Using model order reduction to compute fast frequency sweeps of vibro-acoustic systems described by indirect boundary element models

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November 28, 2014

Outline

Using model order reduction to compute...

What is model order reduction?

= mathematical technique to reduce the complexity of dynamical systems.

- First used in control (controller's complexity same as that of the system to be controlled). Problems:
  - storage
  - accuracy
  - computational speed

- Later used for speeding up simulations and decrease the time-to-market of products when parallelization is not feasible

- MOR offers a trade-off between accuracy and complexity
  - For non-minimal systems, the reduction to minimal system is error-free
  - There are systems for which MOR is not suitable.

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Using MOR to compute fast frequency sweeps...

Why frequency, why frequency sweeps and why fast sweeps?

Analyzing systems in the frequency domain allows one to infer properties regarding resonances (e.g., vibro-acoustic systems), filtering properties (e.g., electrical systems), etc.

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A frequency sweep amounts to solving a linear system $\boldsymbol{A}(f)\boldsymbol{x}(f) = \boldsymbol{b}(f)$ (e.g., $\boldsymbol{A}(f) = j2\pi f\boldsymbol{I} - \boldsymbol{A}$) at many frequencies.

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A fast frequency sweep avoids solving the large linear system for each frequency by using extrapolation.

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Using... of vibro-acoustic systems...

What are vibro-acoustic systems?

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$$\nabla^2 p + k^2 p = 0 \quad \text{in the domain } \Omega, \text{ where}$$

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$$\nabla^2 p + k^2 p = 0 \text{ in the domain } \Omega, \text{ where}$$

- $\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$
- p denotes the complex amplitude of the pressure representing a time harmonic variation given by $p' = \mathrm{Re}(p e^{\mathrm{i}\omega t})$
- $k = \omega / c$, with ω the angular frequency, c the speed of sound
- domain Ω containing an inviscid compressible fluid.

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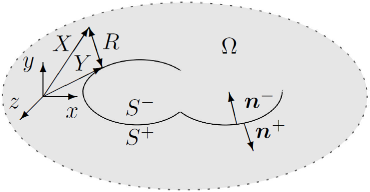
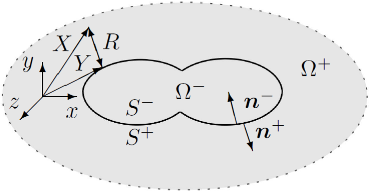
Using … by indirect boundary element models

What is IBEM?

Indirect formulation is employed for interior and exterior problems.



(a) Acoustic system with an open boundary surface.



(b) Combined interior/exterior acoustic system with a closed boundary surface.

In IBEM, the unknowns are $\sigma = \frac{\partial p^+}{\partial n^+} - \frac{\partial p^-}{\partial n^-}$ (single layer potential) and $\mu = p^+ - p^-$ (double layer potential). Acoustic pressure at field point X is

$$p(X) = \int_S \left(G(X,Y)\sigma(Y) - \frac{\partial G(X,Y)}{\partial n(Y)} \mu(Y) \right) dS.$$

$G(X,Y) = \frac{\exp(-ikR)}{4\pi R}$, $R = |X - Y|$ is the 3D Green's function.

What is IBEM?

The surface S is discretized into boundary elements $S \cong \sum_e S^e$. The unknowns are expressed at the discretization points (nodes) as

$$\mu(X) = \boldsymbol{N}_\mu \cdot \boldsymbol{\mu}, \quad \sigma(X) = \boldsymbol{N}_\sigma \cdot \boldsymbol{\sigma}$$

with $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, vectors of nodal double and single layer potentials, and $\boldsymbol{N}_\mu$ and $\boldsymbol{N}_\sigma$, shape functions. This yields the system of equations of size N_{DOF}:

$$\underbrace{\begin{bmatrix} \boldsymbol{A}_{\sigma\sigma} & \boldsymbol{A}_{\sigma\mu} \\ \boldsymbol{A}_{\sigma\mu}^H & \boldsymbol{A}_{\mu\mu} \end{bmatrix}}_{\boldsymbol{A}(f)} \underbrace{\begin{bmatrix} \boldsymbol{\sigma} \\ \boldsymbol{\mu} \end{bmatrix}}_{\boldsymbol{x}(f)} = \underbrace{\begin{bmatrix} 0 \\ \boldsymbol{b}_\mu \end{bmatrix}}_{\boldsymbol{b}(f)}$$

with the matrix $\boldsymbol{A}$ being complex and symmetric.

