Analysis of control of LPV systems with piecewise constant parameters

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Department of Biosystems Science and Engineering
CONTROL THEORY & SYSTEMS BIOLOGY

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Introduction

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LPV systems

$$\begin{array}{rcl} \dot{x}(t) & = & A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) & = & C(\rho(t))x(t) + D(\rho(t))u(t) \\ x(0) & = & x_0 \end{array} \tag{1}$$

where

- x, u and y are the state of the system, the (control) input and the output
- ρ is the parameter vector
- Matrix-valued functions $A(\cdot)$, $B(\cdot)$, $C(\cdot)$ and $D(\cdot)$ “nice enough"

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LPV systems

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where

- x, u and y are the state of the system, the (control) input and the output
- ρ is the parameter vector
- Matrix-valued functions $A(\cdot)$, $B(\cdot)$, $C(\cdot)$ and $D(\cdot)$ “nice enough"

Commonly considered parameters

- Bounded differentiable trajectories
- Discontinuous bounded trajectories
- Periodic, switched and Markov jump systems can also be seen as LPV systems

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What is the rationale for LPV systems?

Stability analysis

- Identical to uncertain time-varying parametric systems
- LPV analysis = robust analysis

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What is the rationale for LPV systems?

Stability analysis

- Identical to uncertain time-varying parametric systems
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LPV design

- We assume in the LPV framework that the parameters are measured/known
- So, we can use them in controllers, observers, etc.
- For instance, a gain-scheduled state-feedback controller would take the form

$$u(t) = K(\rho(t))x(t) \tag{2}$$

- LPV control $>$ robust control
- But LPV controllers are more difficult to design!

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Stability analysis of LPV systems

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Quadratic stability

Theorem

The LPV system

$$\begin{array}{rcl}\dot{x}(t) & = & A(\rho(t))x(t) \\ x(0) & = & x_0\end{array} \tag{3}$$

is quadratically stable if and only if there exists a matrix $P \in \mathbb{S}^n_{\succ 0}$ *such that the LMI*

$$A(\theta)^T P + PA(\theta) \prec 0 \tag{4}$$

holds for all $\theta \in \mathcal{P}$.

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holds for all $\theta \in \mathcal{P}$*.*

Remarks

- Common Lyapunov function $V(x) = x^T Px$
- All possible trajectories for the parameters are considered (with the restriction of existence of solutions)
- Quadratic stability $\Longrightarrow A(\rho)$ Hurwitz stable for all $\rho \in \mathcal{P}$
- Semi-infinite dimensional LMI problem

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Robust stability

Theorem

The LPV system

$$\begin{array}{rcl}\dot{x}(t) & = & A(\rho(t))x(t) \\ x(0) & = & x_0\end{array} \tag{5}$$

with $\rho \in \{f : \mathbb{R}_{\geq 0} \to \mathcal{P} \subset \mathbb{R}^N,\ f'(t) \in \mathcal{D},\ t \geq 0\}$ *is robustly stable if and only if there exists a differentiable matrix-valued function* $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the LMI*

$$\sum_{i=1}^{N} \theta_i' \frac{\partial}{\partial \theta_i} P(\theta) + A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \tag{6}$$

holds for all $\theta \in \mathcal{P}$ *and all* $\theta' \in \mathcal{D}$.

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Robust stability

Theorem

The LPV system

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) \\ x(0) &= x_0 \end{aligned} \tag{5}$$

with $\rho \in \{f : \mathbb{R}_{\geq 0} \to \mathcal{P} \subset \mathbb{R}^N,\ f'(t) \in \mathcal{D},\ t \geq 0\}$ *is robustly stable if and only if there exists a differentiable matrix-valued function* $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the LMI*

$$\sum_{i=1}^{N} \theta_i' \frac{\partial}{\partial \theta_i} P(\theta) + A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \tag{6}$$

holds for all $\theta \in \mathcal{P}$ *and all* $\theta' \in \mathcal{D}$*.*

Remarks

- Parameter-dependent Lyapunov function $V(x) = x^T P(\rho)x$
- Trajectories of the parameters are continuously differentiable
- $A(\rho)$ Hurwitz stable for all $\rho \in \mathcal{P}$ is necessary and sufficient for robust stability provided that the parameters vary sufficiently slowly
- Infinite-dimensional LMI problem

