Réunion pour début de thèse de Luis Henrique Benetti Ramos

Université de Bordeaux

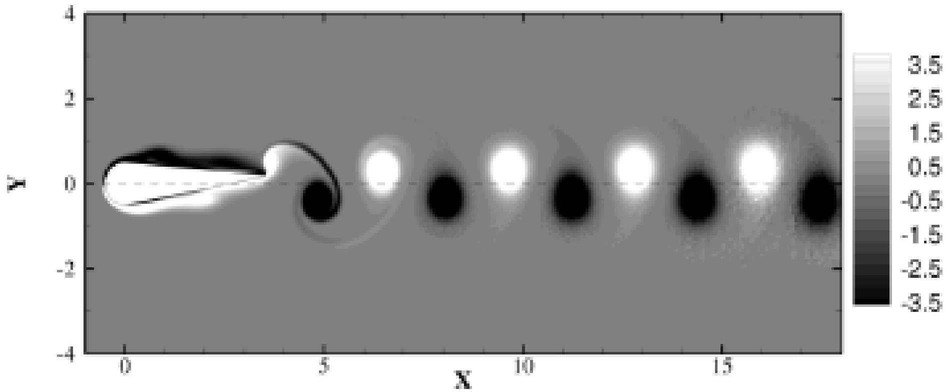
28-29 septembre 2017

Ordre du jour

- Rappel des discussions précédentes
- Organisation des séjours à Bordeaux
- Le contenu scientifique
  - « hydrodynamic resonance theory »
  - IBM temporal simulations
  - Fluid-structure instability in IBM

Hydrodynamic resonance theory

Unsteady propulsion of rigid/flexible foils



- What is the connexion between the wake's structure and the hydrodynamic forces ?

- Does a linear stability analysis of the wake gives information on the propulsion effcicieny?

Hydrodynamic resonance theory

Triantafyllou et al. (1993):

- 2D rigid pitching foils (A: amplitude, f: frequency)

- Reverse Von-Karman vortex street (2S wakes)

- Propulsive efficiency: $\eta = \frac{\bar{\boldsymbol{T}} U_\infty}{\bar{\boldsymbol{P}}_{in}}$

  $\bar{T}$: time-averaged net thrust

  $\bar{\boldsymbol{P}}_{in}$: time-averaged power input to the fluid

- A single peak in propulsive efficiency occurs for

$$0.25 \leq St = \frac{fA}{U_\infty} \leq 0.35$$

Hydrodynamic resonance theory

Triantafyllou et al. (1993):

They proposed that this peak in propulsive efficiency occurs at the frequency of maximum spatial growth rate of the instability of the jet

work is termed the hydrodynamic resonant frequency of the jet. At a fixed location downstream of the trailing edge of a flapping fin, they suggested that the perturbation waves will have the largest amplification (per unit input energy) at the resonant frequency of the jet profile (where there is maximum spatial growth), and that this will cause an expedient shear layer rollup and entrainment, and result in the strongest momentum jet for a given input energy. Thus a peak in the propulsive efficiency is expected when the fin is driven at the resonant frequency of the jet.

from Moored et al. (2012)

Hydrodynamic resonance theory

Lewin and Haj-Hariri (2003):

- 2D rigid heaving foils (A: amplitude, f: frequency)

- Mutiple peaks in efficiency

- Driving frequency match the resonant frequency
  (obtained by stability analysis)

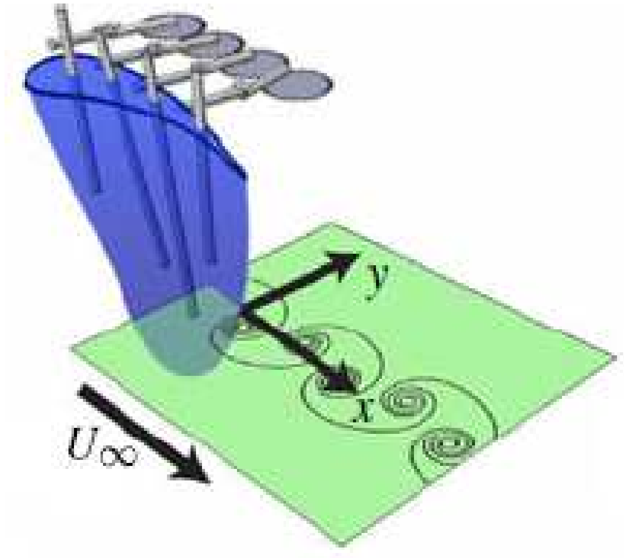
- Need to introduced the reduced frequency

$$\omega = \frac{2\,\pi f L}{U_\infty}$$ with L: the chord length

Hydrodynamic resonance theory

Dewey et al. (2001)

- Three-dimensional ray-like pectoral fin



- Travelling wave motion of the fin

- Non-dimensional wave length

$$\lambda^* = \frac{\lambda}{L} = 4\ or\ 6$$

- Muliple peaks in efficiency observed as St is varied

- Transition from 2P to 2S wakes when increasing St

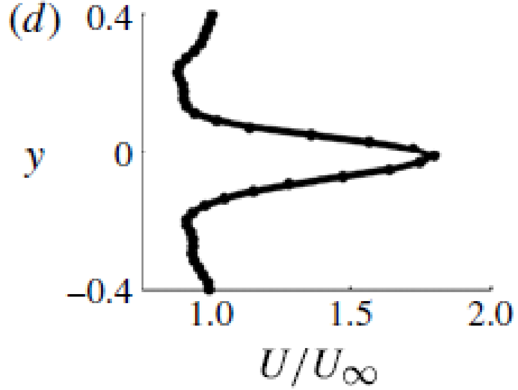
Hydrodynamic resonance theory

Moored et al. (2012):

- Local spatial stability analysis:

Assumption: the flow is weakly non-parallel ($\partial_x \ll \partial_y$)

$U(y)$ $\quad + \quad$ $u'(x,y,t) = \hat{u}(y)e^{i\,(\alpha x - \omega t)}$



velocity profile at a station x

ω: frequency (real parameter)

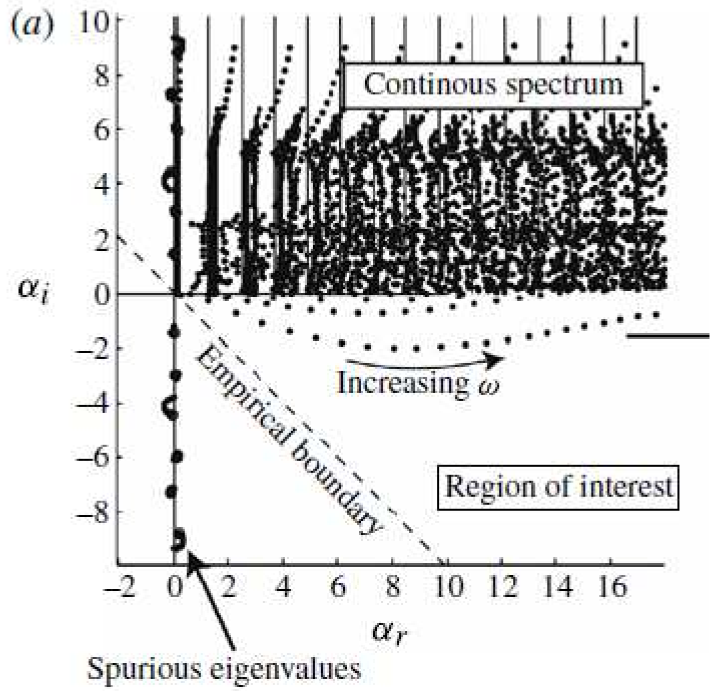
α: complex wavenumber (unknown)

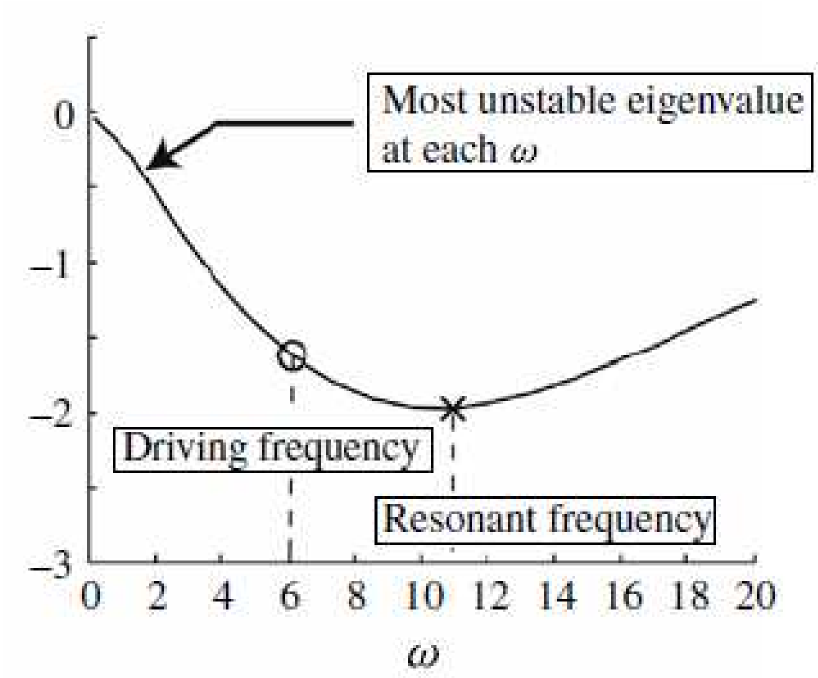
$-\Im(\alpha)$: spatial growth rate

$\Re(\alpha)$: spatial wavenumber

Hydrodynamic resonance theory

Moored et al. (2012):





Eigenvalue spectra obtained for

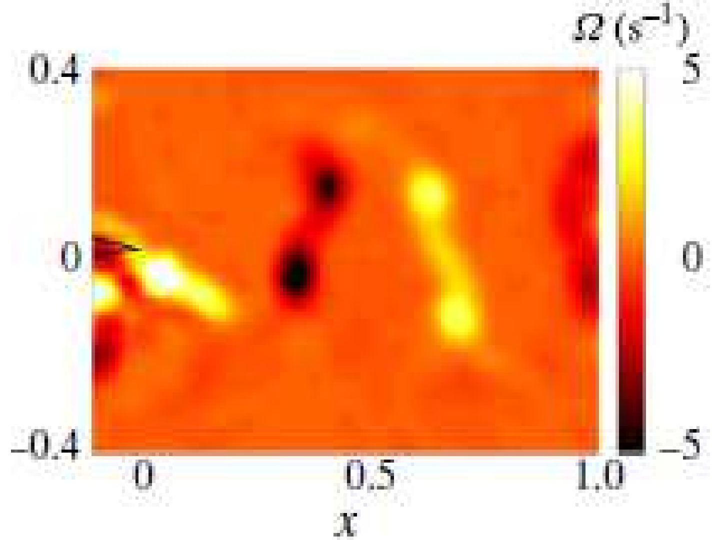
- one velocity profile
- and 40 values of the frequency ω

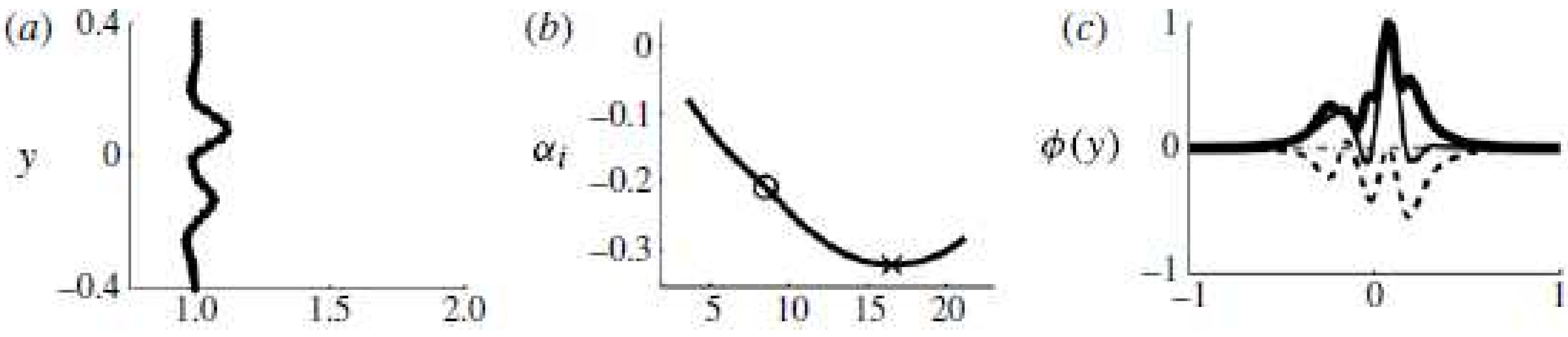
When the driving frequency matches the resonant frequency, a peak in efficiency is expected

Hydrodynamic resonance theory

Moored et al. (2012):

Analysis of 2P wake



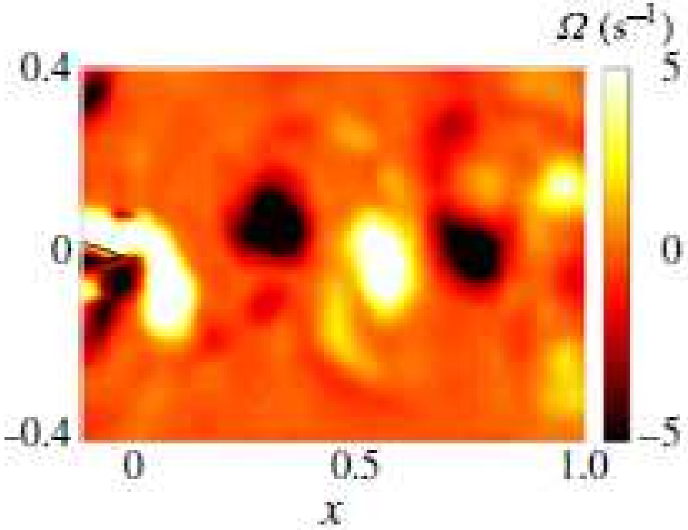


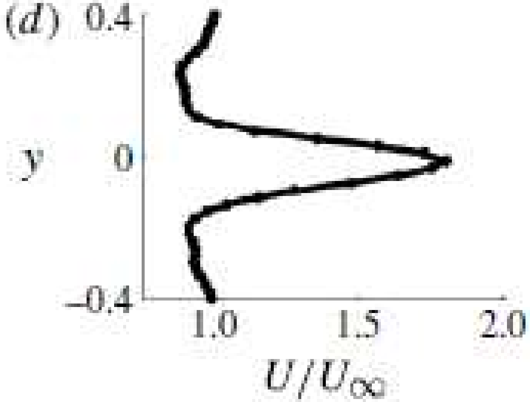
Velocity profile   Growth rate   Eigenmode

Hydrodynamic resonance theory

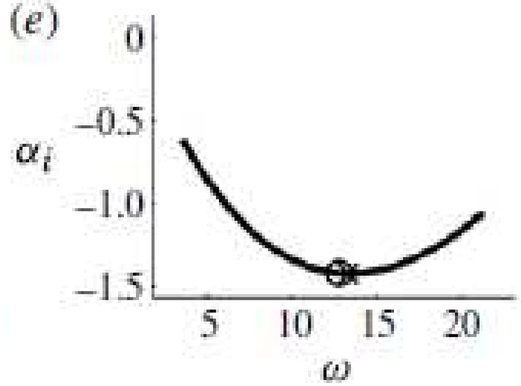
Moored et al. (2012):

Analysis of 2S wake

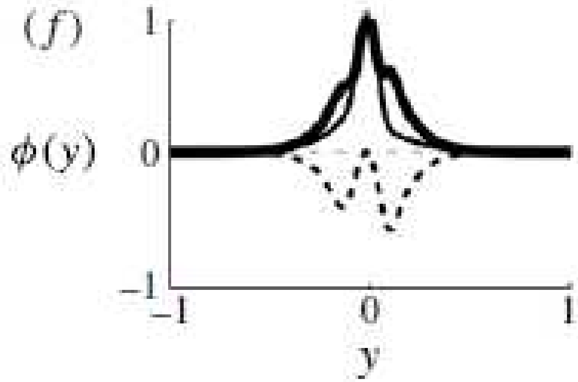




Velocity profile



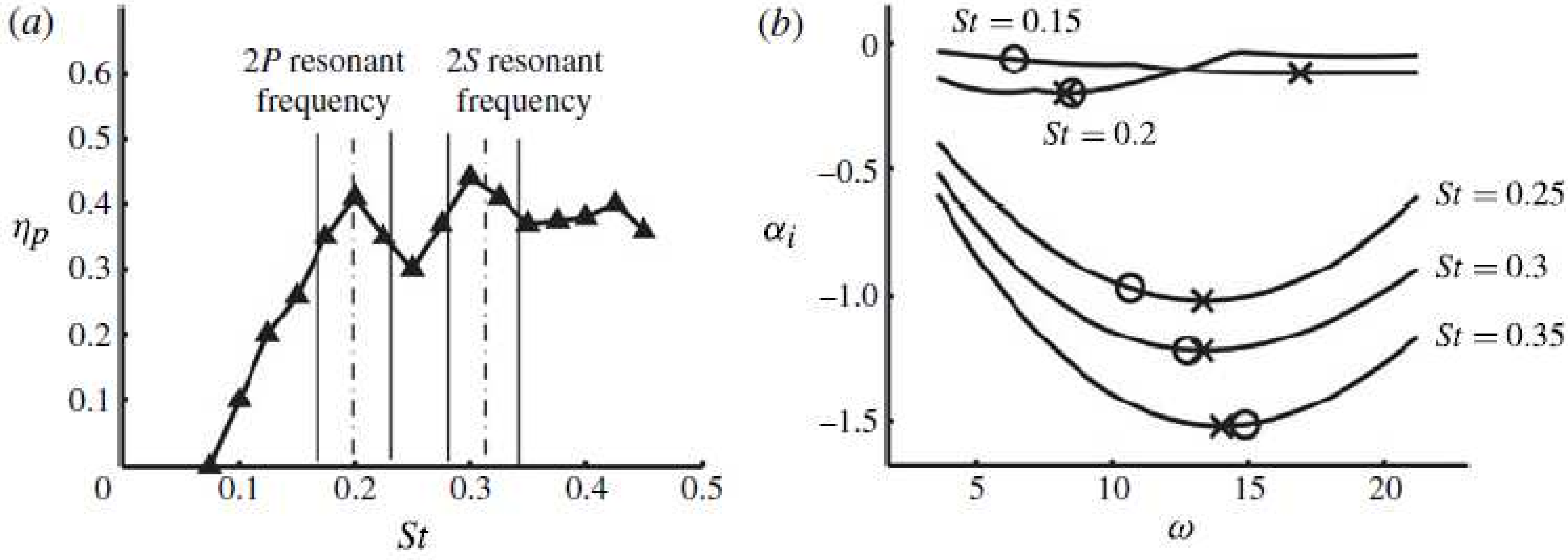
Growth rate



Eigenmode

Hydrodynamic resonance theory

Moored et al. (2012):



Experimental results

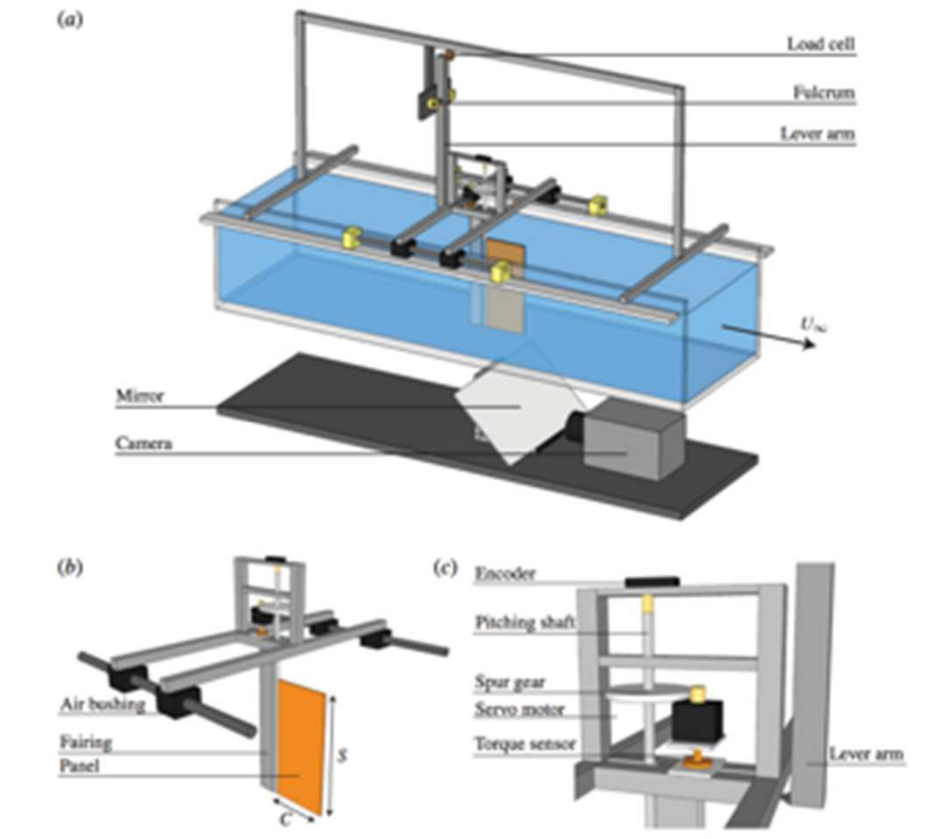
(Propulsive efficiency)

Spatial stability results

Moored et al 2014

- Extends wake resonance theory to flexible propulsors;
- Tuning the structural resonant frequency to a wake resonant frequency improves the propulsive efficiency;
- The entrainment of momentum into the time-averaged jet is seen as a physical explanation to this phenomenon

Dewey et al (2013)

