A state feedback input constrained control design for a 4-semi-active damper suspension system: a quasi-LPV approach

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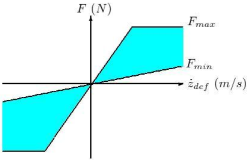
MOSAR Meeting

Institut franco-allemand de recherches de Saint-Louis
April, 23th 2015

Outline

Problem statement

Motivation



Semi-active suspension control problem:

- Main challenge: the dissipativity constraint of semi-active damper.
- LPV approach both for linear and nonlinear model of the damper:
  - Linear modeling *[Poussot-Vassal et al., 2008], [DO et al., 2011]*

$$F_{damper} = c\dot{z}_{def}$$

where the damping coefficient $c \geq 0$ and $c \in [c_{min}, c_{max}]$.

  - Nonlinear modeling *[DO et al., 2010]*

$$F_{damper} = c_0\dot{z}_{def} + k_0 z_{def} + f_I \tanh\left(c_1\dot{z}_{def} + k_1 z_{def}\right)$$

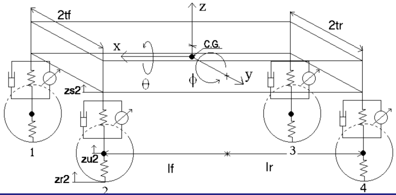
where c_0, k_0, c_1 and k_1 are constant parameters; $f_I \geq$ and $f_I \in [f_{Imin}, f_{Imax}]$

but validated only on the quarter car model.

Problem: A 7dof full vehicle model equipped 4 semi-active dampers

- Use a linear model for the damper $\rightarrow$ 4 scheduling parameters
- The dissipative conditions of the semi-active dampers are recast as saturation conditions on the control inputs.
- Motivate a state feedback input constrained control problem

Vehicle Modelling



A 7 dof full vertical vehicle model:

$$\left\{\begin{array}{ll} m_s \ddot{z}_s & = -F_{sfl} - F_{sfr} - F_{srl} - F_{srr} + F_{dz} \\ I_x \ddot{\theta} & = (-F_{sfr} + F_{sfl})t_f + (-F_{srr} + F_{srl})t_r + mha_y + M_{dx} \\ I_y \ddot{\phi} & = (F_{srr} + F_{srl})l_r - (F_{sfr} + F_{sfl})l_f - mha_x + M_{dy} \\ m_{us} \ddot{z}_{us_{ij}} & = -F_{s_{ij}} + F_{tz_{ij}} \end{array}\right. \quad (1)$$

Suspension force:

$$F_{s_{ij}} = k_{ij}(z_{s_{ij}} - z_{us_{ij}}) + F_{damper_{ij}} \quad (2)$$

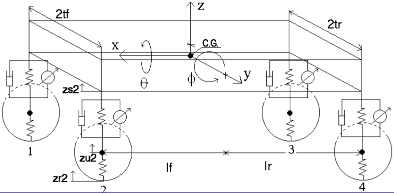
where $F_{damper_{ij}}$ is the semi-active controlled damper force and $\rho_{ij} = \dot{z}_{def_{ij}}$:

$$F_{damper_{ij}} = c_{ij}(.)\dot{z}_{def_{ij}} = c_{ij}(.)(\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}) = c_{nom_{ij}}\dot{z}_{def_{ij}} + u_{ij}^{H_\infty}\rho_{ij} \quad (3)$$

Tire force:

$$F_{tz_{ij}} = -k_{t_{ij}}(z_{us_{ij}} - z_{r_{ij}}) \quad (4)$$

Vehicle Modelling



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Rewrite (1) in the state space representation form:

$$\dot{x}_g(t) = A_g x_g(t) + B_{1g} w(t) + B_{2g}(\rho) u(t) \quad (4)$$

where:

