Hierarchical Performance Analysis of Uncertain Large Scale Systems

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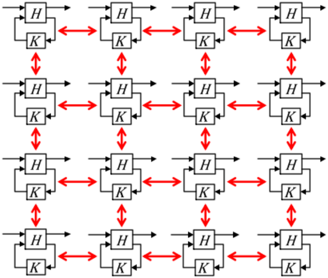
Context : PLL network

Large Scale Systems (LSS) : Phase Locked Loop (PLL) network

- PLL network to deliver clock signal to synchronous multi-core processors
- How to guarantee synchronization ?



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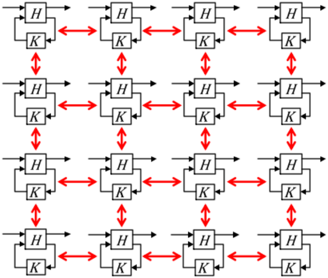

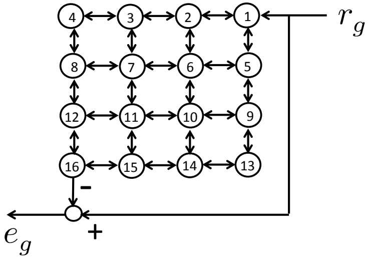

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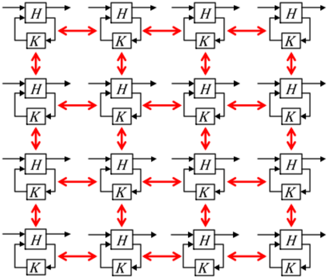
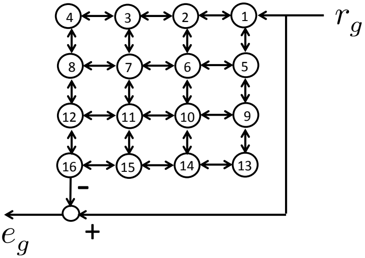
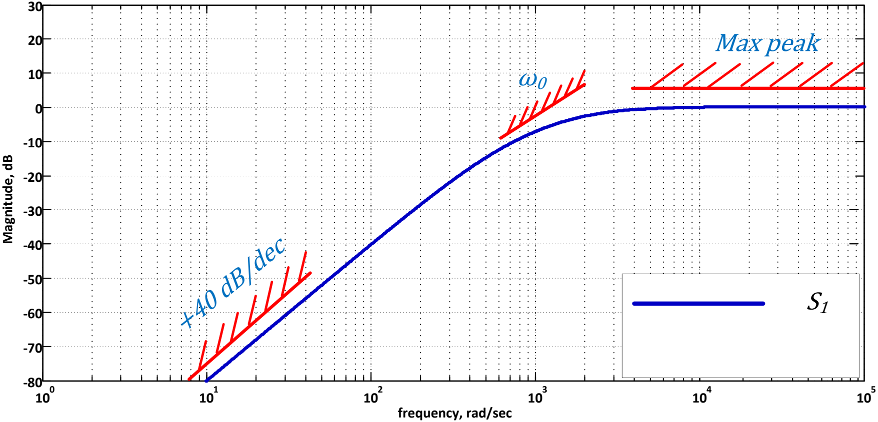




- Introduce global synchronization error
- Synchronization specifications (performance) are guaranteed if $T_{r_g \longrightarrow e_g}$ satisfies some frequency constraints

Context : Performance

Performance is expressed in frequency domain.



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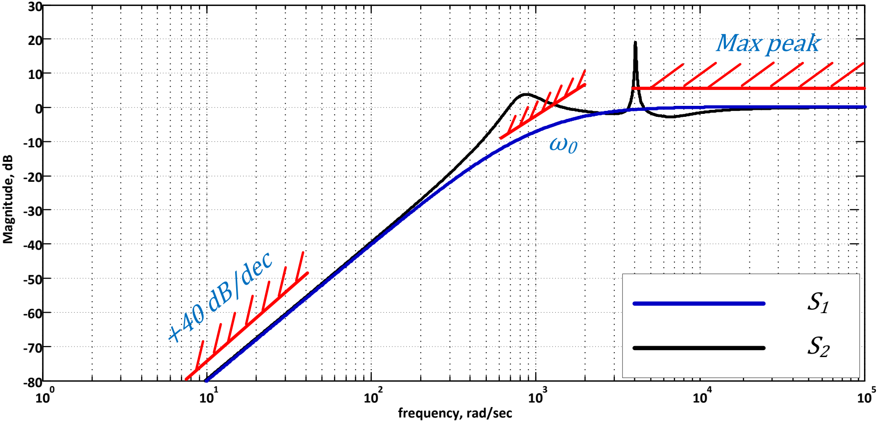

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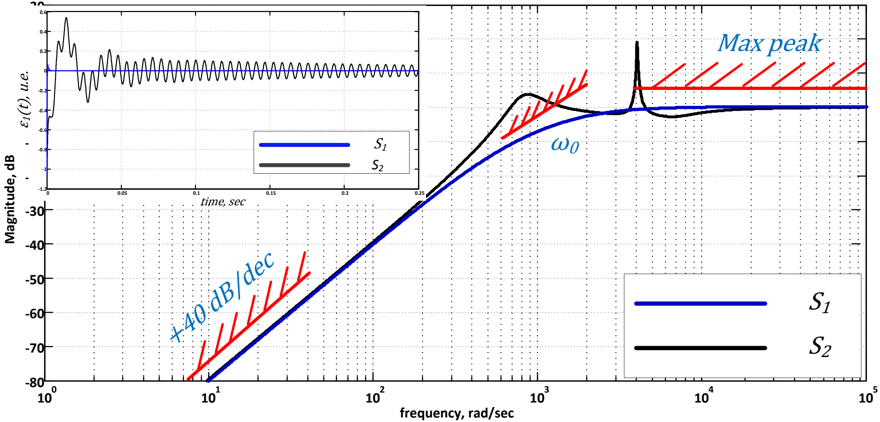