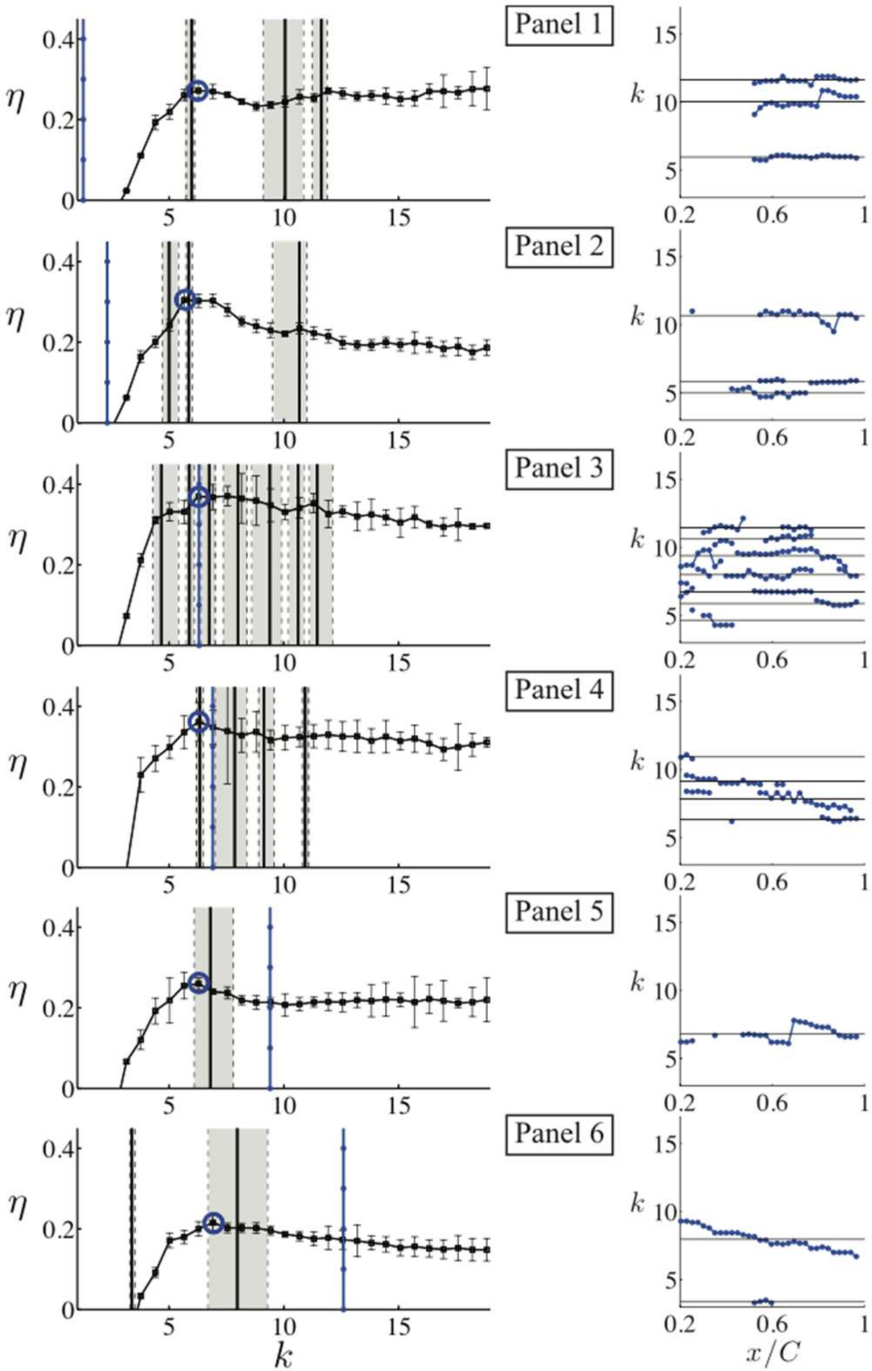


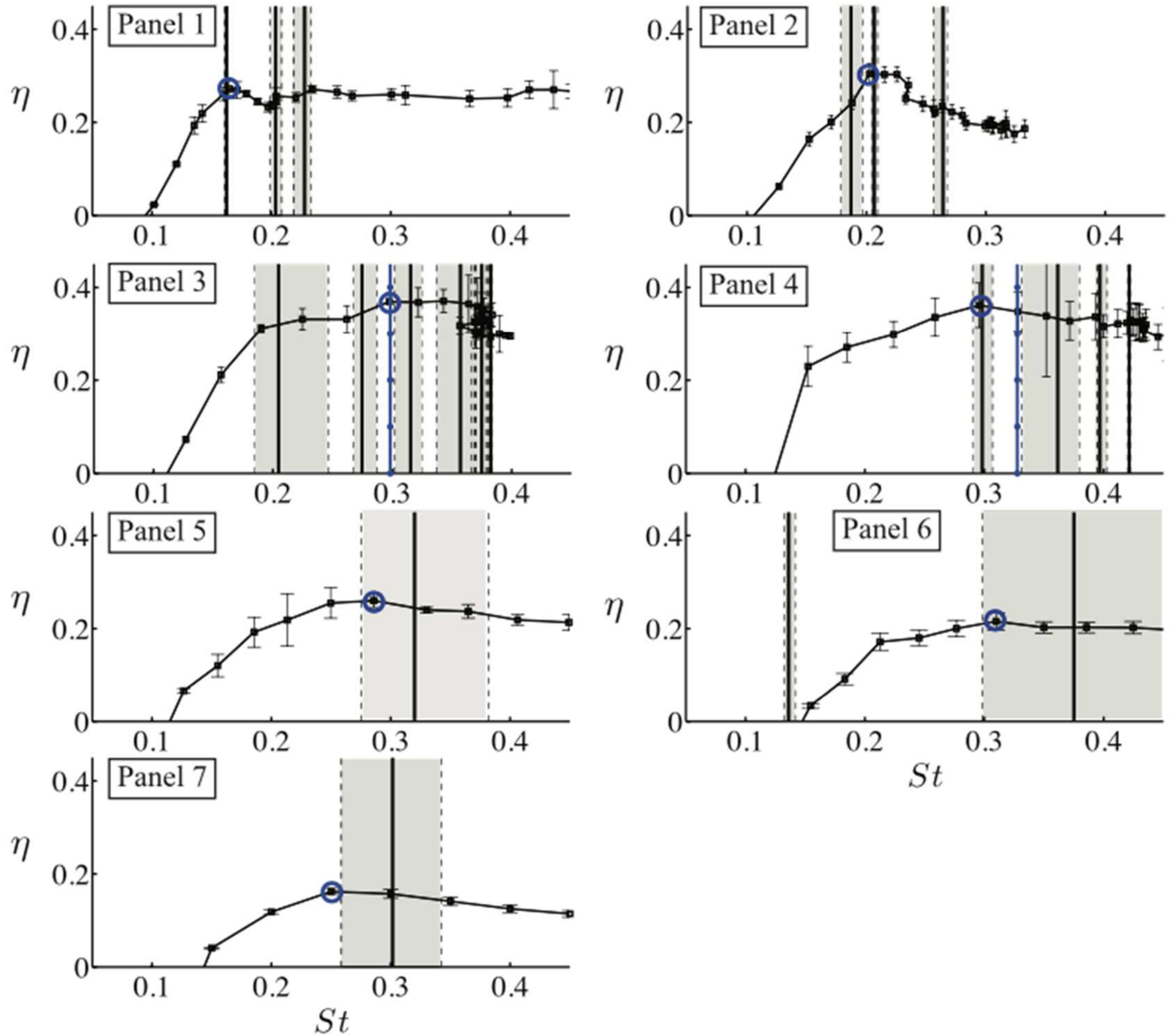
- Re = 7200
- U = 0.06 m/s
- C = 120 mm
- W = 280 mm

- 6 flexible panels and rigid one from previous article.
- EI = 4.2 - 1100 * 10E-4 Nm2

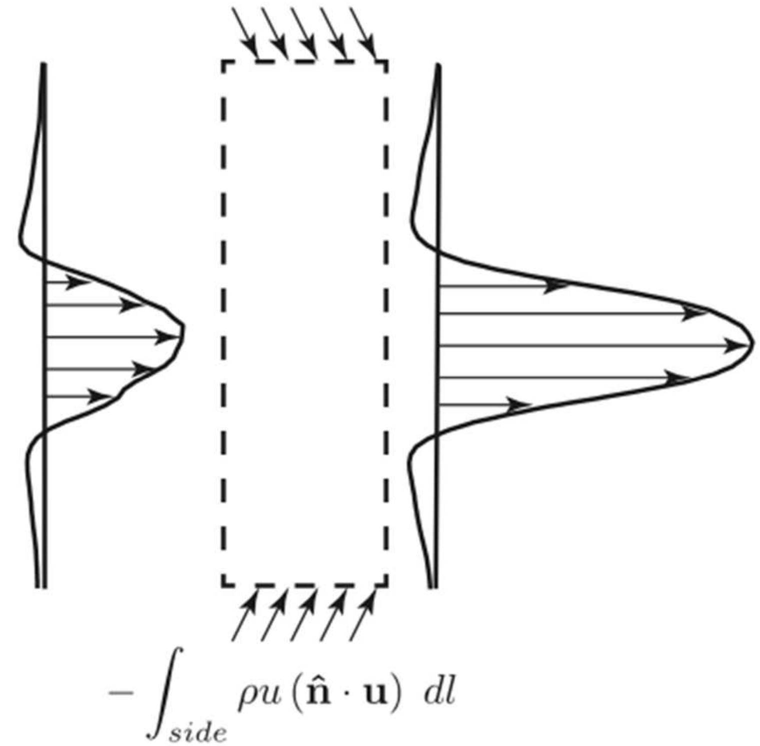
Efficiency

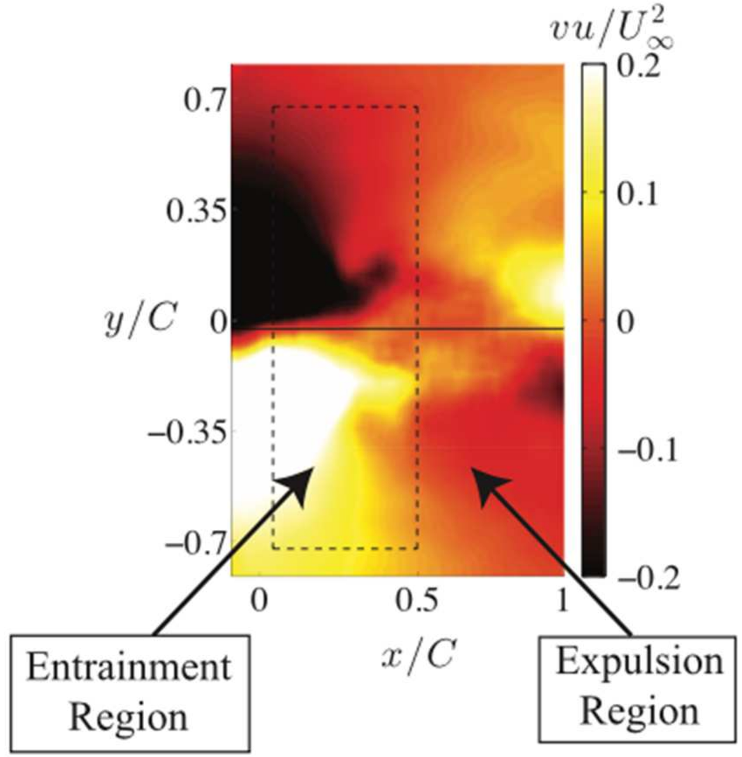
EI increases

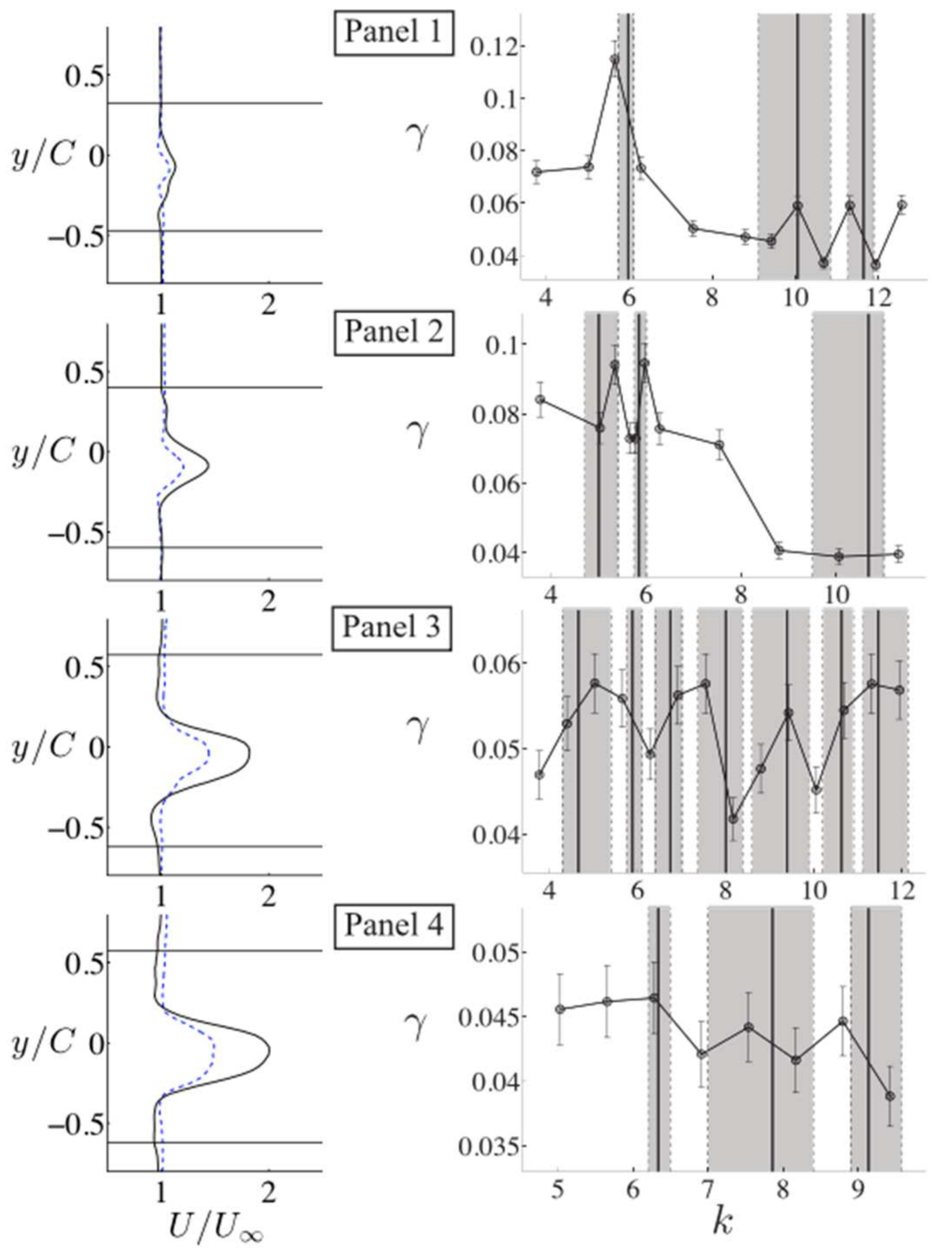



Panel 1
Panel 2
Panel 3
Panel 4
Panel 5
Panel 6
Panel 7
η
St
0
0.2
0.4
0.1
0.3

Momentum entrainment







$$\gamma = \frac{\int_{side} u v \, dl}{\int_{in} u^2 \, dl},$$

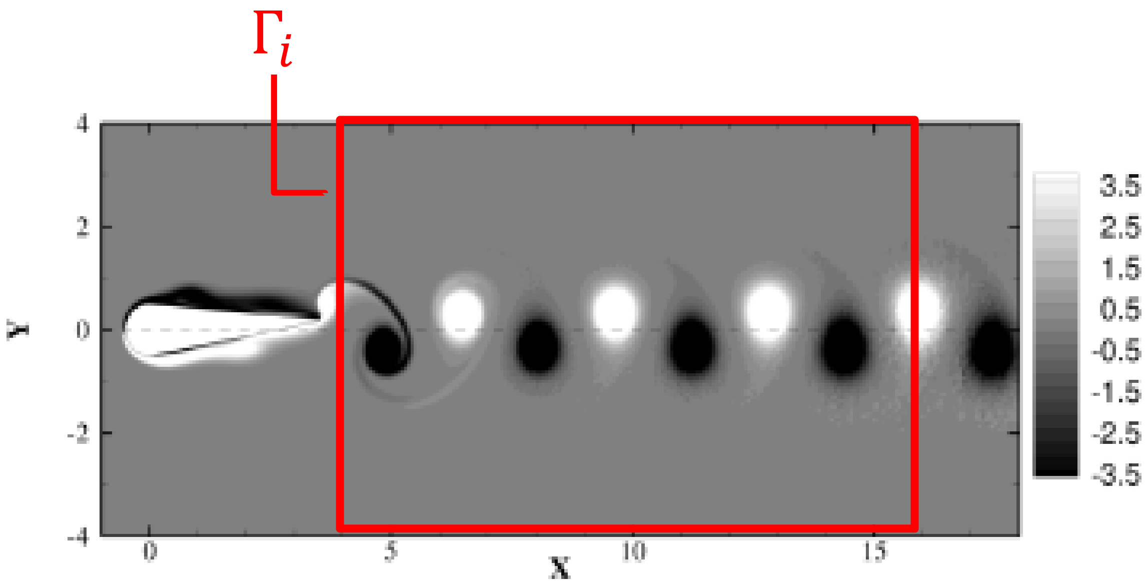
Objectives:

1 - Extend the spatial stability analysis to non-parallel flows based on the resolvent operator

2 - With a momentum balance analysis, better understand the link between the time-averaged power injected in the fluid (fluctuation equation) and the mean drag force(mean flow equation).

3 - Reveal the role of the linear amplification of perturbation in the wake structure .

Back to Navier-Stokes equations



$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot \big(\boldsymbol{u} \otimes \boldsymbol{u} + \boldsymbol{\sigma}(\boldsymbol{u}, p)\big) = 0 \;\; ; \;\; \nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{\sigma}(\boldsymbol{u}, p) = -p\, I + \frac{1}{Re}\left(\boldsymbol{\nabla}\boldsymbol{u} + \boldsymbol{\nabla}\boldsymbol{u}^T\right)$$

$$\boldsymbol{u}(y,t) = \boldsymbol{u_i}(y,t) \text{ on } \boldsymbol{\Gamma_i}$$

Reynolds decomposition of the wake

$$\boldsymbol{u}(\boldsymbol{x},t) = \overline{\boldsymbol{u}}(\boldsymbol{x}) + \boldsymbol{u}'(\boldsymbol{x},t)$$

$$p(\boldsymbol{x},t) = \bar{p}(\boldsymbol{x}) + p'(\boldsymbol{x},t)$$

$$\boldsymbol{u}_i(y,t) = \overline{\boldsymbol{u}_{\boldsymbol{i}}}(y) + \boldsymbol{u}_{\boldsymbol{i}}{}'(y,t)$$

Time-averaged (mean) flow equations

$$\nabla \cdot \left(\overline{\boldsymbol{u}} \otimes \overline{\boldsymbol{u}} + \boldsymbol{\sigma}(\overline{\boldsymbol{u}},\bar{p})\right) = -\nabla \cdot \overline{(\boldsymbol{u}' \otimes \boldsymbol{u}')}$$

$$\overline{\boldsymbol{u}}(y,t) = \overline{\boldsymbol{u}}_{\boldsymbol{i}}(y,t) \text{ on } \boldsymbol{\Gamma_i}$$

Fluctuating flow (momentum) equations

$$\frac{\partial \boldsymbol{u}'}{\partial t} + \nabla \cdot (\boldsymbol{u}' \otimes \overline{\boldsymbol{u}} + \overline{\boldsymbol{u}} \otimes \boldsymbol{u}' + \boldsymbol{\sigma}(\boldsymbol{u}',p')) = -\nabla \cdot (\boldsymbol{u}' \otimes \boldsymbol{u}') + \nabla \cdot \overline{(\boldsymbol{u}' \otimes \boldsymbol{u}')}$$

$$\boldsymbol{u}'(y,t) = \boldsymbol{u}_{\boldsymbol{i}}'(y,t) \text{ on } \boldsymbol{\Gamma_i}$$

Nonlinear fluctuation equation

$$\frac{\partial \boldsymbol{u}'}{\partial t} + \nabla \cdot (\boldsymbol{u}' \otimes \overline{\boldsymbol{u}} + \overline{\boldsymbol{u}} \otimes \boldsymbol{u}' + \boldsymbol{\sigma}(\boldsymbol{u}', p')) = -\nabla \cdot (\boldsymbol{u}' \otimes \boldsymbol{u}') + \nabla \cdot \overline{(\boldsymbol{u}' \otimes \boldsymbol{u}')}$$

$L(\overline{\boldsymbol{u}})$: linear operator (around the mean flow)

$$\frac{\partial \boldsymbol{u}'}{\partial t} + L(\overline{\boldsymbol{u}})(\boldsymbol{u}', p') = \boldsymbol{f}(\boldsymbol{x}, t)\,; \quad \nabla \cdot \boldsymbol{u}' = 0$$

$$\overline{\boldsymbol{u}}(y, t) = \overline{\boldsymbol{u}}_{\boldsymbol{i}}(y, t) \text{ on } \boldsymbol{\Gamma}_{\boldsymbol{i}}$$

Nonlinear fluctuation equation

$$\boldsymbol{q}' = (\boldsymbol{u}', p')^T$$

$$B\frac{\partial \boldsymbol{q}'}{\partial t} + J(\overline{\boldsymbol{u}})\,\boldsymbol{q}' = P_f\,\boldsymbol{f}(\boldsymbol{x}, t)$$

$$\boldsymbol{u}'(y, t) = \boldsymbol{u'}_{\boldsymbol{i}}(y, t) \text{ on } \boldsymbol{\Gamma_i}$$

$J(\overline{\boldsymbol{u}})$: Jacobian operator (around the mean flow)

P_f: Prolongation operator ($P_f\,\boldsymbol{f} = (\boldsymbol{f}, 0)^T$)