Challenges and goals

Challenges for IBEM:

- system matrix $\boldsymbol{A}(f)$ is dense
- assembling and solving are equally expensive
- complicated frequency dependency because of $G(X, Y) = \frac{\exp(-\mathrm{i}kR)}{4\pi R}$.

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Goals for FFS:

- avoid assembling the system matrices at each frequency: perform polynomial interpolation on appropriate frequency scaled matrices
- avoid solving a linear system at each frequency: employ Padé approximations

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Frequency dependency of the system matrices

Motivation: understand this to be able to design well-suited strategies to approximate $\boldsymbol{A}(f)$ by interpolating appropriate frequency-scaled quantities.

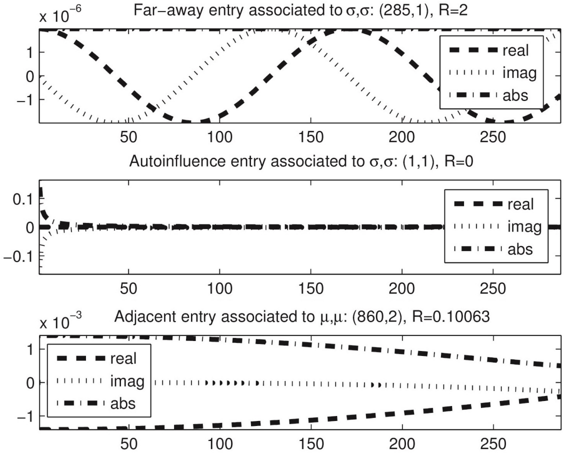
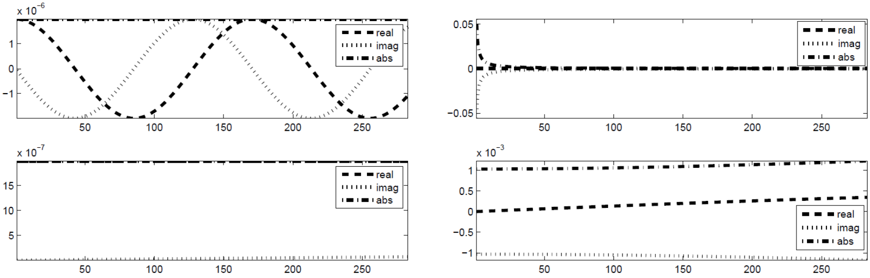


Figure : Frequency behavior of various entries

Scaling the system matrices

The scaled entries are $\hat{A}_{[m,n]} = \begin{cases} e^{ikR_{[m,n]}} A_{[m,n]} \\ kA_{[m,n]} \end{cases}, \quad m,n = 1,\ldots,N_{DOF}.$



(a) Entry multiplied by $e^{ikR_{[m,n]}}$

(b) Entry multiplied by k

Figure : The effect of applying the scaling factor on two matrix entries

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Frequency interpolation of the system matrices

The scaled matrices are interpolated by Lagrange polynomials:

$$\check{A}_{[m,n]}(k) = \sum_{j=1}^{N+1} P_j(k)\hat{A}_{[m,n]}(k_j), \qquad P_j(k) = \prod_{\substack{i=1 \\ i \neq j}}^{N+1} \frac{k - k_i}{k_j - k_i}$$

with $P_j(k) = 1$, for $k = k_j$, and $P_j(k) = 0$, for $k \neq k_j$.
We call the Lagrange nodes k_j the master wavenumbers (frequencies).

The interpolation order N can be 1 (linear interpolation as in [Benthien, 1989]), 2, or higher. This approach requires assembling and storing $N + 1$ system matrices, so one needs to find a trade-off.