$x_g = [z_s \; \theta \; \phi \; z_{usfl} \; z_{usfr} \; z_{usrl} \; z_{usrr} \; \dot{z}_s \; \dot{\theta} \; \dot{\phi} \; \dot{z}_{usfl} \; \dot{z}_{usfr} \; \dot{z}_{usrl} \; \dot{z}_{usrr}]^T$,

$$w = [F_{dz} \; M_{dx} \; M_{dy} \; z_{rfl} \; z_{rfr} \; z_{rrl} \; z_{rrr}]^T,$$
$$u = [u_{fl}^{H\infty}, u_{fr}^{H\infty}, u_{rl}^{H\infty}, u_{rr}^{H\infty}]^T.$$

Input and State constraints

- The dissipativity constraint of the semi-active damper:

$$0 \leqslant c_{min_{ij}} \leqslant c_{ij}(.) \leqslant c_{max_{ij}} \tag{5}$$

$$\begin{aligned} c_{min_{ij}}\dot{z}_{def_{ij}} &\leq F_{damper_{ij}} \leq c_{max_{ij}}\dot{z}_{def_{ij}} \quad \text{if} \quad \dot{z}_{def_{ij}} > 0 \\ c_{max_{ij}}\dot{z}_{def_{ij}} &\leq F_{damper_{ij}} \leq c_{min_{ij}}\dot{z}_{def_{ij}} \quad \text{if} \quad \dot{z}_{def_{ij}} \leq 0 \end{aligned} \tag{6}$$

  The dissipativity constraint is now recast into:

$$\begin{aligned} c_{min_{ij}}\dot{z}_{def_{ij}} &\leq c_{nom_{ij}}\dot{z}_{def_{ij}} + u_{ij}^{H\infty}\dot{z}_{def_{ij}} \leq c_{max_{ij}}\dot{z}_{def_{ij}} \quad \text{if} \quad \dot{z}_{def_{ij}} > 0 \\ c_{max_{ij}}\dot{z}_{def_{ij}} &\leq c_{nom_{ij}}\dot{z}_{def_{ij}} + u_{ij}^{H\infty}\dot{z}_{def_{ij}} \leq c_{min_{ij}}\dot{z}_{def_{ij}} \quad \text{if} \quad \dot{z}_{def_{ij}} \leq 0 \end{aligned}$$

  Because of $c_{nom_{ij}} = \dfrac{(c_{max_{ij}} + c_{min_{ij}})}{2}$, then we must guarantee the Input constraint:

$$|u_{ij}^{H\infty}| \leq \frac{(c_{max_{ij}} - c_{min_{ij}})}{2} \tag{7}$$

- It should be noted that $|\rho_{ij}| = |\dot{z}_{def_{ij}}| = |\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}| \leq 1$. Thus, to ensure the constraints on the scheduling parameter $|\rho_{ij}| \leq 1$, we must ensure also a *state constraint* which will be rewritten later as:

$$|H.x| \leq 1 \tag{8}$$

  where x being the generalized system state and H is state constraint matrix.

Control problem

Problem Statement: Design a suspension control in order to reduce the roll motion of the vehicle equipped with 4 semi-active dampers. The suspension control must satisfy the input saturation constraints (7) and the state constraint (8).
To tackle this problem, we consider an LPV approach detailed in the sequence.

LPV Control in the presence of input saturation

System description

Consider a quasi-LPV system $\mathfrak{S}_\rho$ with input saturation and disturbance:

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1(\rho)w + B_2u \\ z &= C_1(\rho)x + D_{11}(\rho)w + D_{12}u \end{aligned} \quad (9)$$

Let us consider the following assumptions:

- ρ is bounded to be able to apply the polytopic approach for LPV system:

$$\rho \in \Omega = \left\{ \rho_i \mid \underline{\rho}_i \leq \rho_i \leq \overline{\rho}_i, i = 1, ..k \right\}$$

- The applied control signal u takes value in the compact set:

$$\mathcal{U} = \{u \in R^m / - u_{0i} \leqslant u_i \leqslant u_{0i}, i = 1, ..., m\} \quad (10)$$