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Summary

Remarks on LPV systems

- Two main classes of parameter trajectories associated with two main stability concepts
- Quadratic stability may be conservative while robust stability too demanding
- Part of the success of periodic, switched and jump systems lies in the “tailoredness” of the tools
- The definition of the parameter trajectories is way too loose to lead to accurate results (e.g. asymptotic stability does not imply quadratic stability)

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Summary

Remarks on LPV systems

- Two main classes of parameter trajectories associated with two main stability concepts
- Quadratic stability may be conservative while robust stability too demanding
- Part of the success of periodic, switched and jump systems lies in the "tailoredness" of the tools
- The definition of the parameter trajectories is way too loose to lead to accurate results (e.g. asymptotic stability does not imply quadratic stability)

Proposal

- What about something in between the set of all possible trajectories and those that are continuously differentiable?
- For instance, we can consider piecewise continuous/constant parameter trajectories
- Quadratic and robust stability not adapted
- Need something new!

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Stability analysis of LPV systems with piecewise constant parameters

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Outline

Two main class of parameters

- Periodic changes $\rightarrow$ constant dwell-time
- Aperiodic changes $\rightarrow$ minimum dwell-time

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Outline

Two main class of parameters

- Periodic changes $\rightarrow$ constant dwell-time
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Stability results

- Discrete-time-like stability conditions
- Lifted conditions

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Discussions

- Connections with quadratic and robust stability
- Connections with switched systems
- Computational considerations
- Example

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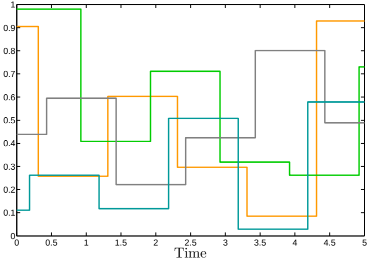
Stability under constant dwell-time

Let us consider the LPV system

$$\dot{x} = A(\rho)x, \; x(0) = x_0 \tag{7}$$

with piecewise constant parameter $\rho \in \mathscr{P}_{\bar{T}}$ where

$$\mathscr{P}_{\bar{T}} = \left\{ \begin{array}{c} \rho : \mathbb{R}_{\geq 0} \to \mathcal{P} : \; \rho(t) = \rho(t_k), \; t \in [t_k, t_{k+1}), \\ t_k = k\bar{T} + \sigma_0, \; 0 \leq \sigma_0 < \bar{T}, \; k \in \mathbb{N} \end{array} \right\} \tag{8}$$



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Theorem

Assume that there exists matrix-valued function $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ such that

$$e^{A(\theta)^T\bar{T}}P(\theta)e^{A(\theta)\bar{T}} - P(\eta) \prec 0 \tag{9}$$

holds for all $\theta, \eta \in \mathcal{P}$.
Then, the LPV system with piecewise constant parameters and constant dwell-time $\bar{T}$ is asymptotically stable.

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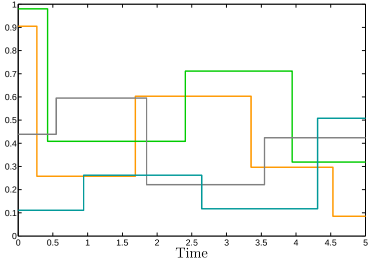
Stability under minimum dwell-time

Let us consider the LPV system

$$\dot{x} = A(\rho)x, \ x(0) = x_0 \tag{10}$$

with piecewise constant parameter $\rho \in \mathscr{P}_{\geqslant \bar{T}}$ where

$$\mathscr{P}_{\geqslant \bar{T}} = \left\{ \begin{array}{c} \rho : \mathbb{R}_{\geq 0} \to \mathcal{P} : \rho(t) = \rho(t_k), t \in [t_k, t_{k+1}) \\ t_{k+1} - t_k \geq \bar{T}, \ k \in \mathbb{N}_0 \end{array} \right\}. \tag{11}$$



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Theorem

Assume that there exists a matrix-valued function $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that*

$$e^{A(\theta)^T \bar{T}} P(\theta) e^{A(\theta)\bar{T}} - P(\eta) \prec 0 \tag{12}$$

and

$$A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \tag{13}$$

holds for all $\theta, \eta \in \mathcal{P}$.
Then, the LPV system with piecewise constant parameters is asymptotically stable with minimum dwell-time $\bar{T}$.