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Inverse scaling of the system matrices

The approximated system matrix entries are obtained by multiplying $\check{A}_{[m,n]}(k)$ with the inverse of the scaling factor:

$$\tilde{A}_{[m,n]}(k) = \begin{cases} e^{-\mathrm{i}kR_{[m,n]}}\check{A}_{[m,n]}(k) = \sum_{j=1}^{N+1} P_j(k) e^{\mathrm{i}(k_j-k)R_{[m,n]}} A_{[m,n]}(k_j) \\ \frac{1}{k}\check{A}_{[m,n]}(k) = \sum_{j=1}^{N+1} P_j(k)\frac{k_j}{k} A_{[m,n]}(k_j). \end{cases}$$

Remark The approximated matrix is equal to the original at k_j: $\tilde{A}(k_j) = A(k_j)$.

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Recap To avoid assembling the system matrix at each f [Benthien, 1989]:

- assemble & store matrices @ master frequencies
- perform the interpolation described above @ slave frequencies.

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Determining the frequency windows

Motivation: A large polynomial order N required when performing interpolation over entire frequency band $\Rightarrow$ smaller intervals.

A few representative matrix entries are carefully chosen and assembled at all frequencies. These entries are interpolated simultaneously by an order N polynomial with an a-priori or user-defined accuracy.

Windows determined as intervals which contain highest possible number of frequencies in ascending order such that the fitting error for the representative entries inside the interval is below the tolerance.

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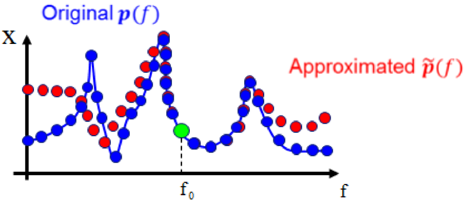
Taylor series



The Taylor series for $x(f)$ around f_0, the expansion frequency:

$$x(f) = x(f_0) + x'(f_0)(f - f_0) + \ldots + x^{(q)}(f_0)\frac{(f - f_0)^q}{q!} + \ldots \quad .$$

Taylor series for vector functions

Recall that we wish to solve $\boldsymbol{A}(f)\mathrm{x}(f) = \boldsymbol{b}(f)$ for many f.

Notation: $\boldsymbol{w}_{q+1} = \frac{\mathrm{x}^{(q)}(f_0)}{q!}, \qquad \boldsymbol{A}_q = \frac{\boldsymbol{A}^{(q)}(f_0)}{q!}, \qquad \boldsymbol{b}_q = \frac{\boldsymbol{b}^{(q)}(f_0)}{q!}.$

$$
\begin{aligned}
\mathrm{x}(f) &= & \boldsymbol{A}_0^{-1}\boldsymbol{b}_0 &= & \boldsymbol{w}_1, \\
\mathrm{x}'(f) &= & \boldsymbol{A}_0^{-1}(\boldsymbol{b}_1 - \boldsymbol{A}_1\boldsymbol{w}_1) &= & \boldsymbol{w}_2, \\
& & \vdots & & \\
\frac{\mathrm{x}^{(q)}(f_0)}{q!} &= & \boldsymbol{A}_0^{-1}\left(\boldsymbol{b}_q - \sum_{i=1}^{q}\boldsymbol{A}_i\boldsymbol{w}_{q-i+1}\right) &= & \boldsymbol{w}_{q+1}.
\end{aligned}
$$

This moments-computation process is ill-conditioned.

Padé approximants

A Padé approximant of order $[q_1/q_2]$ of a scalar $g(f)$ is a rational function

$$\frac{a_0 + a_1(f - f_0) + \ldots + a_{q_1}(f - f_0)^{q_1}}{1 + b_1(f - f_0) + \ldots + b_{q_2}(f - f_0)^{q_2}},$$

whose Taylor expansion around f_0 matches the first $q = q_1 + q_2 + 1$ terms in the Taylor series of $g(f)$.