- The input disturbances w are supposed to be bounded in amplitude i.e w belongs to a set $\mathcal{W}$:

$$\mathcal{W} = \left\{ w \in R^q / w^T w < \delta \right\} \quad (11)$$

- The state vector is assumed to be known (measured or estimated) and the trajectories of system must belong to a region $\mathcal{X}$ defined as follows:

$$\mathcal{X} = \{x \in R^n / |H_i x| \leq h_{0i}, i = 1, ..., k\} \quad (12)$$

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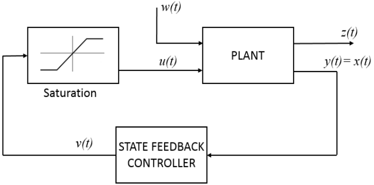

Figure: State feedback control with input saturation

A state feedback control law is considered (Fig.1) and the control signal $v(t)$ computed by the state feedback controller is given by:

$$v(t) = K(\rho)x(t)$$

where $K(\rho) \in \mathcal{R}^{m\times n}$ is a parameter dependent state feedback matrix gain.
Then, the applied control u to system (9) is a saturated one, i.e:

$$u(t) = sat(v(t)) = sat(K(\rho)x(t)) \quad (13)$$

where the saturated function $sat(.)$ is defined by:

$$sat(v_i(t)) = \begin{cases} u_{0i} & if \quad v_i(t) > u_{0i} \\ v_i(t) & if \quad -u_{0i} \leq v_i(t) \leq u_{0i} \\ -u_{0i} & if \quad v_i(t) < -u_{0i} \end{cases} \quad (14)$$

The closed-loop system obtained from the application of (13) in (9) reads as follows:

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1(\rho)w + B_2 sat(K(\rho)x) \\ z &= C_1(\rho)x + D_{11}(\rho)w + D_{12} sat(K(\rho)x) \end{aligned} \quad (15)$$

Let us define now the vector-valued dead-zone function $\phi(K(\rho)x)$:

$$\phi(K(\rho)x) = sat(K(\rho)x) - K(\rho)x \quad (16)$$

From (16), the closed-loop system can therefore be re-written as follows:

$$\begin{aligned} \dot{x} &= (A(\rho) + B_2K(\rho))x + B_2\phi(K(\rho)x) + B_1(\rho)w \\ z &= (C_1(\rho) + D_{12}K(\rho))x + D_{12}\phi(K(\rho)x) + D_{11}(\rho)w \end{aligned} \quad (17)$$

Problem definition

We propose the design of a state feedback $K(\rho)$ for the LPV system (15) in order to satisfy the following conditions:

- When the control input signal is saturated, the nonlinear behavior of the closed-loop system must be considered and the stability has to be guaranteed both internally as well as in the context of input to state, that is:
  - for $w \in \mathcal{W}$, the trajectories of the closed-loop system must be bounded.
  - if $w(t) = 0$ for $t > t_1 > 0$ then the trajectory of the system converge asymptotically to the origin.
- The control performance objective consists in minimizing the upper bound for the L_2 gain from the disturbance w to the controlled output z, i.e $Min \ \gamma > 0$, such that:

$$sup \ \frac{\|z\|_2}{\|w\|_2} < \gamma \quad (18)$$

Remark:

In order to reduce the conservatisme, the L_2 performance problem is solved only in the case that the input saturation is not activated.

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Controller design

Stability analysis

Let us first define the following polyhedral set (saturation model validity region):

$$\mathcal{S}_\rho(K, G, u_0) = \{x \in R^m \mid -u_0 \preceq (K(\rho) - G(\rho))x \preceq u_0\} \quad (19)$$

where this inequality stands for each input variable.

Lemma 1: Sector condition ([Gomes da Silva, 2005])

If $x \in \mathcal{S}_\rho(K, G, u_0)$, then the deadzone function ϕ satisfies the following inequality:

$$\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] \leqslant 0 \quad (20)$$

for any diagonal and positive definite matrix $T(\rho) \in \mathcal{R}^{m \times m}$.