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Difficulties

Conditions

$$e^{A(\theta)^T\bar{T}}P(\theta)e^{A(\theta)\bar{T}} - P(\eta) \prec 0, \quad A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0$$

Verification of the conditions

- Infinite-dimensional LMIs

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Conditions

$$e^{A(\theta)^T\bar{T}}P(\theta)e^{A(\theta)\bar{T}} - P(\eta) \prec 0, \quad A(\theta)^TP(\theta) + P(\theta)A(\theta) \prec 0$$

Verification of the conditions

- Infinite-dimensional LMIs
- Nonconvex exponential terms $e^{A(\theta)\bar{T}}$
- Not easy to check exactly even if the dependence if affine
- Gridding possible but inaccurate and computationally expensive

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Verification of the conditions

- Infinite-dimensional LMIs
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- Not easy to check exactly even if the dependence if affine
- Gridding possible but inaccurate and computationally expensive

Control design

- Nonconvex at all since we would have that $e^{(A(\theta)+B(\theta)K(\theta))\bar{T}}$

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Lifted conditions - Constant DT

Theorem

The following statements are equivalent:

(a) *There exists a matrix-valued function $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ such that the condition*

$$e^{A(\theta)^T \bar{T}} P(\theta) e^{A(\theta)\bar{T}} - P(\eta) \prec 0 \tag{14}$$

holds for all $\theta, \eta \in \mathcal{P}$.

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holds for all $\theta,\eta\in\mathcal{P}$.

(b) *There exists a matrix-valued function* $S:[0,\bar{T}]\times\mathcal{P}\to\mathbb{S}^n$, $S(\bar{T},\theta)\succ 0$, *such that the conditions*

$$\partial_\tau S(\tau,\theta)+\mathrm{Sym}[S(\tau,\theta)A(\theta)]\preceq 0 \tag{15}$$

and

$$S(0,\theta)-S(\bar{T},\eta)\prec 0 \tag{16}$$

hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in\mathcal{T}:=[0,\bar{T}]$.

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(b) *There exists a matrix-valued function $S : [0, \bar{T}] \times \mathcal{P} \to \mathbb{S}^n$, $S(\bar{T}, \theta) \succ 0$, such that the conditions*

$$\partial_\tau S(\tau, \theta) + \mathrm{Sym}[S(\tau, \theta) A(\theta)] \preceq 0 \quad (15)$$

and

$$S(0, \theta) - S(\bar{T}, \eta) \prec 0 \quad (16)$$

hold for all $\theta, \eta \in \mathcal{P}$ and all $\tau \in \mathcal{T} := [0, \bar{T}]$.

Moreover, when one of the above statements holds, the LPV system with piecewise constant parameters and constant dwell-time $\bar{T}$ is asymptotically stable.

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Sketch of the proof

(b) ⇒ (a)

- Integrating $\partial_\tau S(\tau,\theta) + \mathrm{Sym}[S(\tau,\theta)A(\theta)] \preceq 0$ over $\tau \in [0,\bar{T}]$ yields

$$e^{A(\theta)^T\bar{T}}S(\bar{T},\theta)e^{A(\theta)\bar{T}} \preceq S(0,\theta).$$

- Using now $S(0,\theta) - S(\bar{T},\eta) \prec 0$ yields the condition

$$e^{A(\theta)^T\bar{T}}S(\bar{T},\theta)e^{A(\theta)\bar{T}} - S(\bar{T},\eta) \prec 0.$$

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Sketch of the proof

(b) ⇒ (a)

- Integrating $\partial_\tau S(\tau,\theta) + \mathrm{Sym}[S(\tau,\theta)A(\theta)] \preceq 0$ over $\tau \in [0,\bar{T}]$ yields

$$e^{A(\theta)^T\bar{T}}S(\bar{T},\theta)e^{A(\theta)\bar{T}} \preceq S(0,\theta).$$

