Toward nonlinear tracking and rejection using LPV control

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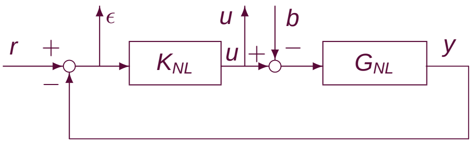
Plan

1. A typical control problem
2. Extension of H_∞ control approach to nonlinear systems
3. Nonlinear $\mathcal{L}_2$ gain control using LPV
4. Limitations of the $\mathcal{L}_2$ gain control
5. Typical specs are ensured using $\mathcal{L}_2$ incremental gain
6. LPV control for ensuring typical nonlinear specs

A typical control problem

Tracking and rejection

- Usual control problem involves both tracking and rejection specifications
- Let us focus on a simple problem: Given a nonlinear plant G_{NL}, find K_{NL} such that



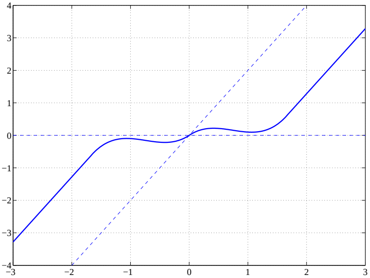
- Typical control specs
  - tracking of step reference with a null static error and a response time ≤ 0.1 s
  - rejection of step disturbance at the plant input
  - limited control energy

Nonlinear plant under consideration

$y = G_{NL}(u)$ with

$$\begin{cases} \dot{x}_1(t) &= -100\varphi(x_1(t)) - 70x_2(t) + 300u(t) \\ \dot{x}_2(t) &= 70x_1(t) - 14x_2(t) \\ y(t) &= x_1(t) \end{cases}$$

with φ defined by



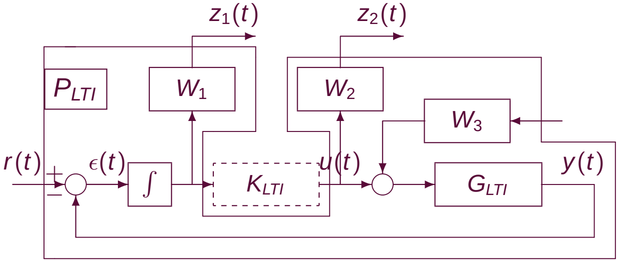
that is

$$0 \leq \varphi(x_1) \leq 2x_1$$

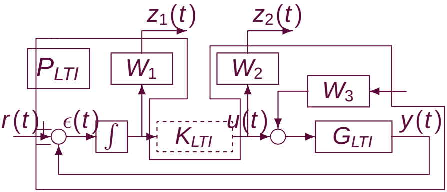
Extension of H_∞ control approach to nonlinear systems

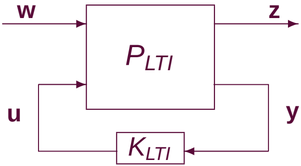
LTI case: H_∞ control approach

- Integral control
- With weighting functions W_1, W_2, W_3 suitable for the specs
- Compute K_{LTI} such that H_∞ norm of the closed loop system less than 1



H_∞ control approach (recall)





Given an (augmented) LTI plant P_{LTI}

$$
\begin{array}{rcccccc}
\dot{x}(t) & = & Ax(t) & + & B_w w(t) & + & B_u u(t) \\
z(t) & = & C_z x(t) & + & D_{zw} w(t) & + & D_{zu} u(t) \\
y(t) & = & C_y x(t) & + & D_{yw} w(t) & + & D_{yu} u(t)
\end{array}
$$

Compute an LTI controller K_{LTI}

$$
\begin{array}{rcccc}
\dot{\overline{x}}(t) & = & A_K \overline{x}(t) & + & B_K y(t) \\
u(t) & = & C_K \overline{x}(t) & &
\end{array}
$$

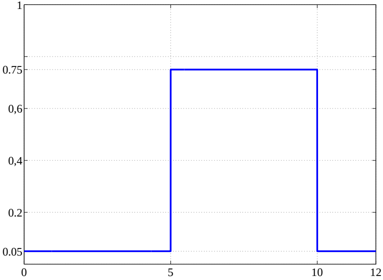
Such that

$$\|T_{w\to z}\|_\infty \leq 1$$

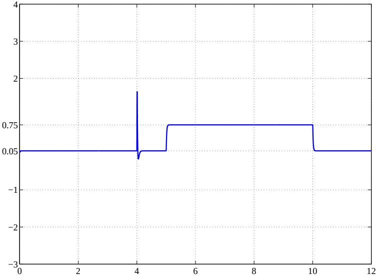
- Efficient solution (Riccati or LMI)