Definition: ([Blanchini, 1999])

The set $\mathcal{E} \subset \mathcal{R}^n$ is said to be W-invariant if $\forall x(t_0) \in \mathcal{E}, \forall w(t) \in \mathcal{W}$ implies that the trajectory $x(t) \in \mathcal{E}$ for all $t \geqslant t_0$.

Remark: ([Boyd et al., 1994])

The quadratic stability of a system can be intepreted in term of the existence of an invariant ellipsoid.

Consider an ellipsoidal set $\mathcal{E}$ associated to a Lyapunov function $V = x^T P x$ with $P = P^T \succ 0$,

$$\mathcal{E}(P) = \left\{x \in \mathcal{R}^n : x^T P x < 1\right\} \quad (21)$$

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$$\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] \leqslant 0 \quad (20)$$

for any diagonal and positive definite matrix $T(\rho) \in \mathcal{R}^{m\times m}$.

Definition: ([Blanchini, 1999])

The set $\mathcal{E} \subset \mathcal{R}^n$ is said to be W-invariant if $\forall x(t_0) \in \mathcal{E}, \forall w(t) \in \mathcal{W}$ implies that the trajectory $x(t) \in \mathcal{E}$ for all $t \geqslant t_0$.

Remark: ([Boyd et al., 1994])

The quadratic stability of a system can be intepreted in term of the existence of an invariant ellipsoid.

Consider an ellipsoidal set $\mathcal{E}$ associated to a Lyapunov function $V = x^T P x$ with $P = P^T \succ 0$,

$$\mathcal{E}(P) = \{x \in \mathcal{R}^n : x^T P x < 1\} \quad (21)$$

Stability analysis

Let us first define the following polyhedral set (saturation model validity region):

$$\mathcal{S}_\rho(K,G,u_0) = \{x \in R^m \mid -u_0 \preceq (K(\rho) - G(\rho))x \preceq u_0\} \tag{19}$$

where this inequality stands for each input variable.

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Theorem 1: Stability condition

If there exist a matrix Q-positive definite, a matrix $S(\rho)$-diagnonal positive definite, matrices $\bar{K}(\rho)$, $\bar{G}(\rho)$ of appropriate dimensions and positive scalar λ_2 such that the following conditions are verified:

-
$$\left[\begin{array}{c|c|c} \bar{\mathcal{M}}(\rho) & (B_2 S(\rho) - \bar{G}(\rho)^T) & B_1(\rho) \\ \hline (S(\rho)B_2^T - \bar{G}(\rho)) & -2S(\rho) & 0 \\ \hline B_1(\rho)^T & 0 & -\lambda_2 I \end{array}\right] < 0 \quad (22)$$
where $\bar{\mathcal{M}}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T B_2^T) + (QA(\rho)^T + \bar{K}(\rho)^T B_2^T)^T + \lambda_1 Q$.
-
$$\left[\begin{array}{c|c} Q & (\bar{K}_i(\rho) - \bar{G}_i(\rho))^T \\ \hline \bar{K}_i(\rho) - \bar{G}_i(\rho) & u_{0i}^2 \end{array}\right] \succeq 0, i = 1, \ldots, m \quad (23)$$
where $\bar{K}_i(\rho)$, $\bar{G}_i(\rho)$ are i^{th} line of $\bar{K}(\rho)$, $\bar{G}(\rho)$ respectively.
-
$$\left[\begin{array}{c|c} Q & QH_i^T \\ \hline H_i Q & h_{0i}^2 \end{array}\right] \geq 0, i = 1, \ldots, k \quad (24)$$
-
$$\lambda_2 \delta - \lambda_1 < 0 \quad (25)$$

Then, with $K(\rho) = \bar{K}(\rho)Q^{-1}$:
a) For any $w \in \mathcal{W}$ and $x(0) \in \mathcal{E}(\mathcal{P})$ the trajectories do not leave $\mathcal{E}(\mathcal{P})$, i.e. $\mathcal{E}(\mathcal{P})$ is an W-invariant domain for the system (15).
b) If $x(0) \in \mathcal{E}(\mathcal{P})$ and $w(t) = 0$ for $t > t_1$, then the corresponding trajectory converge asymptotically to the origin , i.e. $\mathcal{E}(\mathcal{P})$ (with $P = Q^{-1}$) is included in the region of attraction of the closed-loop system (15).

