Sliding mode control application on UAVs under external disturbances

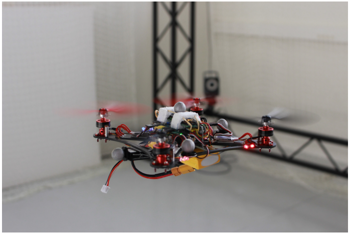
Gabriele Perozzi

PhD student at Onera: the French aerospace lab, DAAP, Lille

Work co-funded by Hauts de France region and Onera

Journées Automatique du GDR MACS, Lille, 16 nov 2016

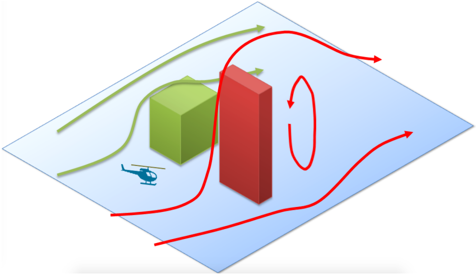
ONERA
THE FRENCH AEROSPACE LAB



RT-MaG Project
Real Time-Marseille Grenoble Project
www.gipsa-lab.fr/projet/RT-MaG

Based on article: *Altitude and attitude sliding mode control of UAV under wind disturbances*. G.Perozzi, D.Efimov, JM.Biannic, L.Planckaert, P.Coton. Submitted to IFAC 2017 Toulouse.

- Action in urban areas (eg: earthquakes like in Italy).
- Fluid obstacle.
- Unpredictable turbulent airflow pattern.

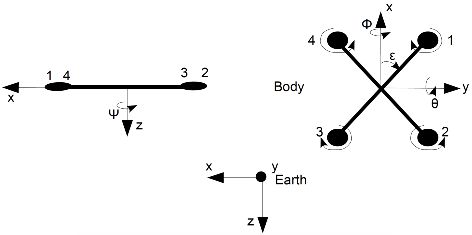


- Aerodynamic model, which takes into account wind disturbances directly inside of UAV dynamics equations;
- Nonlinear control law which considers realistic assumptions on external disturbances of quadrotors.

Why sliding mode control?

- SMC is an efficient tool to design robust controllers for nonlinear systems operating under uncertainty conditions

UAV configuration



- Rotational matrix

$$R = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\phi + c_\psi s_\theta s_\phi & s_\phi s_\psi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}$$

- Passage from earth frame ($\mathcal{R}_0$) to body frame($\mathcal{R}$)

$$[X^T]_{\mathcal{R}} = [X^T]_{\mathcal{R}_0} \cdot R$$

- Traslational dynamics in the body frame

$$m\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} + m\begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \begin{bmatrix}u\\ v\\ w\end{bmatrix} = \begin{bmatrix}F_{Xaero}\\ F_{Yaero}\\ F_{Zaero}\end{bmatrix} + m\begin{bmatrix}-g\sin\theta\\ g\cos\theta\sin\phi\\ g\cos\theta\cos\phi\end{bmatrix}$$

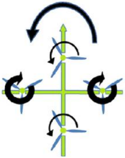
- Rotational dynamics with respect to inertial earth frame

$$I\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = -\begin{bmatrix}p\\ q\\ r\end{bmatrix} \times I\begin{bmatrix}p\\ q\\ r\end{bmatrix} + \begin{bmatrix}L_{aero}\\ M_{aero}\\ N_{aero}\end{bmatrix}$$

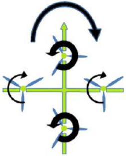
- Relationship between angular velocities and eulear angles

$$\begin{aligned}\dot{\phi} &= p + \tan\theta(q\sin\phi + r\cos\phi)\\ \dot{\theta} &= q\cos\phi - r\sin\phi\\ \dot{\psi} &= \frac{q\sin\phi + r\cos\phi}{\cos\theta}\end{aligned}$$

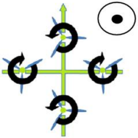
- UAV desired movements are obtained changing rotors speed in a proper way (altitude and attitude)



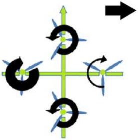
Rotate left



Rotate right



Going up



Move right

- Aerodynamic forces and momenta for each rotor

$$\begin{aligned}
F_{X_j} &= -\rho A R^2 \frac{u_j - u_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{H_j} \omega_j^2 \\
F_{Y_j} &= -\rho A R^2 \frac{v_j - v_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{H_j} \omega_j^2 \\
F_{Z_j} &= -\rho A R^2 C_{T_j} \omega_j^2 \\
L_j &= -\operatorname{sign} \omega_j \rho A R^3 \frac{u_j - u_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Rm_j} \omega_j^2 \\
M_j &= -\operatorname{sign} \omega_j \rho A R^3 \frac{v_j - v_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Rm_j} \omega_j^2 \\
N_j &= -\operatorname{sign} \omega_j \rho A R^3 C_{Q_j} \omega_j^2
\end{aligned}$$