Typical approach for LTI plant and controller

Reference signal

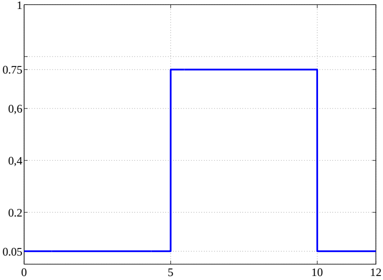


Output signal

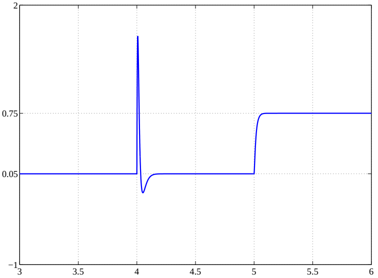


Typical approach for LTI plant and controller

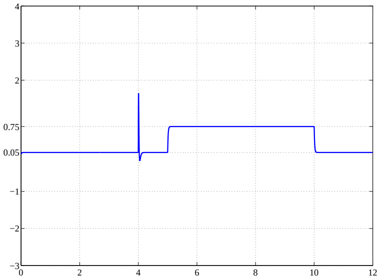
Reference signal



Output signal

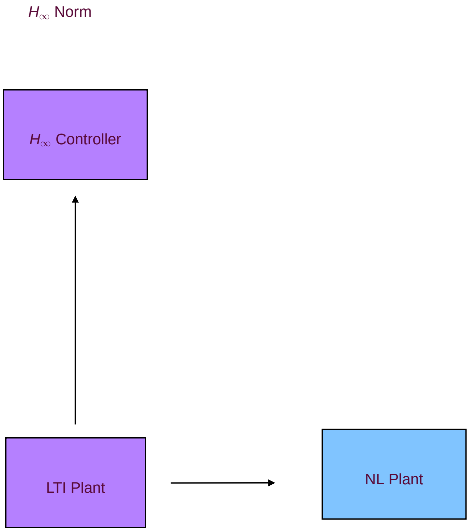


Comments on the result

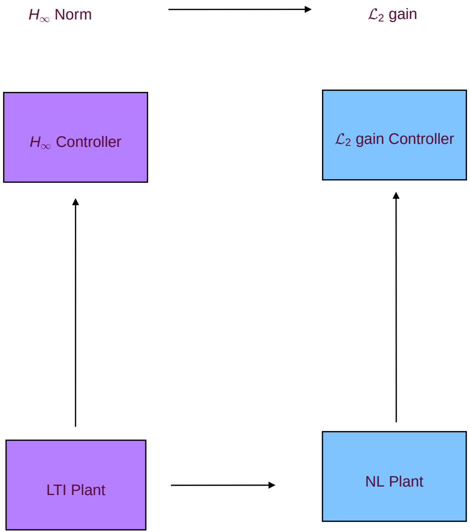


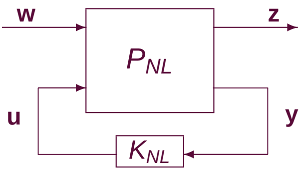
- Nice steady state behaviour
  - Stability of LTI system $\Rightarrow$ for constant input, output $\longrightarrow$ constant
  - Stability + integral control $\Rightarrow$ null static error
- Nice transient behavior
  - Inequality on the weighted H_∞ norm of the closed loop system

First possible extension using the $\mathcal{L}_2$ gain



First possible extension using the $\mathcal{L}_2$ gain





Given an (augmented) nonlinear plant P_{NL}

$$\begin{cases} \dot{x}(t) & = & f(x(t), w(t), u(t)) \\ z(t) & = & g(x(t), w(t), u(t)) \\ y(t) & = & h(x(t), w(t)) \end{cases}$$

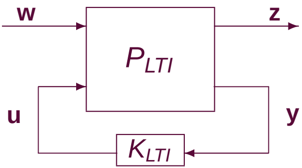
Compute a nonlinear controller K_{NL}

$$\begin{aligned} \dot{\overline{x}}(t) & = f_K(\overline{x}(t), y(t)) \\ u(t) & = g_K(\overline{x}(t), y(t)) \end{aligned}$$

Such that the $\mathcal{L}_2$ gain of the closed loop system is less than 1: for all w

$$\forall\, T > 0, \quad \int_0^T z(t)^T z(t)\, dt \leq \int_0^T w(t)^T w(t)\, dt$$

$\mathcal{L}_2$ gain & H_∞ norm



- For LTI system, $\|T_{w\to z}\|_\infty \leq 1$ is equivalent to the $\mathcal{L}_2$ gain is less than 1: for all w

$$\forall\, T > 0,\ \int_0^T z(t)^T z(t)\, dt \leq \int_0^T w(t)^T w(t)\, dt$$

- A natural idea is then to extend the H_∞ control to nonlinear systems by the $\mathcal{L}_2$ gain control: usually refered to as “nonlinear H_∞ control”

$\mathcal{L}_2$ gain control problem

Two questions

1. How to compute a solution (nonlinear controller) to the $\mathcal{L}_2$ gain control problem?

   No efficient direct approach $\Rightarrow$ indirect approach: Quasi LPV control

2. Does the $\mathcal{L}_2$ gain controller ensures nice tracking and rejection properties?

   See application on the illustrative example

$\mathcal{L}_2$ gain control problem

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   See application on the illustrative example

Nonlinear $\mathcal{L}_2$ gain control using LPV

LPV control problem

Given a Linear Parameter Varying (LPV) plant G_{LPV}

$$\begin{array}{rcl} \dot{x}(t) & = & \mathbf{A}(\theta(t))x(t) + \mathbf{B}_1(\theta(t))w(t) + \mathbf{B}_2(\theta(t))u(t) \\ z(t) & = & \mathbf{C}_1(\theta(t))x(t) + \mathbf{D}_{11}(\theta(t))w(t) + \mathbf{D}_{12}(\theta(t))u(t) \\ y(t) & = & \mathbf{C}_2(\theta(t))x(t) + \mathbf{D}_{21}(\theta(t))w(t) + \mathbf{D}_{22}(\theta(t))u(t) \end{array}$$

- $\theta(t)$ = vector of time varying parameters, measured in real-time, which belong to a given interval
- $\mathbf{A}(.)$, $\mathbf{B}_1(.)$,... rational function of $\theta_i(t)$

Compute an LPV controller K_{LPV}

$$\begin{array}{rcl} \dot{\overline{x}}(t) & = & \mathbf{A}_K(\theta(t))\overline{x}(t) + \mathbf{B}_K(\theta(t))y(t) \\ u(t) & = & \mathbf{C}_K(\theta(t))\overline{x}(t) \end{array}$$

Such that the $\mathcal{L}_2$ gain of the closed loop system is less than 1: for all w

$$\forall\, T > 0, \quad \int_0^T z(t)^T z(t)\, dt \leq \int_0^T w(t)^T w(t)\, dt$$

Interest of the LPV control problem

- Solutions of the LPV control problem can be computed using LMI optimization

- A strong motivation of the LPV control problem is to propose, in contrast with the gain scheduling control, a rigorous solution to the nonlinear $\mathcal{L}_2$ gain control problem[1]

[1] W. J. Rugh and J. S. Shamma, “Research on gain scheduling,” *Automatica*, vol. 36, pp. 1401–1425, 2000.