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Asymptotic Waveform Evaluation(AWE)
Given derivatives of $g(f)$ up to order q, a linear system with a Hankel matrix is solved for the coefficients $a_0, \ldots, a_{q_1}$ and $b_1, \ldots, b_{q_2}$.
For vector functions, such an approximant must be computed for each component of the solution vector $\mathbf{x}(f)$.

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Very ill-conditioned and time consuming!

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Galerkin Asymptotic Waveform Evaluation

Galerkin AWE amounts to forming the moment-matching subspace $\boldsymbol{W}_q = [\boldsymbol{w}_1 \boldsymbol{w}_2 \ldots \boldsymbol{w}_q] \in \mathbb{C}^{N_{\mathrm{DOF}} \times q}$ and imposing that the residual is perpendicular to $\boldsymbol{W}_q$, yielding the following solution vector

$$\boldsymbol{x}_q(f) = \boldsymbol{W}_q \left(\boldsymbol{W}_q^H \boldsymbol{A}(f) \boldsymbol{W}_q \right)^{-1} \left(\boldsymbol{W}_q^H \boldsymbol{b}(f) \right).$$

It can be proven that the approximated vector $\boldsymbol{x}_q(f)$ matches the solution, as well as the value of $q-1$ derivatives around f_0.

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Advantages of GAWE:

- a much smaller linear system needs to be solved, namely $\left(\boldsymbol{W}_q^H \boldsymbol{A}(f) \boldsymbol{W}_q \right)^{-1} \left(\boldsymbol{W}_q^H \boldsymbol{b}(f) \right)$ where $\boldsymbol{W}_q^H \boldsymbol{A}(f) \boldsymbol{W}_q$ is of size $q \times q$
- yields the Padé approximant of the entire vector $\boldsymbol{x}(f)$.

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WCAWE [Slone et al., 2003]

Uses GAWE with the moments computed in a well conditioned manner.

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Before: $\boldsymbol{w}_{q+1} = \boldsymbol{A}_0^{-1} \left(\boldsymbol{b}_q - \sum_{i=1}^{q} \boldsymbol{A}_i \boldsymbol{w}_{q-i+1} \right)$.

WCAWE: $$\widetilde{\boldsymbol{w}}_{q+1} = \boldsymbol{A}_0^{-1} \left(\sum_{i=1}^{q} \boldsymbol{b}_i c_i - \boldsymbol{A}_1 \mathbf{w}_q - \sum_{i=2}^{q} \boldsymbol{A}_j \boldsymbol{W}_{q-i+1} \boldsymbol{d}_i \right),$$

where c_i, $\boldsymbol{d}_i$ are correction factors.

Moreover, they are orthonormalized via a modified Gram-Schmidt process:

$$\begin{aligned}
&\text{for } i = 1, \ldots, q-1 \\
&\quad U_{[i,q]} = \mathbf{w}_i^H \widetilde{\mathbf{w}}_q \\
&\quad \widetilde{\mathbf{w}}_q = \widetilde{\mathbf{w}}_q - U_{[i,q]} \mathbf{w}_j \\
&U_{[q,q]} = \|\widetilde{\mathbf{w}}_q\|,\ \mathbf{w}_q = \widetilde{\mathbf{w}}_q U_{[q,q]}^{-1}.
\end{aligned}$$

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Derivatives of the system matrix

The q^{th} derivative at the expansion wave number k_0:

$$\left.\frac{\partial^q \tilde{A}_{[m,n]}(k)}{\partial k^q}\right|_{k=k_0} = \sum_{j=1}^{N+1} \frac{\partial^q}{\partial k^q}\left(\frac{P_j(k)}{k}\right)\bigg|_{k=k_0} k_j A_{[m,n]}(k_j),$$

for entries scaled by k, and

$$\left.\frac{\partial^q \tilde{A}_{[m,n]}(k)}{\partial k^q}\right|_{k=k_0} = \sum_{j=1}^{N+1} \frac{\partial^q}{\partial k^q}\left(P_j(k) e^{-\mathrm{i}kR_{[m,n]}}\right)\bigg|_{k=k_0} e^{\mathrm{i}k_j R_{[m,n]}} A_{[m,n]}(k_j)$$

otherwise.

