PROPOSITION DE SUJET DE THESE

Intitulé : Development of SIAC-reconstructed Discontinuous Galerkin schemes for unstructured meshes

Référence : **SNA-DAAA-2026-03**
(à rappeler dans toute correspondance)

Starting date : October 2026	**Date limite de candidature :**

Keywords

High-order methods for CFD, Discontinuous Galerkin, SIAC reconstruction, unstructured meshes, compressible flows, turbulent flows

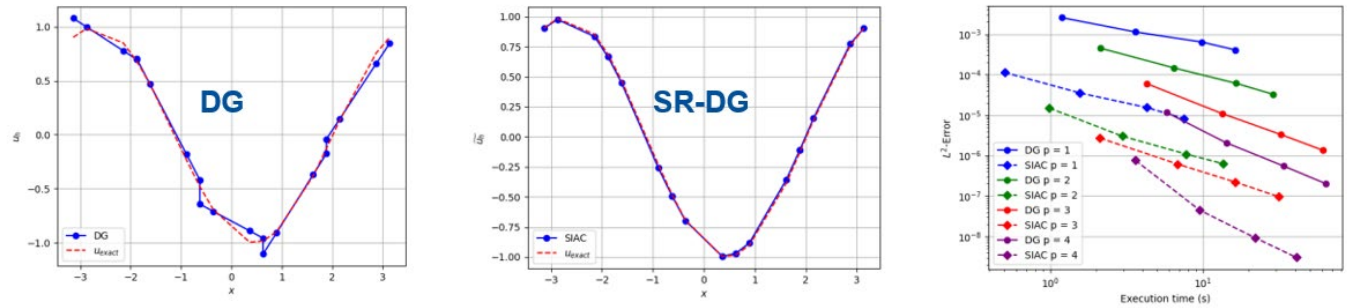
Profil et compétences recherchées

Strong interest in numerical analysis and CFD, fluid dynamics, programming in python and C++

The simulation of flow problems in aerodynamic applications is particularly challenging due to the complexity of the aircraft geometries and the related intricate flow physics: high-Reynolds compressible turbulent flows yield very thin boundary layers around flying bodies, complex multi-scale vortex motions and wakes, as well as steep shock waves. The present research aims to develop novel numerical schemes for these applications, that deliver high accuracy, robustness and computational efficiency, while also accommodating unstructured grids suitable for discretizing fluid domains with complex geometries.

ONERA has been involved for several years in the development of Discontinuous Galerkin (DG) solvers for complex applications [3,4]. DG schemes combines concepts from both Finite Volume and Finite Element methods to achieve arbitrarily high orders of accuracy on unstructured, multi-element meshes [1,2]. However, they still exhibit several limitations. In particular, they may produce unphysical jumps in between elements when coarse meshes are used. And they remain computationally expensive, with strong CFL constraint that limit their efficiency. We are currently developing improvements of DG schemes to address these shortcomings, by replacing the DG polynomial reconstruction with a SIAC convolution within the DG spatial approximation. SIAC filters (for Smoothness Increasing Accuracy Conserving), are a particular class of polynomial-based filters that both increase the smoothness and accuracy of DG solutions [6]. The new scheme based on SIAC reconstruction, referred to as SR-DG, has been applied to linear and non-linear scalar convection-diffusion equations. It has demonstrated clear advantages over standard DG schemes regarding smoothness, accuracy, and overall efficiency [5], see Figure 1 which illustrates the gains in smoothness and efficiency for the Burgers equations. More recently, we have extended the scheme to Navier-Stokes equations in two spatial dimensions, considering structured cartesian grids.

The present work aims at proposing an extension of the SR-DG scheme to multi-elements unstructured meshes, which is key to consider applications involving complex geometries. In particular, the challenge lies in defining filtering kernels and accurate reconstructions for meshes containing highly skewed and anisotropic simplex elements, such as triangles and tetrahedra. Once a robust formulation has been developed for 2D and 3D meshes, it will be tested on a prototype code to demonstrate the advantages of the new scheme over standard DG methods. Subsequently, the methodology will be ported to CODA, a DG software co-developed by ONERA, Airbus and DLR, to address complex applications including 2D and 3D configurations representative of aircraft Geometries. This PhD work will also address broader developments of the SR-DG scheme, including shock capturing and regularization techniques, theoretical analyses to gain a deeper understanding of the numerical dissipation and dispersion errors associated with the choice of SIAC kernel, and extensions towards implicit time integration methods.



(a) u_h, $N = 5$ elements, $p = 1$ (b) $\widetilde{u_h}$, $N = 5$ elements, $p = 1$ (c) L^2-Error VS T_{final}

Figure 1 Smoothness and accuracy enhancing properties of the SR-DG scheme over standard DG scheme for the 1D Burgers equations

The tentative research work plan is as follows:

- Read and understand the existing literature for DG schemes, and our recent SR-DG related work, handle the existing SR-DG prototype in 1D and structured 2D
- Establish a 2D formulation on triangles for Euler and Navier-Stokes equations for the SR-DG, carry on the implementation in a prototype and perform systematic validation and comparison against standard DG schemes on similar grids
- Establish a 3D formulation and carry on the same steps as in 2D, then prepare an integration in the CODA flow solver
- Depending on the progress and the applicant preferences, carry out theoretical analysis of the scheme allowing for a relevant choice of SIAC kernel parameters, develop shock capturing techniques for the SR-DG scheme on unstructured meshes and exploit the SR-DG formalism to establish filtered Navier-Stokes equations with application to Large-Eddy Simulation.

The applicant must have a strong interest for research and numerical developments, as well as a solid record in numerical analysis and fluid dynamics.

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Collaborations envisagées

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