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PROPOSITION DE SUJET DE THESE

Cell-centered Finite Volume schemes for the approximation of compressible gas dynamics on polyhedral meshes

Référence : **SNA-DAAA-2024-33**
(à rappeler dans toute correspondance / to be included in all correspondence)

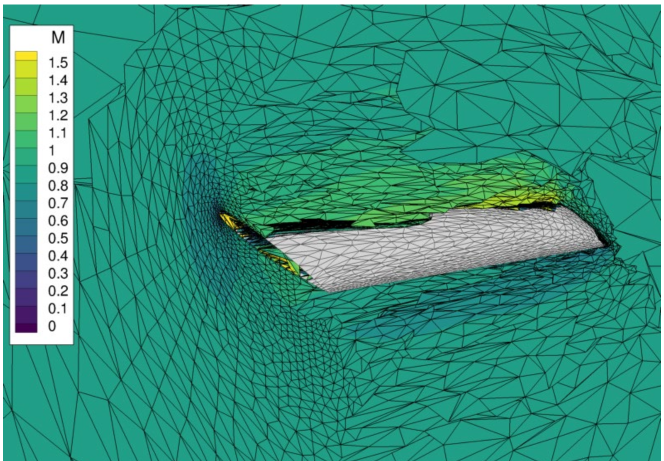
Start of contract: September 2024 | **Application deadline:** May 2024

Keywords: Finite volume method, high-order reconstruction, polyhedral mesh, anisotropic mesh, hyperbolic conservation laws

Profile and required skills: A solid background in computational mechanics (numerical analysis of PDEs, scientific computing), programming skills and motivation to learn are required. Ideally with M.Sc. degree in applied mathematics, mechanics or a related discipline, with excellent academic records

Context: The Finite Volume (FV) method is a discretization method for solving partial differential equations (PDEs) where the degrees of freedom (DOFs) approximate the average of the PDE solution over control volumes that constitute a partition of the domain (the mesh) [EGH]. Nowadays, FV methods are used routinely in industrial PDE solvers. FV methods allow for unstructured meshes, making them well adapted to problems with complex geometries, or in presence of rapid variations in the solution (e.g., through sharp boundary or interior layers). Such configurations may however require highly anisotropic meshes, which may affect the performances of the FV method in terms of accuracy and robustness.

The goal of this PhD project is the design, implementation and theoretical and practical analyses of new high-resolution FV schemes that prove to be robust and stable on polyhedral and highly anisotropic meshes. Applications will concern compressible flow problems, which are characterized by the propagation of wave-like structures at finite speed and whose solutions may develop discontinuities, such as shock and contact waves.



Example of anisotropic mesh and associated Mach number contours obtained from the simulation of the transonic flow over a wing.

Scientific approach: We will consider cell-centered FV schemes using an accurate reconstruction of the space derivatives of the solution (e.g., MUSCL [vL] and WENO [S09] reconstructions), which provide accurate numerical solutions and are well adapted to the capture of discontinuities. The method will be associated with either time explicit or time implicit integration via a method of lines.

We are interested in the design and theoretical analysis of the scheme when applied to highly anisotropic polyhedral meshes. High-aspect ratio control volumes may affect its performances in terms of accuracy and robustness mainly due to inaccurate solution reconstructions. One important direction of research will thus be the design of robust and accurate reconstruction of the solution. Particular attention will be paid to ensure, at the discrete level, some fundamental physical principles (e.g., conservation, positivity of some quantities, second law of thermodynamics) and keep these properties when the scheme becomes singular due to vanishing control volumes.

Description of work: The successful candidate will investigate different ways of solution reconstruction (e.g., gradient or polynomial reconstructions) in terms of convergence, robustness, accuracy and efficiency. He will in particular consider different approaches of using high-order MUSCL reconstructions [PN], additional DOFs [H00,BLS,DE], a larger set of DOFs [GCLM], multiple gradient reconstructions [H12], etc. The candidate will also analyze the properties of the scheme for the approximation of boundary conditions. The analyses will be systematically performed for forward and backward Euler time integrations.

Numerical experiments in an unstructured code developed at Onera will be conducted to assess the performances of the method. The results will be the subject of publications in journals and scientific conferences.

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