To the (augmented) nonlinear plant P_{NL}

$$\begin{cases} \dot{x}(t) = f(x(t), w(t), u(t)) \\ z(t) = g(x(t), w(t), u(t)) \\ y(t) = h(x(t), w(t)) \end{cases} \quad (1)$$

is associated an LPV plant P_{LPV}

$$\begin{array}{rclclcl} \dot{x}(t) & = & \mathbf{A}(\theta(t))x(t) & + & \mathbf{B}_1(\theta(t))w(t) & + & \mathbf{B}_2(\theta(t))u(t) \\ z(t) & = & \mathbf{C}_1(\theta(t))x(t) & + & \mathbf{D}_{11}(\theta(t))w(t) & + & \mathbf{D}_{12}(\theta(t))u(t) \\ y(t) & = & \mathbf{C}_2(\theta(t))x(t) & + & \mathbf{D}_{21}(\theta(t))w(t) & + & \mathbf{D}_{22}(\theta(t))u(t) \end{array} \quad (2)$$

such that with

$$\Omega_{NL} = \left\{ \begin{pmatrix} x & z & y & w & u \end{pmatrix} \mid (1) \text{ is satisfied} \right\}$$

and

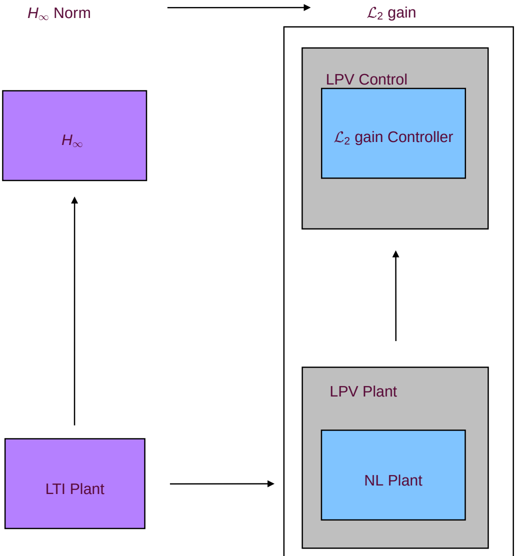
$$\Omega_{LPV} = \left\{ \begin{pmatrix} x & z & y & w & u \end{pmatrix} \mid (2) \text{ is satisfied} \right\}$$

we have

$$\Omega_{NL} \subset \Omega_{LPV}$$

Extension ot H_∞ to nonlinear systems: $\mathcal{L}_2$ gain?

An LPV model is a differential inclusion



Limitations of the $\mathcal{L}_2$ gain control

To the nonlinear plant G_{NL}:

$$\begin{cases} \dot{x}_1(t) &= -100\varphi(x_1(t)) - 70x_2(t) + 300u(t) \\ \dot{x}_2(t) &= 70x_1(t) - 14x_2(t) \\ y(t) &= x_1(t) \end{cases}$$

we associate the LPV plant G_{LPV}:

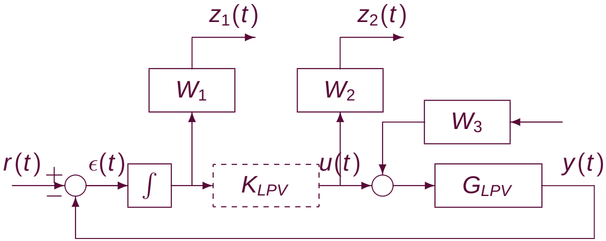
$$\begin{cases} \dot{x}(t) &= A_G(\theta(t))x(t) + \begin{bmatrix} 300 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}, \quad \theta(t) \in [0,\ 2]$$

with[2]

$$A_G(\theta(t)) = \begin{bmatrix} 0 & -70 \\ 70 & -14 \end{bmatrix} + \theta(t) \begin{bmatrix} -100 & 0 \\ 0 & 0 \end{bmatrix}$$

[2] $\theta(t) = \frac{\varphi(y(t))}{y(t)}$ with $0 \leq \varphi(y) \leq 2y$

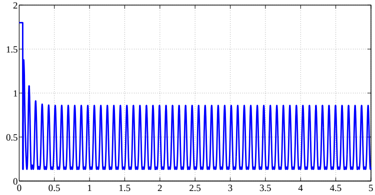
- For step tracking and step rejection, an LPV controller is computed using the augmented plant defined as follows.



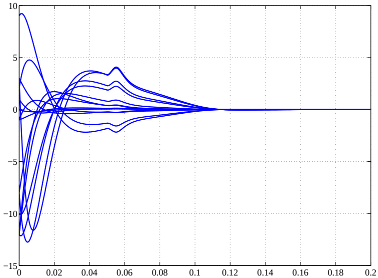
- Thanks to the embedding process, this controller is a solution to the nonlinear $\mathcal{L}_2$ gain control problem
- Does the controller ensure satisfying tracking and rejection?

Behaviour of the LPV closed loop system with respect to initial conditions & zero inputs

For a given function $\theta(t) \in [0,\ 2]$



Output $y(t)$ for different initial conditions x_0, $r = 0$, $b = 0$



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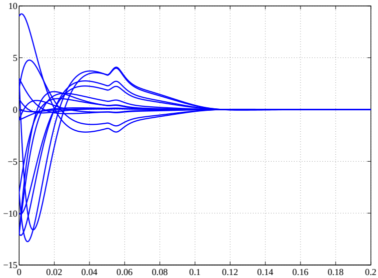
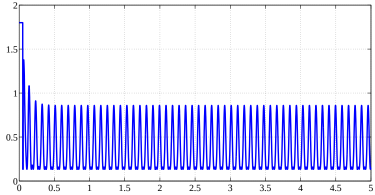