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Active clock distribution network

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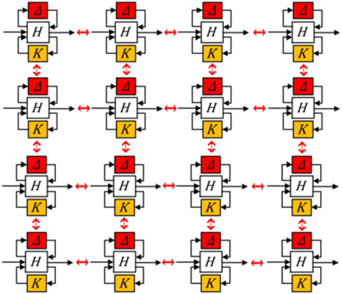

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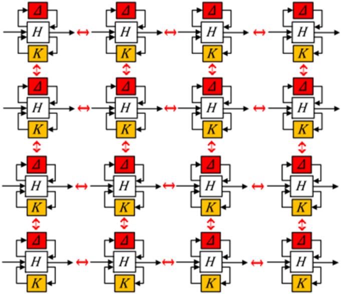

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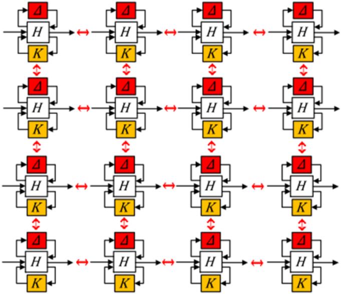

- Uncertain Network
- Robustness analysis :
  Perform the worst case robustness analysis for all the uncertainties Δ_i

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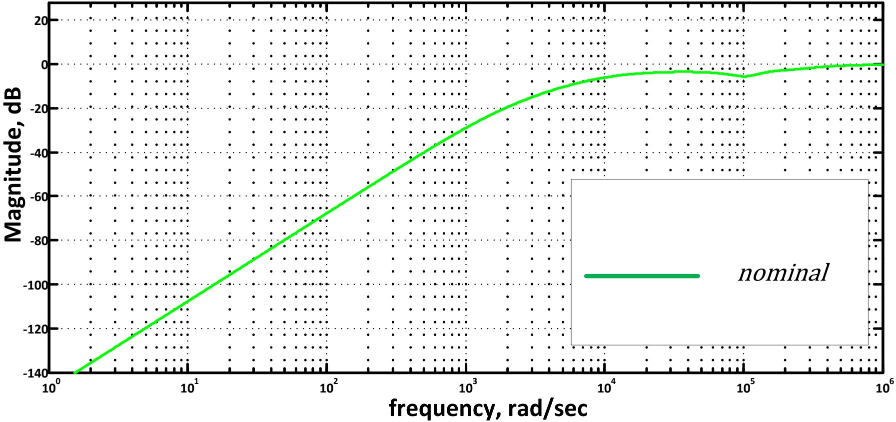

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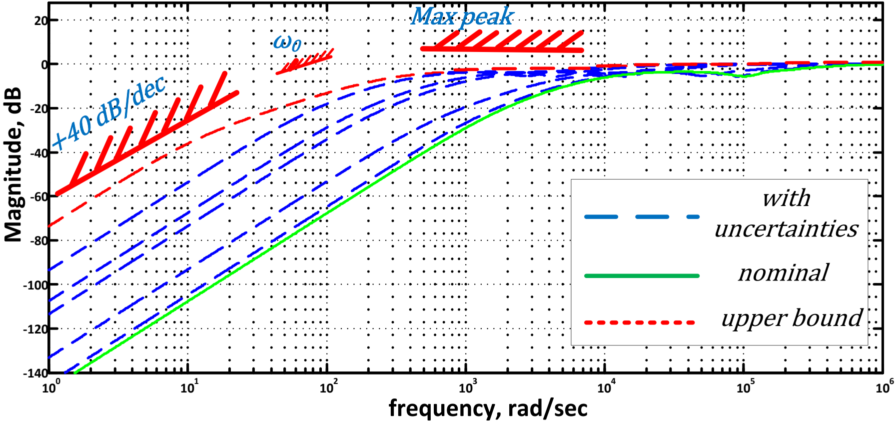
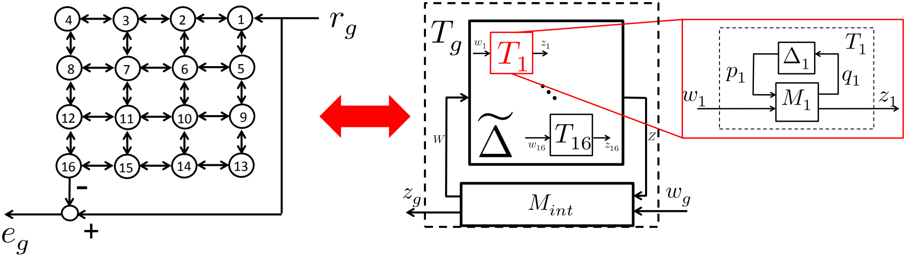


Synchronization specifications (performance) are guaranteed if the upper bound satisfies the frequency constraints

PLL network Performance

- 16 PLLs mutually synchronized



- Two uncertain parameters for every PLL $\Longrightarrow$ 32 uncertain parameters
- Nowadays networks : 100 PLLs $\Longrightarrow$ 200 uncertain parameters
  $\Longrightarrow$ classic method is not applicable
- 16 PLL network to show classic method results

Objective Compute an upper bound on $\|T_{r_g \to e_g}\|$ for all the uncertainties

Problem analysis

Large scale robustness analysis : two aspects problem

1. Robustness analysis : IQC based analysis (input-output description)
2. Large scale : decomposition techniques from graph theory