Proof:

- Idea: Demonstrate that $\mathcal{E}(\mathcal{P})$ is a W-invariant set for the system $\forall w(t) \in \mathcal{W}$.
  This condition can be satisfied if there exist scalars $\lambda_1 > 0$ and $\lambda_2 > 0$, such that

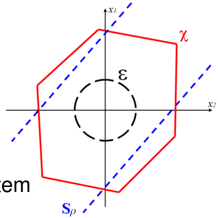
$$\dot{V} + \lambda_1(\xi^T P \xi - 1) + \lambda_2(\delta - w^T w) < 0 \quad (26)$$

  From "Lemma 1": $\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] \leqslant 0$, then (26) is satisfied if:

$$\dot{V} + \lambda_1(x^T P x - 1) + \lambda_2(\delta - w^T w) - 2\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] < 0 \quad (27)$$

  Then we obtain: (22),(25).

- Then, to ensure that $x(t)$ belongs effectively to $\mathcal{S}_\rho(K, G, u_0)$ and that the state constraints are not violated, we must ensure that $\mathcal{E}(P) \subset \mathcal{S}_\rho(K, G, u_0) \cap \mathcal{X}$, i.e $\mathcal{E}(P) \subset \mathcal{S}_\rho(K, G, u_0)$ and $\mathcal{E}(P) \subset \mathcal{X}$.
  It leads to (23),(24).



- Finally, if $w(t) = 0$, it follows: $\dot{V}(x(t)) \leq -\lambda_1 x^T P x < 0$.
  i.e $V(x(t)) \leq e^{-\lambda_1 t} V(x(0))$, it means that the trajectories of the system converge asymptotically to the origin. □

Performance objective

Disturbance attenuation

$$\dot{V}(x(t)) + \frac{1}{\gamma} z^T z - \gamma w^T w < 0 \quad (28)$$

In linear mode, $sat(K(\rho)x) = K(\rho)x$, the closed loop system (15) becomes:

$$\begin{aligned} \dot{x} &= (A(\rho) + B_2 K(\rho))x + B_1(\rho)w \\ z &= (C_1(\rho) + D_{12} K(\rho))x + D_{11}(\rho)w \end{aligned} \quad (29)$$

Then, condition (28) holds if the following inequality is satisfied:

$$\left[\begin{array}{c|c|c} N(\rho) & PB_1(\rho) & (C_1(\rho) + D_{12}K(\rho))^T \\ \hline B_1(\rho)^T P & -\gamma I & D_{11}^T \\ \hline C_1(\rho) + D_{12}K(\rho) & D_{11} & -\gamma I \end{array}\right] < 0 \quad (30)$$

where $N(\rho) = (A(\rho) + B_2 K(\rho))^T P + P(A(\rho) + B_2 K(\rho))$.
Pre and post-multiplying (30) by $diag(P^{-1}, I, I)$, and with $P^{-1} = Q$ one obtains:

$$\left[\begin{array}{c|c|c} \bar{N}(\rho) & B_1(\rho) & (QC_1(\rho)^T + \bar{K}(\rho))^T D_{12}^T \\ \hline B_1(\rho)^T & -\gamma I & D_{11}^T \\ \hline C_1(\rho)Q + D_{12}\bar{K}(\rho) & D_{11} & -\gamma I \end{array}\right] < 0 \quad (31)$$

where $\bar{N}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T B_2^T) + (QA(\rho)^T + \bar{K}(\rho)^T B_2^T)^T$

Controller computation

The state feedback gain $K(\rho)$ that satisfies the stability condition for the saturated system and the disturbance attenuation for the unsaturated system can be derived by solving the following optimization problem:

$$\begin{aligned} &\min_{Q,S,\bar{K},\bar{G},\lambda_2} \gamma \\ &\text{subject to} \qquad (22, 23, 24, 25, 31), \\ &\qquad\qquad\qquad Q, S > 0, \lambda_2 > 0. \end{aligned} \tag{32}$$

