PROPOSITION DE SUJET DE THESE

Intitulé : Robust high-order implicit DGSEM schemes for the compressible RANS equations

Référence : **SNA-DAAA-2023-17**
(à rappeler dans toute correspondance)

Début de la thèse : Octobre 2023	**Date limite de candidature : Mai 2023**

Mots clés : Computational fluid dynamics, turbulent flows, discontinuous Galerkin methods, RANS modeling

Thématique : Méthodes numériques pour la mécanique des fluides

Profil et compétences recherchées : A solid background in Computational Mechanics (numerical analysis of PDEs), programming skills and motivation to learn are required. Ideally with M.Sc. degree in Applied Mathematics, Mechanics or a related discipline, with excellent academic records

Présentation du projet doctoral, contexte et objectif

Context: In the designing stage of an aircraft, being able to predict and simulate its behavior in turbulent conditions is pivotal. The Reynolds-Averaged Navier-Stokes (RANS) turbulence modeling enjoys to this day a great popularity thanks to its efficiency and simplicity of usage, while still ensuring accurate results. Research on the subject is still ongoing and concerns researchers as well as engineers and developers. The drastic increase of computing resources over the last decades stirred the interest towards high-order methods which have become nowadays well wide-spread and available in almost all commercial CFD solvers. Moreover, due to their intrinsic low dissipation, high-order schemes are more adapted to deal with turbulence than low-order ones, and indeed they can often overcome the issues encountered by low-order schemes in some critical flow configurations, such as vortex-dominated ones. Among others high-order schemes, discontinuous Galerkin (DG) methods, a subset of finite element schemes which relaxes the continuity constraint at mesh interfaces, have gained their fair share of popularity. This project aims to develop robust and efficient high-order DG schemes for the RANS turbulent equations.

Project: DG methods consist in high-order numerical schemes that can be used on generic mesh elements and which approximate the solution with element-wise polynomials. The discontinuous Galerkin spectral element method (DGSEM) is a sub-class of DG schemes, whose salient features are colocation of quadrature- and interpolation-nodes and tensorisation. DGSEM have been extensively studied and several works have highlighted performance advantages with respect to classical DG methods.

We aim at designing a robust method with high physical fidelity. On the numerical side, this requirement translates for the method to be entropy-stable (able to capture the physically relevant solution) and positivity-preserving (in order to avoid pathological computational results, such as negative densities).

In the literature, there exist some examples of improvement of the DGSEM, or more generally DG or spectral methods, enjoying the above-mentioned properties. Most of these works, see for instance [1], address schemes with explicit time-stepping for hyperbolic systems. There also exist, however, some ways to bridge results obtained on hyperbolic systems to viscous equations, see for instance [2] for the entropy-stability: although usually applied to other numerical schemes, one may work along the same lines to applied those ideas to DGSEM.

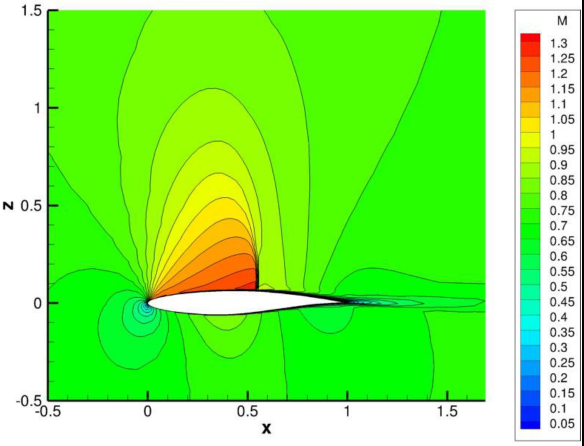

Figure 1: High-order accurate DGSEM simulation of the flow around a RAE2822 airfoil in transonic turbulent flow conditions (color levels: Mach number contours)

We will also consider implicit-time integration. Indeed, while being easy to address numerically, explicit time-stepping has the main drawback of being limited by the Courant-Friedrichs-Lewy constraint. This often leads to having to consider small time-steps, which, in turn, may be quite time-consuming if dealing with steady-state problems. With implicit time-integration, while one could use larger time-steps, one should still pay attention to ensure that the solution remains physically relevant. Works such as the positivity-preserving limiter for 1D DG discretizations proposed in [3] will be adapted to our needs: indeed, one can take advantage of the intrinsic tensorization of DGSEM to use similar tools.

Considering implicit schemes will lead to face resolutions of algebraic systems. Investigations will concern also improvements of the resolution of the algebraic systems issued from the numerical scheme. Indeed, because of the tensorisation and the choice of quadrature points considered in DGSEM, the resulting linear system are extremely sparse and two adjacent elements are related only via the degrees of freedom of their shared interface: some techniques taking into account this particular structure (e.g. adapted storage) might lead to substantial gains in computational performances.

In addition to the theoretical analysis, the student will work on Aghora [4], a CFD code based on DG methods and developed at ONERA and/or CODA [5], an industrial CFD solver, to implement the proposed methods and conduct numerical experiences to assess the performances of the methods. The results obtained during this PhD work will be the subject of publications in academic journals and scientific conferences.

The tasks of the project are:

1. Characterize and analyze theoretically the framework of the 1D time-implicit Navier-Stokes equations with positivity-preserving entropy-stable DGSEM
2. Develop the schemes a DG solver and assess via numerical simulations the developed framework in terms of compliance with expected properties (orders of convergences, positivity, stability,…)
3. Extend and assess the framework to the multidimensional case
4. (According to progression) Devise algebraic optimizations adapted to the DGSEM framework

Bibliography:

1. Entropy stable, robust and high-order DGSEM for the compressible multicomponent Euler equations, Renac, 2022, JCP, doi 10.1016/j.jcp.2021.110584
2. Entropy stable high order discontinuous Galerkin methods with suitable quadrature rules for hyperbolic conservation laws, Chen and Shu, 2017, JCP, doi 10.1016/j.jcp.2017.05.025

3. Implicit positivity-preserving high-order discontinuous Galerkin methods fro conservation laws, Qin and Shu, 2018, SIAM J. Sci. Comput, 10.1137/17M112436X
4. Aghora: A High-Order DG Solver for Turbulent Flow Simulations, F. Renac, M. de la Llave Plata, E. Martin, J. B. Chapelier, and V. Couaillier. In: Kroll, N., Hirsch, C., Bassi, F., Johnston, C., Hillewaert, K. (eds) IDIHOM: Industrialization of High-Order Methods - A Top-Down Approach. Notes on Numerical Fluid Mechanics and Multidisciplinary Design, vol 128. Springer, Doi 10.1007/978-3-319-12886-3_15
5. DLR-Project Digital-X-Next Generation CFD Solver Flucs, Leicht, Jägersküpper, Vollmer, Schwöppe, Hartmann, Fiedler and Schlauch, 2016

Collaborations envisagées

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