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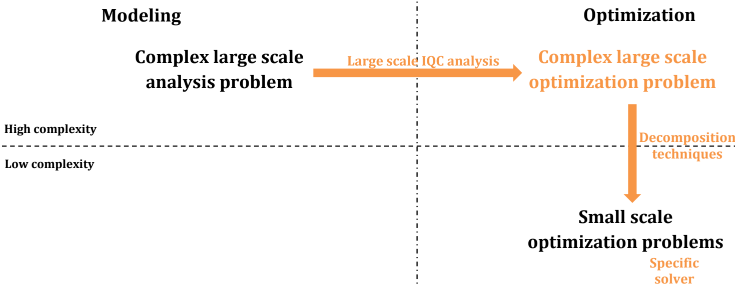
Few methods combining the two aspects : [Andersen et al., 2014]



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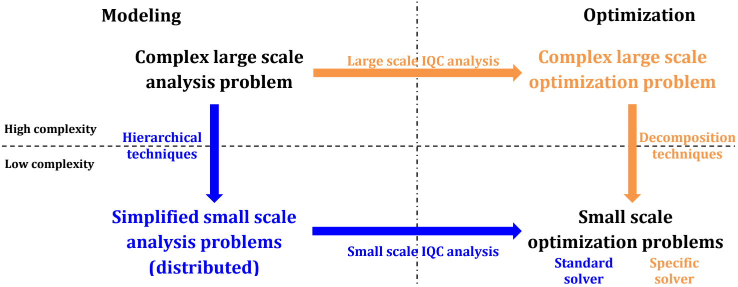

Integral Quadratic Constraints (IQC)

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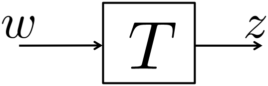
$$\int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \Phi_P(j\omega) \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$



- Possibility to cover classical characterizations of performance
  - $\mathcal{L}_2$ gain

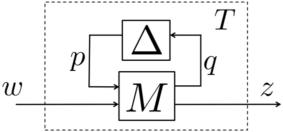
$$\int_{0}^{+\infty} \|z(t)\|_2 \; dt \leq \gamma^2 \int_{0}^{+\infty} \|w(t)\|_2 \; dt \Longleftrightarrow \int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \begin{pmatrix} -I & 0 \\ 0 & \gamma^2 \end{pmatrix} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$

  - Passivity

$$\int_{0}^{+\infty} z(t)^T w(t) dt \geq 0 \Longleftrightarrow \int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$

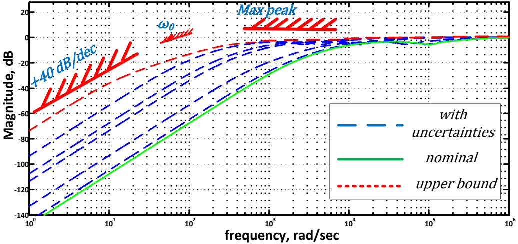
Proposed approach

Linear Time Invariant Systems



- $z(j\omega) = T(j\omega)w(j\omega)$ and QC based analysis
- Frequency domain : frequency response at ω_0
- Performance : compute an upper bound on the frequency response $(\bar{\sigma}(T) \leq \gamma)$

$$\min_{\gamma} \quad \gamma \quad \text{s.t.} \quad \begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} -I & 0 \\ 0 & \gamma^2 I \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0$$



- General performance $\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0$

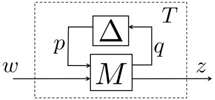
Proposed approach : Robust Performance Theorem (LTI systems)

QC for performance and uncertainty : Classical interpretation

Theorem (Robust Performance Theorem)

T is $\{X, Y, Z\}$ dissipative $i.e.$

$$\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0 \quad \forall\, \Delta \in \underline{\Delta} \quad \Longrightarrow \text{QC of } T$$



if and only if

1) $\begin{pmatrix} \Delta \\ I \end{pmatrix}^* \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^* & \Phi_{22} \end{pmatrix} \begin{pmatrix} \Delta \\ I \end{pmatrix} \geq 0 \quad \forall \Delta \in \underline{\Delta} \quad \Longrightarrow$ *QC of* Δ

2) $\begin{pmatrix} M \\ I \end{pmatrix}^* \begin{pmatrix} -\Phi_{22} & 0 & -\Phi_{12}^* & 0 \\ 0 & X & 0 & Y \\ -\Phi_{12} & 0 & -\Phi_{11} & 0 \\ 0 & Y^* & 0 & Z \end{pmatrix} \begin{pmatrix} M \\ I \end{pmatrix} > 0$

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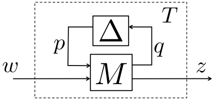

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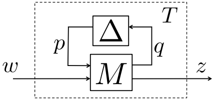

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- Condition 1) : infinite dimensional
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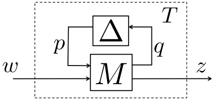

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QC for performance and uncertainty : Classical interpretation

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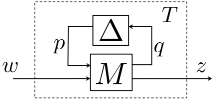

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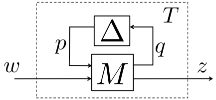

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- Conservatism depends on Φ_{Δ}

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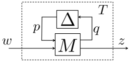
QC for performance and uncertainty : New interpretation



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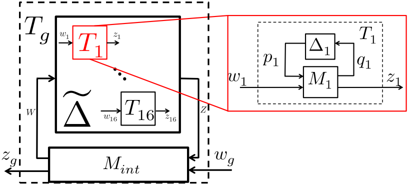

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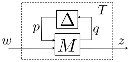

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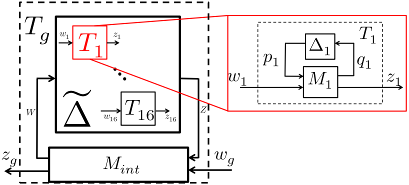