- Using now $S(0,\theta) - S(\bar{T},\eta) \prec 0$ yields the condition

$$e^{A(\theta)^T\bar{T}}S(\bar{T},\theta)e^{A(\theta)\bar{T}} - S(\bar{T},\eta) \prec 0.$$

(a) ⇒ (b)

- Assume that there exists $P(\theta)$ such that $e^{A(\theta)^T\bar{T}}P(\theta)e^{A(\theta)\bar{T}} - P(\eta) \prec 0$
- Pick $S^*(\tau,\theta) = e^{-A(\theta)^T\tau}S^*(0,\theta)e^{-A(\theta)\tau}$
- Then, we have that $\partial_\tau S^*(\tau,\theta) + \mathrm{Sym}[S^*(\tau,\theta)A(\theta)] = 0$
- Moreover, we have that

$$S^*(0,\theta) - S^*(\bar{T},\eta) = e^{A(\theta)^T\bar{T}}S^*(\bar{T},\theta)e^{A(\theta)\bar{T}} - S^*(\bar{T},\eta) \prec 0 \qquad (17)$$

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Lifted conditions - Minimum DT

Theorem

The following statements are equivalent:

(a) *There exists a matrix-valued function* $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the conditions*

$$A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \tag{18}$$

$$e^{A(\theta)^T \bar{T}} P(\theta) e^{A(\theta)\bar{T}} - P(\eta) \prec 0 \tag{19}$$

hold for all $\theta, \eta \in \mathcal{P}$.

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hold for all $\theta,\eta\in\mathcal{P}$.

(b) *There exists a matrix-valued function* $S:[0,\bar{T}]\times\mathcal{P}\to\mathbb{S}^n$, $S(\bar{T},\theta)\succ 0$, *such that the conditions*

$$A(\theta)^T S(\bar{T},\theta)+S(\bar{T},\theta)A(\theta)\prec 0 \tag{20}$$

$$\partial_\tau S(\tau,\theta)+\mathrm{Sym}[S(\tau,\theta)A(\theta)]\preceq 0 \tag{21}$$

$$S(0,\theta)-S(\bar{T},\eta)\prec 0 \tag{22}$$

hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in\mathcal{T}:=[0,\bar{T}]$.

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hold for all $\theta,\eta\in\mathcal{P}$*.*

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hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in\mathcal{T}:=[0,\bar{T}]$*.*

Moreover, when one of the above statements holds, the LPV system with piecewise constant parameters and minimum dwell-time $\bar{T}$ *is asymptotically stable.*

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Connection with quadratic and robust stability

Theorem (Quadratic stability)

When $\bar{T} \to 0$ in the minimum dwell-time theorem, then the quadratic stability condition

$$A(\theta)^T P + PA(\theta) \prec 0 \tag{23}$$

is recovered.

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Connection with quadratic and robust stability

Theorem (Quadratic stability)

When $\bar{T} \to 0$ *in the minimum dwell-time theorem, then the quadratic stability condition*

$$A(\theta)^T P + PA(\theta) \prec 0 \tag{23}$$

is recovered.

Theorem (Robust stability)

When $\bar{T} \to \infty$ *in the minimum dwell-time theorem, then the robust stability condition*

$$A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \tag{24}$$

for constant parametric uncertainties is recovered.

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Connection with quadratic and robust stability

Theorem (Quadratic stability)

When $\bar{T} \to 0$ in the minimum dwell-time theorem, then the quadratic stability condition

$$A(\theta)^T P + PA(\theta) \prec 0 \tag{23}$$

is recovered.

Theorem (Robust stability)

When $\bar{T} \to \infty$ in the minimum dwell-time theorem, then the robust stability condition

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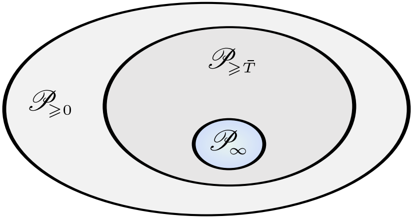
for constant parametric uncertainties is recovered.