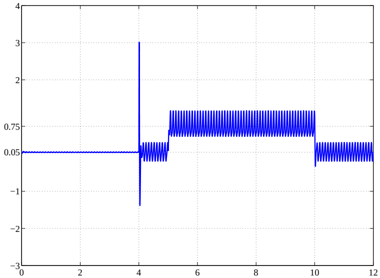
($\mathcal{L}_2$ gain) stability ensures convergence to 0 for different initial conditions

Behaviour of the LPV closed loop system with respect to step reference & disturbance

For a given function $\theta(t) \in [0,\ 2]$

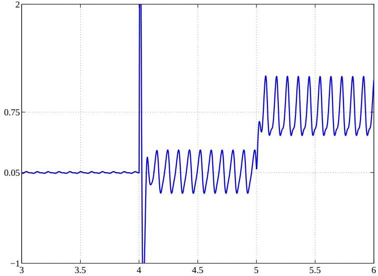


Output $y(t)$ for $x_0 = 0$, a step disturbance at 4s and a square reference signal



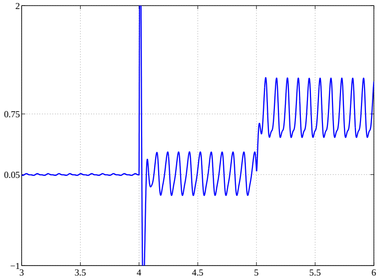
Behaviour of the LPV closed loop system with respect to step reference & disturbance

Output $y(t)$ for $x_0 = 0$, a step disturbance at 4s and a square reference signal



Behaviour of the LPV closed loop system with respect to step reference & disturbance

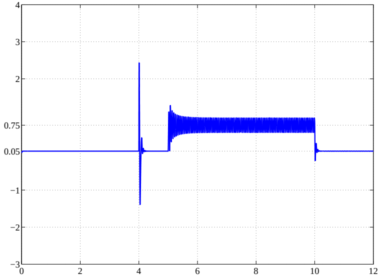
Output $y(t)$ for $x_0 = 0$, a step disturbance at 4s and a square reference signal



($\mathcal{L}_2$ gain) stability + integral control do not ensure step tracking/rejection for LTV system

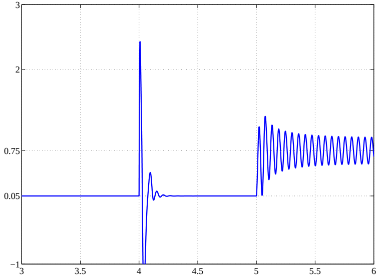
Behaviour of the nonlinear closed loop system with respect to step reference & disturbance

Output $y(t)$ for $x_0 = 0$, a step disturbance at 4s and a square reference signal



Behaviour of the nonlinear closed loop system with respect to step reference & disturbance

Output $y(t)$ for $x_0 = 0$, a step disturbance at 4s and a square reference signal



($\mathcal{L}_2$ gain) stability + integral control do not ensure step tracking/rejection for nonlinear system

Except perhaps for inputs close to 0

Discussion on the illustrative example

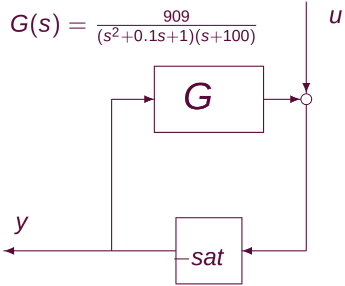
- For inputs close to 0, the $\mathcal{L}_2$ gain control solution reduces to the H_∞ one[3]
- Null static errors for step reference & disturbance by integral control depend on the property that for constant inputs, the system signals tend to a constant
- Unfortunately, this property is not ensured by ($\mathcal{L}_2$ gain) stability

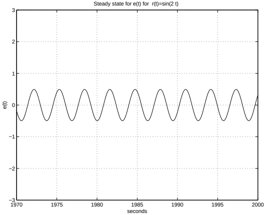
[3] A. J. van der Schaft, "$\mathcal{L}_2$-gain analysis of nonlinear systems and nonlinear state feedback H_∞ control," *IEEE Trans. Automatic Control*, vol. 37, no. 6, pp. 770–784, June 1992

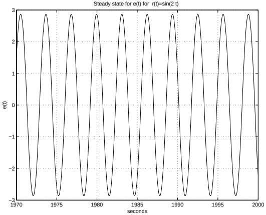
($\mathcal{L}_2$ gain) stability does not ensure a good behavior

- For periodic inputs: different steady states for input $u(t) = \sin(2t)$

$$G(s) = \frac{909}{(s^2+0.1s+1)(s+100)}$$







How to ensure a good behavior?

Typical specs are ensured using $\mathcal{L}_2$ incremental gain

How to ensure a good behaviour? Use the $\mathcal{L}_2$ incremental gain

Nonlinear plant G_{NL}

$$\begin{aligned} \dot{x}(t) &= f(x(t), w(t)) \\ z(t) &= g(x(t), w(t)) \end{aligned}$$

- ($\mathcal{L}_2$ gain) stability if $\exists \gamma \geq 0$, $\forall\, w$,

$$\forall\, T > 0, \;\; \int_0^T z(t)^T z(t)\, dt \leq \gamma^2 \int_0^T w(t)^T w(t)\, dt$$

  $\mathcal{L}_2$ gain of G_{NL} ($\|G_{NL}\|_{i-2}$) = the smallest value of such γ

- ($\mathcal{L}_2$) incremental (gain) stability if stability and $\exists \eta \geq 0$, $\forall\, T > 0$, $\forall\, w_1$, $\forall\, w_2$,

$$\int_0^T (z_1(t) - z_2(t))^T (z_1(t) - z_2(t)) dt \leq \eta^2 \int_0^T (w_1(t) - w_2(t))^T (w_1(t) - w_2(t)) dt$$

  Incremental $\mathcal{L}_2$ gain of G_{NL} ($\|G_{NL}\|_{\Delta}$) = the smallest value of such η

- For an LTI system, H_∞ norm = $\mathcal{L}_2$ gain = $\mathcal{L}_2$ incremental gain

Why incremental ($\mathcal{L}_2$) gain is nice for control performance?