Fourier decomposition

$$\boldsymbol{q}' = \left(\boldsymbol{q_1}(\boldsymbol{x})\, e^{i\,\omega t} + c.c.\ \right) + \left(\boldsymbol{q_2}(\boldsymbol{x})\, e^{i\,2\omega t} + c.c.\ \right) + \cdots$$

$$\boldsymbol{f}' = \left(\boldsymbol{q_1}(\boldsymbol{x})\, e^{i\,\omega t} + c.c.\ \right) + \left(\boldsymbol{q_2}(\boldsymbol{x})\, e^{i\,2\omega t} + c.c.\ \right) + \cdots$$

$$\boldsymbol{u}_{\boldsymbol{i}}' = \left(\boldsymbol{u_{i1}}(\boldsymbol{x})\, e^{i\,\omega t} + c.c.\ \right) + \left(\boldsymbol{u_{i2}}(\boldsymbol{x})\, e^{i\,2\omega t} + c.c.\ \right) + \cdots$$

$$\left(i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\ \boldsymbol{q_1} = P_f\ \boldsymbol{f}_1 \qquad \boldsymbol{u_1}(y) = \boldsymbol{u_{i1}}(\boldsymbol{y}) \text{ on } \boldsymbol{\Gamma_i}$$

$$\left(2i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\ \boldsymbol{q_2} = P_f\ \boldsymbol{f_2} \qquad \boldsymbol{u_2}(y) = \boldsymbol{u_{i2}}(\boldsymbol{y}) \text{ on } \boldsymbol{\Gamma_i}$$

Decompostion into forced and inlet problems

$$\left(i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\,\boldsymbol{q}_{\mathbf{1}} = P_f\,\boldsymbol{f}_1 \qquad \widehat{\boldsymbol{u}}_{\mathbf{1}}(y) = \boldsymbol{u}_{i1}(\boldsymbol{y}) \text{ on } \boldsymbol{\Gamma}_{\boldsymbol{i}}$$

We assume that $\boldsymbol{f}_1$ and $\boldsymbol{u}_{i1}(\boldsymbol{y})$ are unknowns and independent of the solution

We decompose the solution as $\boldsymbol{q}_1 = \boldsymbol{q}_1^f + \boldsymbol{q}_1^i$

Forced problem

$$\left(i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{f}} = P_f\,\boldsymbol{f}_1 \qquad \boldsymbol{u}_{\mathbf{1}}^{\boldsymbol{f}}(y) = \mathbf{0} \text{ on } \boldsymbol{\Gamma}_{\boldsymbol{i}}$$

Inlet problem

$$\left(i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{i}} = \mathbf{0} \qquad \boldsymbol{u}_{\mathbf{1}}^{\boldsymbol{i}}(y) = \boldsymbol{u}_{i1}(\boldsymbol{y}) \text{ on } \boldsymbol{\Gamma}_{\boldsymbol{i}}$$

Forced problem and resolvent modes

$$\hat{\boldsymbol{q}}^f = R(\omega)\, P_f\, \hat{\boldsymbol{f}} = \left(i\, \omega\, B + J(\overline{\boldsymbol{u}})\right)^{-1} P_f\, \hat{\boldsymbol{f}}$$

Resolvent operator with boundary condition

$$\hat{\boldsymbol{u}}(y) = \mathbf{0} \text{ on } \boldsymbol{\Gamma_i}$$

$$\max_{\hat{f}}\left(\frac{\hat{\boldsymbol{q}}^{fH}\, B\, \hat{\boldsymbol{q}}^f}{\hat{\boldsymbol{f}}^H\, \hat{\boldsymbol{f}}}\right) ?$$

$$\hat{\boldsymbol{q}}^{fH}\, B\, \hat{\boldsymbol{q}}^f = \left(R\, P_f\, \hat{\boldsymbol{f}}\right)^H B\, \left(\, R\, P_f\, \hat{\boldsymbol{f}}\,\right) = \hat{\boldsymbol{f}}^{\boldsymbol{H}} P_f^T R^H B\, R\, P_f\, \hat{\boldsymbol{f}}$$

Forced problem and resolvent modes

Eigenvalue problem

$$P_f^T R^{\,H} B\, R\, P_f\, \hat{\boldsymbol{f}}_{\boldsymbol{k}} = \lambda_k^2\, \hat{\boldsymbol{f}}_{\boldsymbol{k}}$$

λ_k^2: positive real eigenvalue ($\lambda_0^2 \geq \lambda_1^2 \geq \cdots$)

$\hat{\boldsymbol{f}}_{\boldsymbol{k}}$: eigenvector

Associated flow solution

$$\hat{\boldsymbol{q}}_{\boldsymbol{k}}^{\boldsymbol{f}} = R(\omega)\, P_f\, \hat{\boldsymbol{f}}_{\boldsymbol{k}} \quad \text{with} \quad \hat{\boldsymbol{u}}_{\boldsymbol{k}}(y) = \boldsymbol{0} \text{ on } \boldsymbol{\Gamma_i}$$

$$\lambda_k^2 = \frac{\hat{\boldsymbol{q}}_{\boldsymbol{k}}^{\boldsymbol{f}\,\boldsymbol{H}}\, B\, \hat{\boldsymbol{q}}_{\boldsymbol{k}}^{\boldsymbol{f}}}{\hat{\boldsymbol{f}}_{\boldsymbol{k}}^{\boldsymbol{H}}\, \hat{\boldsymbol{f}}_{\boldsymbol{k}}} = \text{energetic ratio}$$

Projection of the forcing

Projection of the harmonic forcing onto the basis

$$\boldsymbol{f}_{\mathbf{1}} = \alpha_1\,\widehat{\boldsymbol{f}}_{\mathbf{1}} + \alpha_2\,\widehat{\boldsymbol{f}}_{\mathbf{2}} + \cdots$$

The flow solution writes then

$$\boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{f}} = R\,(\omega) P_f\,\boldsymbol{f}_1 = R(\omega)\,P_f\left(\alpha_1\,\widehat{\boldsymbol{f}}_{\mathbf{1}} + \alpha_2\,\widehat{\boldsymbol{f}}_{\mathbf{2}} + \cdots\right)$$
$$= \alpha_1\,\widehat{\boldsymbol{q}}_{\mathbf{1}}^{\boldsymbol{f}} + \alpha_2\,\widehat{\boldsymbol{q}}_{\mathbf{2}}^{\boldsymbol{f}} +$$

$$E_c\left(\boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{f}}\right) = \boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{fH}} B\,\boldsymbol{q}_{\mathbf{1}}^{\boldsymbol{f}} = \alpha_1\,\lambda_1^2 + \alpha_2\lambda_2^2 + \cdots$$

Inlet problem and resolvent modes

$$\widehat{\boldsymbol{q}}^{\boldsymbol{i}} = R(\omega)\, P_i\, \widehat{\boldsymbol{u}}_{\boldsymbol{i}}$$

where P_i : operateur de relèvement de la condition aux limites

and the forcing term vanishes $\widehat{\boldsymbol{f}}(\boldsymbol{x}) = \mathbf{0}$ in Ω

$$\max_{\widehat{\boldsymbol{u}}_{\boldsymbol{i}}}\left(\frac{\widehat{\boldsymbol{q}}^{iH}\ B\ \widehat{\boldsymbol{q}}^{i}}{\widehat{\boldsymbol{u}}_{\boldsymbol{i}}^{\boldsymbol{H}}\widehat{\boldsymbol{u}}_{\boldsymbol{i}}}\right)?$$

$$\widehat{\boldsymbol{q}}^{iH}\ B\ \widehat{\boldsymbol{q}}^{i} = (R\ P_i\widehat{\boldsymbol{u}}_{\boldsymbol{i}})^H\ B\ (\ R\ P_i\ \widehat{\boldsymbol{u}}_{\boldsymbol{i}}\ ) = \widehat{\boldsymbol{u}}_{\boldsymbol{i}}{}^{\boldsymbol{H}} P_i^T R^H B\ R\ P_i\ \widehat{\boldsymbol{u}}_{\boldsymbol{i}}$$

Inlet problem and resolvent modes

Eigenvalue problem

$$P_i^T R^{\,H} B\; R\; P_i\; \boldsymbol{\hat{u}_{ik}} = \sigma_k^2\; \boldsymbol{\hat{u}_{ik}}$$

σ_k^2: positive real eigenvalue ($\sigma_0^2 \geq \sigma_1^2 \geq \cdots$)

$\boldsymbol{\hat{u}_{ik}}$: eigenvector

Associated flow solution

$$\boldsymbol{\hat{q}_k^i} = R(\omega)\; P_i\; \boldsymbol{\hat{u}_{ik}} \quad \text{with} \quad \boldsymbol{\hat{f}_k} = \boldsymbol{0} \text{ in } \Omega$$

$$\sigma_k^2 = \frac{\boldsymbol{\hat{q}_k^{iH}}\, B\, \boldsymbol{\hat{q}_k^i}}{\boldsymbol{\hat{u}_{ik}^H}\, \boldsymbol{\hat{u}_{ik}}} = \text{energetic ratio}$$

Projection of the inlet

Projection of the harmonic inlet velocity profile onto the basis

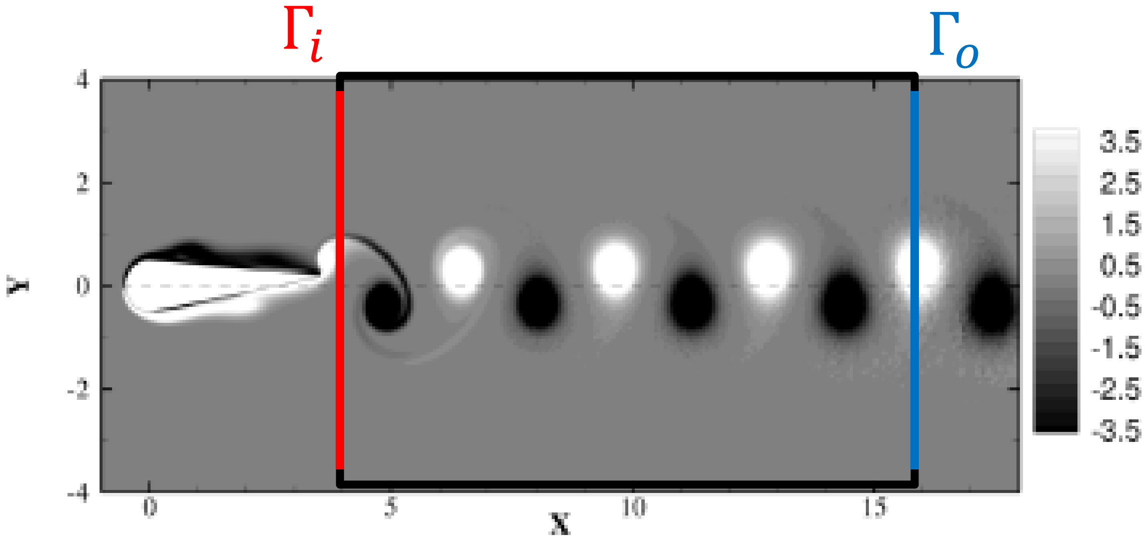
$$\boldsymbol{u_{i1}} = \beta_1\, \widehat{\boldsymbol{u}}_{\boldsymbol{i1}} + \beta_2\, \widehat{\boldsymbol{u}}_{\boldsymbol{i2}} + \cdots$$

The flow solution then writes

$$\boldsymbol{q_1^i} = R\,(\omega)P_i\, \boldsymbol{u}_{i1} = R(\omega)\, P_i\, (\beta_1\, \widehat{\boldsymbol{u}}_{\boldsymbol{i1}} + \beta_2\, \widehat{\boldsymbol{u}}_{\boldsymbol{i2}} + \cdots)$$
$$= \beta_1\, \widehat{\boldsymbol{q}}_{\boldsymbol{1}}^{\boldsymbol{i}} + \beta_2\, \widehat{\boldsymbol{q}}_{\boldsymbol{2}}^{\boldsymbol{i}} +$$

$$E_c\left(\boldsymbol{q_1^i}\right) = \boldsymbol{q_1^{iH}} B\, \boldsymbol{q_1^i} = \beta_1\, \sigma_1^2 + \beta_2 \sigma_2^2 + \cdots$$

New resolvent problems - 1



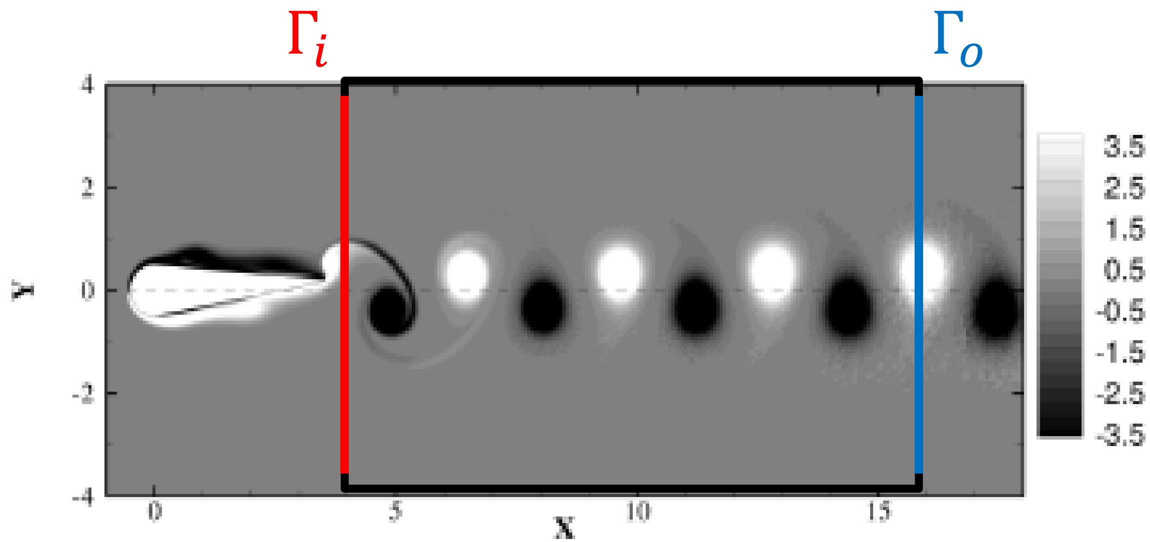
$$\left(i\,\omega\,B + J(\overline{\boldsymbol{u}})\right)\widehat{\boldsymbol{q}}^{\boldsymbol{i}} = \boldsymbol{0}$$

$\widehat{\boldsymbol{u}}^{\boldsymbol{i}}(x,y) = \widehat{\boldsymbol{u}}_{\boldsymbol{i}}(\boldsymbol{y})\ ;\ x \in \boldsymbol{\Gamma}_{\boldsymbol{i}}$

$\widehat{\boldsymbol{u}}^{\boldsymbol{i}}(x,y) = \widehat{\boldsymbol{u}}_{\boldsymbol{o}}(\boldsymbol{y})\ ;\ x \in \boldsymbol{\Gamma}_{\boldsymbol{o}}$

$$\max_{\widehat{\boldsymbol{u}}_{\boldsymbol{i}}}\left(\frac{\widehat{\boldsymbol{u}}_{\boldsymbol{o}}^{\boldsymbol{H}}\widehat{\boldsymbol{u}}_{\boldsymbol{o}}}{\widehat{\boldsymbol{u}}_{\boldsymbol{i}}^{\boldsymbol{H}}\widehat{\boldsymbol{u}}_{\boldsymbol{i}}}\right)\ ?$$

New resolvent problems - 1

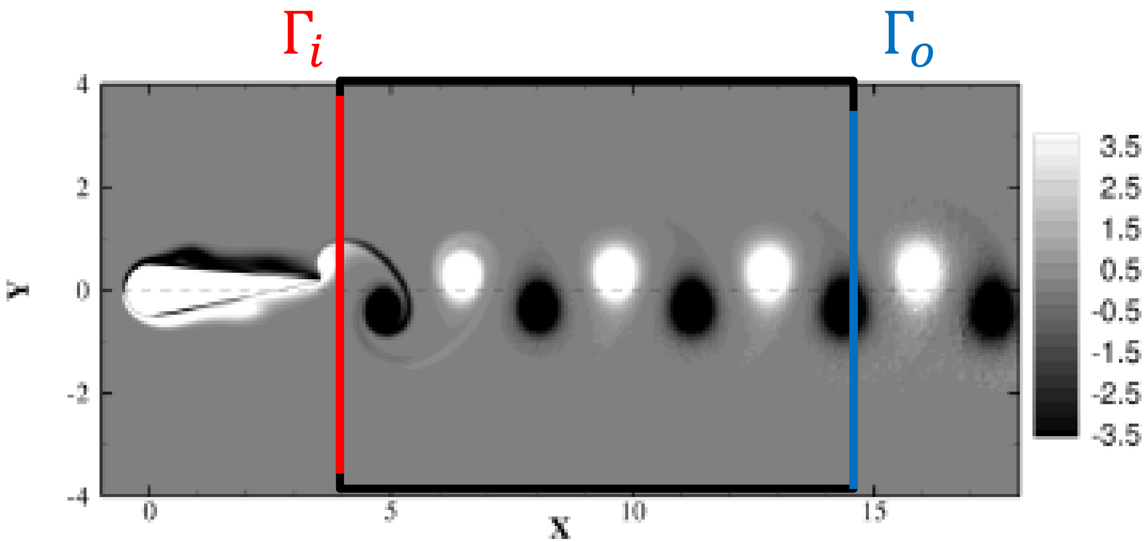


$$\hat{\boldsymbol{q}}^{\boldsymbol{i}} = R(\omega) P_i\, \hat{\boldsymbol{u}}_{\boldsymbol{i}} \qquad \hat{\boldsymbol{u}}_{\boldsymbol{o}} = P_o^T\, \hat{\boldsymbol{q}}^{\boldsymbol{i}}$$

$$\hat{\boldsymbol{u}}_{\boldsymbol{o}}^{\boldsymbol{H}}\, \hat{\boldsymbol{u}}_{\boldsymbol{o}} = \hat{\boldsymbol{q}}^{\boldsymbol{i}^{\boldsymbol{H}}} P_o P_o^T \hat{\boldsymbol{q}}^{\boldsymbol{i}} = \hat{\boldsymbol{u}}_{\boldsymbol{i}}^{\boldsymbol{H}}\, \left(P_i^H R(\omega)^H P_o P_o^T R(\omega) P_i \right)\, \hat{\boldsymbol{u}}_{\boldsymbol{i}}$$

$$P_i^H R(\omega)^H P_o P_o^T R(\omega) P_i\, \hat{\boldsymbol{u}}_{\boldsymbol{i}} = \lambda^2\, \hat{\boldsymbol{u}}_{\boldsymbol{i}}$$

Effect on the mean flow



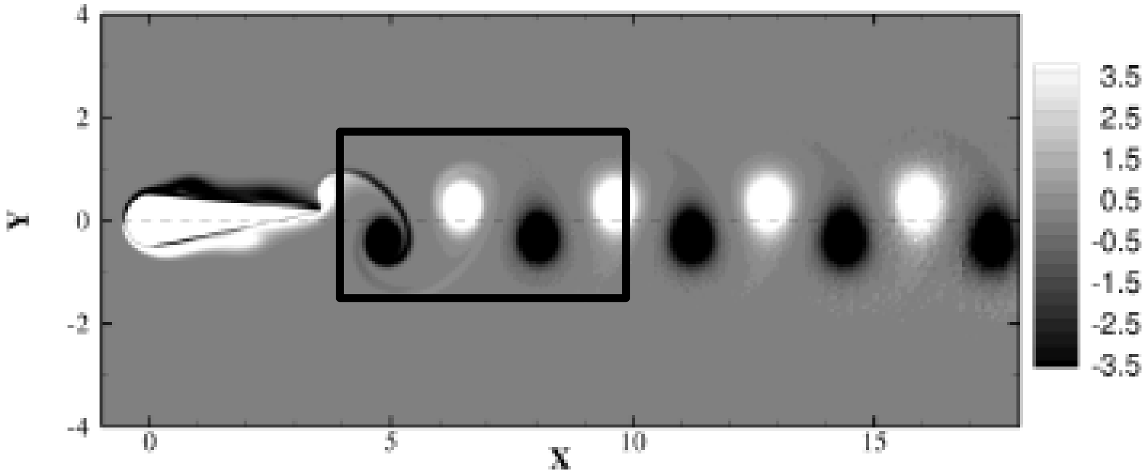
$$\hat{\boldsymbol{q}} = R(\omega) P_i\, \hat{\boldsymbol{u}}_{\boldsymbol{i}} \qquad \overline{\boldsymbol{F}} = -\nabla \cdot (\hat{\boldsymbol{u}}^* \otimes \hat{\boldsymbol{u}} + \hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{u}}^*)$$

Examine the mean flow force (in the streamwise direction) induced by the optimized harmonic response

$$\bar{F}_x = -2\, \Re(\, \hat{u}^* \partial_x \hat{u} + \hat{v}^* \partial_y \hat{u})$$

