Robustness of an adaptive predictor-based output feedback for a wave PDE in presence of in-domain viscous damping

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GT MOSAR/SAR
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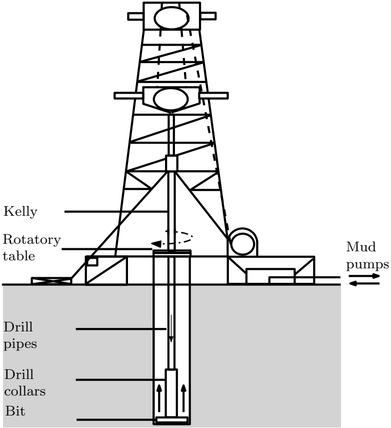
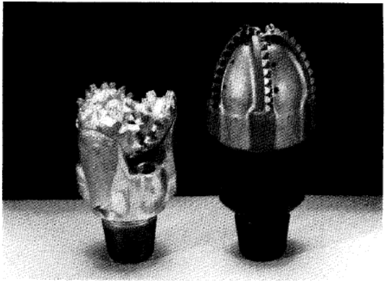
Facilities/vibrations

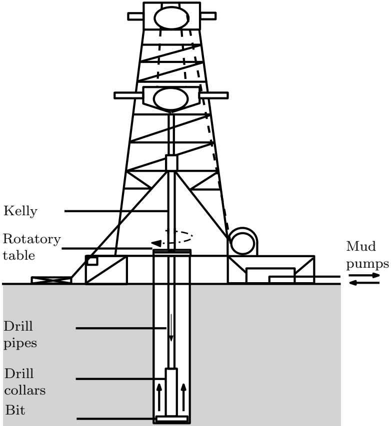




Source : J.D. Jansen, *Nonlinear dynamics of oilwell drillstrings*, PhD thesis, Delft University of Technology, 1993

Well length between 500m and 5km

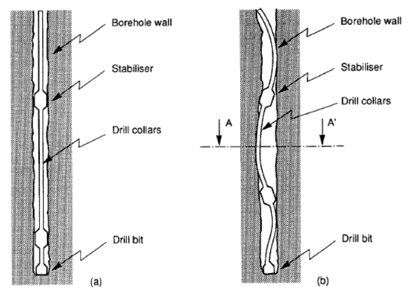
Facilities/vibrations



Well length between 500m and 5km

Vibrations types

- whirl oscillations



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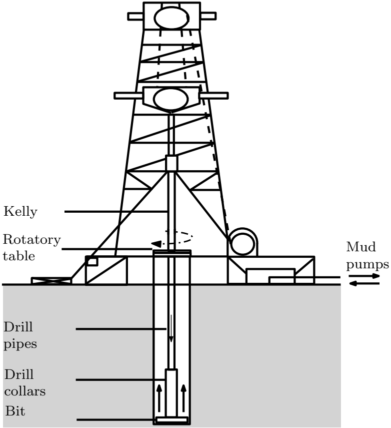
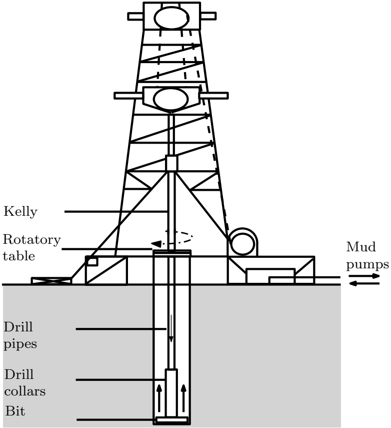


Vibrations types

- whirl oscillations
- vertical vibrations (bit bounce)