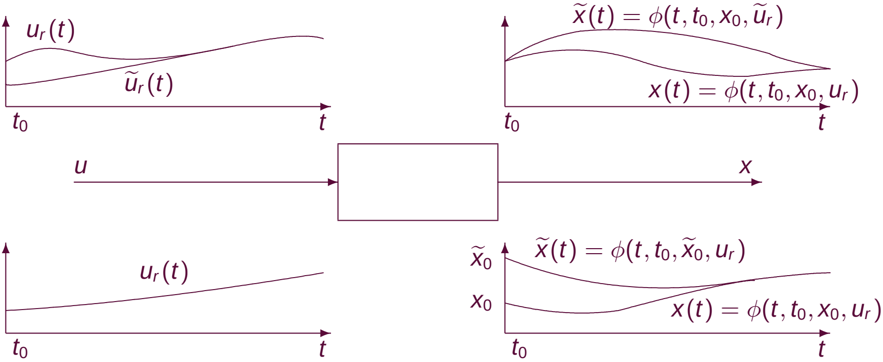
	LTI	NL	NL
$\downarrow$ Specs $\setminus$ Norm $\rightarrow$	H_∞	$\mathcal{L}_2$ gain	incremental gain
Unique steady state	YES	NO	YES
Convergence of the unperturbed motions	YES	NO	YES
Constant input $\longrightarrow$ constant output	YES	NO	YES
T periodic input $\longrightarrow$ T periodic output	YES	NO	YES
Quantitative perf.	YES	NO	YES
Robustness	YES	YES	YES

V. Fromion and S. Monaco and D. Normand-Cyrot, The weighted incremental norm approach: from linear to nonlinear H_∞ control, Automatica 2001

V. Fromion and G. Scorletti. The behavior of incrementally stable discrete time systems, System and Control Letters 2002

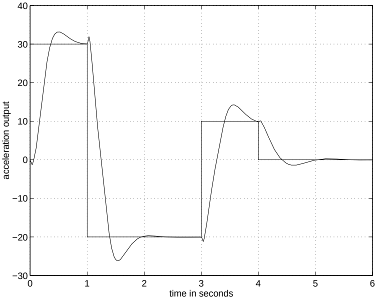
V. Fromion, Some results on the behavior of Lipschitz continuous systems, ECC 97

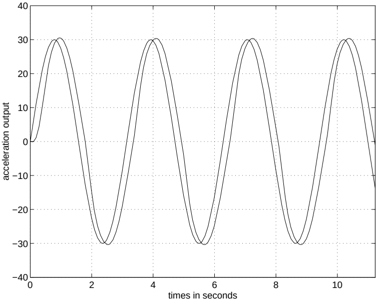
Unique steady state



Convergence of the unperturbed motions

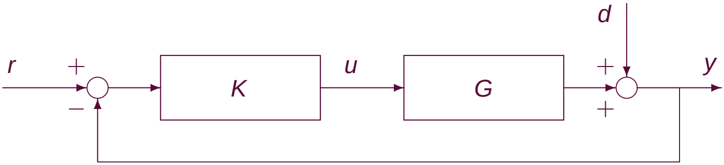
Constant (periodic) input $\longrightarrow$ Constant (periodic) output





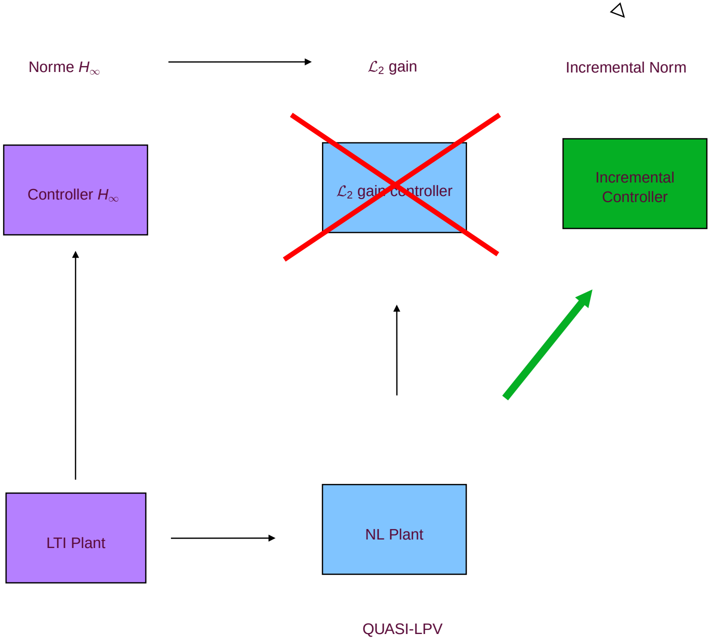
Quantitative specs: Performance

Disturbance attenuation of a set of perturbation d, for any initial condition



for d such that $\|W_p^{-1}(d)\|_{2,T} \leq \|d\|_{2,T} \Rightarrow \|y\|_{2,T} \leq \alpha$

Extension of H_∞ to nonlinear systems



The incremental norm as a rigorous extension of the H_∞ norm

Given an (augmented) nonlinear plant P_{NL}

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & f(x(t), w(t), u(t)) \\ z(t) & = & g(x(t), w(t), u(t)) \\ y(t) & = & h(x(t), w(t)) \end{array}\right.$$

Compute a nonlinear controller K_{NL}

$$\begin{array}{rcl} \dot{\overline{x}}(t) & = & f_K(\overline{x}(t), y(t)) \\ u(t) & = & g_K(\overline{x}(t), y(t)) \end{array}$$

Such that the $\mathcal{L}_2$ **incremental** gain of the closed loop system is less than 1:
for all w_1, w_2

$$\forall T > 0, \int_0^T (z_1(t) - z_2(t))^T (z_1(t) - z_2(t)) dt < \int_0^T (w_1(t) - w_2(t))^T (w_1(t) - w_2(t)) dt$$

- As for $\mathcal{L}_2$ gain control, no efficient direct method for solving this problem