Reynolds stress tensor

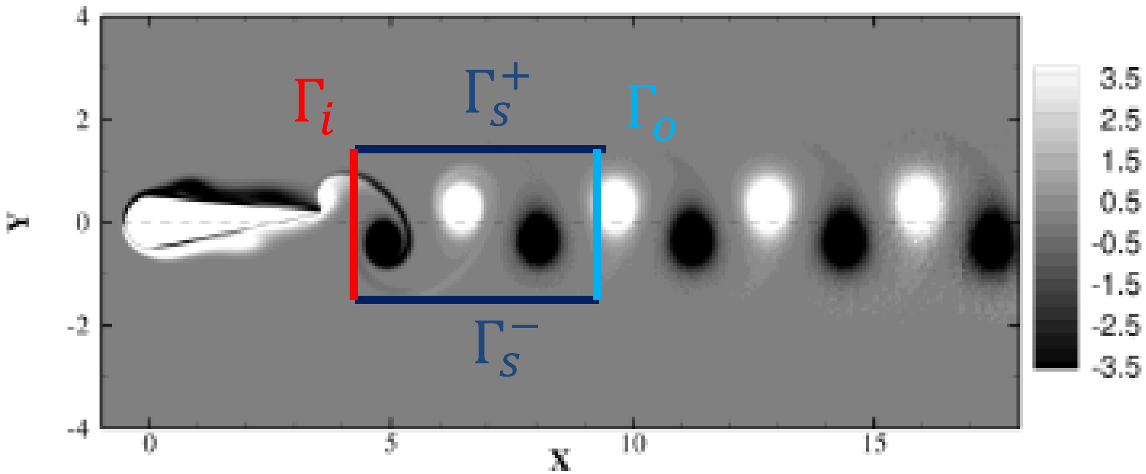
$$\overline{\boldsymbol{F}} = -\nabla \cdot (\widehat{\boldsymbol{u}}^{*} \otimes \widehat{\boldsymbol{u}} + \widehat{\boldsymbol{u}} \otimes \widehat{\boldsymbol{u}}^{*}) = \boldsymbol{\nabla} \cdot \begin{pmatrix} 2\,\hat{u}^{*}\hat{u} & 2\Re\,(\hat{u}^{*}\hat{v}) \\ 2\,\Re\,(\hat{v}^{*}\hat{u}) & 2\,\hat{v}^{*}\hat{v} \end{pmatrix}$$



$$\int_{\Omega} \overline{F}\, \mathrm{d}\Omega = \oint_{\partial\Omega} 2(\widehat{\boldsymbol{u}}^{*} \otimes \widehat{\boldsymbol{u}} + \widehat{\boldsymbol{u}} \otimes \widehat{\boldsymbol{u}}^{*}) \cdot \boldsymbol{n}\, dS$$

Reynolds stress tensor

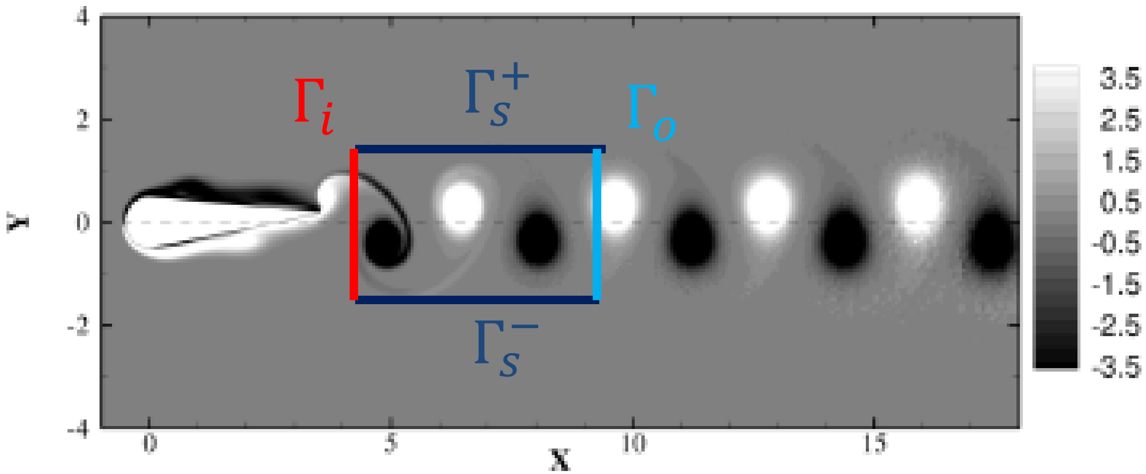
$$\overline{\boldsymbol{F}} = -\nabla \cdot (\widehat{\boldsymbol{u}}^* \otimes \widehat{\boldsymbol{u}} + \widehat{\boldsymbol{u}} \otimes \widehat{\boldsymbol{u}}^*) = \boldsymbol{\nabla} \cdot \begin{pmatrix} 2\, \hat{u}^* \hat{u} & 2\Re\,(\hat{u}^* \hat{v}) \\ 2\,\Re\,(\hat{u}^* \hat{v}) & 2\, \hat{v}^* \hat{v} \end{pmatrix}$$



$$\int_\Omega \overline{F}_x \, \mathrm{d}\Omega = \int_{\Gamma_o} (2\hat{u}^* \hat{u})\, dy \; - \int_{\Gamma_i} (2\hat{u}^* \hat{u})\, dy \; + \int_{\Gamma_s^+} 2\Re\,(\hat{u}^* \hat{v}) dx \; - \int_{\Gamma_s^-} 2\Re\,(\hat{u}^* \hat{v}) dx$$

Reynolds stress tensor

$$\overline{\boldsymbol{F}} = -\nabla \cdot (\widehat{\boldsymbol{u}}^* \otimes \widehat{\boldsymbol{u}} + \widehat{\boldsymbol{u}} \otimes \widehat{\boldsymbol{u}}^*) = \boldsymbol{\nabla} \cdot \begin{pmatrix} 2\,\hat{u}^*\hat{u} & 2\Re\,(\hat{u}^*\hat{v}) \\ 2\,\Re\,(\hat{u}^*\hat{v}) & 2\,\hat{v}^*\hat{v} \end{pmatrix}$$

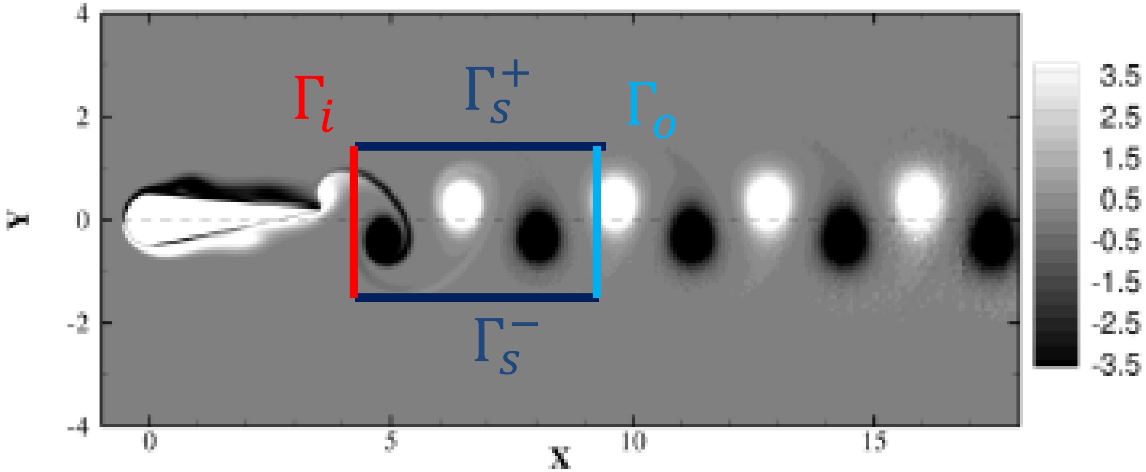


$$\int_\Omega \overline{F}_y\, \mathrm{d}\Omega = \int_{\Gamma_o} 2\Re\,(\hat{u}^*\hat{v})\, dy \;-\int_{\Gamma_i} 2\Re\,(\hat{u}^*\hat{v})\, dy \;+\int_{\Gamma_s^+} 2(\hat{v}^*\hat{v})dx \;-\int_{\Gamma_s^-} 2\,(\hat{v}^*\hat{v})dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

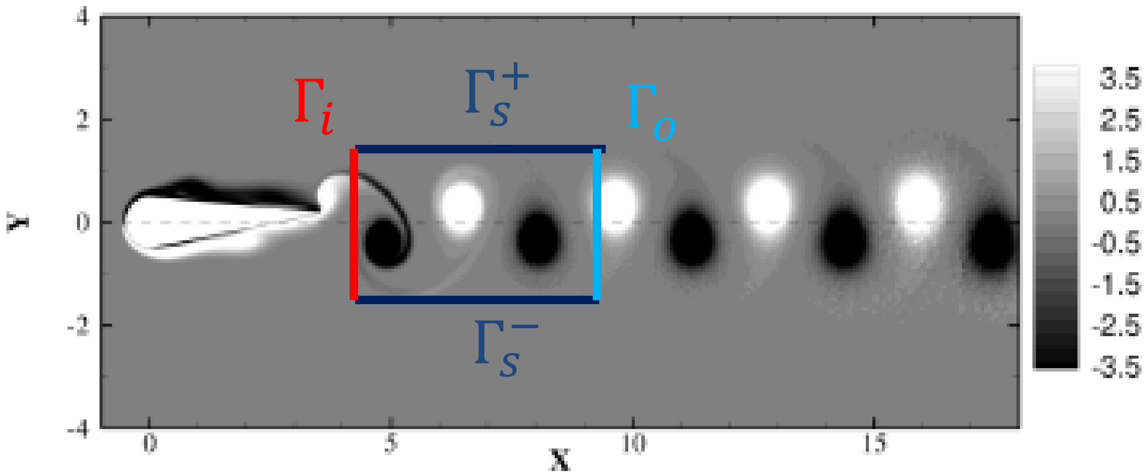


$$\int_{\Omega} \overline{F_x}\, \mathrm{d}\Omega = \int_{\Gamma_o} (2\hat{u}^*\hat{u})\, dy \; - \int_{\Gamma_i} (2\hat{u}^*\hat{u})\, dy \; + 2\int_{x_i}^{x_o} \Re\,(\hat{u}^*\hat{v})(y_s) - \Re\,(\hat{u}^*\hat{v})(-y_s) dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

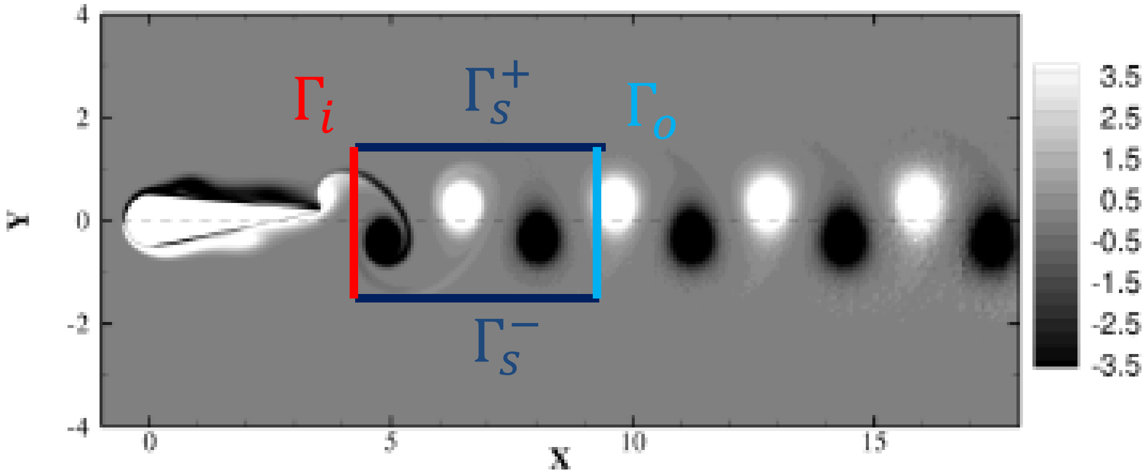


$$\int_\Omega \overline{F_x}\, \mathrm{d}\Omega = 2\int_{-y_s}^{y_s} \big((\hat{u}^*\hat{u})(x_o) - (\hat{u}^*\hat{u})(x_i)\big)dy \; + 4\int_{x_i}^{x_o} \Re\,(\hat{u}^*\hat{v})(y_s)dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

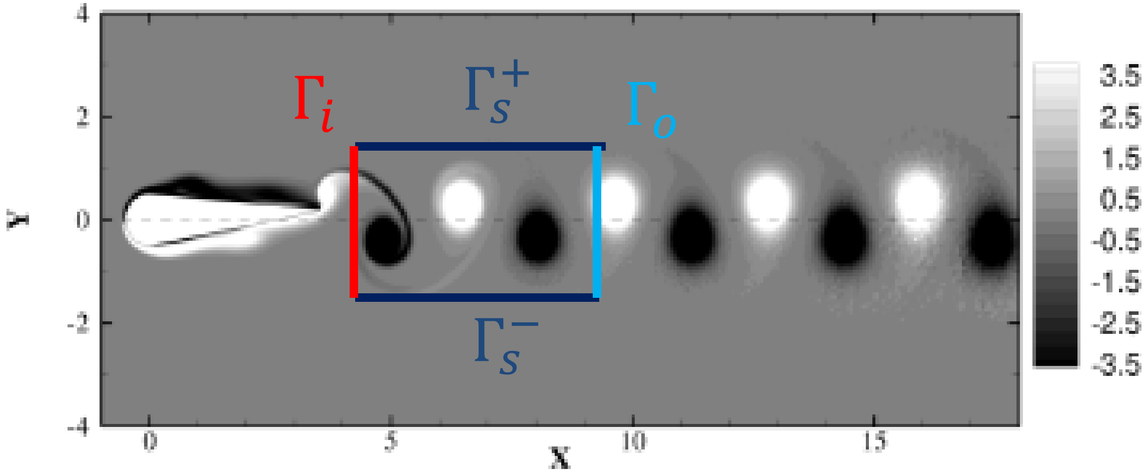


$$\hat{u}^*\hat{u}(x,-y) = \hat{u}^*\hat{u}(x,y)$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

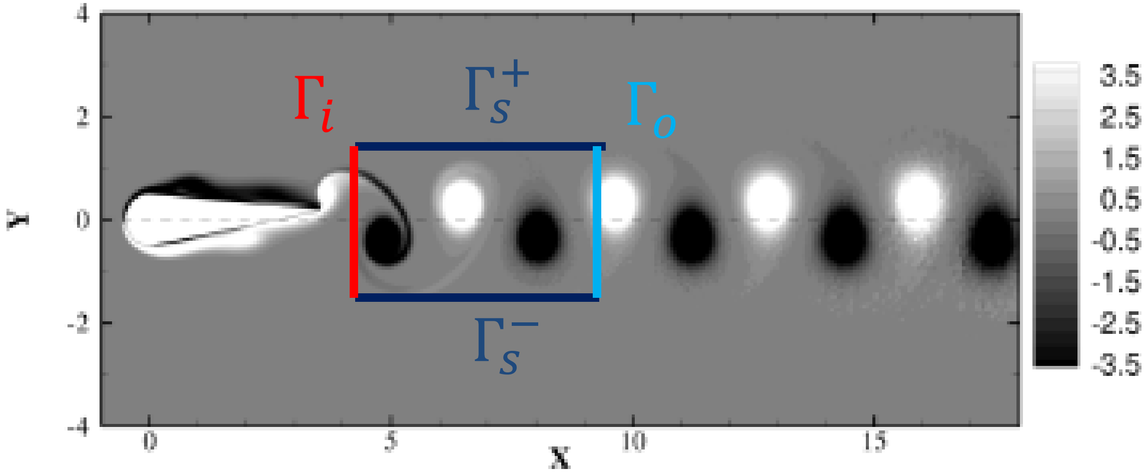


$$\int_{\Omega} \bar{F_x}\, \mathrm{d}\Omega = 4\int_0^{y_s}\big((\hat{u}^*\hat{u})(x_o) - (\hat{u}^*\hat{u})(x_i)\big)dy \;\; + 4\int_{x_i}^{x_o} \Re\,(\hat{u}^*\hat{v})(y_s)dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

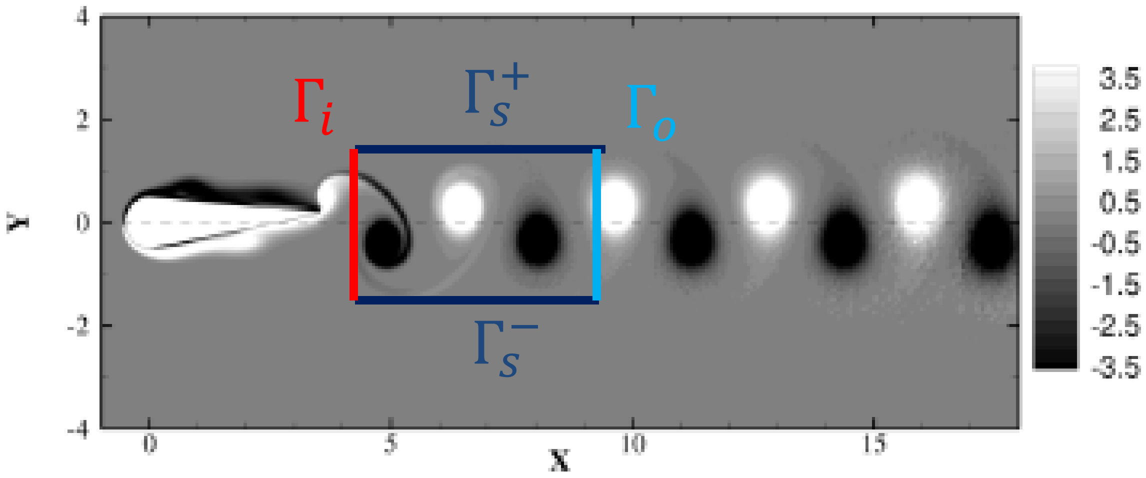


$$\int_\Omega \overline{F_y}\, \mathrm{d}\Omega = 2\int_{-y_s}^{y_s} (\Re\, (\hat{u}^*\hat{v})(x_o) - \Re\, (\hat{u}^*\hat{v})(x_i))dy \;+ \int_{x_i}^{x_o} 2(\hat{v}^*\hat{v})(y_s) - 2(\hat{v}^*\hat{v})(-y_s)dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

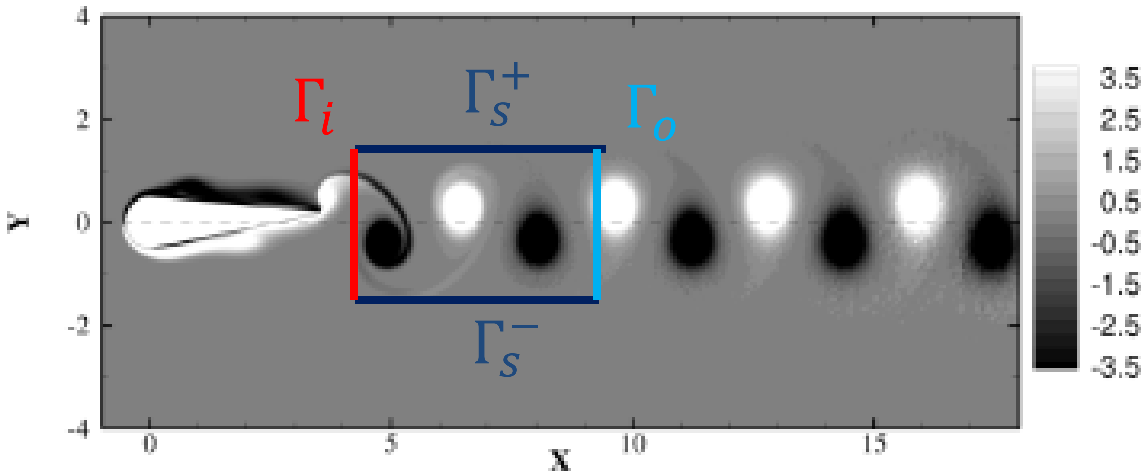


$$\int_\Omega \bar{F}_y \, \mathrm{d}\Omega = 2\int_{-y_s}^{y_s} (\Re\,(\hat{u}^*\hat{v})(x_o) - \Re\,(\hat{u}^*\hat{v})(x_i))dy \;\; + \int_{x_i}^{x_o} 2(\hat{v}^*\hat{v})(y_s) - 2(\hat{v}^*\hat{v})(y_s)dx$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

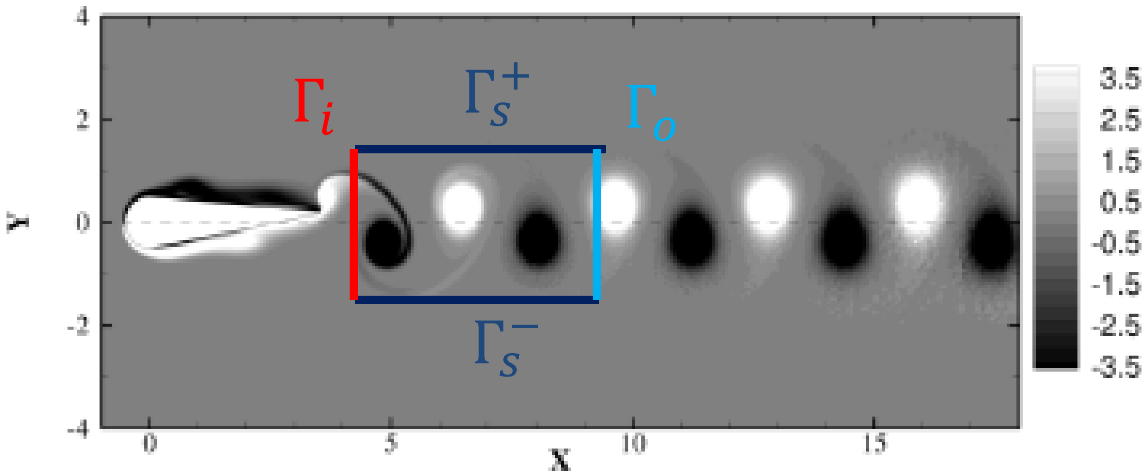


$$\int_{\Omega} \overline{F_y}\, \mathrm{d}\Omega = 2\int_{-y_s}^{y_s} (\Re\,(\hat{u}^*\hat{v})(x_o) - \Re\,(\hat{u}^*\hat{v})(x_i))dy$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$