Well length between 500m and 5km

Facilities/vibrations

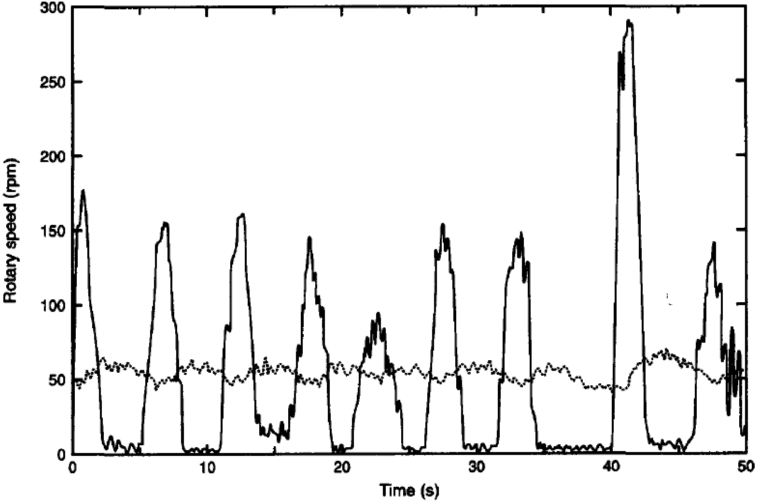


Vibrations types

- whirl oscillations
- vertical vibrations (bit bounce)
- torsional oscillations (stick-slip)

Well length between 500m and 5km

Stick-slip oscillations



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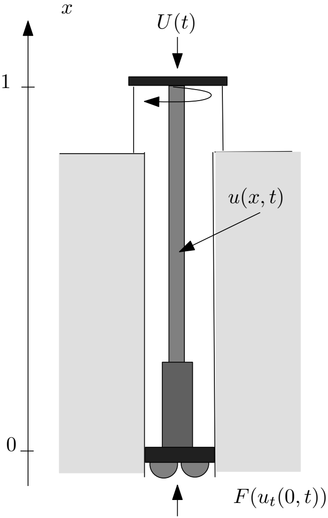
Modeling of the angular displacement $u(x,t)$

Damped wave equation + friction

$$u_{tt}(x,t) = u_{xx}(x,t) + \lambda u_t(x,t)$$



Modeling of the angular displacement $u(x,t)$

Damped wave equation + friction

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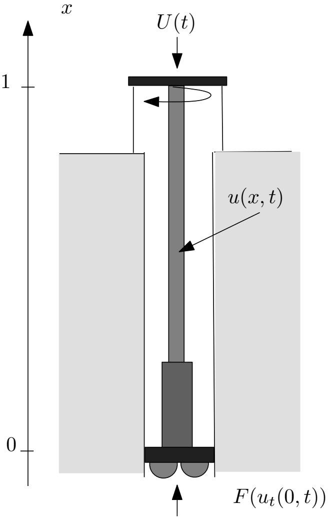
$$u_x(1,t) = U(t) \text{ (applied torque)}$$



Modeling of the angular displacement $u(x,t)$

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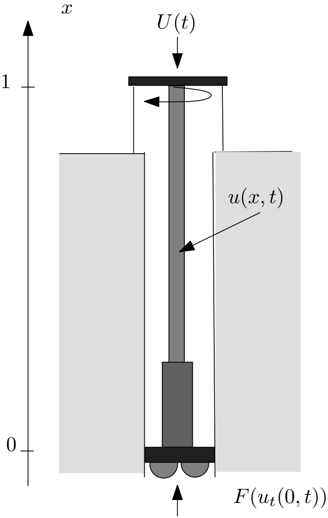
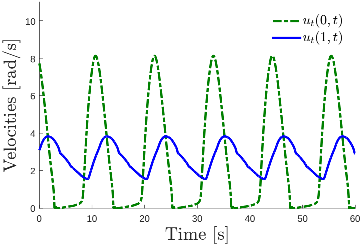
$u_x(1,t) = U(t)$ (applied torque)

$$u_{tt}(0,t) = a\underbrace{F(u_t(0,t))}_{\text{friction}} + a u_x(0,t)$$

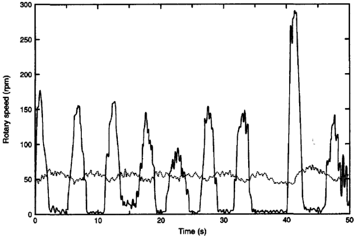


Modeling of the angular displacement $u(x, t)$

- Modeling



- Experimental data



Modeling of the angular displacement $u(x,t)$

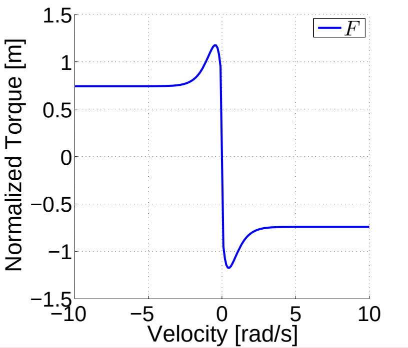
Damped wave equation + friction

$$u_{tt}(x,t) = u_{xx}(x,t) + \lambda u_t(x,t)$$

$$u_x(1,t) = U(t) \text{ (applied torque)}$$

$$u_{tt}(0,t) = \underbrace{a\,F(u_t(0,t))}_{\text{friction}} + a u_x(0,t)$$

Rock-on-the-bit friction term



Very uncertain, depends on

- the nature of the rock ;
- the weight on the bit...