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Proposed algorithm

1. Choose a few representative matrix entries, assemble at all frequencies
2. Apply polynomial interpolation of order N to scaled entries with deviation $d_{\text{tol}} = 10^{-4} \Rightarrow$ frequency windows
3. Each frequency window contains $N + 1$ master frequencies $\Rightarrow$ set the middle one as the expansion frequency
4. Apply WCAWE inside each window by matching moments at the expansion frequency
   1. Start with a small moment subspace
   2. Add new vectors to the moments subspace as long as residual $r(f) = \frac{\left\|\tilde{\boldsymbol{A}}(f)x_{\boldsymbol{q}}(f)-\boldsymbol{b}(f)\right\|_2}{\|\boldsymbol{b}(f)\|_2}$ is larger than $\varepsilon_{\text{tol}} = 10^{-3}$

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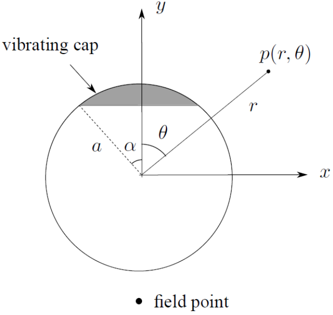
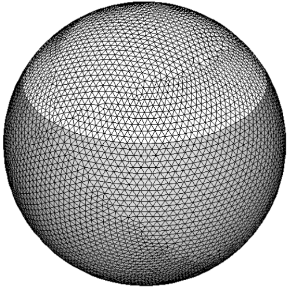
Sphere with rigid cap

The pressure outside the sphere verifies:

$$p(r,\theta) = \frac{-i\rho c v_0(f)}{2} \sum_{n=0}^{\infty} \left[\widetilde{P}_{n-1}(\cos\alpha) - \widetilde{P}_{n+1}(\cos\alpha) \right] \frac{h_n(kr)}{h_n'(ka)} \widetilde{P}_n(\cos\theta),$$

r, distance to evaluation point, h_n, spherical Hankel functions of first kind, $\widetilde{P}_n$, Legendre polynomials, $v_0(f)$, uniform normal velocity of spherical cap, and a, radius of sphere. The infinite summation truncated at $2k$.





Parameters for the problem

- sphere radius $a = 0.6\ \mathrm{m}$, angle defining the vibrating cap $\alpha = \pi/3$ rad
- sound speed is $c = 340\ \mathrm{m/s}$, fluid density is $\rho = 1.225\ \mathrm{kg/m^3}$
- cap normal velocity $v_0(f)$ is taken as the response of a classical 3 DOF mass-spring-damper: $M_1 = 60$, $M_2 = 40$, $M_3 = 20$ (kg); $K_{1,2,3} = 2.7 \times 10^5\ (\mathrm{N/m})$; $C_{1,2,3} = 20\ (\mathrm{Ns/m}) \Rightarrow 3$ resonances
- mesh with 8 653 nodes, 17 302 triangular elements $\Rightarrow N_{\mathrm{DOF}} = 15\ 136$
- $F = [200, 1000]$ Hz with 1 Hz increment (1101 individual frequencies)
- $N = 2$ interpolation for 8 representative matrix entries $\Rightarrow$ 10 frequency windows (4 min, 95% on assembly)

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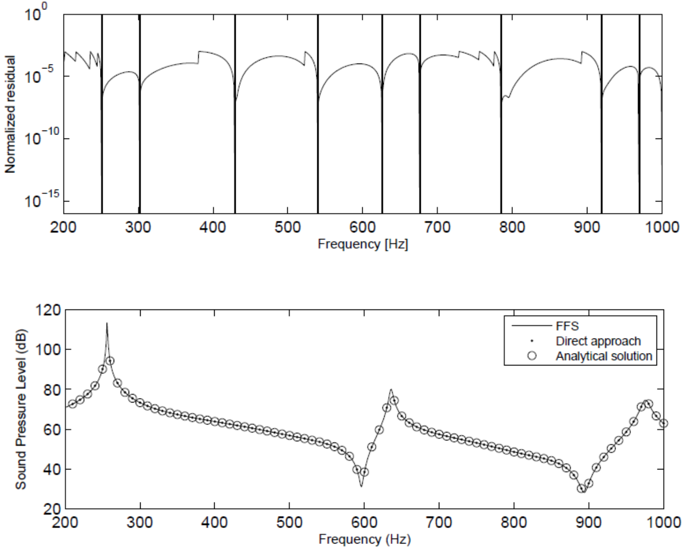
Results