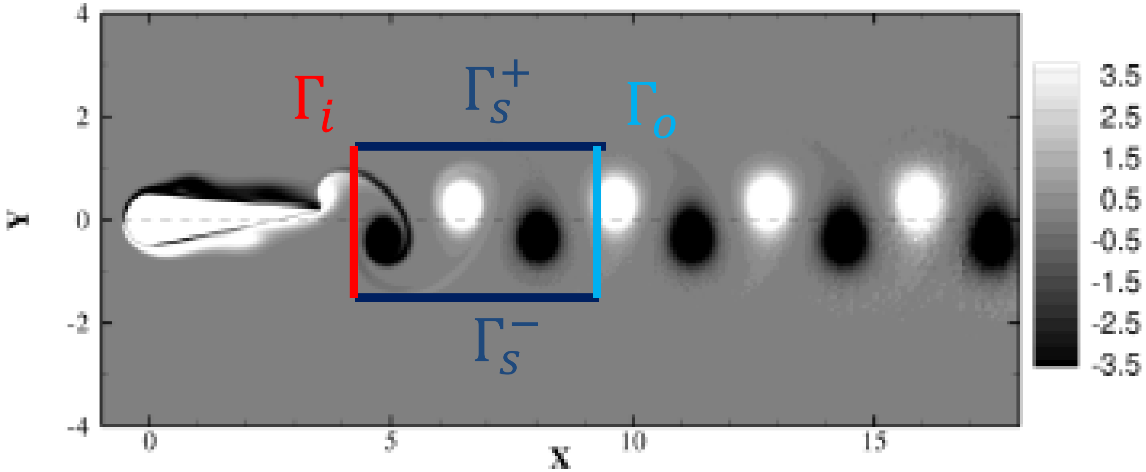


$$\Re\,(\hat{u}^*\hat{v})(x,-y) = -\Re\,(\hat{u}^*\hat{v})(x\ )$$

Reynolds stress tensor

Symmetry

$$\hat{u}(-y) = -\hat{u}(y) \quad \text{and} \quad \hat{v}(-y) = \hat{v}(y)$$



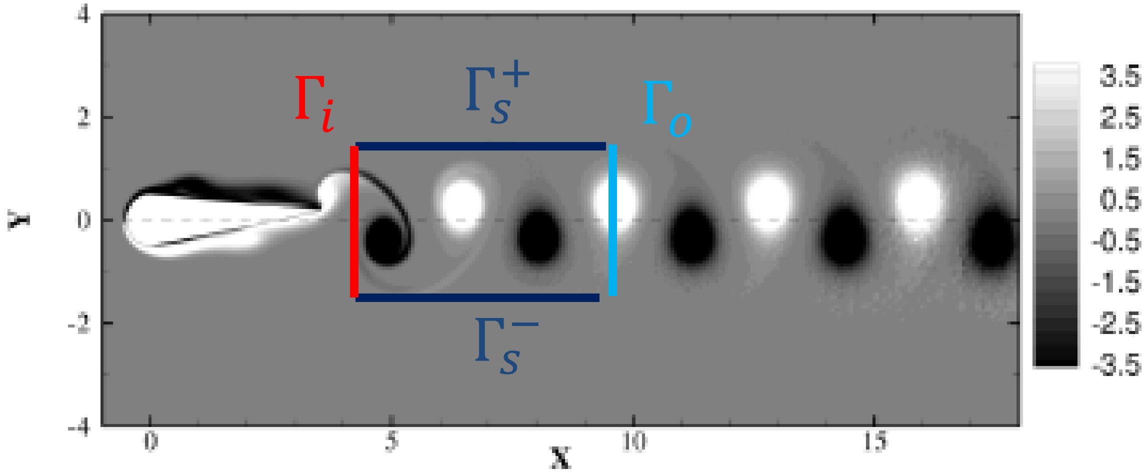
$$\int_{\Omega} \bar{F}_y \, \mathrm{d}\Omega = 0$$

Resolvent for entrainement

- Design a basis to maximize the entrainement

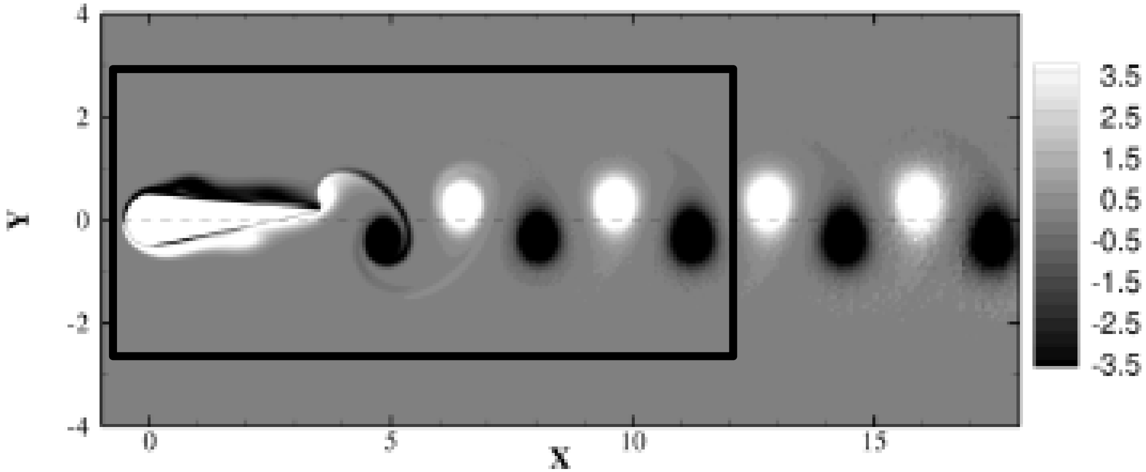
$$\int_{\Omega} \overline{F}_x \, \mathrm{d}\Omega = 2\int_{-y_s}^{y_s} \big((\hat{u}^*\hat{u})(x_o) - (\hat{u}^*\hat{u})(x_i)\big)dy \;\; + 4\int_{x_i}^{x_o} \Re \, (\hat{u}^*\hat{v})(y_s)dx$$

Vertical entrainement



$$\frac{\int_{-y_s}^{y_s}\big((\hat{u}^*\hat{u})(x_o)\big)dy + \int_{x_i}^{x_o} \Re \, (\hat{u}^*\hat{v})(y_s)dx}{\int_{-y_s}^{y_s}\big((\hat{u}^*\hat{u})(x_i)\big)dy}$$

Resolvent and body



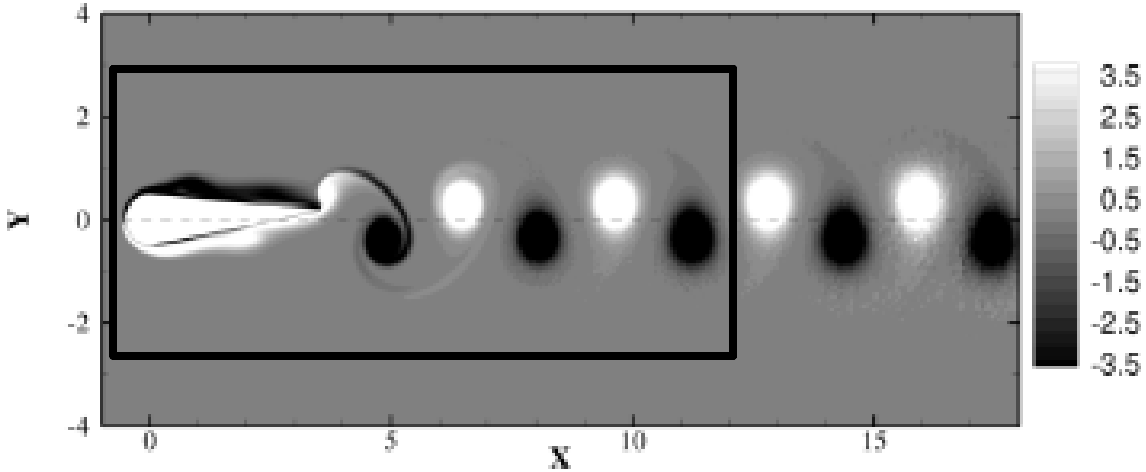
$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u}) + \nabla \cdot \big(\boldsymbol{\sigma}(\boldsymbol{u}, p)\big) = 0 \ ; \ \nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{\sigma}(\boldsymbol{u}, p) = (1-\chi)\boldsymbol{\sigma_f} + \chi \boldsymbol{\sigma_s}$$

$\chi = 1$: solid $\quad$ $\chi = 0$: fluid

$$\boldsymbol{\sigma_f}(\boldsymbol{u}, p) = -p\, I + \frac{1}{Re}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$$

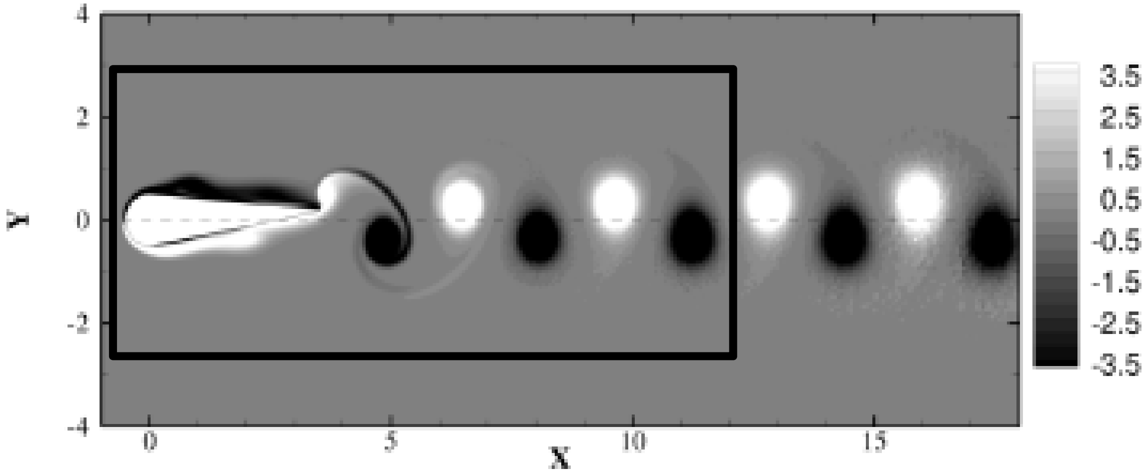
Resolvent and body



$$\nabla \cdot \left(\boldsymbol{\sigma}\,(\boldsymbol{u}, p)\right) = \nabla \cdot \left((1-\chi)\boldsymbol{\sigma_f} + \chi\boldsymbol{\sigma_s}\right)$$
$$= (1-\chi)\,\nabla \cdot \boldsymbol{\sigma_f} + \chi\,\nabla \cdot \boldsymbol{\sigma_s} + (\boldsymbol{\sigma_s} - \boldsymbol{\sigma_f}) \cdot \boldsymbol{\nabla}\chi$$

$$(1-\chi)\,\nabla \cdot \boldsymbol{\sigma_f} = (1-\bar{\chi}-\chi')\,\nabla \cdot \left(\overline{\boldsymbol{\sigma}}_{\boldsymbol{f}} + \boldsymbol{\sigma}'_{\boldsymbol{f}}\right)$$
$$= (1-\bar{\chi})\,\nabla \cdot \left(\overline{\boldsymbol{\sigma}}_{\boldsymbol{f}}\right) + (1-\bar{\chi})\,\nabla \cdot \left(\boldsymbol{\sigma}'_{\boldsymbol{f}}\right) + \chi'\nabla \cdot \left(\overline{\boldsymbol{\sigma}}_{\boldsymbol{f}}\right) + \chi'\,\nabla \cdot \left(\boldsymbol{\sigma}'_{\boldsymbol{f}}\right)$$

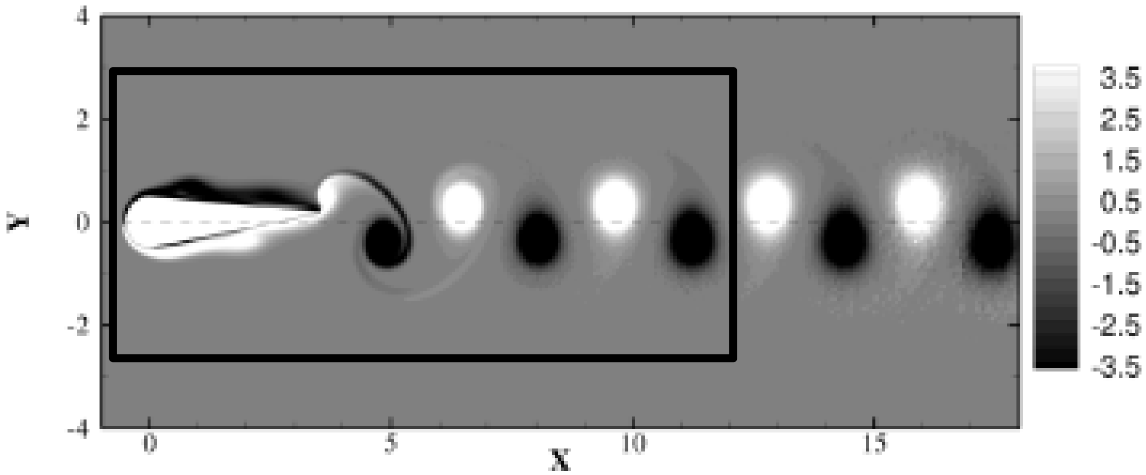
Resolvent and body



$$\overline{(1-\chi)\,\nabla\cdot\boldsymbol{\sigma_f}} = (1-\bar{\chi})\,\nabla\cdot\left(\overline{\boldsymbol{\sigma}}_{\boldsymbol{f}}\right) + \overline{\chi'\,\nabla\cdot\left(\boldsymbol{\sigma}'_{\boldsymbol{f}}\right)}$$

$$\left((1-\chi)\,\nabla\cdot\boldsymbol{\sigma_f}\right)' = (1-\bar{\chi})\,\nabla\cdot\left(\boldsymbol{\sigma}'_{\boldsymbol{f}}\right) + \chi'\nabla\cdot\left(\overline{\boldsymbol{\sigma}}_{\boldsymbol{f}}\right)$$

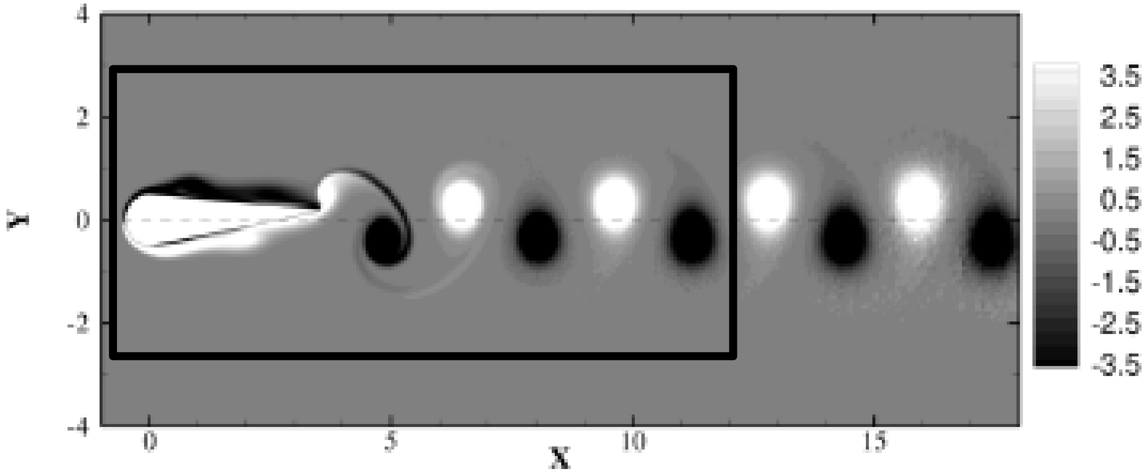
Resolvent and body



$$\overline{\chi\, \nabla \cdot \boldsymbol{\sigma_s}} = \bar{\chi}\, \nabla \cdot (\overline{\boldsymbol{\sigma}}_{\boldsymbol{s}}) + \overline{\chi'\, \nabla \cdot (\boldsymbol{\sigma}'_{\boldsymbol{s}})}$$

$$(\chi\, \nabla \cdot \boldsymbol{\sigma_s})' = \bar{\chi}\, \nabla \cdot (\boldsymbol{\sigma}'_{\boldsymbol{s}}) + \chi' \nabla \cdot (\overline{\boldsymbol{\sigma}}_{\boldsymbol{s}})$$

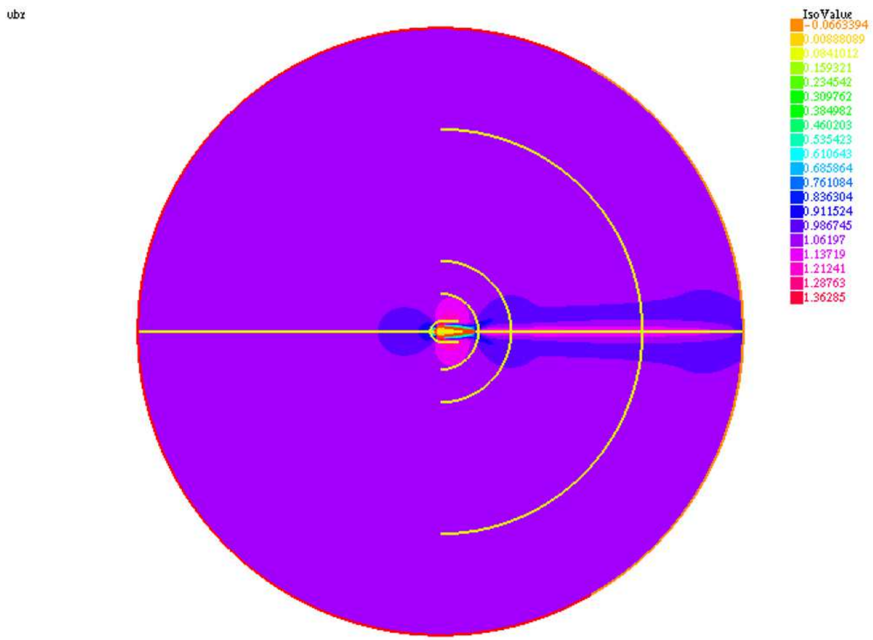
Resolvent and body

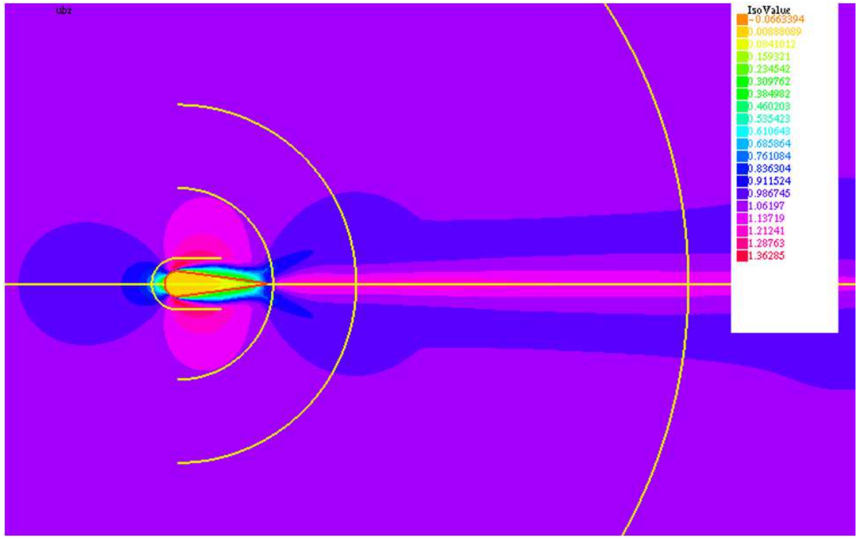


$$\overline{(\boldsymbol{\sigma_s} - \boldsymbol{\sigma_f})\boldsymbol{\nabla}\chi} = \overline{(\boldsymbol{\sigma_s} - \boldsymbol{\sigma_f})}\ \overline{\boldsymbol{\nabla}\chi} + \overline{(\boldsymbol{\sigma_s'} - \boldsymbol{\sigma_f'})\boldsymbol{\nabla}\chi'}$$

$$(\boldsymbol{\sigma_s} - \boldsymbol{\sigma_f})\boldsymbol{\nabla}\chi(\chi\, \nabla \cdot \boldsymbol{\sigma_s})' = \bar{\chi}\, \nabla \cdot (\boldsymbol{\sigma_s'}) + \chi' \nabla \cdot (\overline{\boldsymbol{\sigma}}_{\boldsymbol{s}})$$

