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THE FRENCH AEROSPACE LAB

PROPOSITION DE SUJET DE THÈSE

Intitulé : Worst case copula for reliability analysis with autoencoders

Référence : **TIS-DTIS-2024-20**
(à rappeler dans toute correspondance)

Début de la thèse : October 2024	**Date limite de candidature :** May 2024

Mots clés : Uncertainty quantification, rare event estimation, variational autoencoder, Dependency modeling

Profil et compétences recherchées

Applied mathematics, Probability / statistics, Python programming.

Présentation du projet doctoral, contexte et objectif

Physical systems may be schematically described by a relationship of the type $Y = \phi(X)$, where the multidimensional input X is assumed to be random with a known density and the output Y is determined via the deterministic function ϕ. A typical application can be found in mechanical engineering where ϕ represents a computational code, such as stress calculations on complex mechanical structures, and X the external conditions under which this computation is performed. One example is a finite element code, the complexity of which makes it impossible to analyse the function ϕ and therefore the output Y analytically.

The reliability analysis of this kind of numerical model mainly consists in the estimation of its failure probability. The failure is often a rare event and thus has a small probability. The high computational cost of an evaluation of the numerical model (several minutes to several days CPU) and the low value of the probability make the usual quadrature methods [1] and Monte Carlo sampling [2] inappropriate to handle this problem, but various techniques reviewed in [3] have been developed to estimate more precisely a such probability at a limited computational cost, including importance sampling [4] for example.

The marginal distributions of the input X are often set accurately with a database or expert opinion. However, the dependency structure between the inputs is much more complex to choose or estimate, especially in high dimensions whereas it can have a significant impact on the output distribution. The effect of this lack of knowledge is thus particularly noticeable on a rare event probability of the output [5]. The order of magnitude of this probability can change significantly depending on the dependency structure chosen [6]. **The objective of this thesis is to determine the input dependency structure that maximizes or minimizes the rare event probability.** Although this problem has already been analysed in the literature [7], the proposed algorithm requires a number of hypotheses and approximations that make it hardly applicable to high dimensional and expensive black box model. A methodology envisaged in this thesis would be to couple importance sampling [8] and copula learning by variational auto encoders (VAE) [9] to bound the effects of a lack of knowledge on the input dependence structure.

The first stage of the thesis will consist of a literature review on sampling methods for reliability and variational autoencoder. In addition, a Master project internship will start in April 2024 on a similar subject and could serve as a starting point for this thesis. Copula learning with VAE will be the priority point to be addressed. Secondly, the PhD student will estimate the rare event probability with importance sampling and seek to optimize the VAE weights to maximize/minimize the rare event probability. Finally, a reliability-oriented sensitivity analysis could find the combinations of inputs that are the most influential on the rare event probability variations.

This PhD will be co-funded by ANITI 2.0 (Artificial and Natural Intelligence Toulouse Institute) and ONERA.

References :

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[2] Rubinstein, R. Y., & Kroese, D. P. (2016). Simulation and the Monte Carlo method. John Wiley & Sons.

[3] Morio, J., & Balesdent, M. (2015). Estimation of rare event probabilities in complex aerospace and other systems: a practical approach. Woodhead publishing.

[4] Bucklew, J. A., & Bucklew, J. (2004). Introduction to rare event simulation (Vol. 5). New York: Springer.

[5] Sarazin, G., Morio, J., Lagnoux, A., Balesdent, M., & Brevault, L. (2021). Reliability-oriented sensitivity analysis in presence of data-driven epistemic uncertainty. Reliability Engineering & System Safety, 215, 107733.

[6] Torre, E., Marelli, S., Embrechts, P., & Sudret, B. (2019). A general framework for data-driven uncertainty quantification under complex input dependencies using vine copulas. Probabilistic Engineering Mechanics, 55, 1-16.

[7] Benoumechiara, N., Bousquet, N., Michel, B., & Saint-Pierre, P. (2020). Detecting and modeling critical dependence structures between random inputs of computer models. Dependence Modeling, 8(1), 263-297.

[8] Balesdent, M., Morio, J., & Brevault, L. (2016). Rare event probability estimation in the presence of epistemic uncertainty on input probability distribution parameters. Methodology and Computing in Applied Probability, 18, 197-216.

[9] Kingma, D. P., & Welling, M. (2019). An introduction to variational autoencoders. Foundations and Trends® in Machine Learning, 12(4), 307-392.

Collaborations envisagées : Institut Mathématiques de Toulouse

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