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Connection with switched systems

Switched systems

Let $\mathcal{P} = \{1, \ldots, M\}$, for some finite $M \in \mathbb{N}$, and define

$$A(\rho) = \sum_{i=1}^{M} \delta_{i,\rho} A_i \tag{25}$$

where $\delta_{i,j}$ is the Kronecker delta; i.e. $\delta_{i,j} = 1$ if $i = j$, and 0 otherwise.

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Connection with switched systems

Switched systems

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where $\delta_{i,j}$ is the Kronecker delta; i.e. $\delta_{i,j} = 1$ if $i = j$, and 0 otherwise.

Corollary ([1])

Assume that there exist matrices $P_i \in \mathbb{S}^n_{\succ 0}$, $i = 1, \ldots, M$, such that the conditions

$$A_i^T P_i + P_i A_i \prec 0 \tag{26}$$

and

$$e^{A_i^T \bar{T}} P_i e^{A_i \bar{T}} - P_j \prec 0 \tag{27}$$

hold for all $i, j = 1, \ldots, M$, $i \neq j$.
Then, the switched system defined for (25) is asymptotically stable with minimum dwell-time $\bar{T}$.

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[1] J. C. Geromel and P. Colaneri. Stability and stabilization of continuous-time switched linear systems, *SIAM Journal on Control and Optimization*, 2006

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Computational aspects - One parameter case

Preliminaries

- The sets $\mathcal{P}$ and $\mathcal{T}$ are defined as

$$\begin{array}{rcl}\mathcal{P} & := & \{\theta \in \mathbb{R} : g(\theta) := (\theta_{max} - \theta)(\theta - \theta_{min}) \geq 0\} \\ \mathcal{T} & := & \left\{\tau \in \mathbb{R} : \ h(\tau) := \tau(\bar{T} - \tau) \geq 0\right\}\end{array}$$

- We say that that a symmetric matrix-valued function $M(\cdot)$ is a matrix sum of squares if there exists a matrix-valued function $N(\cdot)$ such that $M(\cdot) = N(\cdot)^T N(\cdot)$.

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Proposition

Let $\varepsilon_1, \varepsilon_2, \bar{T} > 0$ be given and assume that there exist polynomial matrix-valued functions $S, \Gamma_j : \mathbb{R}^2 \to \mathbb{S}^n, j = 1, \ldots, 4$ and $\Gamma : \mathbb{R} \to \mathbb{S}^n$ such that

- $\Gamma, \Gamma_j, \ j = 1, \ldots, 4$*, are SOS matrix polynomials*

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- $S(\bar{T}, \theta) - \Gamma(\theta) g(\theta) - \varepsilon_1 I_n$ *is SOS*

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Computational aspects - One parameter case

Preliminaries

- The sets $\mathcal{P}$ and $\mathcal{T}$ are defined as

$$\begin{array}{rcl}\mathcal{P} & := & \{\theta \in \mathbb{R} : g(\theta) := (\theta_{max} - \theta)(\theta - \theta_{min}) \geq 0\} \\ \mathcal{T} & := & \left\{\tau \in \mathbb{R} :\ h(\tau) := \tau(\bar{T} - \tau) \geq 0\right\}\end{array}$$

- We say that that a symmetric matrix-valued function $M(\cdot)$ is a matrix sum of squares if there exists a matrix-valued function $N(\cdot)$ such that $M(\cdot) = N(\cdot)^T N(\cdot)$.

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Then the LPV system with piecewise constant parameters and constant dwell-time $\bar{T}$ is asymptotically stable.

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Example

- Let us consider here an LPV system with the matrix

$$A(\theta) = \begin{bmatrix} 0 & 1 \\ -2-\theta & -1 \end{bmatrix} \tag{28}$$

where $\theta \in [0, \bar{\theta}]$, $\bar{\theta} > 0$.

- It is known that this system is quadratically stable if and only if $\bar{\theta} \leq 3.828$.