Then the state feedback gain matrix $K(\rho)$ can be computed by:

$$K(\rho) = \bar{K}(\rho)P = \bar{K}(\rho)Q^{-1} \tag{33}$$

where:

$$K(\rho) = \sum_{j=1}^{2^k} \alpha_j(\rho)K_j, \quad \sum_{j=1}^{2^k} \alpha_j(\rho) = 1.$$

Application of LPV approach to the full vehicle

Performance objective: reduce the roll motion of the vehicle.
$\rightarrow$ Minimizing the effect of the road disturbance w to the controlled output z ($z = \theta$) while taking into account the actuator saturation.
The H_∞ framework is used to solve this objective, the weighting function W_θ on θ is added:

$$W_\theta = k_\theta \frac{s^2 + 2\xi_{11}\Omega_1 s + {\Omega_1}^2}{s^2 + 2\xi_{12}\Omega_1 s + {\Omega_1}^2}. \tag{34}$$

Noting that 7 DOF vertical model:

$$\dot{x}_g(t) = A_g x_g(t) + B_{1g} w(t) + B_{2g}(\rho) u$$

has the parameter dependent input matrix $B_{2g}(\rho) \rightarrow$ add a low pass filter to obtain the parameter independent input matrix.
The interconnection between the 7 DOF vertical model, W_θ, and the low pass filter gives the following parameter dependent suspension generalized plant ($\Sigma_{gv}(\rho)$):

$$\Sigma_{gv}(\rho) : \begin{cases} \dot{x} = A(\rho)x + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \end{cases} \tag{35}$$

where $x = [x_g^T \; x_{wf}^T \; x_f^T]^T$, x_g, x_{wf}, x_f are the vertical model, weighting function and filter states respectively.

Context of simulation: Full nonlinear vehicle model, validated in a real car "Renault Mégane Coupé " coll. MIPS lab [Basset, Pouly and Lamy]:

- The varying parameter $\rho_{ij} = \dot{z}_{def_{ij}} \in [-1\ \ 1]$
- The damping coefficients vary as follows:
  -For the front dampers: $c_{min_f} = 660\ Ns/m, c_{max_f} = 3740\ Ns/m$.
  -For the rear dampers: $c_{min_r} = 1000\ Ns/m, c_{max_r} = 8520\ Ns/m$.
  Thus, the input constraints (7) lead to:
  $[|u_{fl}^{H\infty}|\ \ |u_{fr}^{H\infty}|\ \ |u_{rl}^{H\infty}|\ \ |u_{rr}^{H\infty}|] \leq [1540\ \ 1540\ \ 3760\ \ 3760]$
- The road profile is chosen in the set $\mathcal{W}$ subject to (11) with $\delta = 0.01\ m^2$.
- The state constraint in (12) is the constraint on suspension deflection speed: $|\dot{z}_{def_{ij}}| = |\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}| = |H_g.x_g| = |[H_g\ \ 0_{wf}\ \ 0_f]x| = |Hx| \leq 1$. where H_g is the matrice that allows to calculate $\dot{z}_{def_{ij}}$ from x_g and $0_{wf}, 0_f$ are zero matrices.

The scenario is proposed:

- The vehicle runs at $90km/h$ in a straight line on a dry road ($\mu = 1$, where μ stands for the adherence to the road).
- A $5cm$ bump occurs on the left wheels (from $t = 0.5s$ to $t = 1s$). A lateral wind disturbance occurs also in this time to excite the roll motion.
- Moreover, a line change that causes also the roll motion is performed from $t = 4s$ to $t = 7s$.

The road profile and steering angle are shown in the Fig. 2 and Fig. 3.