2 h 07 min vs 58 h ⇒ speed up factor of 27.4

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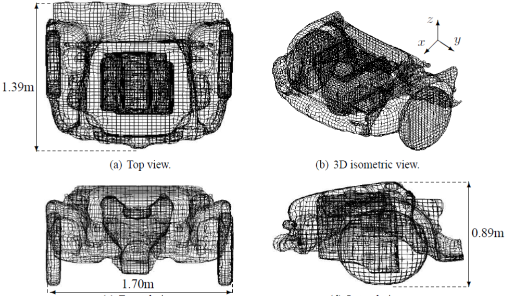
Car engine compartment

Motivation:

- Vehicles should comply with noise emission regulations
- Engine is a major contributor to vehicle pass by noise
- Acoustic treatments in various locations of engine compartment (e.g., under-bonnet, dash, firewall, floor, etc) are employed
- Interior/exterior acoustics problem: cavity with interior resonances and acoustic radiation in free field.



(a) Top view. (b) 3D isometric view.

(c) Frontal view. (d) Lateral view.

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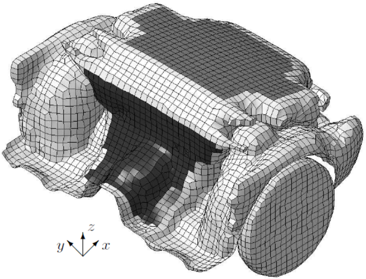
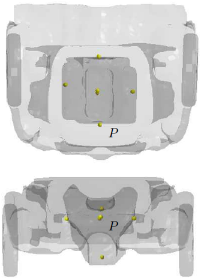
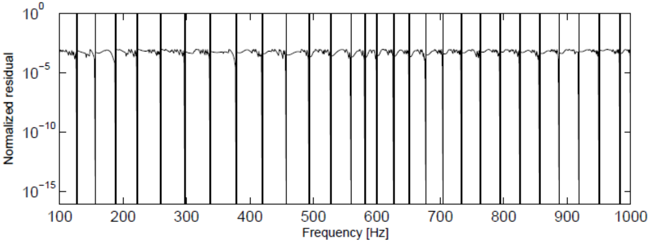
Parameters for the problem

- Mesh with 9 326 nodes, 18 408 elements $\Rightarrow N_{\text{DOF}} = 10\ 151$
- Discontinuous impedance is applied on the internal sides of the bonnet (light grey elements) and the firewall (dark grey elements)
- Remaining elements in white are considered acoustically rigid
- 6 field points measured by microphones
- A spherical point source is located at ($x = 3$ m, $y = 7$ m, $z = 0$ m)
- $F = [100, 1000]$ Hz with 1 Hz frequency increment (901 frequencies)
- 29 frequency windows

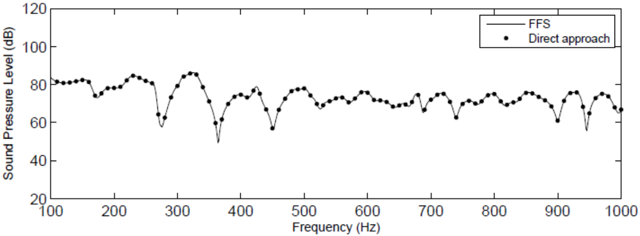




Results



(b) Residual and interpolation frequency windows (vertical lines).



5 h 07 min vs 55 h $\Rightarrow$ speed up factor of 10

Outline

Conclusion

MOR for computing FFS for IBEM:

- avoids assembling and storing the system matrix at each frequency
- avoids solving the linear system at each frequency

Current work: compute moments at several expansion frequencies per window and combine these subspaces (multi-point or rational approach).

Thank you for your attention!

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