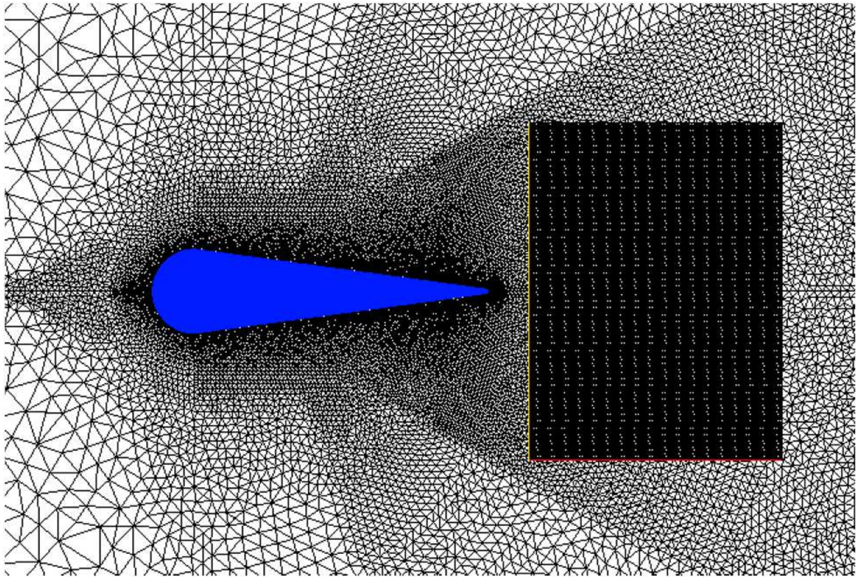
Temporal simulation





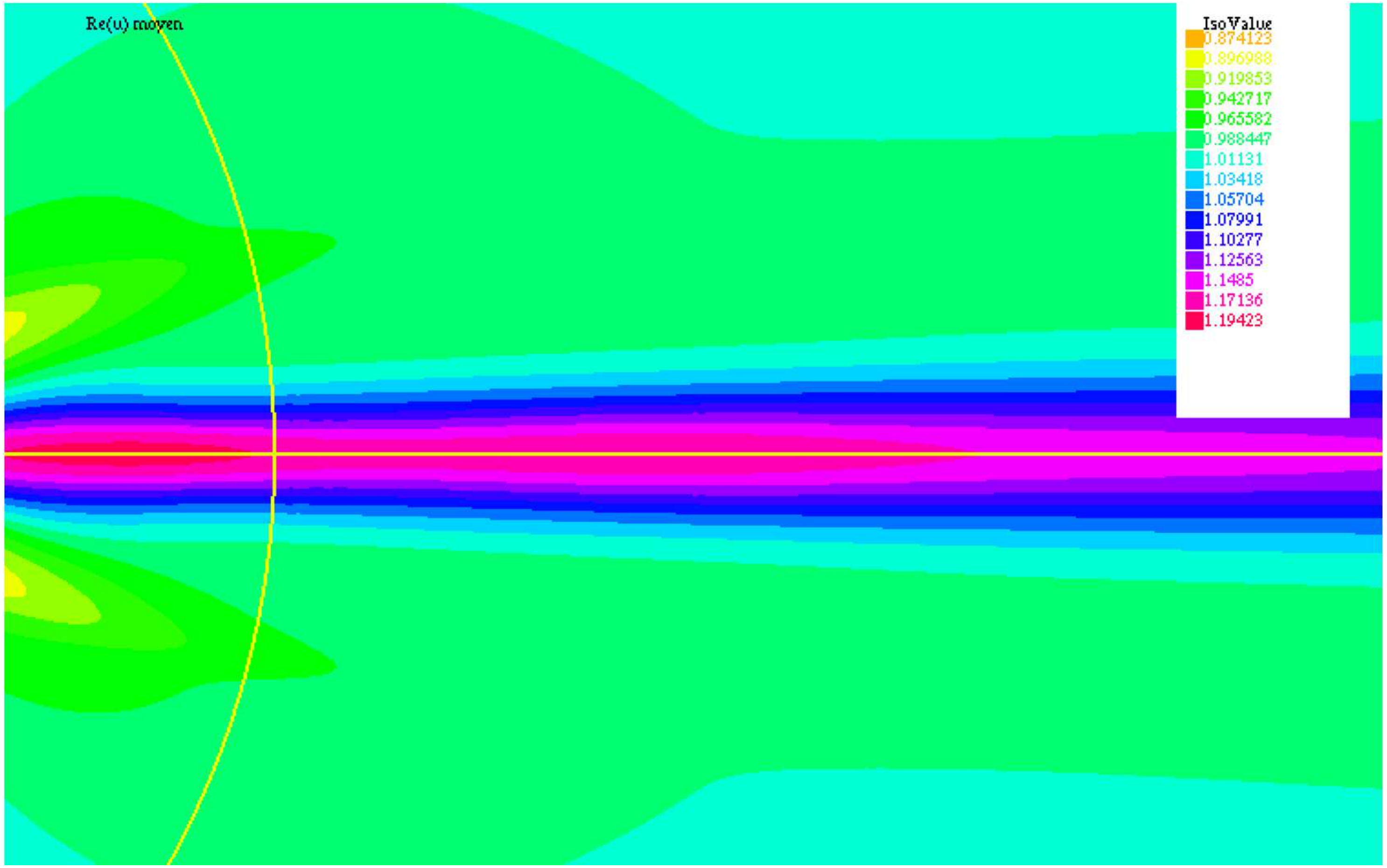
Jallas et al. (2017)

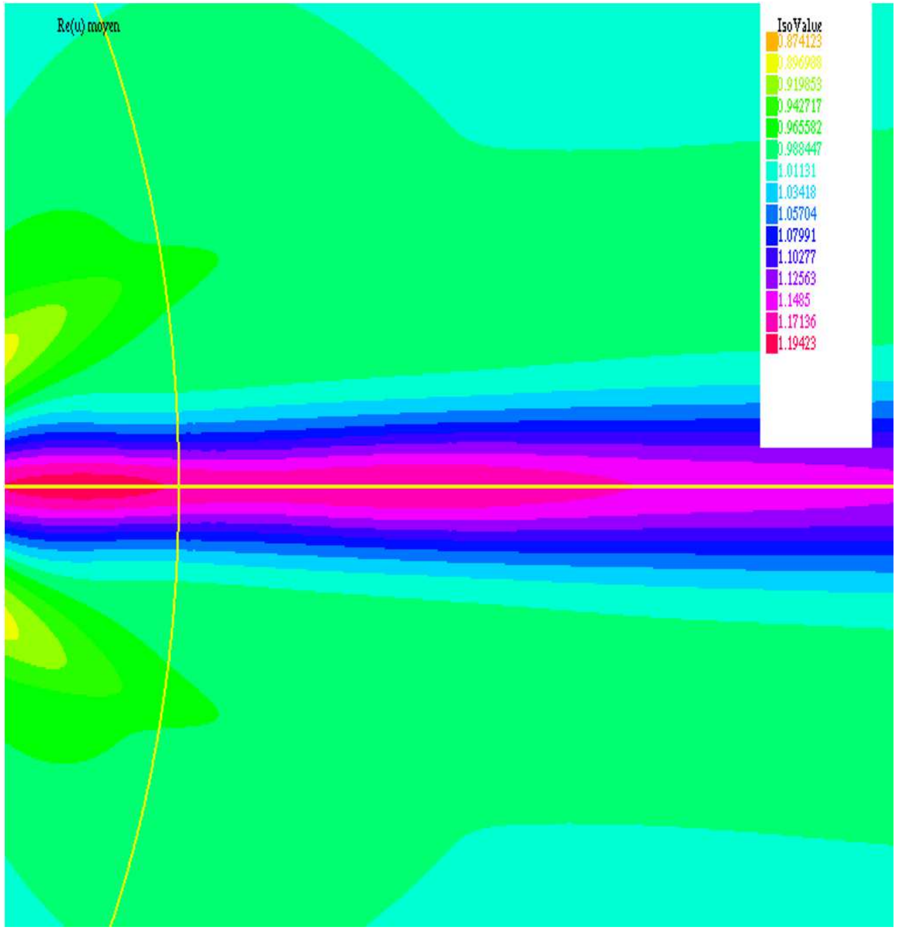
Analyse dans une section



- Comparaison résolvant x Analyse locale de Moored et al (2012);
- Obtention des fluctuations à partir des réponses optimales de l'analyse résolvant, utilisant des données du champ moyen de la DNS.

U moyen

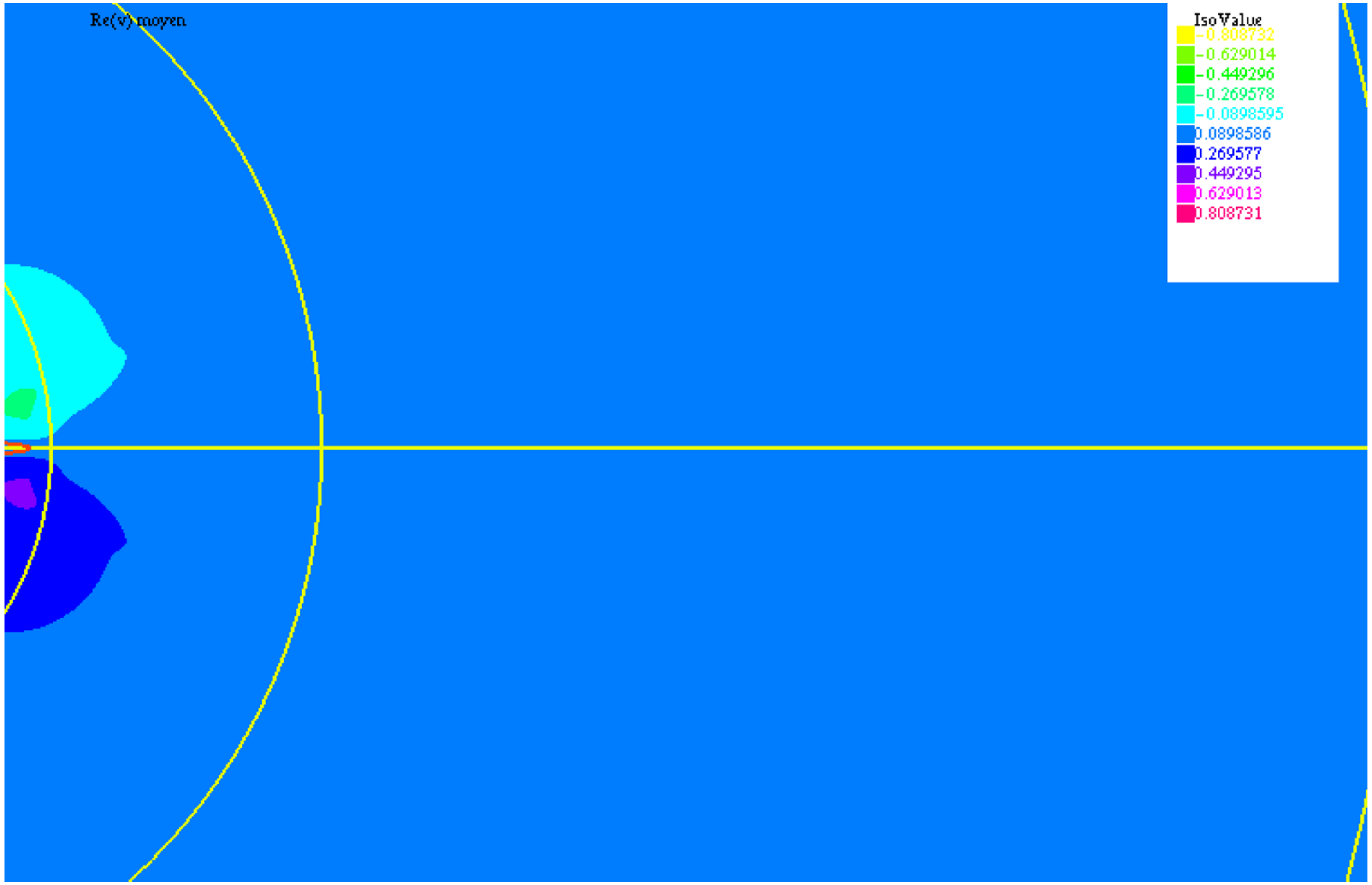

Re(u) moyen
IsoValue
0.874123
0.896988
0.919853
0.942717
0.965582
0.988447
1.01131
1.03418
1.05704
1.07991
1.10277
1.12563
1.1485
1.17136
1.19423



- Profil moyen d'un jet comme analysé par Moored et al;

- Comparer analyse resolvent, modes de Fourier et analyse faite par Moored et al.

V moyen

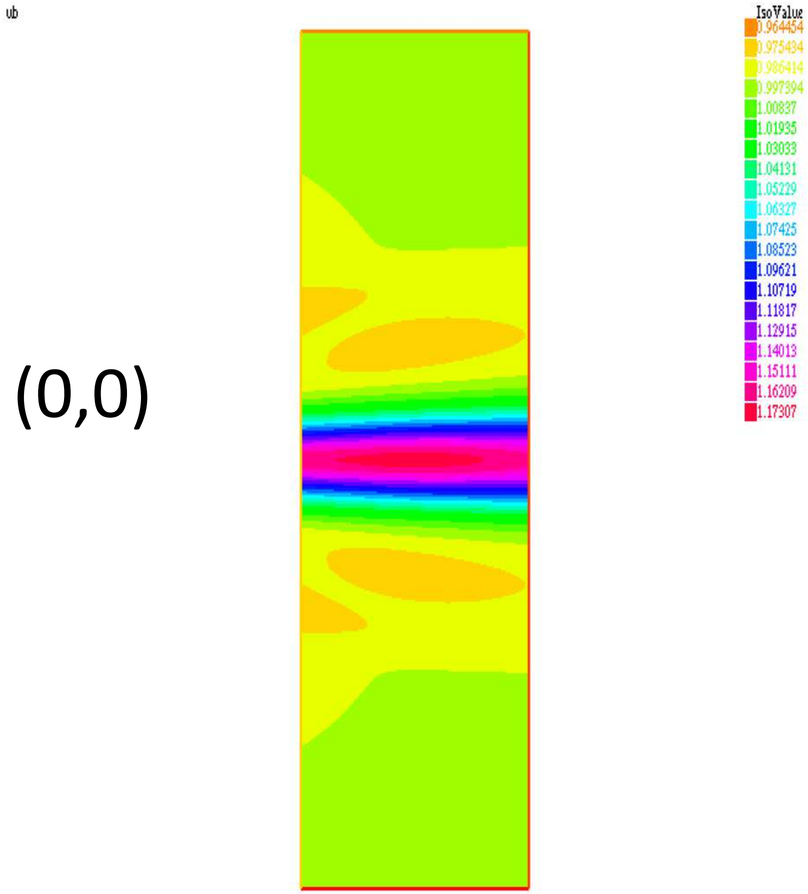


Re(v) moyen
IsoValue
-0.808732
-0.629014
-0.449296
-0.269578
-0.0898595
0.0898586
0.269577
0.449295
0.629013
0.808731

Problème forçage

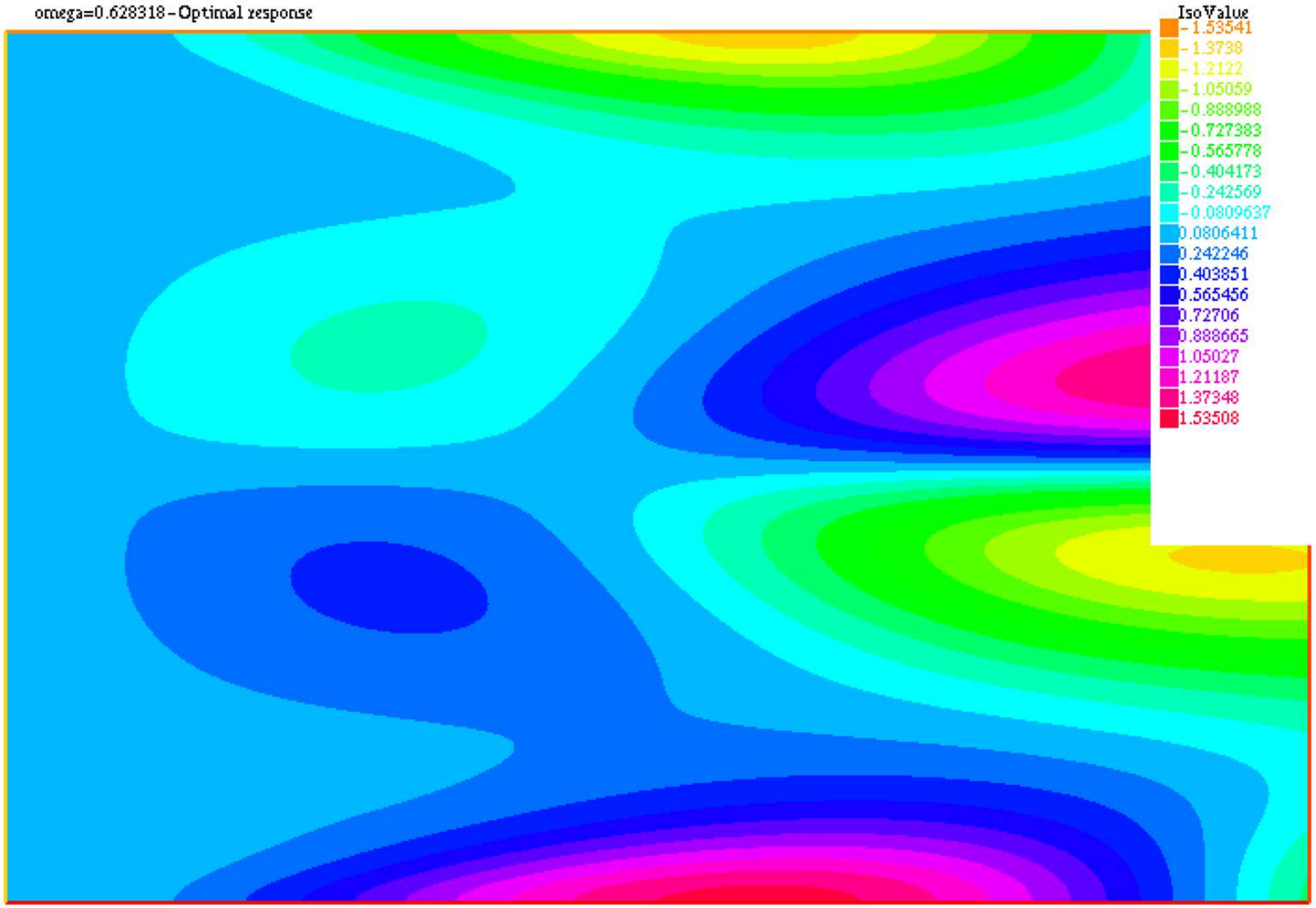
- v = 0

- (u,v) = (0,0)

