PROPOSITION DE SUJET DE THESE

Intitulé : Adjoint analysis of nonlinear hyperbolic systems with non-conservative products

Référence : SNA-DAAA-2019-19
(à rappeler dans toute correspondance)

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Keywords: non-conservative hyperbolic systems, kinetic relations, adjoint equations, finite volume method, discontinuous Galerkin method, multiphase flows.

Context: In this work, we consider the adjoint equations of first-order hyperbolic non-conservative systems, i.e. systems of first-order quasi-linear PDEs that cannot be written into divergence form. Methods based on adjoint equations are of great importance in computational fluid dynamics (CFD) for shape optimization, flow control, receptivity-sensitivity-stability analyzes, uncertainty quantification, data assimilation, error analysis, etc. These methods are often used for the linear analysis of nonlinear conservation laws (hereafter referred to as the primal equations) where the adjoint is defined as the dual form of the linearized equations governing infinitesimal perturbations on solutions of the primal problem. One usually considers either continuous adjoint methods coming from the linearization of the formal primal equations, or discrete adjoint methods coming from the linearization of the numerical scheme for the discretization of the primal equations. The latter methods are often preferred as they allow the use of automatic differentiation tools for the linearization of the numerical scheme used for the discretization of the primal problem.

In the case of hyperbolic equations, this raises the question of the validity of the linearization around discontinuities in the primal solution because the adjoint equations are linear ones with discontinuous coefficients for which the Cauchy problem is not well posed in general. The analysis for the continuous adjoint must include the linearization of the jump relations at the discontinuity, while discrete adjoint methods associated to conservative and convergent schemes have been successfully applied to the linear analysis of conservation laws.

Nevertheless no theoretical analyzes exist on the adjoint equations for hyperbolic systems in non-conservative form in presence of discontinuous primal solutions (with shock and contact waves), though those systems include many physical models (shock-turbulence interaction, multiphase flows, shallow water flows, plasma physics, MHD, etc.). Mathematical difficulties are inherent in those systems because the theory of distributions do not apply to define the non-conservative products and further
theories are needed to generalize the notion of weak solutions from the setting of conservation laws. These theories induce two main difficulties in the adjoint analysis of discontinuous primal solutions [Coq18]:

- lack of differentiability in the definition of the non-conservative product across discontinuities in the primal solution;
- consistency of the discrete adjoint scheme difficult to prove due to the non explicit consistency of the primal numerical scheme.

In this work, we propose to address those difficulties from the theoretical and numerical points of view with the objective to analyze the properties of the associated discrete adjoint methods. Applications will concern the adjoint analysis of compressible multiphase flow models [BN86].

**Description of work:** The proposed work will first extend the analysis from [Coq18] with a different definition of the non-conservative terms using kinetic relations [Ber12]. This definition allows the formal linearization of the associated jump relations of the primal solution at a discontinuity, thus extending the theoretical analysis from the conservative setting introduced in [Maj83]. This will constitute a key aspect of the continuous adjoint analysis of non-conservative hyperbolic systems.

At the discrete level, we will consider both first-order finite volume (FV) methods and high-order discontinuous Galerkin (DG) methods [Ren18] for the discretization of the primal problem. The concept of kinetic relations again allows the design of numerical schemes for FV methods that can also be used in the DG methods. Moreover, it provides a natural framework for the consistency analysis of the discrete adjoint method and thus constitutes an advantage compared to the analysis in [Coq18].

Numerical experiments in a DG code developed at ONERA will concern sensitivity-receptivity analyses [Luch14] of compressible multiphase flows to assess the relevance of the discrete adjoint method in the context of flow control and optimization.

The results obtained during this PhD work will be the subject of publications in academic journals and scientific conferences.

**Bibliography:**


**Collaborations extérieures:** CMAP, École Polytechnique, 91128 Palaiseau Cedex

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**PROFIL DU CANDIDAT**

**Formation:** M.Sc. degree in Applied Mathematics, Mechanics or a related discipline, with excellent academic records.

**Spécificités souhaitées:** A solid background in Computational Mechanics (numerical analysis of PDEs), programming skills and motivation to learn are required.