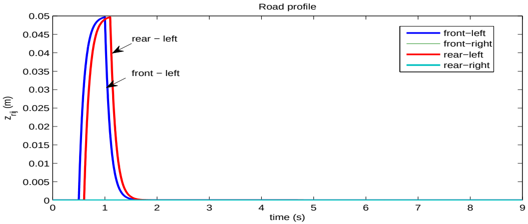


Figure: Road profile

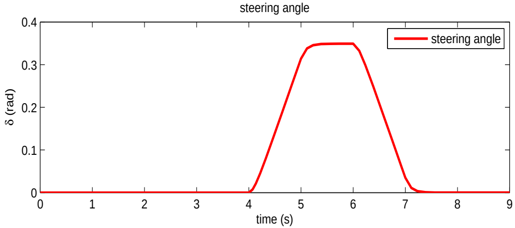


Figure: Steering angle

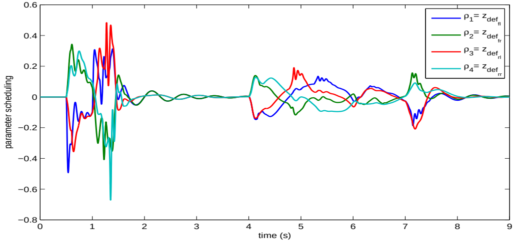


Figure: Scheduling parameters satisfy the condition $|\rho_{ij}| \leq 1$

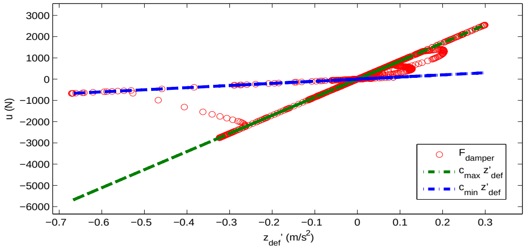


Figure: Force/deflection speed

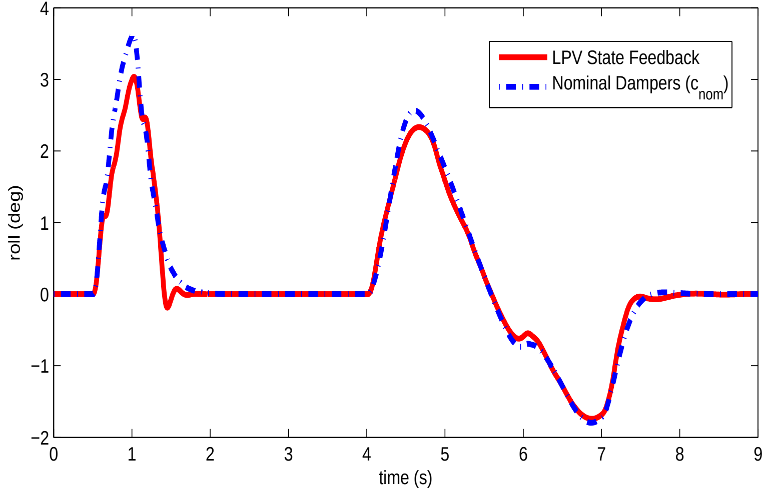


Figure: Comparasion of roll motion

Conclusion

Conclusions

- Application of an LPV/H_∞ State Feedback approach subject to input saturation to the problem of semi-active suspension control for a full vehicle equipped with 4 semi-active dampers.

Future works

- Consider different performance objectives: comfort, road holding or suspension stroke...
- Reduce the conservatism of the solution (for example, use two different Lyapunov functions for stability and performance)
- To implement this strategy on a test bed, available at Gipsa-lab Grenoble.

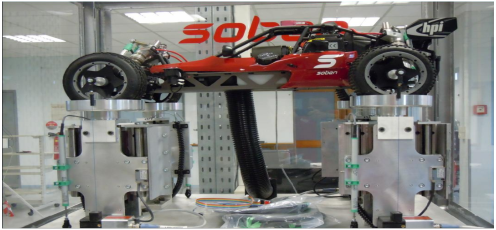


Figure: Car equipped by 4 semi-active suspension dampers.

THANKS FOR YOUR ATTENTION