1. Adaptive feedback without distributed damping
2. Robustness to distributed damping

Linearized model

Wave equation + friction

$$u_{tt}(x,t) = u_{xx}(x,t)$$

$$u_x(1,t) = U(t)$$

$$u_{tt}(0,t) = aF(u_t(0,t)) + au_x(0,t)$$

Desired equilibrium : uniform velocity u_t^r

$$\begin{cases} u^r(x,t) = u_t^r t - F(u_t^r)x + u_0 \ (u_0 \in \mathbb{R}) \\ U^r = -F(u_t^r) \\ u_x^r = U^r \end{cases}$$

Linearized model

Wave equation + friction

$$u_{tt}(x,t) = u_{xx}(x,t)$$

$$u_x(1,t) = U(t)$$

$$u_{tt}(0,t) = aF(u_t(0,t)) + au_x(0,t)$$

$\Rightarrow$

Linearized model for $\tilde{u} = u - u^r$

$$\tilde{u}_{tt}(x,t) = \tilde{u}_{xx}(x,t)$$

$$\tilde{u}_x(1,t) = U(t) - U^r$$

$$u_{tt}(0,t) = aq\tilde{u}_t(0,t) + a\tilde{u}_x(0,t)$$

Desired equilibrium : uniform velocity u_t^r

$$\begin{cases} u^r(x,t) = u_t^r t - F(u_t^r)x + u_0 \ (u_0 \in \mathbb{R}) \\ U^r = -F(u_t^r) \\ u_x^r = U^r \end{cases}$$

with $q = \frac{dF}{du_t}(u_t^r) > 0$
for high velocities

Problem at stake

PDE + ODE

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t) \\ u_x(1,t) = U(t) \\ u_{tt}(0,t) = aqu_t(0,t) + a(u_x(0,t) - d) \end{cases}$$

Adaptive controller : $q \to \hat{q}(t)$ and $d \to \hat{d}(t)$

Assumptions

- There exist known constants $\underline{q}, \overline{q}, \underline{d}$ and $\overline{d}$ such that $\underline{q} > \overline{q}$, $\underline{d} < \overline{d}$ and $q \in [\underline{q}, \overline{q}]$, $d \in [\underline{d}, \overline{d}]$.
- We measure $u_t(0,t)$ and $u_t(1,t)$ for all $t \geq 0$.

Reformulation

(Modified) Riemman
variables

$$\begin{cases} \zeta(x,t) = & u_t(x,t) + u_x(x,t) - \hat{d}(t) \\ \omega(x,t) = & u_t(x,t) - u_x(x,t) - \hat{d}(t) \end{cases}$$

$$W(t) = u_t(1,t) + U(t) - \hat{d}(t)$$

Reformulation

(Modified) Riemman variables

$$\left\{ \begin{array}{ll} \zeta(x,t) = & u_t(x,t) + u_x(x,t) - \hat{d}(t) \\ \omega(x,t) = & u_t(x,t) - u_x(x,t) - \hat{d}(t) \end{array} \right.$$

$$W(t) = u_t(1,t) + U(t) - \hat{d}(t)$$

PDEs + ODE Cascade (with d known)

$$\left\{ \begin{array}{l} u_{tt}(0,t) = aqu_t(0,t) + a(u_x(0,t) - d) \\ u_{tt}(x,t) = u_{xx}(x,t) \\ u_x(1,t) = U(t) \end{array} \right.$$

$$\hat{d}(t) = d \quad \Rightarrow \quad \left\{ \begin{array}{l} \left. \begin{array}{l} u_{tt}(0,t) = a(q-1)u_t(0,t) + a\zeta(0,t) \\ \zeta_t(x,t) = \zeta_x(x,t) \\ \zeta(1,t) = W(t) \end{array} \right\} \text{input-delay system} \\ \left. \begin{array}{l} \omega_t(x,t) = -\omega_x(x,t) \\ \omega(0,t) = 2u_t(0,t) - \zeta(0,t) \end{array} \right\} \text{transport PDE (stable)} \end{array} \right.$$