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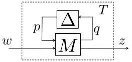

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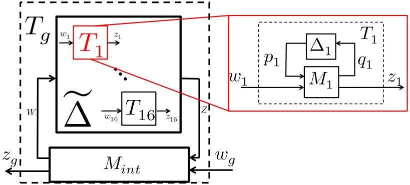

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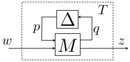

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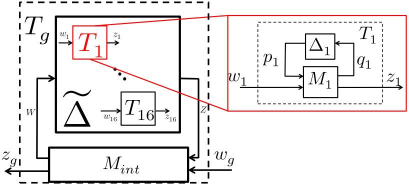

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- Local step : find simple QC for every $T_i \Longrightarrow$ reduce the complexity
- T_i are seen as uncertainty Δ_i

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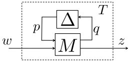

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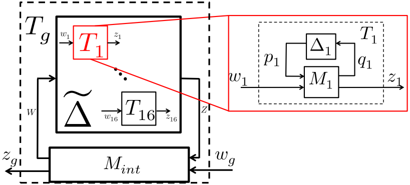

- Local step : find simple QC for every $T_i \Longrightarrow$ reduce the complexity
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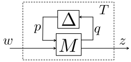
Proposed approach : Robust Performance Theorem (LTI systems)

QC for performance and uncertainty : New interpretation



Theorem (Robust Performance Theorem)

T is $\{X, Y, Z\}$ dissipative $i.e.$

$$\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0 \quad \forall\, \Delta \in \underline{\Delta}$$

if (and only if)

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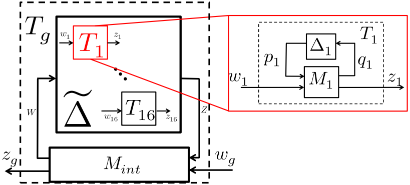

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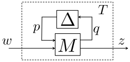

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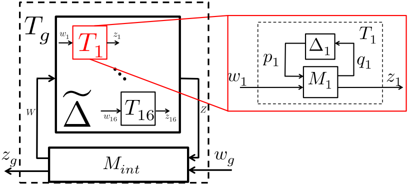

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- Global step : use local QC to find global QC $\Longrightarrow$ conservative results

$\Longrightarrow$ create a basis for QC of T_i (to use as Φ_Δ in global step)

Proposed approach : Robust Performance Theorem (LTI systems)

Classical interpretation :

For given X, Y and Z find Φ from basis Φ_Δ

New interpretation :

- Find basis for X, Y and Z from given $\Phi \in \Phi_\Delta$
- Propagate the old basis into the new basis

$\Longrightarrow$ QC propagation

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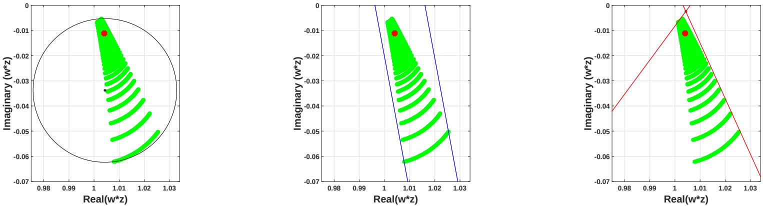
$\Longrightarrow$ QC propagation

Difficulties

- Size : not too big/small
- Quality : describes the best the uncertain system
- Efficient computation : convex

Robustness Analysis : QC classes

- Some classes of QC with geometric interpretations
  - disc [Dinh et al., 2013]
  - band [Dinh et al., 2014]
  - cone [Laib et al., 2015]



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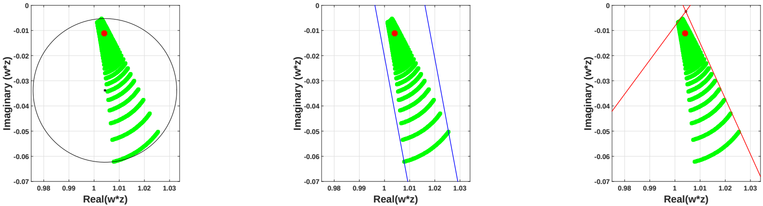

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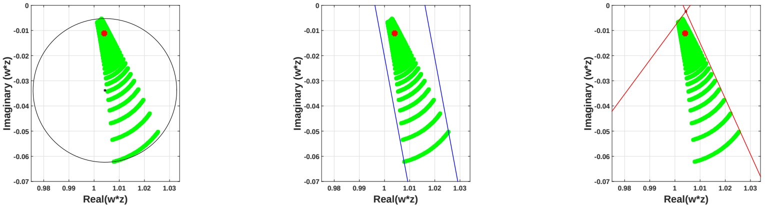

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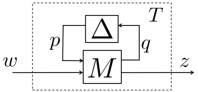
Robustness Analysis : Numerical Range

- For a given a frequency response Γ of a system T



- The numerical range $\mathcal{N}(\Gamma)$

$$\mathcal{N}(\Gamma) = \{w^* z \mid z = \Gamma w, w \in \mathbb{C}^{n_w} \text{ and } \|w\| = 1\}$$

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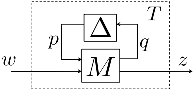

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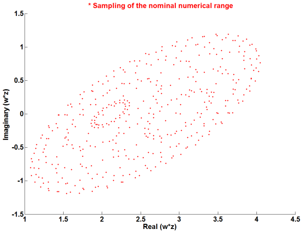
Certain numerical range



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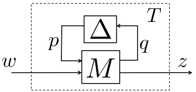