LPV control for ensuring typical nonlinear specs

Equivalence between local properties and global properties

NL Plant

$$y = G_{NL}(u) : \begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases} \qquad (1)$$

$$\downarrow$$

(Gâteaux Derivative) TV Linearizations of G_{NL} at $u_r \in \mathcal{L}_2$

$$\bar{y} = DG_{NL}[u_r](\bar{u}) : \begin{cases} \dot{\bar{x}}(t) & = & \bar{A}(t)\bar{x}(t) & + & \bar{B}(t)\bar{u}(t) \\ \bar{y}(t) & = & \bar{C}(t)\bar{x}(t) & + & \bar{D}(t)\bar{u}(t) \end{cases}$$

with

$$\begin{bmatrix} \bar{A}(t) & \bar{B}(t) \\ \bar{C}(t) & \bar{D}(t) \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x}(x_r(t), u_r(t)) & \frac{\partial f}{\partial u}(x_r(t), u_r(t)) \\ \frac{\partial g}{\partial x}(x_r(t), u_r(t)) & \frac{\partial g}{\partial u}(x_r(t), u_r(t)) \end{bmatrix}$$

where $x_r(t)$ is the solution of (1) for the input $u(t) \equiv u_r(t)$

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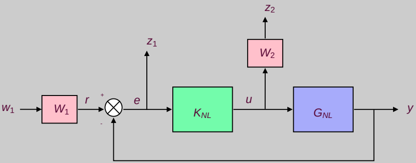
Mean Value Theorem in Norm

$$||G_{NL}||_{\Delta} \leq \gamma \quad \Leftrightarrow \quad ||DG_{NL}[u_r]||_{i,2} \leq \gamma, \ \forall\, u_r \in \mathcal{L}_2$$

Equivalence between local properties and global ones

Global Properties

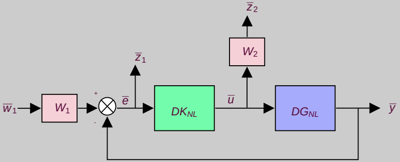
$\mathcal{L}_2$ **incremental gain** prop.
of the NL system



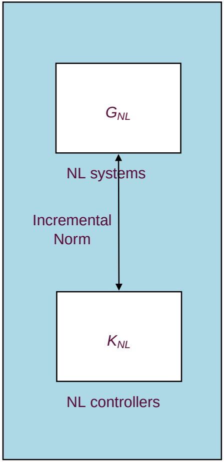
$\Leftrightarrow$

Local Properties

$\mathcal{L}_2$ **gain** prop.
of TV linearisations



An LPV approach for incremental synthesis



K_{NL} such that $||\mathcal{F}_l(G_{NL}, K_{NL})||_\Delta < \gamma$

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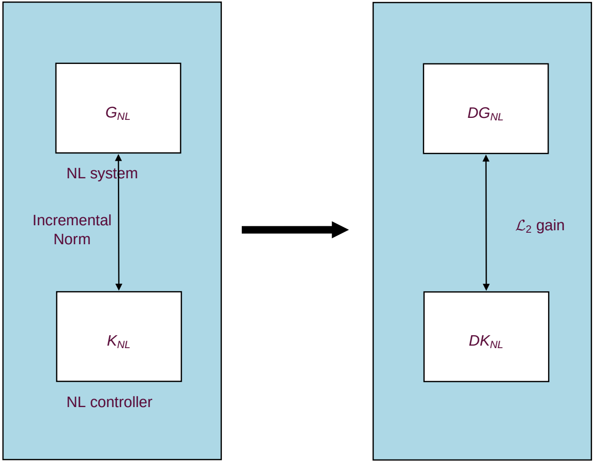

K_{NL} such that $||\mathcal{F}_l(G_{NL}, K_{NL})||_\Delta < \gamma$

DK_{NL} such that $||\mathcal{F}_l(DG_{NL}, DK_{NL})||_{i,2} < \gamma$

Two questions

1. How to compute for any $u_r \in \mathcal{L}_2$, $DK_{NL}[u_r]$?

   $\hookrightarrow$ Use an LPV method with G_{LPV} which embeds $DG_{NL}[u_r]$ for any u_r

2. From $DK_{NL}[u_r]$, defined for any $u_r \in \mathcal{L}_2$, how to compute K_{NL}?

   $\hookrightarrow$ focus on a special class of nonlinear control problem with the appropriated LPV control method

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LPV model embeds time varying linearizations

To the time varying linearizations $DP_{NL}[w_r, u_r]$

$$\left\{ \begin{array}{rcccccc} \dot{\overline{x}}(t) & = & A(t)\overline{x}(t) & + & B_w(t)\overline{w}(t) & + & B_u(t)\overline{u}(t) \\ \overline{z}(t) & = & C_z(t)\overline{x}(t) & + & D_{zw}(t)\overline{w}(t) & + & D_{zu}(t)\overline{u}(t) \\ \overline{y}(t) & = & C_y(t)\overline{x}(t) & + & D_{yw}(t)\overline{w}(t) & + & D_{yu}(t)\overline{u}(t) \end{array} \right. \quad (3)$$

is associated an LPV plant

$$\begin{array}{rcccccc} \dot{x}(t) & = & \mathbf{A}(\theta(t))x(t) & + & \mathbf{B}_1(\theta(t))w(t) & + & \mathbf{B}_2(\theta(t))u(t) \\ z(t) & = & \mathbf{C}_1(\theta(t))x(t) & + & \mathbf{D}_{11}(\theta(t))w(t) & + & \mathbf{D}_{12}(\theta(t))u(t) \\ y(t) & = & \mathbf{C}_2(\theta(t))x(t) & + & \mathbf{D}_{21}(\theta(t))w(t) & + & \mathbf{D}_{22}(\theta(t))u(t) \end{array} \quad (4)$$