Control law

ODE

$$u_{tt}(0,t) = a(q-1)u_t(0,t) + a[W(t) - \tilde{d}(t)]$$

Adaptive PI controller for the error $e(t) = u_t(0,t) - u_t^r$

$$U(t) = -u_t(1,t) + \hat{d}(t) + W(t)$$
$$W(t) = -(c_0 + \hat{q}(t) - 1)\Omega(t)$$

$$\Omega(t) = \qquad e(t)$$

Control law

ODE

$$u_{tt}(0,t) = a(q-1)u_t(0,t) + a[W(t-1) - \tilde{d}(t)]$$

Adaptive PI controller for the **predicted** error $e(t+1) = u_t(0,t+1) - u_t^r$

$$\begin{aligned} U(t) &= -u_t(1,t) + \hat{d}(t) + W(t) \\ W(t) &= -(c_0 + \hat{q}(t) - 1)\Omega(t) \\ \Omega(t) &= \underbrace{e^{a(\hat{q}(t)-1)}e(t) + a\int_{t-1}^{t} e^{a(\hat{q}(t)-1)(t-\tau)}[U(\tau) + u_t(1,\tau) - \hat{d}(t)]d\tau}_{=\text{ adaptive predictor of } e(t+1)} \end{aligned}$$

Only needs

- the speed of the bit $u_t(0,t)$ and the top velocity $u_t(1,t)$
- the top input $U(t)$ (torque applied to the rotatory table)

(storage in memory over a length equal to the propagation time)

Update laws and stability result

$$\begin{aligned}\dot{\hat{q}}(t) =& \mathrm{Proj}_{[\underline{q},\overline{q}]}\left\{\frac{\gamma_q}{1+N(t)} e(t)\Upsilon(t), \hat{q}(t)\right\}\\ \dot{\hat{d}}(t) =& -\mathrm{Proj}_{[\underline{d},\overline{d}]}\left\{\frac{\gamma_d}{1+N(t)}\Upsilon(t), \hat{d}(t)\right\}\end{aligned}$$

Theorem [ACC14]

For all $c_0 > 0$, there exist $\gamma_q^*, \gamma_d^* > 0$ such that, if $\gamma_q < \gamma_q^*$ and $\gamma_d < \gamma_d^*$, there exist $R > 0$ and $\rho > 0$ such that

$$\Gamma(t) \leq R(e^{\rho\Gamma(0)} - 1)$$

$$\Gamma(t) = \underbrace{u_t(0,t)^2 + \int_0^1 (u_x(x,t) - d)^2 dx + \int_0^1 u_t(x,t)^2 dx}_{\text{Potential + kinetic energy}} + \tilde{q}(t)^2 + \tilde{d}(t)^2$$

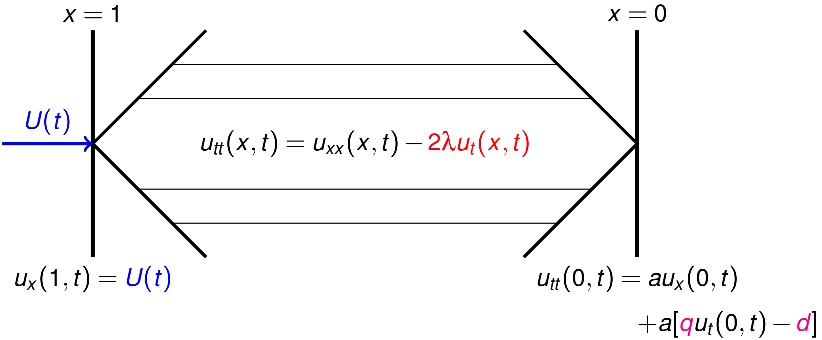
Besides, the regulation in $\mathcal{L}_2$-norm is achieved.