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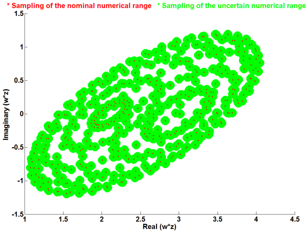
Certain numerical range   Uncertain numerical range

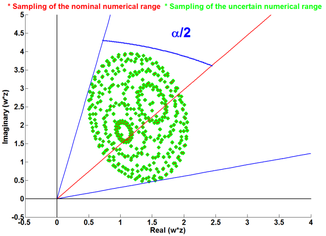


Robustness Analysis : Cone QC [Laib et al., 2015]

Theorem

Given the frequency response of an uncertain system T

Finding the smallest α :



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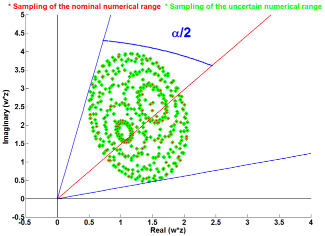

Finding the smallest α :

- *Quasiconvex optimisation problem*
- *LMI constraints*

Robustness Analysis : Cone QC [Laib et al., 2015]

Theorem

- *Given* $T = \Delta \star M$, *let* $\lambda = \cot \frac{\alpha}{2}$

$$\min_{\substack{\lambda,\,\Omega \\ \widehat{D}_1,\,\widehat{G}_1,\,\widetilde{D}_1,\,\widetilde{G}_1 \\ \widehat{D}_2,\,\widehat{G}_2,\,\widetilde{D}_2,\,\widetilde{G}_2}} \quad \lambda$$

s.t :

$$\lambda \begin{pmatrix} \widehat{D}_1 & 0 \\ 0 & \widehat{D}_2 \end{pmatrix} + \begin{pmatrix} \widetilde{D}_1 & 0 \\ 0 & -\widetilde{D}_2 \end{pmatrix} > 0$$

$$\lambda \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix}^* \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix} + \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix}^* \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix} > 0$$

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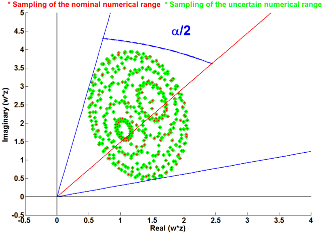
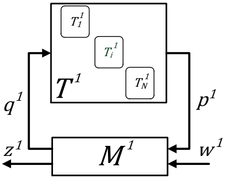


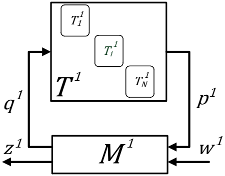
$\Longrightarrow$ Efficient tools to solve the problem

Resume



Level 1

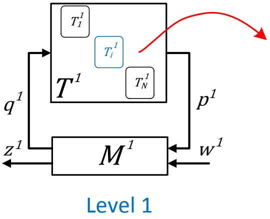
Resume



Level 1

1. Consider hierarchical structure of the system

Resume



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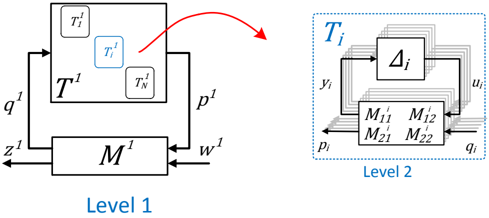

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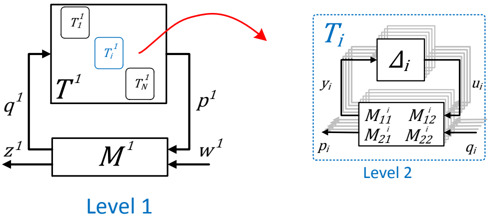
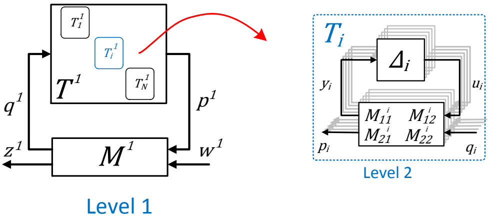


1. Consider hierarchical structure of the system
   - Find basis (QC description) for T_i with Robust Performance Theorem

Resume



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   - Propagate this basis to the global level

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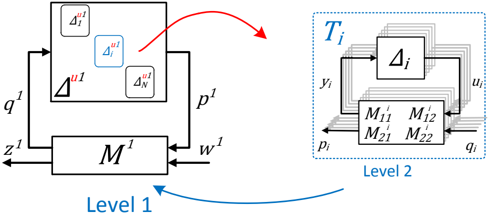
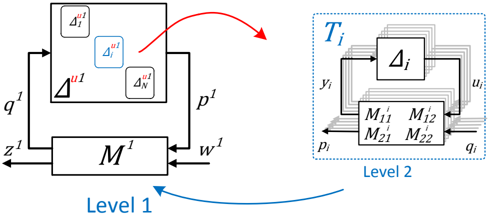


1. Consider hierarchical structure of the system
   - Find basis (QC description) for T_i with Robust Performance Theorem
   - Propagate this basis to the global level
2. For global hierarchical level, investigate the performance with Robust Performance Theorem

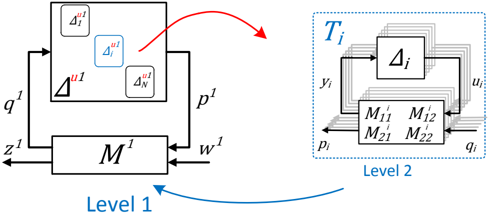
Resume



Computation time is reduced however conservatism may appear

- robustness of feedbacks loops $\Longrightarrow$ simple set may be sufficient
- combination of several simple sets $\Longrightarrow$ increase of the computation time

