PROPOSITION DE SUJET DE THESE

Efficient and robust spectral high-order schemes for the simulation of compressible flows on polyhedral meshes

Référence : **SNA-DAAA-2026-02**
(à rappeler dans toute correspondance / to be included in all correspondence)

Start of contract: October 2026	**Application deadline:** May 2026

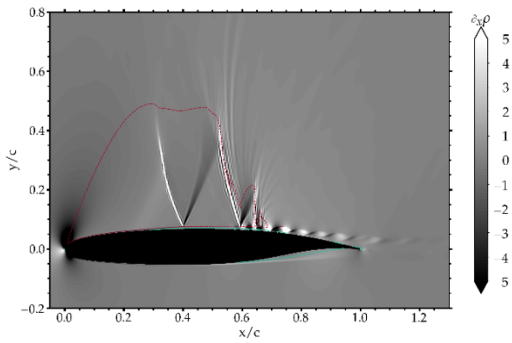
Keywords: discontinuous Galerkin method, spectral method, polyhedral mesh, hyperbolic conservation laws, turbulent flows, transonic flows

Profile and required skills: A solid background in computational mechanics (numerical analysis of PDEs, scientific computing), programming skills and motivation to learn are required. Ideally, with M.Sc. degree in applied mathematics, mechanics or a related discipline, with excellent academic records.

Eligibility conditions: the candidate should not have spent more than 12 months in France in the last 3 years prior to the recruitment date (last trimester of 2026) and they cannot have more than 4 years of experience.

Context of the PhD work and collaborations: This PhD thesis will be part of the project TRIMS (TRansItion and control Methods for industrial flow Simulations) funded by the Marie Skłodowska-Curie Actions Doctoral Networks, involving 10 partner organizations in Europe, and starting in October 2026. The remuneration is quite attractive and several research visits of few months are planned in other research institutes. Interested candidates should be aware of the eligibility conditions before applying to this PhD position.

Overall objective: improve the maturity of high-order discontinuous spectral schemes for the approximation of compressible turbulent flows by extending them to complex industrial applications. This will be achieved through two main goals: their extension to heterogeneous meshes with polyhedral elements and the improvement of their robustness and stability.



Instantaneous numerical Schlieren of the transonic turbulent flow past the OALT25 airfoil obtained with a high-order DGSM [Pla, PRMT].

Context: The discontinuous Galerkin spectral method (DGSM) is a compact discontinuous spectral element method which looks for a piecewise polynomial approximate solution. The method results in very low dispersion and dissipation errors, which are essential features for the simulation of turbulent flows (see figure). These methods also combine the flexibility of unstructured meshes with efficient algorithms based on tensor-product of one-dimensional operators and have been implemented in industrial-level CDF solvers [VCSJC]. The methods have however several drawbacks. First, they lose their tensor-product structure on polyhedral elements, which affect their efficiency. Second, their robustness is worsening with high-order accuracy, mainly due to spurious oscillations associated to the Gibbs phenomenon around discontinuities such as shock waves, and strong CFL restriction on the time step. This PhD project aims at addressing these issues.

Scientific approach: To extend the DGSM to heterogeneous meshes with polyhedral elements, we will consider tensor product operators on a generic mapping from any type of physical element to the reference cube. The mapping will aim at allowing: (i) the tensor-product structure of the discretization operators to leverage efficient sum-factorization algorithms; (ii) a discrete analogue of integration by parts (known as summation-by-parts property), which constitutes an essential ingredient for stabilizing the numerical scheme [MZa, MZb]. We will then make use of the tensor-product structure of the scheme to design robust and efficient time implicit solvers that remove the strong CFL condition.

While it is possible to prove nonlinear stability and robustness of the DGSM on quadrangular and hexahedral meshes [GWK, Ren], the analysis is by far less complete for general meshes. The goal of this PhD project is the design, implementation and theoretical and practical analyses of new high-resolution DGSM that prove to be robust and stable on polyhedral meshes. Applications will concern compressible flow problems, which are characterized by the propagation of wave-like structures at finite speed and whose solutions may develop discontinuities, such as shock and contact waves. Particular attention will be paid to ensure, at the discrete level, some fundamental physical principles (e.g., conservation, positivity of some quantities, second law of thermodynamics).