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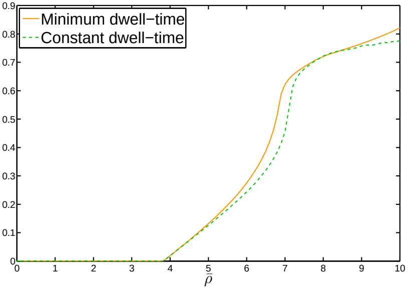
- We use polynomials of order 4 and we get the following results:



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Control of LPV systems with piecewise constant parameters

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Setup

System

- Let us consider the LPV system

$$\begin{array}{rcl} \dot{x}(t) & = & A(\rho(t))x(t) + B(\rho(t))u(t) \\ x(0) & = & x_0 \end{array}$$

where $\{t_k\}_{k\in\mathbb{N}_0}$ is the sequence of time instants at which the parameter vector changes value.

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Control laws

- Constant dwell-time case

$$u(t) = K(t - t_k, \rho(t_k))x(t), \ t \in [t_k, t_{k+1}) \tag{29}$$

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$$u(t) = K(t - t_k, \rho(t_k))x(t), \; t \in [t_k, t_{k+1}) \tag{29}$$

- Minimum dwell-time case

$$u(t) = \begin{cases} K(t - t_k, \rho(t_k))x(t), & t \in [t_k, t_k + \bar{T}) \\ K(\bar{T}, \rho(t_k))x(t), & t \in [t_k + \bar{T}, t_{k+1}) \end{cases} \tag{30}$$

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State-feedback control - Constant DT

Theorem

The following statements are equivalent:

(a) *There exists a matrix-valued function* $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the condition*

$$\Phi_\theta(\bar{T})^T P(\theta) \Phi_\theta(\bar{T}) - P(\eta) \prec 0 \quad (31)$$

holds for all $\theta, \eta \in \mathcal{P}$ *where*

$$\Phi_\theta'(s) = (A(\theta) + B(\theta)K(s,\theta))\Phi_\theta(s),\ \Phi_\theta(0) = I,\ s \in [0, \bar{T}]. \quad (32)$$

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(b) *There exists a matrix-valued function* $\tilde{S}:[0,\bar{T}]\times\mathcal{P}\to\mathbb{S}^n$, $\tilde{S}(\bar{T},\theta)\succ 0$, *such that the conditions*

$$-\partial_\tau\tilde{S}(\tau,\theta)+\mathrm{Sym}[A(\theta)\tilde{S}(\tau,\theta)+B(\theta)U(\tau,\theta)]\preceq 0 \tag{33}$$

and

$$\tilde{S}(\bar{T},\eta)-\tilde{S}(0,\theta)\prec 0 \tag{34}$$

hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in[0,\bar{T}]$.

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hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in[0,\bar{T}]$.

Moreover, when one of the above statements holds, then the closed-loop LPV system is asymptotically stable with constant dwell-time $\bar{T}$ *and a suitable controller gain can be computed using* $K(\tau,\theta)=U(\tau,\theta)\tilde{S}(\tau,\theta)^{-1}$.

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State-feedback control - Minimum DT

Theorem

Assume that there exists a matrix-valued function $\tilde{S} : [0, \bar{T}] \times \mathcal{P} \to \mathbb{S}^n$, $\tilde{S}(\bar{T}, \theta) \succ 0$, *such that the conditions*

$$\mathrm{Sym}[A(\theta)\tilde{S}(\bar{T}, \theta) + B(\theta)U(\bar{T}, \theta)] \prec 0, \tag{35}$$

$$-\partial_\tau \tilde{S}(\tau, \theta) + \mathrm{Sym}[A(\theta)\tilde{S}(\tau, \theta) + B(\theta)U(\tau, \theta)] \preceq 0 \tag{36}$$

and

$$\tilde{S}(\bar{T}, \eta) - \tilde{S}(0, \theta) \prec 0 \tag{37}$$

hold for all $\theta, \eta \in \mathcal{P}$ *and all* $\tau \in [0, \bar{T}]$.

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State-feedback control - Minimum DT

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hold for all $\theta,\eta\in\mathcal{P}$ *and all* $\tau\in[0,\bar{T}]$.