Resume



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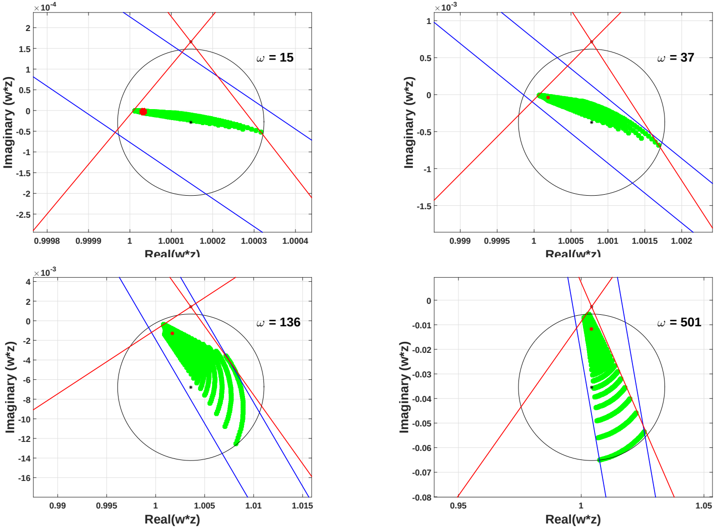
$\Longrightarrow$ trade-off conservatism/computation time

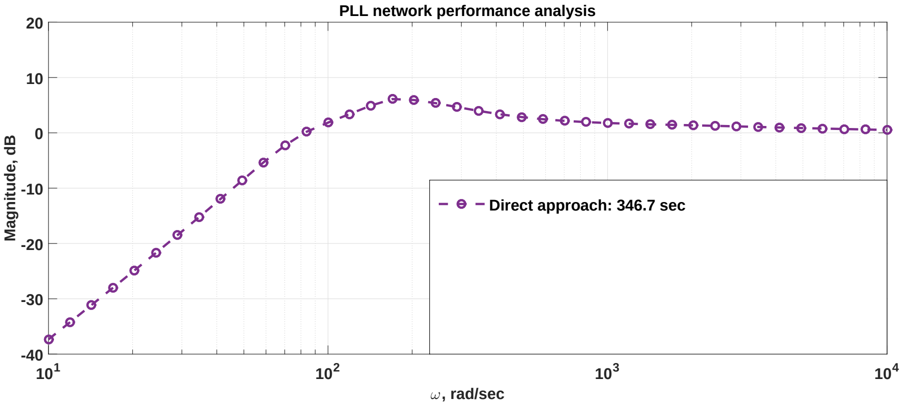
PLL network : Local Step

Characterize each PLL with QC with : disc, band and cone



PLL network : Global Step

Compute an upper bound on $T_{r_g \to e_g}$ for all the uncertainties



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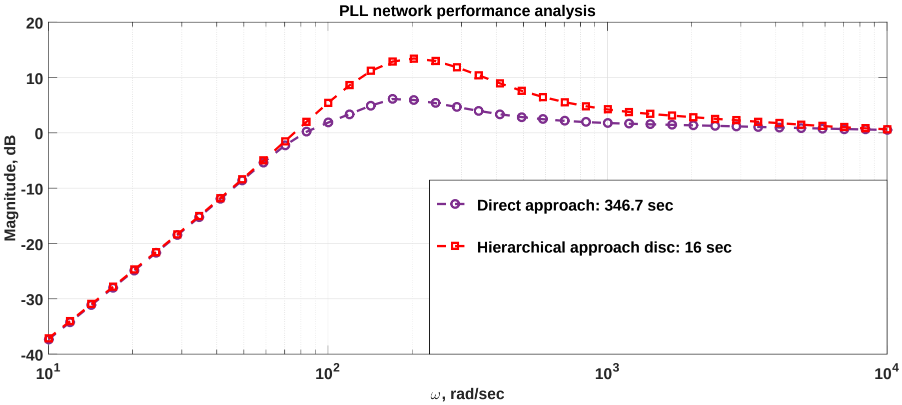

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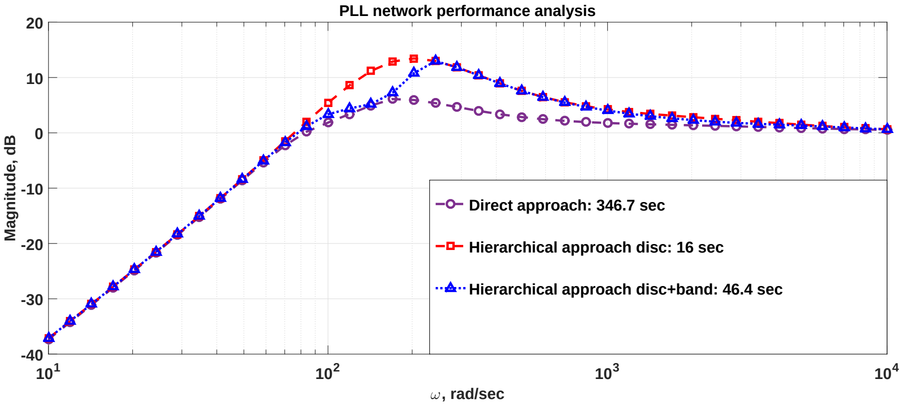