Contributions

+ Pointwise velocity regulation
+ Adaptive control wrt the friction with the rock
+ Only measurement of $u_t(1, t)$ and $u_t(0, t)$
- **in-domain damping neglected,** $\lambda = 0$

1. Adaptive feedback without distributed damping
2. Robustness to distributed damping

Distributed damping introduces coupling



$x = 1$

$x = 0$

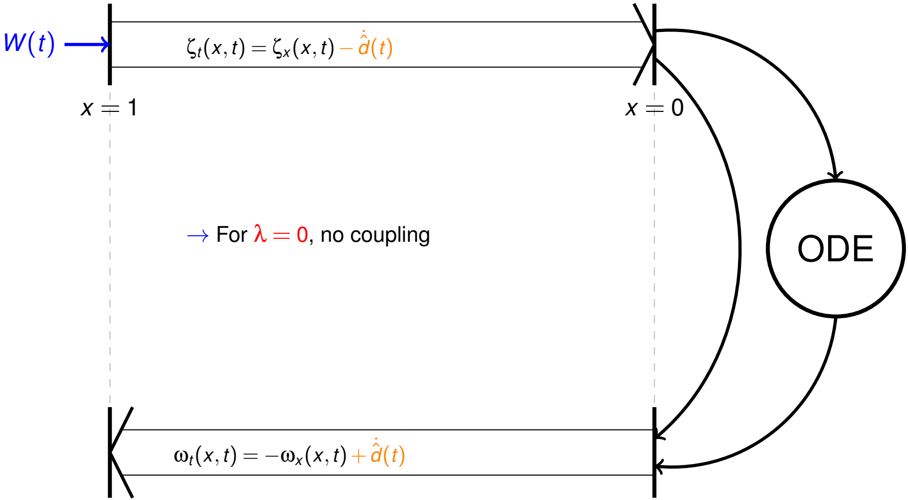
$U(t)$

$$u_{tt}(x,t) = u_{xx}(x,t) - 2\lambda u_t(x,t)$$

$$u_x(1,t) = U(t)$$

$$u_{tt}(0,t) = au_x(0,t) + a[qu_t(0,t) - d]$$

Distributed damping introduces coupling



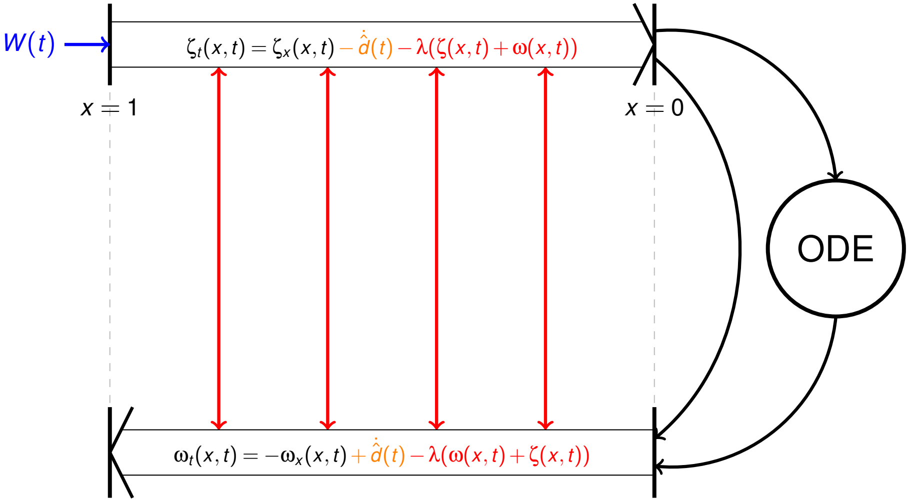
$$\zeta_t(x,t) = \zeta_x(x,t) - \dot{\hat{d}}(t)$$

→ For $\lambda = 0$, no coupling

$$\omega_t(x,t) = -\omega_x(x,t) + \dot{\hat{d}}(t)$$

ODE $\quad \dot{y}(t) = a(q-1)y(t) + a[\zeta(0,t) - \tilde{d}(t)]$

Distributed damping introduces coupling



$W(t)$

$$\zeta_t(x,t) = \zeta_x(x,t) - \dot{\hat{d}}(t) - \lambda(\zeta(x,t) + \omega(x,t))$$