Description of work: The research activities to be conducted include:

- Design, analyze and implement orthonormal DGSM basis functions for any type of mesh element through an orthonormalization procedure. Care needs to be taken to get a tensor-product structure that allows efficient sum-factorization techniques for the implementation of the discretization operators in multiple space dimensions.
- Design modifications of the scheme to make it satisfy essential properties of conservation, preservation of uniform states, entropy stability [MZb,ACCC], robustness [Ren], accuracy, etc.
- Consider time implicit discretization to remove the restriction on the time step and design efficient and robust preconditioners in quasi-Newton iterative solvers.
- Design and analyze the implementation of boundary conditions to keep the discrete properties of the scheme for boundary value problems.
- Conduct numerical experiments in a DGSM code developed at Onera [Agh] to assess the performances of the method. The targeted applications will be relevant for the aerospace industry and characterized by compressible turbulent flows modeled by the RANS and URANS equations in multiple space dimensions. As a validation, the schemes will be used to perform linear (stability-sensitivity-receptivity analysis) and nonlinear analyses of such flows.
- The results will be the subject of publications in journals and scientific conferences.

Bibliography:

[MZa] T. Montoya, D.W. Zingg, Efficient tensor-product spectral-element operators with the summation-by-parts property on curved triangles and tetrahedra, SIAM J. Sci. Comput., 46 (2024) A2270–A2297, https://doi.org/10.1137/23M1573963.

[MZb] T. Montoya, D.W. Zingg, Efficient entropy-stable discontinuous spectral-element methods using tensor-product summation-by-parts operators on triangles and tetrahedra, J. Comput. Phys., 516 (2024), 113360, https://doi.org/10.1016/j.jcp.2024.113360.

[ACCC] L. Alberti, E. Carnevali, A. Colombo, A. Crivellini, An entropy conserving/stable discontinuous Galerkin solver in entropy variables based on the direct enforcement of entropy balance, J. Comput. Phys., 508 (2024), 113007, https://doi.org/10.1016/j.jcp.2024.113007.

[Abg] R. Abgrall, A general framework to construct schemes satisfying additional conservation relations. Application to entropy conservative and entropy dissipative schemes, J. Comput. Phys., 372 (2018), 640-666, https://doi.org/10.1016/j.jcp.2018.06.031.

[PRMT] M. Plath, F. Renac, O. Marquet, C. Tenaud, A high-order local correlation based transition model for transonic airfoil flows, Comput. Fluids (2024), 106461, https://doi.org/10.1016/j.compfluid.2024.106461.

[Pla] M. Plath, Numerical aeroelastic stability of transonic laminar airfoils, Thèse de doctorat de l'Université Paris-Saclay.

[GWK] G. J. Gassner, A. R. Winters, D. A. Kopriva, Split form nodal discontinuous Galerkin schemes with summation-by-parts property for the compressible Euler equations, J. Comput. Phys., 327 (2016), 39-66, https://doi.org/10.1016/j.jcp.2016.09.013.

[Ren] F. Renac, Entropy stable, robust and high-order DGSEM for the compressible multicomponent Euler equations, J. Comput. Phys., 445 (2021), 110584, https://doi.org/10.1016/j.jcp.2021.110584.

[VCSJC] P. Volpiani, J.-B. Chapelier, A. Schwöppe, J. Jägersküpper, S. Champagneux, Simulating the Common Research Model using the new CFD software from ONERA, DLR and Airbus, AIAA AVIATION 2023 Forum, doi 10.2514/6.2023-3275.

[Agh] F. Renac, M. de la Llave Plata, E. Martin, J.-B. Chapelier, V. Couaillier, Aghora : A high-order DG solver for turbulent flow simulations, in N. Kroll et al. (Eds.), Notes on Numerical Fluid Mechanics and Multidisciplinary Design, 128 (2015), Springer Verlag.

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