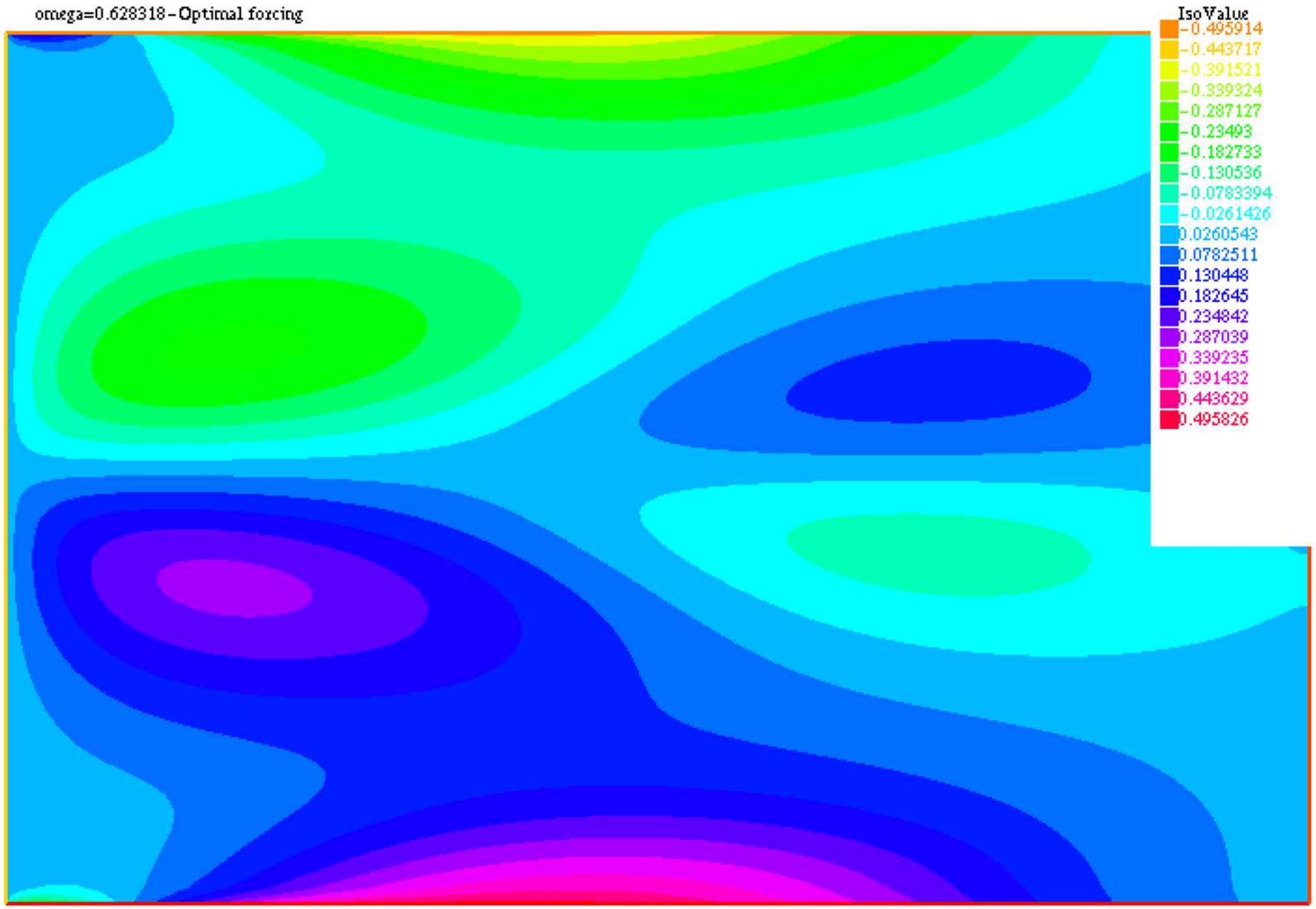


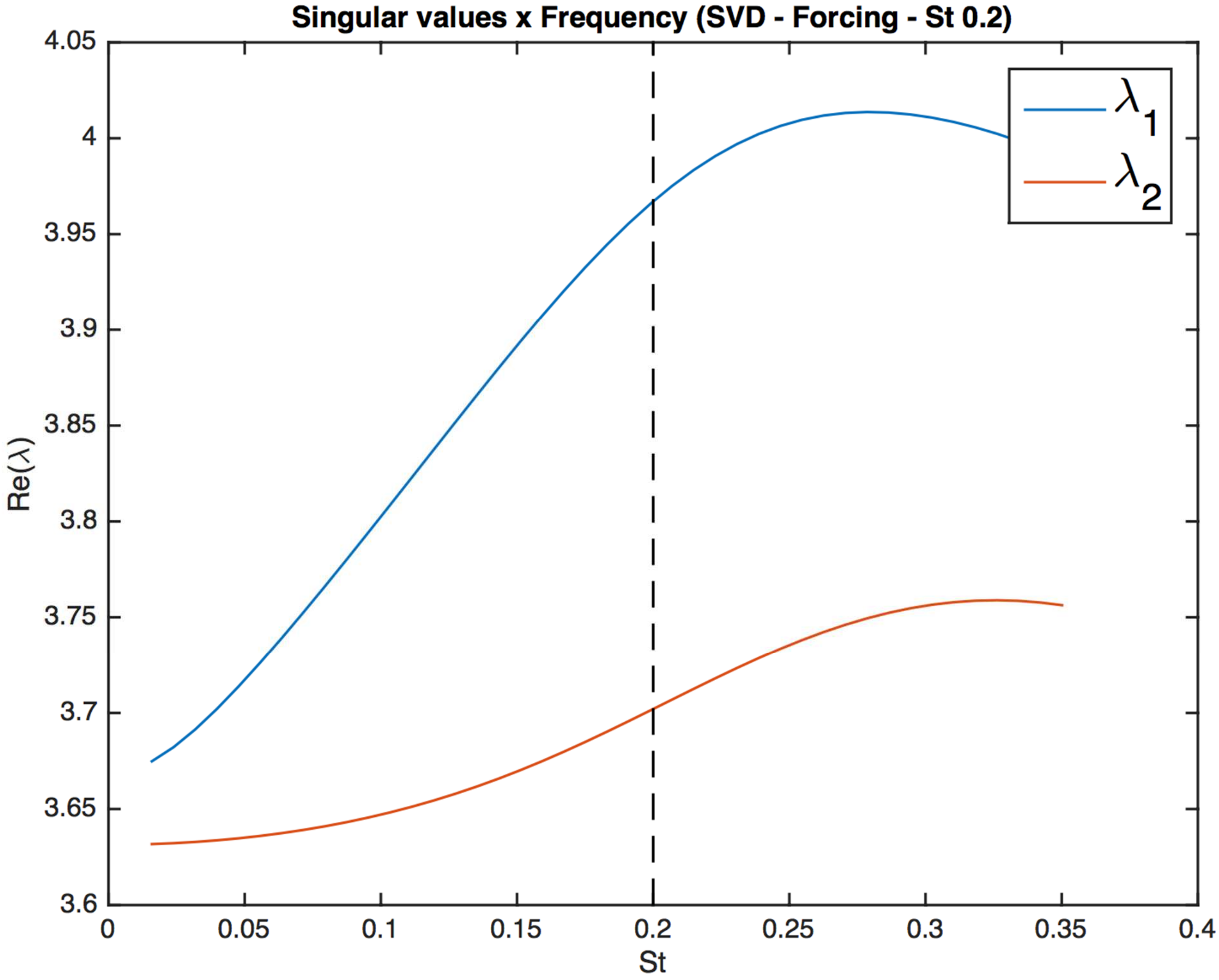
- v = 0

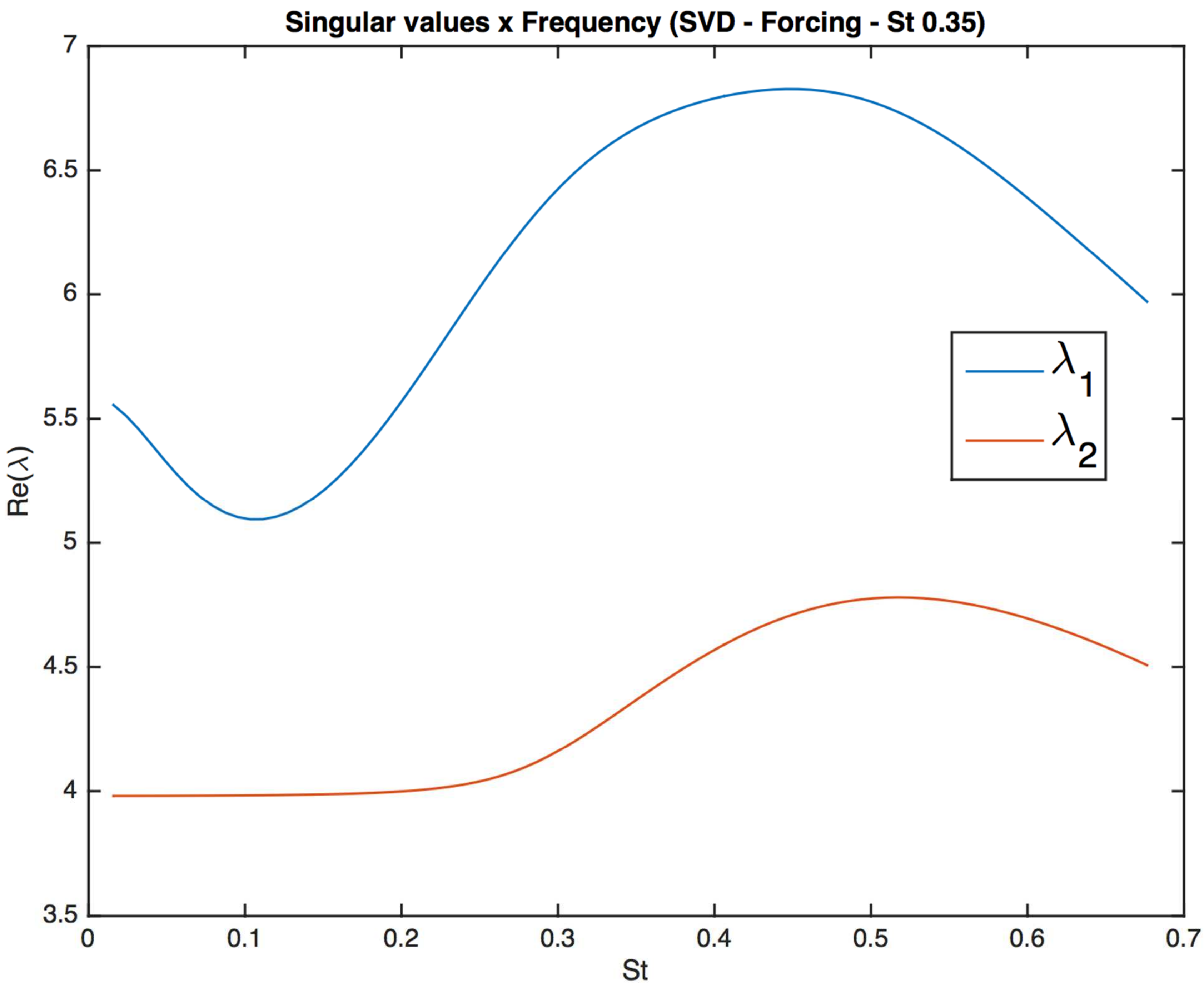
Réponse optimale



Force optimale




Singular values x Frequency (SVD - Forcing - St 0.2)
Re(λ)
St
λ1
λ2
4.05
4
3.95
3.9
3.85
3.8
3.75
3.7
3.65
3.6
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

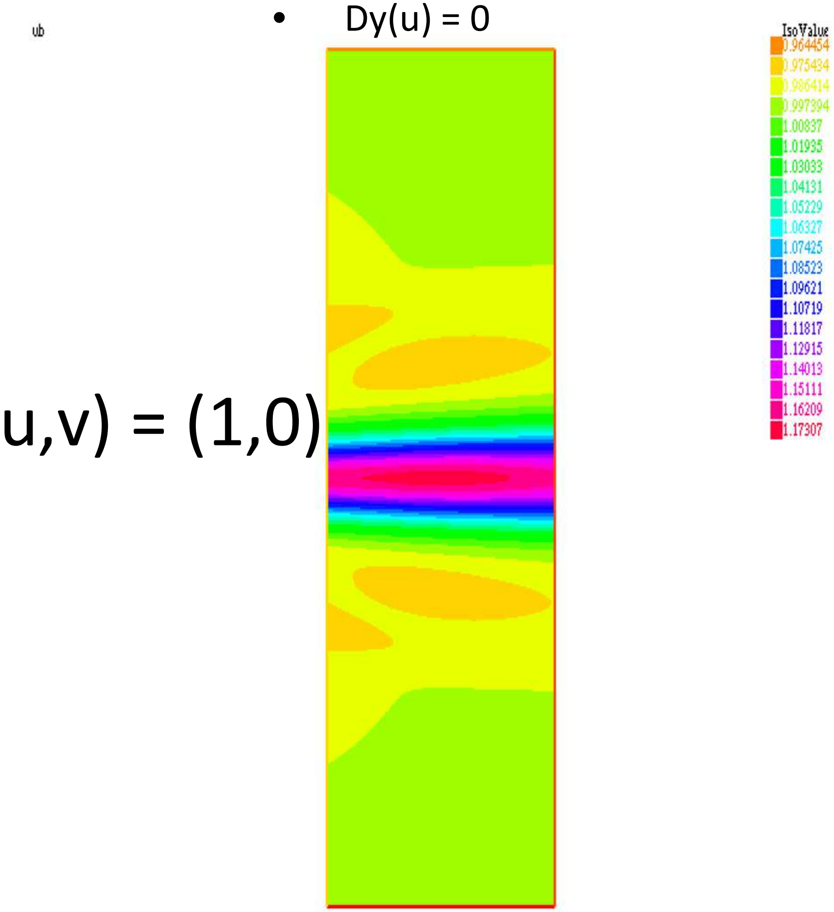

Singular values x Frequency (SVD - Forcing - St 0.35)
Re(λ)
St
λ1
λ2
7
6.5
6
5.5
5
4.5
4
3.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

- Dans une première analyse: pics après la fréquence de battement;
- Avec l'augmentation de la fréquence de battement les pics s'éloingnent de la même;
- Si on suit le raisonnement de Moored et al, la fréquence de réssonance hydrodynamique se retrouve au dessous de St = 0.2.

Problème inflow

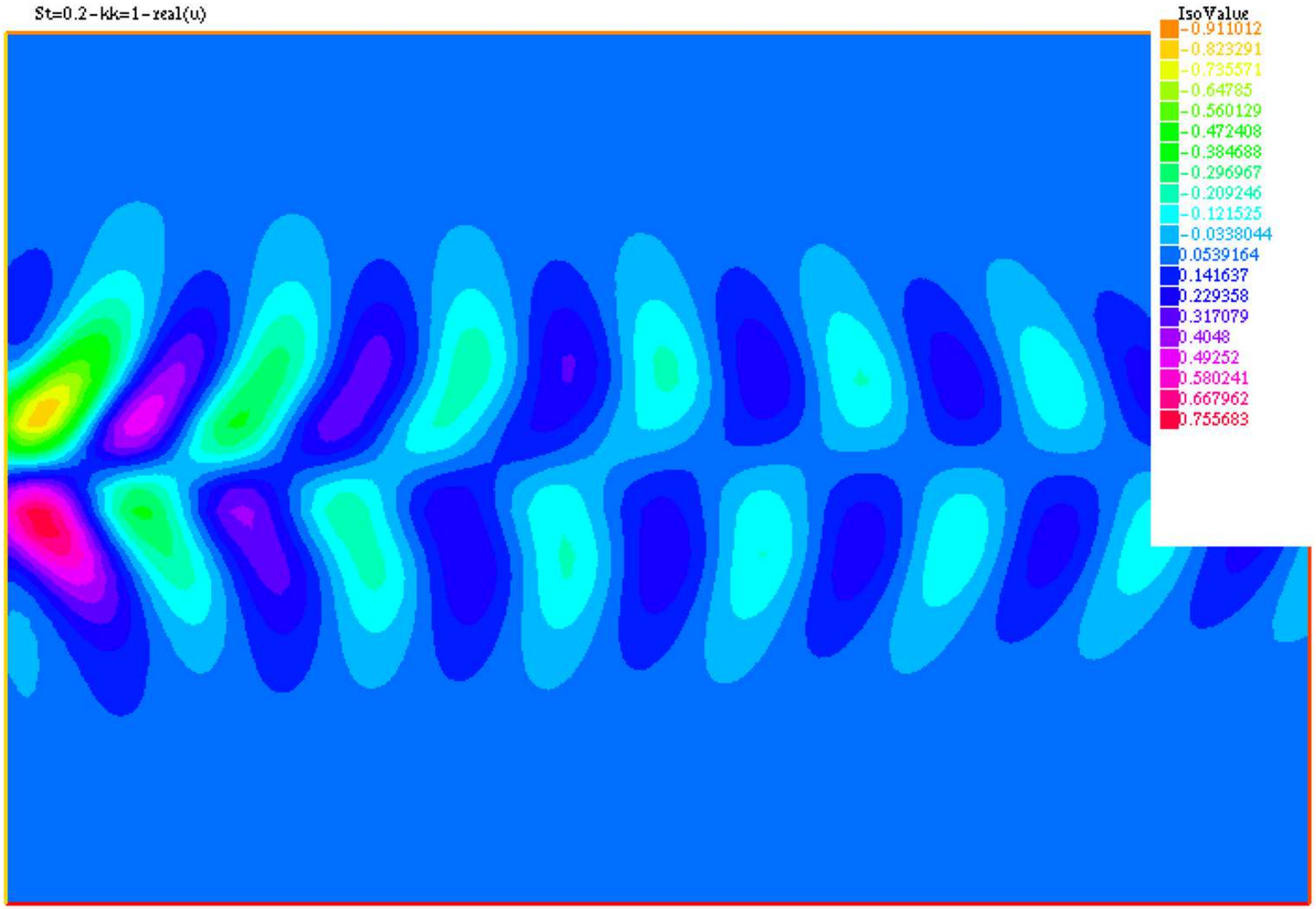
- v = 0
- Dy(u) = 0

- (u,v) = (1,0)

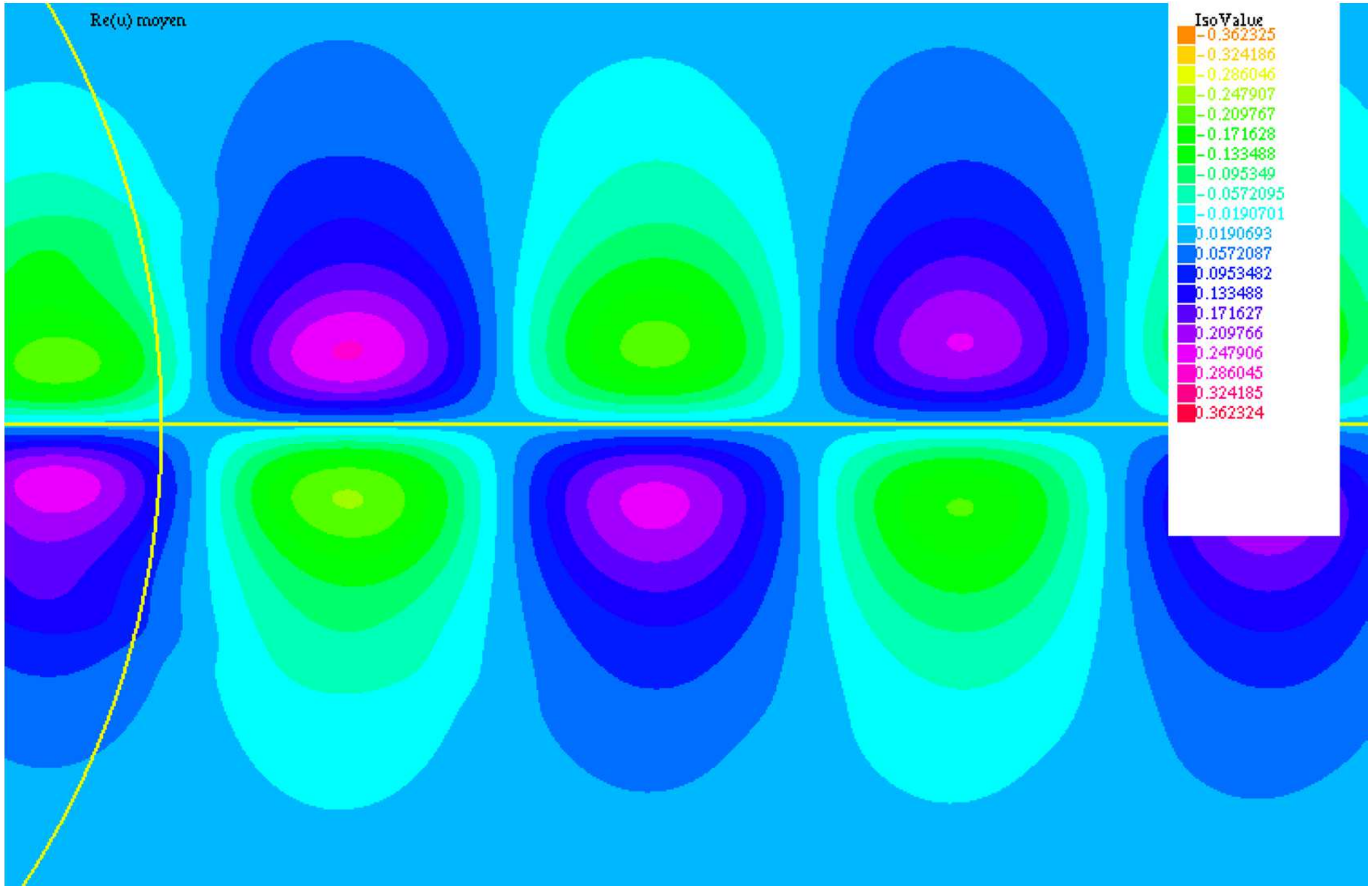


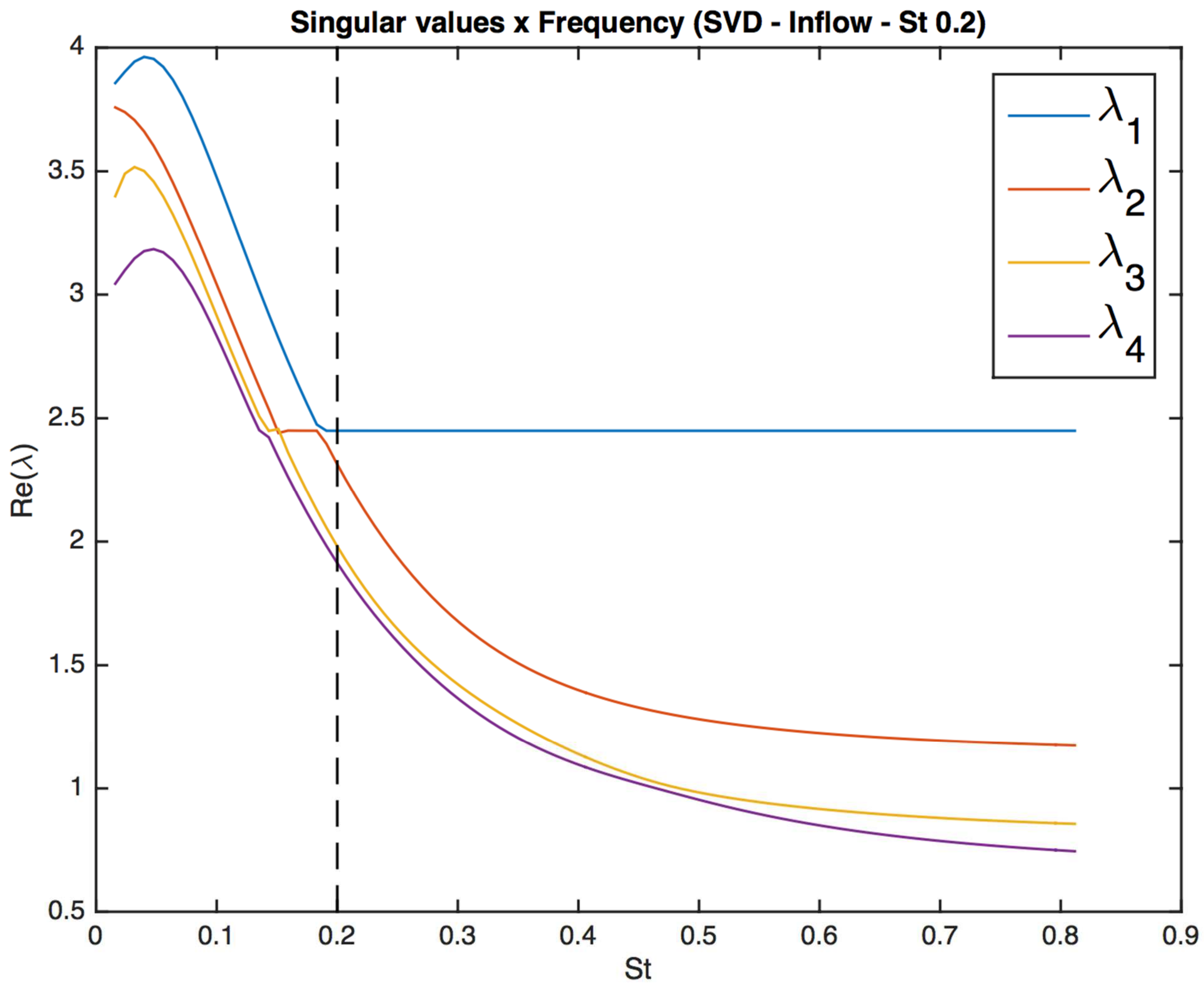
- v = 0
- Dy(u) = 0

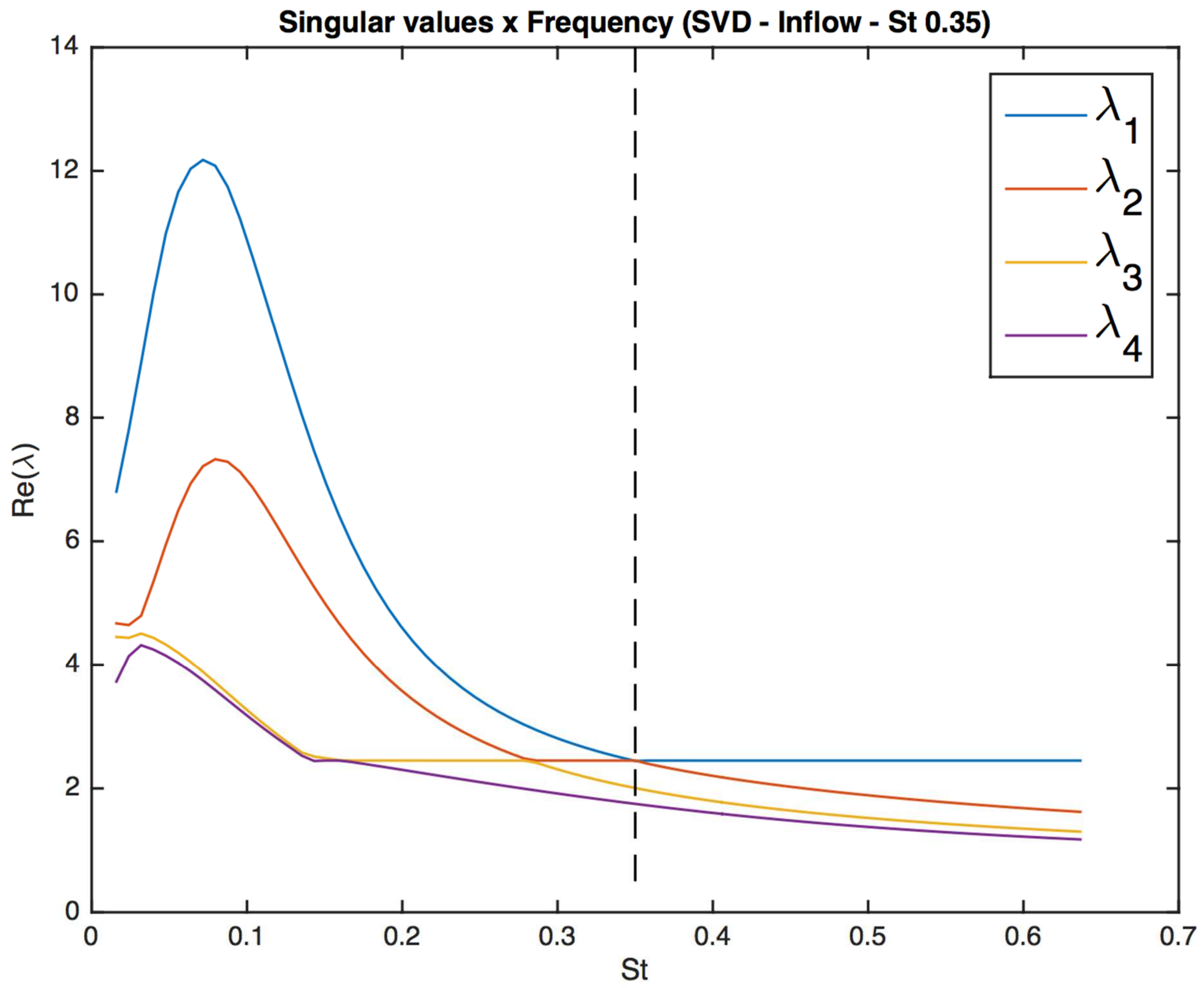
Réponse optimale (1er mode)