such that with

$$\Omega_{DNL} = \left\{ \left(\begin{array}{ccccc} \overline{x} & \overline{z} & \overline{y} & \overline{w} & \overline{u} \end{array} \right) \mid \exists u_r, w_r, (3) \text{ is satisfied} \right\}$$

and

$$\Omega_{LPV} = \left\{ \left(\begin{array}{ccccc} x & z & y & w & u \end{array} \right) \mid (4) \text{ is satisfied} \right\}$$

we have

$$\Omega_{DNL} \subset \Omega_{LPV}$$

- Roughly speaking, nonlinear system of the form

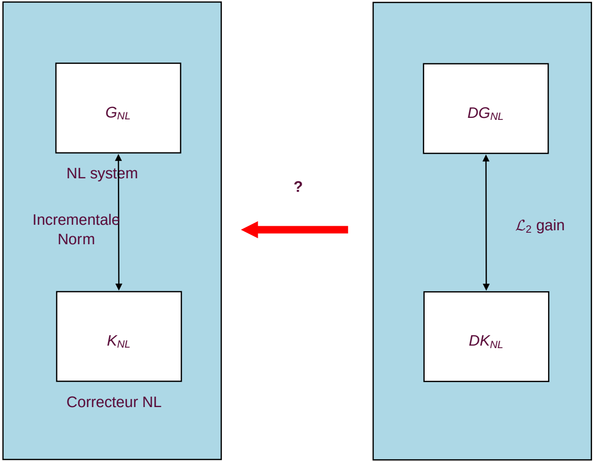
$$\begin{cases} \dot{x}(t) &= Ax(t) + B_2u(t) + \widetilde{f}(x(t)) \\ x(0) &= x_0 \end{cases}$$

  - with $\widetilde{f}(x(t)) = B_0 p(t)$
  - where $p(t)$ is measured on-line or where the components of $x(t)$, $w(t)$ and $u(t)$ necessary for the computation of $p(t)$ are measured, that is, there exists a function α such that

$$p(t) = \alpha(x(t), w(t), u(t))$$

- More details in S. de Hillerin, G. Scorletti, and V. Fromion, "Reduced-Complexity Controllers for LPV Systems: Towards Incremental Synthesis," *Proc. IEEE Conf. on Decision and Control*, dec 2011

An LPV approach for incremental synthesis



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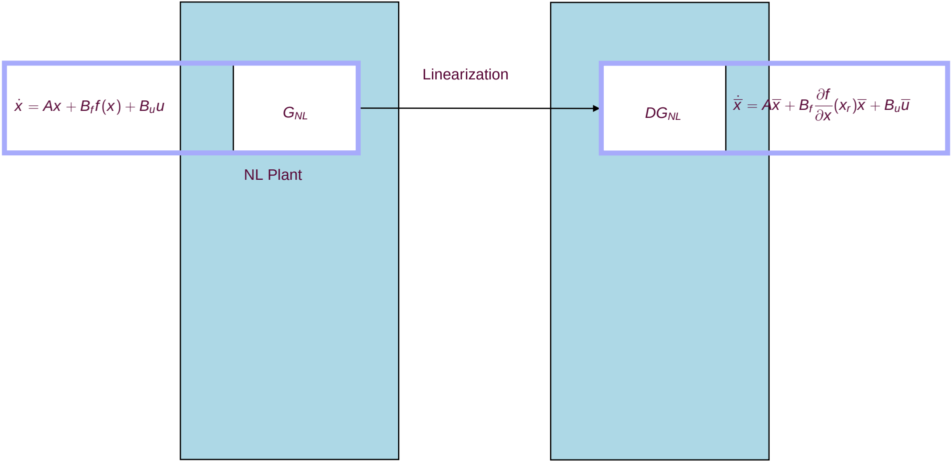

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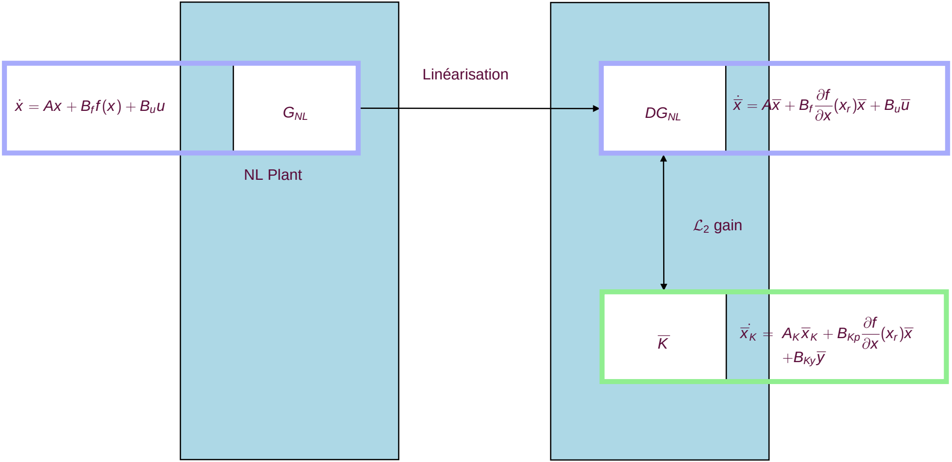

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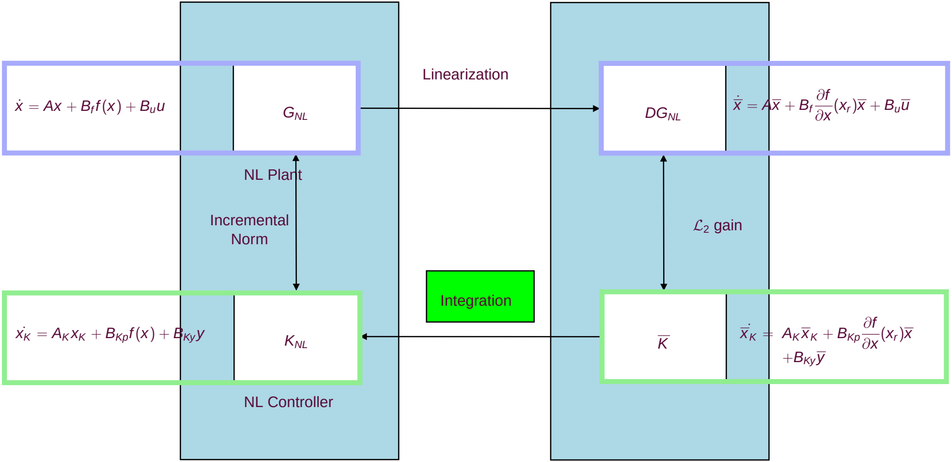