Then the closed-loop LPV system is asymptotically stable with minimum dwell-time $\bar{T}$ *and a suitable controller gain is moreover given by*

$$K(\tau,\theta)=U(\tau,\theta)\tilde{S}(\tau,\theta)^{-1}. \tag{38}$$

Example

System

$$\dot{x} = \begin{bmatrix} 3-\theta & 1 \\ 1-\theta & 2+\theta \end{bmatrix} x + \begin{bmatrix} 1 \\ 1+\theta \end{bmatrix} u,\ \theta \in [0,1]. \quad (39)$$

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Proposition

No control law of the form $u = K(\theta)x$ can quadratically stabilize the system (39).

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Proof

- Quadratically stabilizable if and only if the LMI

$$L(\theta) := B_\perp(\theta)[A(\theta)P + PA(\theta)^T]B_\perp(\theta)^T \prec 0$$

is feasible for all $\theta \in [0,1]$ where $B_\perp(\theta) = \begin{bmatrix} 1+\theta & -1 \end{bmatrix}$.

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- Assume it is stabilizable, then $L(0) \prec 0$ and $L(1) \prec 0$.
- This implies that there exists a $p \in \mathbb{R}$ such that

$$f_1(p) = p^2 - 3p + 2 < 0 \quad \text{and} \quad f_2(p) = p^2 - 6p + 8 < 0. \tag{40}$$

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- But $f_1(p) < 0 \Leftrightarrow p \in (1,2)$ and $f_2(p) < 0 \Leftrightarrow p \in (2,4)$.

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- This implies that the system is not quadratically stabilizable.

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Example

- We pick $\bar{T} = 0.05$, polynomials S, U of order 1, and polynomials Γ's of order 2.

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Example

- We pick $\bar{T} = 0.05$, polynomials S, U of order 1, and polynomials Γ's of order 2.
- Primal/dual variables: 551/120; computation time: less than 2sec

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Example

- We pick $\bar{T} = 0.05$, polynomials S, U of order 1, and polynomials Γ's of order 2.
- Primal/dual variables: 551/120; computation time: less than 2sec
- We find

$$K(\tau,\theta) = \frac{1}{\mathrm{den}(\tau,\theta)} \begin{bmatrix} K_1(\tau,\theta) & K_2(\tau,\theta) \end{bmatrix}$$

where

$$\begin{array}{rcl} K_1(\tau,\theta) & = & 76.930 - 1109.596\tau + 14.343\theta + 1569.878\tau^2 + 170.469\tau\theta - 9.158\theta^2 \\ K_2(\tau,\theta) & = & 24.445 - 739.302\tau - 17.004\theta + 1136.874\tau^2 + 159.427\tau\theta + 3.174\theta^2 \\ \mathrm{den}(\tau,\theta) & = & -23.189 + 483.241\tau - 0.934\theta - 947.359\tau^2 + 3.140\tau\theta + 1.066\theta^2 \end{array}$$

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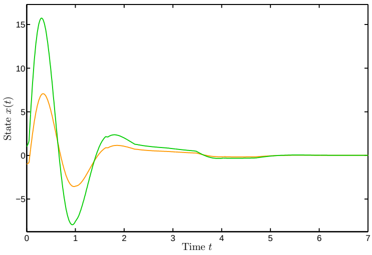

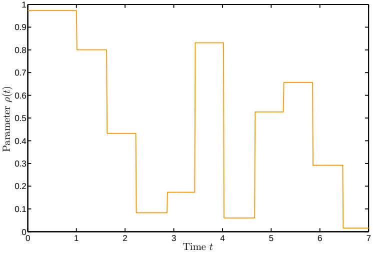

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Concluding remarks

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Conclusions

Concluding statements

- Tractable conditions for analysis and control of LPV systems with piecewise constant parameters
- Extend quadratic and robust stability

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Possible extensions

- Piecewise differentiable parameters (underway)
- Dynamic output feedback?
- Performance analysis, e.g. L_2-performance
- Nonlinear systems
- Homogeneous Lyapunov functions (non-conservative[1])

1 F. Wirth. A converse Lyapunov theorem for linear parameter-varying and linear switching systems, *SIAM Journal on Control and Optimization*, 2005

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Thank you for your attention