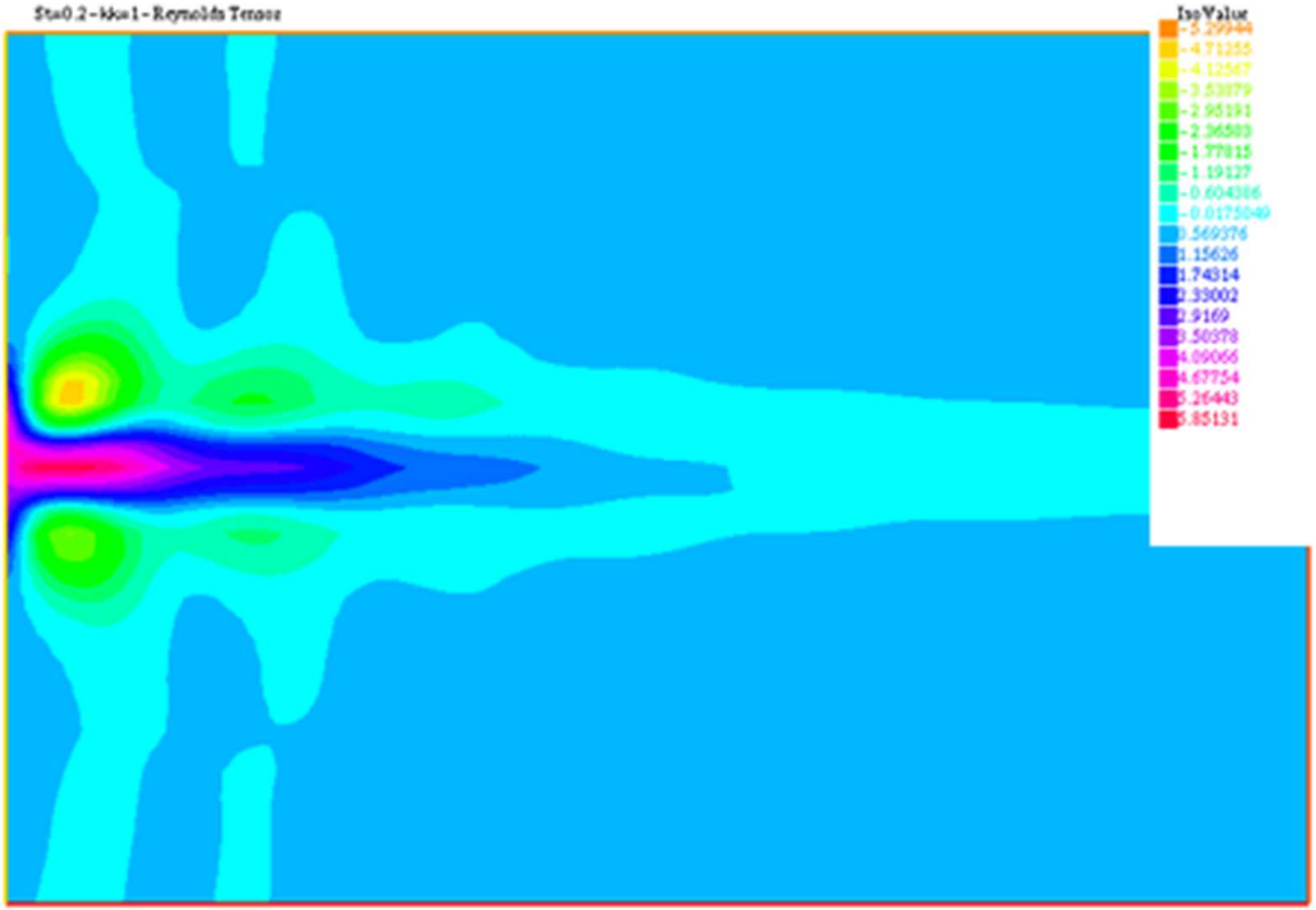
Fluctuation Fourier



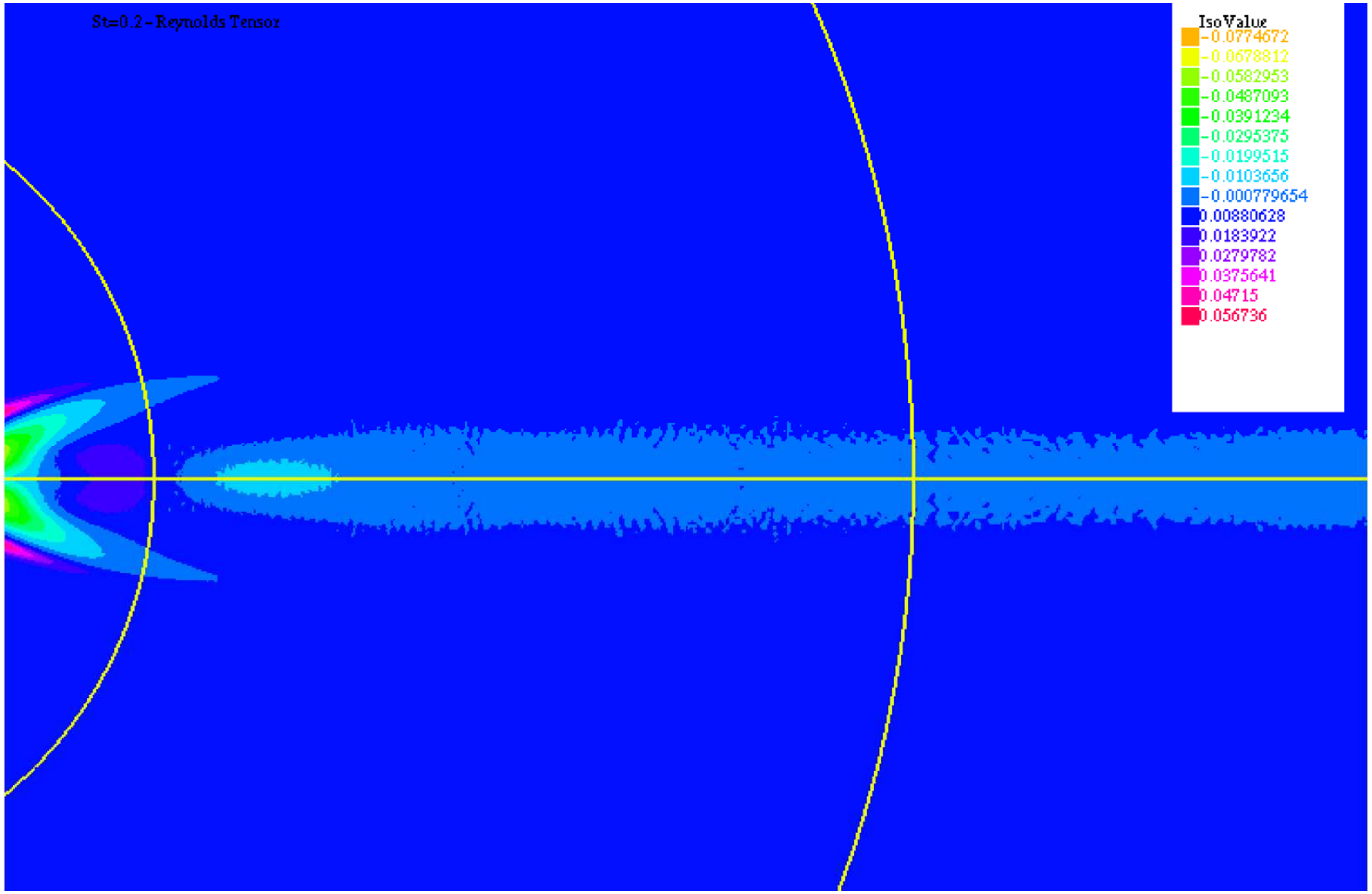

Singular values x Frequency (SVD - Inflow - St 0.2)
Re(λ)
St
λ1
λ2
λ3
λ4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.5
1
1.5
2
2.5
3
3.5
4


Singular values x Frequency (SVD - Inflow - St 0.35)
Re(λ)
St
λ1
λ2
λ3
λ4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
2
4
6
8
10
12
14

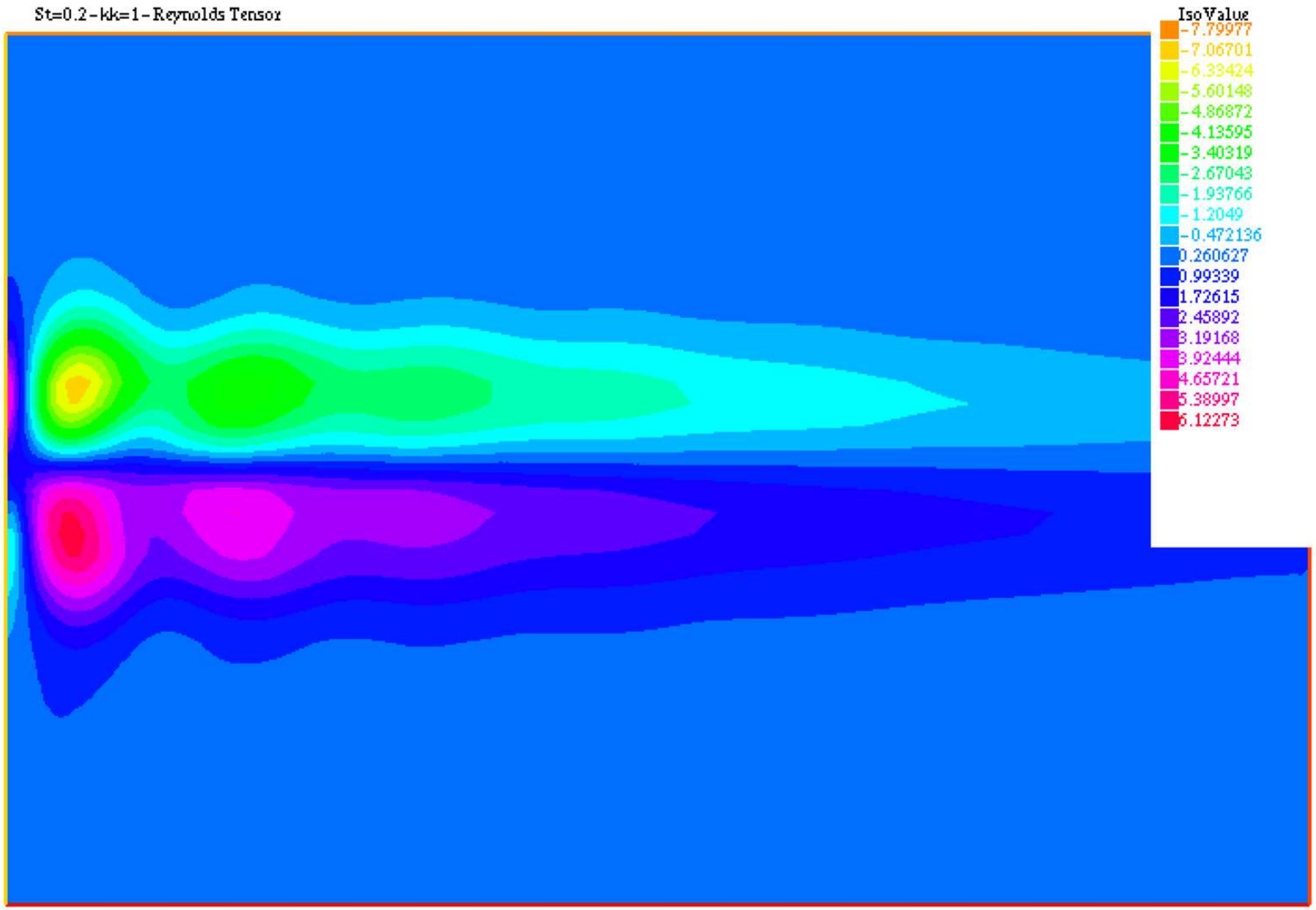
T. de Reynolds horizontal Réponse



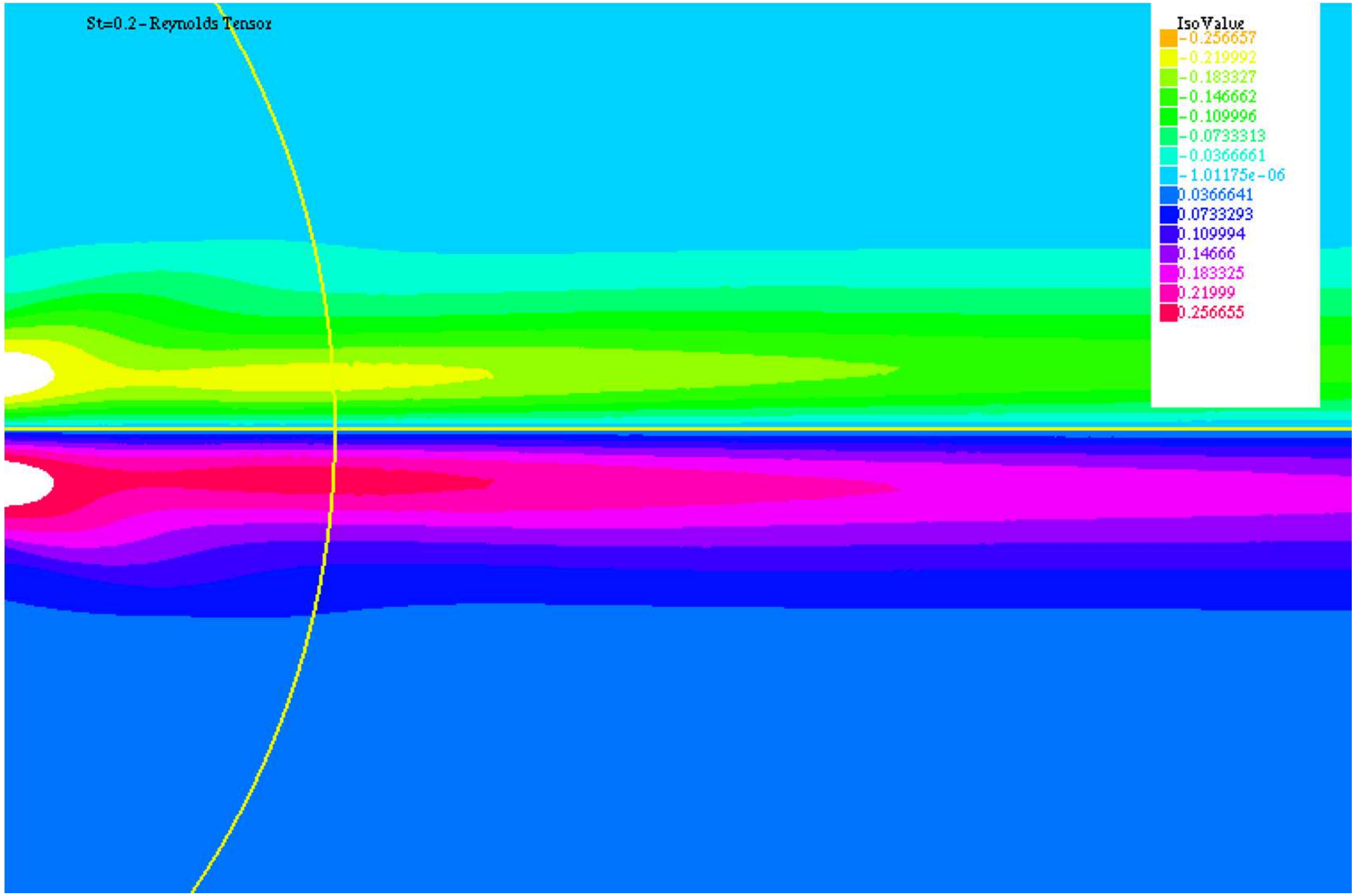
T. de Reynolds horizontal DNS



T. de Reynolds vertical - Réponse

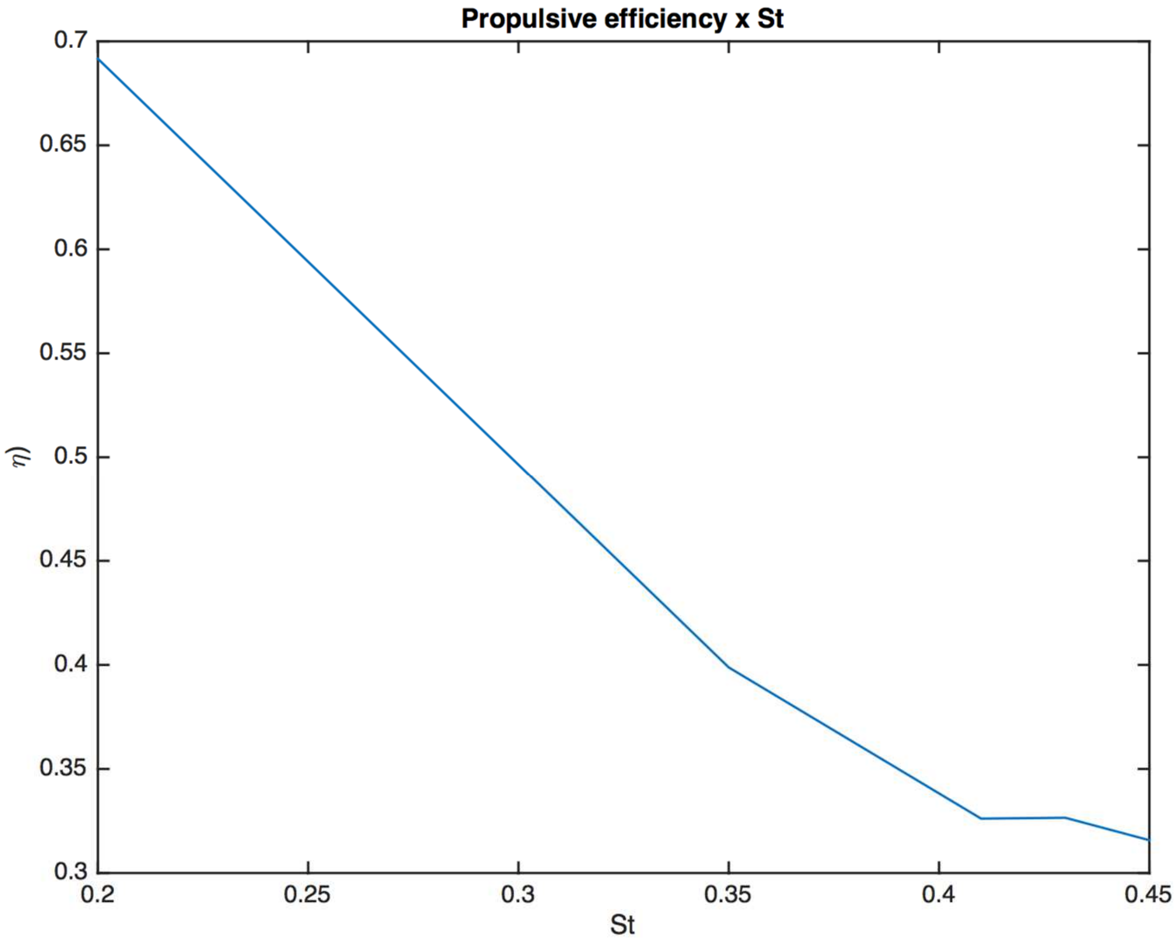


T. de Reynolds vertical DNS



- Les pics de gain apparaissent dans une fréquence petite et moins important que la fréquence de battement.
- La forme des réponses optimales, ainsi que le tenseur de Reynolds, construits à partir du premier mode du profil de vitesse d'entrée optimal possèdent une forme semblable à ceux obtenus avec la DNS.
- Comme on n'observe pas la séparation des modes par rapport au gain, on pourrait utiliser d'autres modes pour obtenir une répresentation plus réelle de la DNS.

Efficacité propulsive



Work in Progress

- Regarder l'effet du domaine sur l'analyse résolvant;
- Composition des réponses avec modes subséquents.