$x = 1$

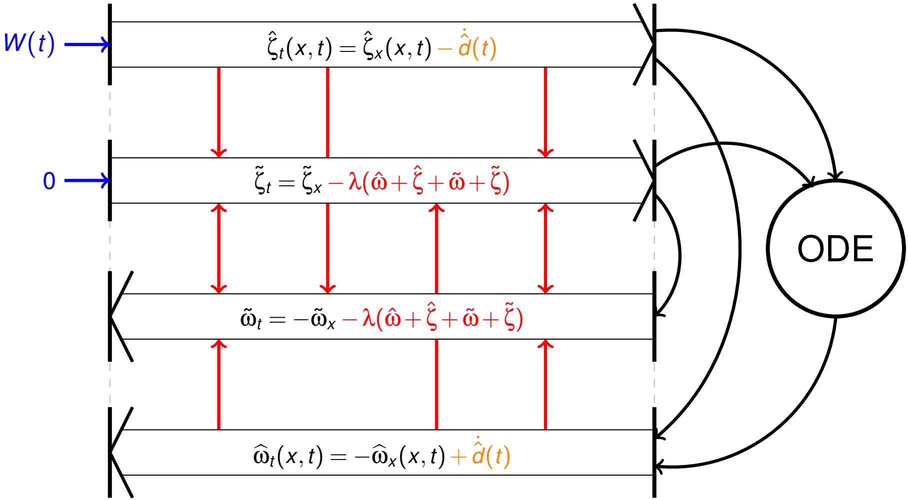
$x = 0$

ODE

$$\omega_t(x,t) = -\omega_x(x,t) + \dot{\hat{d}}(t) - \lambda(\omega(x,t) + \zeta(x,t))$$

ODE $$\dot{y}(t) = a(q-1)y(t) + a[\zeta(0,t) - \tilde{d}(t)]$$

Elements of analysis : Reformulation



$$\zeta(x,t) = \widehat{\zeta}(x,t) + \widetilde{\zeta}(x,t), \qquad \omega(x,t) = \widehat{\omega}(x,t) + \widetilde{\omega}(x,t)$$

Robustness of [BPK2014]

Theorem [ACC16]

Consider the closed-loop system with in-domain damping and the previous control law developed in [ACC14], define the functional

$$\Gamma(t) = u_t(0,t)^2 + \int_0^1 u_t(x,t)^2 dx + \int_0^1 (u_x(x,t) - d)^2 dx + (q - \hat{q})^2 + (d - \hat{d})^2$$

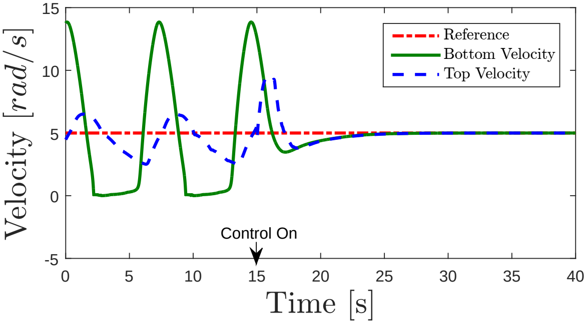
There exists $\lambda^* > 0$ such that, if $\lambda < \lambda^*$, there are $R > 0$ and $\rho > 0$ such that

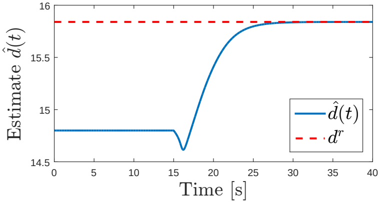
$$\Gamma(t) \leqslant R(e^{\rho \max_{s\in[0,2]} \Gamma(-s)} - 1), \quad \forall t \geqslant 0$$

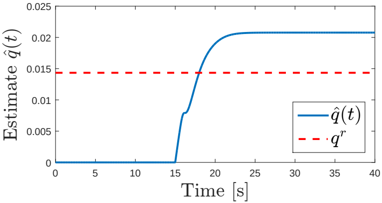
and the regulation in L_2-norm follows

$$\lim_{t\to\infty} u_t(0,t) = \lim_{t\to\infty} \|u_t(t)\| = \lim_{t\to\infty} \|u_x(t) - d\| = \lim_{t\to\infty} (\hat{d} - d) = 0$$

Simulations on the nonlinear model for $\lambda \neq 0$







Directions of future works

Contributions

- \+ Pointwise velocity regulation
- \+ Adaptive control w.r.t. the friction with the rock
- \+ Only measurement of $u_t(1,t)$ and $u_t(0,t)$
- \+ **robustness to in-domain damping,** $\lambda \neq 0$

On-going and future works

- for larger values of λ, extend the prediction approach for coupled transport equations (first step : tailored observer-based backstepping controller, CDC16)
- extend this work to the measurement of $u_t(1,t)$ only
- develop a general observation strategy for 1D hyperbolic equations cascaded with an ODE