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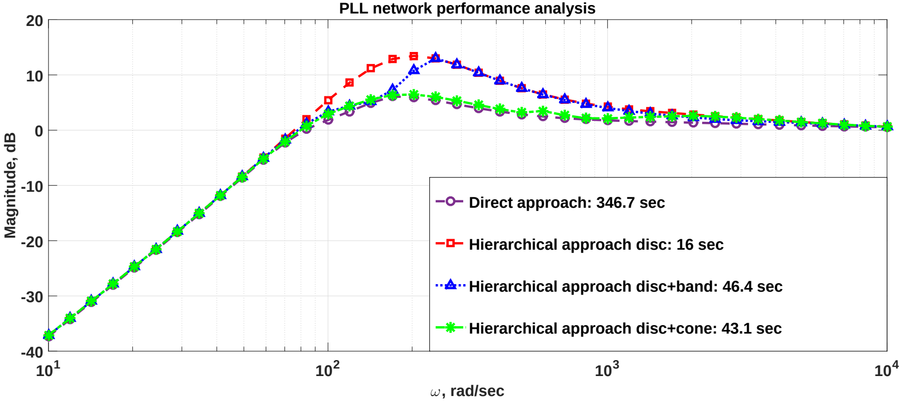
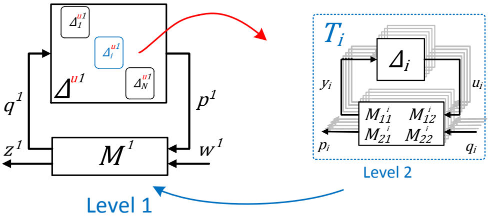


General Hierarchical Approach

Hierarchical approach



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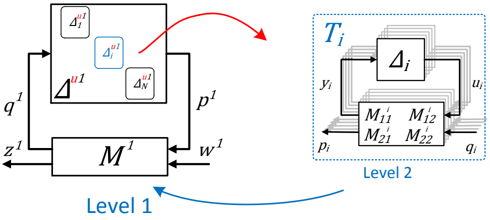
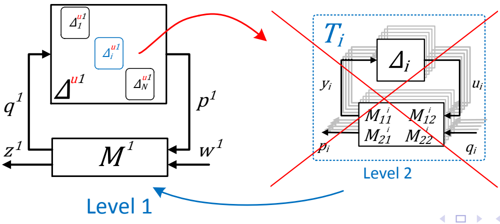


Special case : Direct approach



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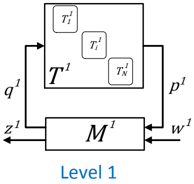

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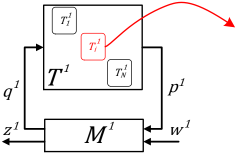

Level 1

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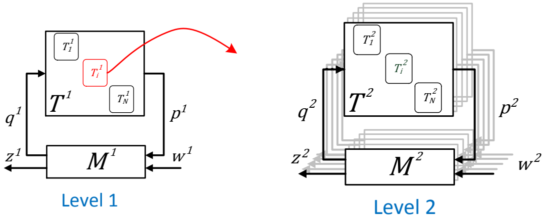

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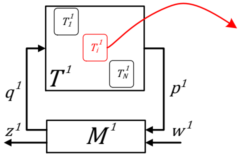

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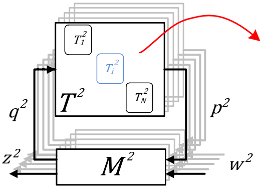
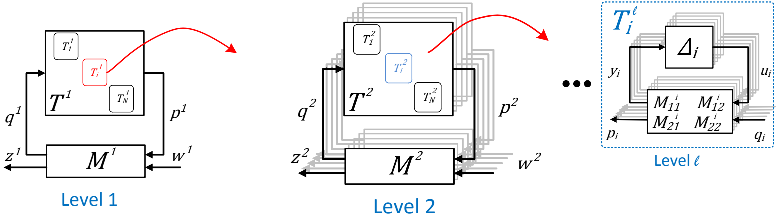


Level 2

General Hierarchical Approach



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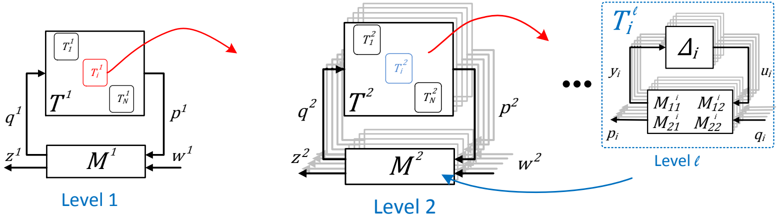

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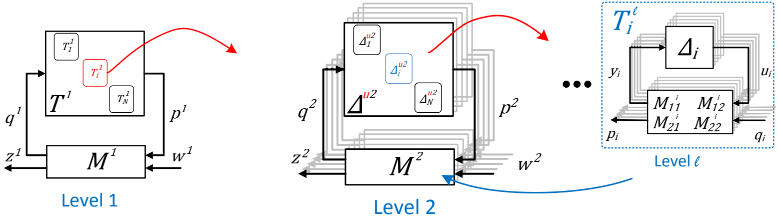

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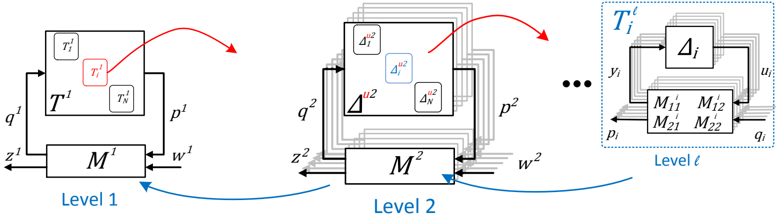

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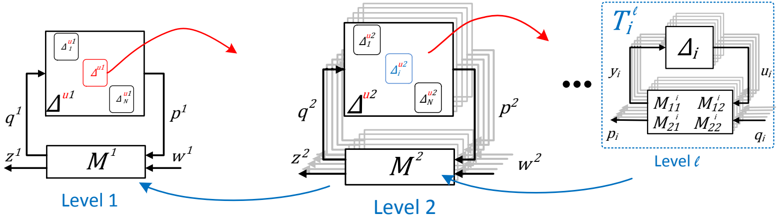