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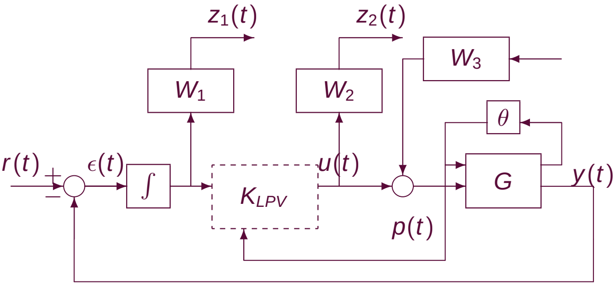
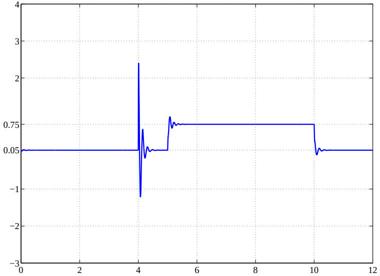
Application to the illustrative example





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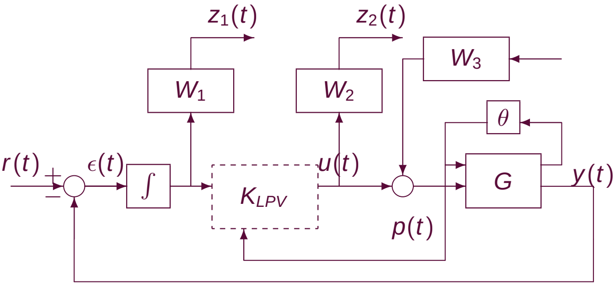

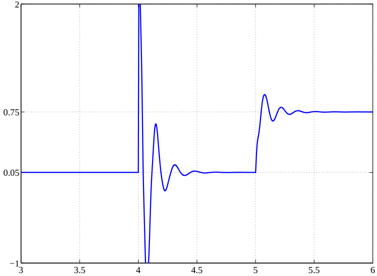

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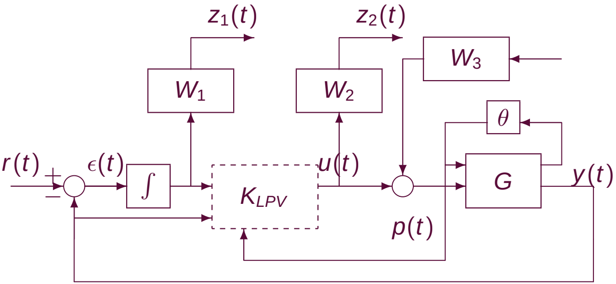

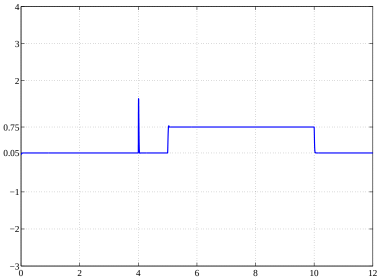

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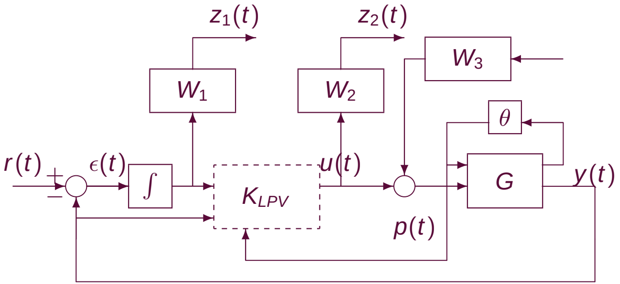

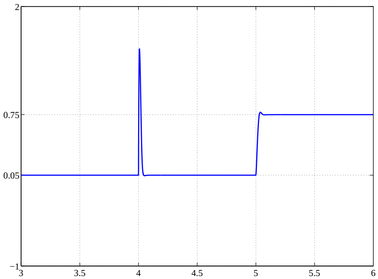

Toward a new approach of nonlinear control using LPV

- Two existing approaches of nonlinear control using LPV
  - Gain scheduling
    - Main idea: LPV model embeds **time invariant linearizations** of nonlinear plant
    - Interest: improve a widespread engineering practise
    - Drawback: few garantees on the closed loop behavior
  - Quasi LPV
    - Main idea: LPV model embeds **nonlinear plant**
    - Interest: stability garantees
    - Drawback: typical specs are not ensured
- Proposition of a third LPV approach
  - LPV for incremental control
    - Main idea: LPV model embeds **time variant linearizations** of nonlinear plant
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Conclusions

- Pave the way to a common LTI/NL framework for performance control, ensuring typical specifications
- A key result of robust control is the translation of performance specs in a well-posed optimisation problem (H_∞ norm)
- Its extension for typical specs is not the $\mathcal{L}_2$ gain / stability approach but the $\mathcal{L}_2$ incremental gain / incremental stability one
- Combined with LPV methods, pave the way to the practical design of nonlinear controllers ensuring typical specifications
- Objective: propose a rigourous alternative to the widespread gain-scheduling control used by the engineers

On-going / Forthcoming projects involving this nonlinear performance approach

- Nonlinear robust performance analysis: extension of the μ analysis with less conservative approach than IQC (S. Waitman, P. Massioni, L. Bako)
- Identication for control: extension to nonlinear systems (X. Bombois)
- Nonlinear control design using LPV
- Design of systems with nonlinearities