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PROPOSITION DE SUJET DE THESE

Intitulé : Nonlinear Low-Dimensional Modelling of Turbulent Flows *via* Resolvent Analysis

Référence : **MFE-DAAA-2026-05**
(à rappeler dans toute correspondance)

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Mots clés

turbulence, RANS, reduced order models, nonlinear dynamics, non-normality, resolvent analysis

Profil et compétences recherchées

Mechanical/Aerospace Engineer (master recherche, grande école généraliste)

Présentation du projet doctoral, contexte et objectif

Despite the high dimensionality of the Navier-Stokes equations, fluid flows are often dominated by large-scale coherent structures exhibiting low-dimensional behavior driven by low-frequency dynamics. This feature enables reduced-order modelling (ROM), offering both computational efficiency and physical insight. Data-driven methods like POD-Galerkin, EDMD, autoencoders, and deep learning have significantly advanced ROMs [1], but they still come with challenges: large data requirements for training, sensitivity to noise, overfitting, limited interpretability and generalizability. Additionally, control inputs can significantly change system dynamics, potentially invalidating models trained on uncontrolled data. Conversely, reduced-order models directly derived from governing equations are far more interpretable, and precise recipes exist to account for nonlinearities. Traditionally, ROMs constructed from first-principles have relied on **modal analysis** [2], projecting weakly nonlinear dynamics onto slowly decaying or nearly marginally stable eigenmodes, to study bifurcations or responses to external inputs. Techniques like multiple-scale expansions, center manifold theory, and spectral submanifolds [3] are well-established for this purpose. However, these methods break down for advection-dominated flows—e.g., spatially developing boundary layers or free jets—where the linearized operator exhibits a fully stable but highly **non-normal** spectrum [4]. In such cases, the non-orthogonal eigenmodes form an inefficient basis, requiring many modes and defeating the purpose of model reduction.

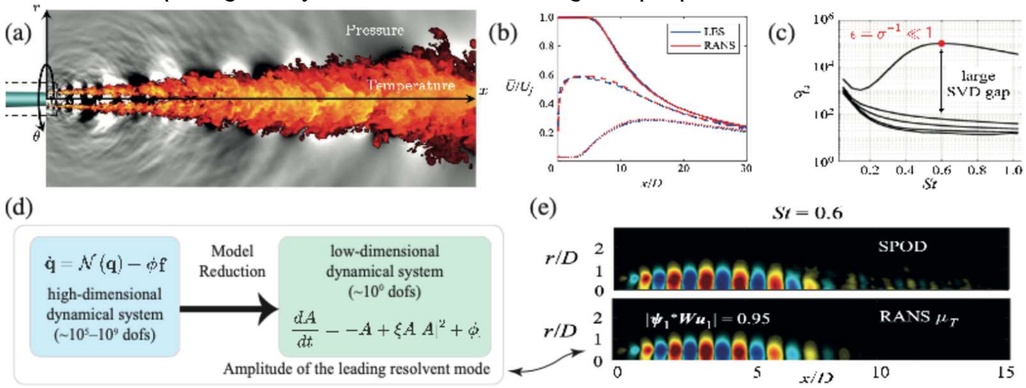

Figure 1: **(a)** Snapshot from LES simulations [8]. **(b)** Mean velocity profiles from LES [8] and RANS (LEVM k-ε) [9] at Mach 0.4. **(c)** Optimal gains (σ) vs. Strouhal number (St) from 'mean-flow' resolvent analysis, showing a large SVD gap between the leading and suboptimal gains. **(d)** Model reduction schematic: due to the gain separation in (c), the system is expected to reduce to a single nonlinear amplitude equation for the leading mode. **(e)** Real part of the coherent pressure fluctuations from spectral proper orthogonal decomposition (SPOD) of the LES data at St = 0.6, Mach 0.4 (top), and corresponding 'mean-flow' resolvent mode (bottom); the reported coefficient indicates strong alignment, capturing coherent Kelvin–Helmholtz structures.

This is where **nonmodal approaches** like **resolvent analysis** come in. Following the interpretation of the linearized Navier-Stokes system as an input-output framework, the resolvent operator acts as a transfer function that reveals pseudo-resonant amplifications hidden in the spectrum [5]. It provides optimal input (forcing) and output (response) orthonormal bases, whose individual contribution can be prioritized on the basis of the associated linear gain. This is ideal for projecting the governing equations and enables systematic and hierarchical model reduction. The new methodology proposed in [6] is precisely based on this resolvent viewpoint. Because of non-normal mechanisms, even small amplitude external forcing (e.g. the action of an actuator) can induce a large response, making nonlinear effects potentially non-negligible. In cases displaying non-normality inducing a strong linear amplification, but where the input amplitude is sufficiently small such that the nonlinear interactions arise but remain weak, one may rigorously construct a nonmodal amplitude equation for this ***weakly nonlinear*** **regime.** These equations are derived from asymptotically expanding the governing equations and Galerkin-projecting onto dominant linear resolvent modes. The resulting low-dimensional models can capture leading-order nonlinearities through amplitude coupling terms in a way that is reminiscent of the amplitude equations obtained from modal analysis [7] (see Fig.1d).

So far, the approach discussed in [6] has only been applied to incompressible flows linearized around laminar fixed points. **The aim of this PhD is to apply and extend this technique to compressible, turbulent flows**, starting from the unsteady Reynolds-Averaged Navier-Stokes (uRANS) closed by a certain turbulence model; both linear eddy viscosity models (LEVMs) and Reynolds stress models (RSMs) will explored for this purpose. In this setup, it is of interest to consider (i) external steady and harmonic forcing mimicking the action of a control actuator, and (ii) stochastic forcing modelling, e.g., the effect of uncoherent turbulent fluctuations on the coherent dynamics.

The **flow configurations consider** for these purposes will be isothermal **turbulent round jets**, spanning different regimes, i.e. from subsonic to supersonic (see Fig.1a). Prior linear studies suggest that a RANS modelling is sufficient to accurately reproduce the actual turbulent mean state (see Fig.1b). Moreover, the linear resolvent operator based on such a mean flow can fairly well predict coherent structures in jets, at least in some range of control parameter and oscillation frequencies, also showing a significant SVD gap in terms of optimal gains (see Figure 1c,e). These evidences suggest that the proposed methodology should apply successfully, hence enabling us to obtain the desired surrogate models for the weakly nonlinear input/output dynamics of such turbulent flows.

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Collaborations envisagées :

Laboratoire d'accueil à l'ONERA
Département : Aérodynamique, Aéroélasticité, Acoustique
Lieu (centre ONERA) : Meudon
Contact :
Alessandro Bongarzone (DAAA-MASH)
Javier Sierra-Ausin (DMPE-STAT)
Tél. : 01 46 73 42 22
Email :
alessandro.bongarzone@onera.fr
javier.sierra_ausin@onera.fr

Directeur de thèse
Nom : Colin Leclercq
Laboratoire : DAAA-MASH, ONERA
Tél. : 01 46 23 51 11
Email : colin.leclercq@onera.fr
Co-Directeur de thèse
Nom : Denis Sipp
Laboratoire : DSG/GDSG, ONERA
Tél : 01 80 38 68 15
Email : denis.sipp@onera.fr