growth rate of unstable modes



- The wake mode is always unstable
- The solid mode is unstable for $\mathcal{K}_B \in [0.015, 0.047]$
- A stationary mode ($\omega = 0$) appears for $\mathcal{K}_B < 0.04$

Effect of $\mathcal{K}_B$ – comparison with DNS

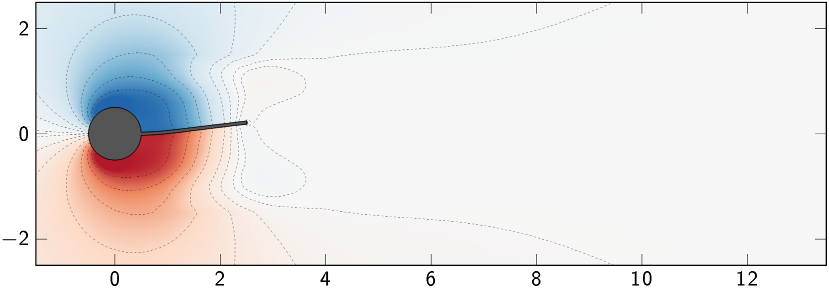
Always vortex shedding in the DNS, and



- $\mathcal{K}_B < 0.05$: solid + fluid mode → vortex shedding + structure disp.
- $\mathcal{K}_B > 0.05$: fluid mode only → vortex shedding only

⇒ Good prediction of bifurcation **threshold** – other things are non-linear !

$\mathcal{K}_\mathrm{B} = 0.02$ – stationary mode

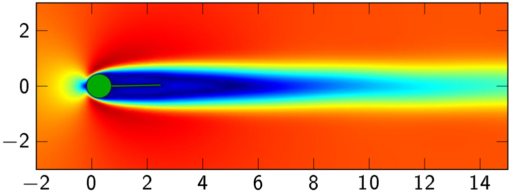


pressure component (real part), $\omega = 0$

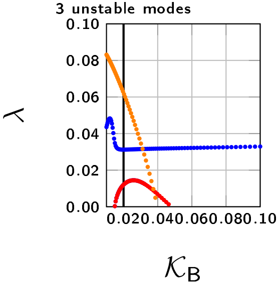
- no time evolution : $\omega = 0$
- shape of a first plate bending mode (one vibration node)

$\mathcal{K}_\mathrm{B} = 0.02$ – comparison with DNS

Non-linear simulation, $\mathcal{K}_\mathrm{B} = 0.02$:



velocity magnitude, $\mathcal{K}_\mathrm{B} = 0.02$



⇒ A three mode interaction is observed non-linearly

Conclusion & perspectives

About the method

- An new approach : linear stability analysis of a fully coupled FSI/ALE problem with exact Jacobian has never been reported
- An alternative to expensive direct simulations for instability threshold prediction
- Next : implement sensibility analysis tools, RANS turbulence model (another PhD student at ONERA), parallelization (Post-doc at ONERA).

About the results

- Apply on a sub-critical case (only one unstable mode, no mode interaction $\rightarrow$ physical picture is easier)
- Next : play with weakly non-linear expansions

Thank you – Questions

References

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Fernández, M. Á. et P. Le Tallec (2003). « Linear stability analysis in fluid-structure interaction with transpiration. Part I : Formulation and mathematical analysis ». In : *Computer methods in applied mechanics and engineering* 192, p. 4805–4835.

Lee, Jinmo et Donghyun You (2013). « Study of vortex-shedding-induced vibration of a flexible splitter plate behind a cylinder ». In : *Physics of Fluids* 25, p. 110811.

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Appendix – parameters

Parameters

$$\mathcal{R}\mathrm{e} = \frac{DU_\infty}{\nu}, \quad \mathcal{K}_\mathrm{B} = \frac{EH^3}{12\rho_f U_\infty^2 L^3}, \quad \mathcal{M}_\rho = \rho_s/\rho_f$$

In this study :

- $H/L \sim 1\%$
- $\mathcal{R}\mathrm{e} \sim 100$
- $\mathcal{K}_\mathrm{B} \sim 10^{-3} - 1$
- $\mathcal{M}_\rho \sim 100 - 1$

Corresponding values in the real world (e.g. in a water chanel) :

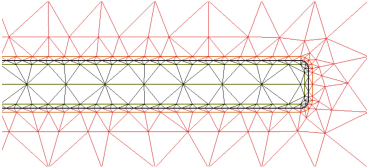
- $D \sim 1$ mm, $U_\infty \sim 0.1$ m/s, $E \sim 0.1 - 100$ MPa (rubber)

To stay in the same $\mathcal{R}\mathrm{e}$ and $\mathcal{K}_\mathrm{B}$ ranges with a less ridiculous size → play with plate thickness

Appendix – About the numerics

Common discretization :

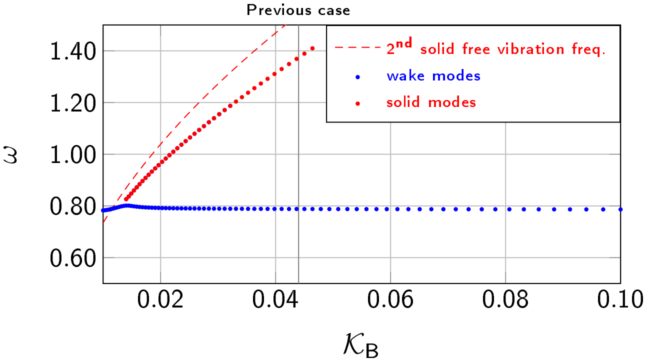
- Finite elements P2 for velocity/displacement, P1 for the pressure
- Domain $x/D \in [-20, 50]$, $y/D \in [-20, 20]$
- Conformal mesh with $\sim 20\,000$ elements $\rightarrow 100\,000$ d.o.f. in the stability problem



Method-specific :

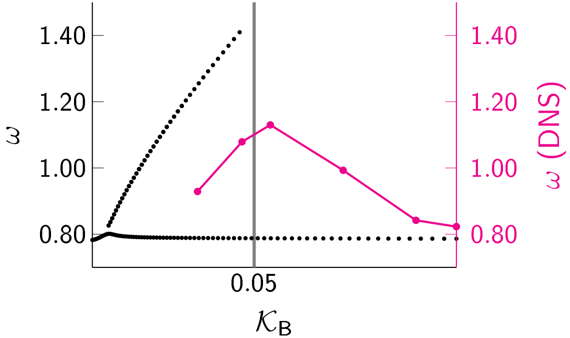
- DNS : FSI/ALE module of *Comsol Multiphysics*, 2nd order in time
- Baseflow : Newton method in `FreeFem++` with exact Jacobian
- Stability : ARPACK & shift/invert in `FreeFem++` with exact Jacobian

Appendix – Effect of $\mathcal{K}_B$, unstable modes frequency



- solid mode frequencies close to beam modal frequencies
- wake mode frequencies almost constant

Appendix – Effect of $\mathcal{K}_B$, frequency, comparison with



- Less good prediction at bifurcation threshold, but two unstable modes $\rightarrow$ non-linear interactions
- Better matching at higher rigidity (vortex shedding onset at $\mathcal{R}e = 92$ when $\mathcal{K}_B \rightarrow +\infty$)