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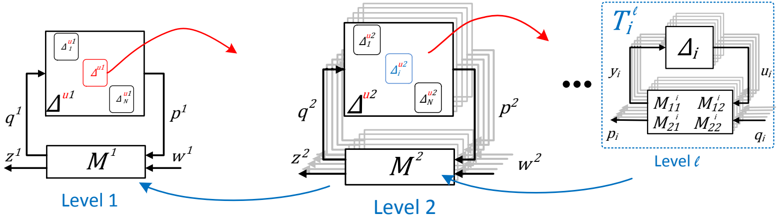
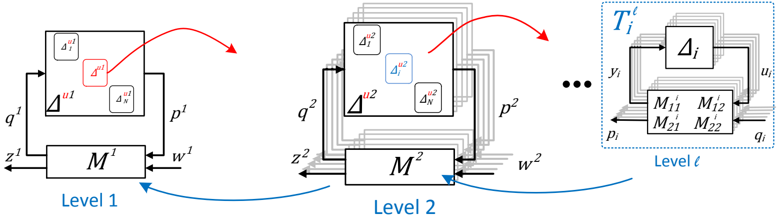


Many degrees of freedom to handle the trade-off conservatism/computation time

General Hierarchical Approach

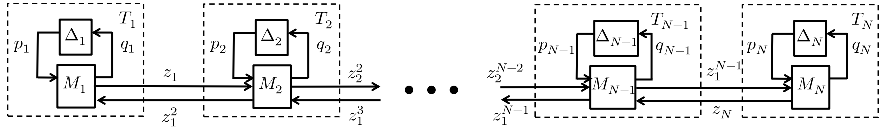


Many degrees of freedom to handle the trade-off conservatism/computation time

- Number of levels
- Number of T_i in each level
- Basis for Δ_i
- Basis for T_i in each level
- Parallel computing

Robust stability

Network with N systems randomly generated [Andersen et al., 2014].



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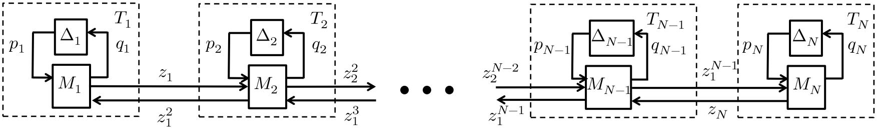
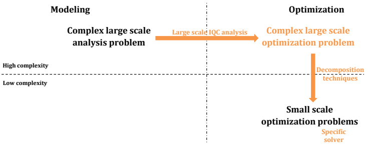
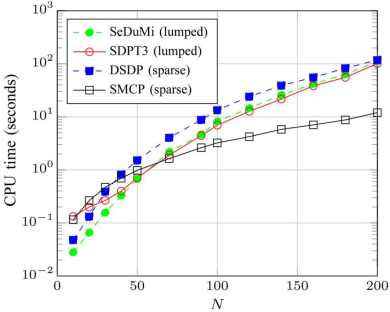


$$\frac{\text{Direct method computation time}}{\text{Proposed method computation time}} = 10 \text{ for } N = 200$$





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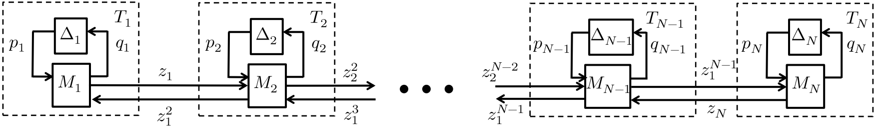
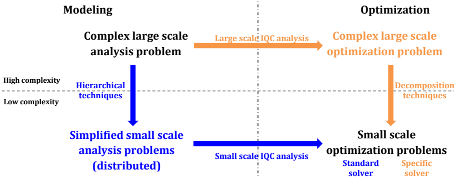
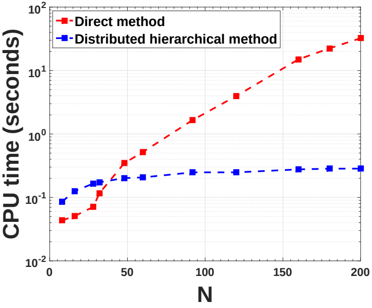


$$\frac{\text{Direct method computation time}}{\text{Proposed method computation time}} = 113 \text{ for } N = 200$$





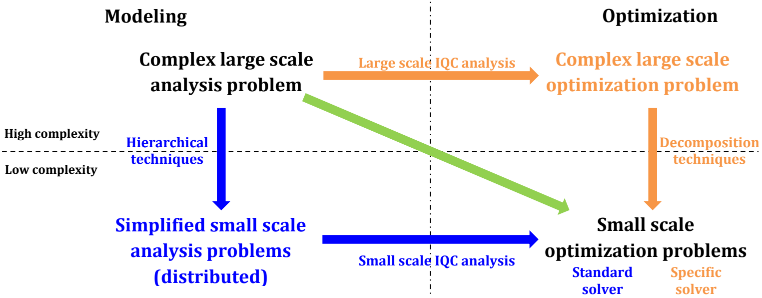
Conclusion

- Performance analysis of uncertain large scale systems
- Important computation time with direct method
- Exploit hierarchical structure using basis (QC) propagation
- General approach with degrees of freedom
- Reduce computation time with possible conservatism
- Trade-off conservatism/computation time

Perspectives

Perspectives

- Systematic decomposition technique using Graph Theory
- Combine hierarchical method with specific solvers



Thank you for your attention

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