- Total aerodynamic forces

$$F_{Xaero} = \sum_{j=1}^{4} F_{X\,j},\; F_{Yaero} = \sum_{j=1}^{4} F_{Y\,j},\; F_{Zaero} = \sum_{j=1}^{4} F_{Z\,j}$$

- Total aerodynamic momenta

$$L_{aero} = \sum_{j=1}^{4} (L_j + F_{Z\,j} l \mathrm{s}_j - h F_{Y\,j})$$

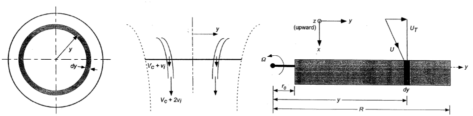
$$M_{aero} = \sum_{j=1}^{4} (M_j - F_{Z\,j} l \mathrm{c}_j + h F_{X\,j})$$

$$N_{aero} = \sum_{j=1}^{4} (N_j + F_{Y\,j} l \mathrm{c}_j - F_{X\,j} l \mathrm{s}_j)$$

$$\mathrm{c}_j = \cos\left(\frac{\pi}{2}(j-1) + \epsilon\right)$$

$$\mathrm{s}_j = \sin\left(\frac{\pi}{2}(j-1) + \epsilon\right)$$

- Aerodynamic coefficients from blade element momentum theory



$$\mu_j = \frac{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}}{R|\omega_j|}$$

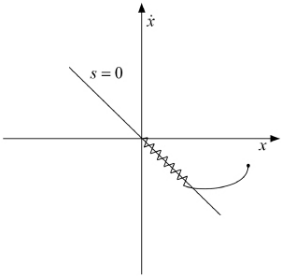
- Simplified coefficients

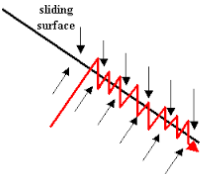
$$\begin{aligned}
\lambda_j =& \lambda_{stat} - \frac{4}{\sigma a} K_z \frac{w_j - w_w}{R|\omega_j|} \\
C_{T\,j} =& C_{T\,stat} + K_z \frac{w_j - w_w}{R|\omega_j|} \\
C_{H\,j} =& K_D \mu_j
\end{aligned}$$

Sliding Mode Control

SMC design is composed of two steps:

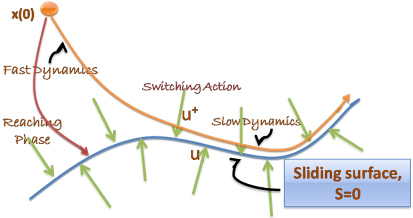
- Design of a surface. While on the sliding surface, the dynamics is restricted to the equations of the surface and is robust against external disturbances.
- Design a feedback control law to provide convergence of the system trajectory to the sliding surface, and to obtain a finite time convergence.





Sliding Mode Control

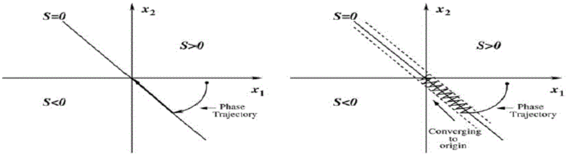
- Reaching phase: the trajectory, starting from a nonzero initial conditions, reaches the sliding surface.
- Sliding surface: the trajectory remains and evolves according to the dynamics specified by the sliding surface.



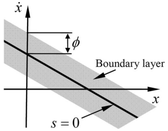
Sliding Mode Control

Chattering issue:

- In theory the trajectory slides along the surface.
- In practice there is high frequency switching called chattering.



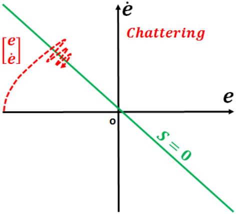
- Solutions have been developed to reduce the chattering so that the trajectory remains in a small neighborhood of the surface (High order SMC, saturation function).



Sliding Mode Control

- Errors

$$\begin{aligned} e_z &= z - z_{des} \\ e_\phi &= \phi - \phi_{des} \\ e_\theta &= \theta - \theta_{des} \\ e_\psi &= \psi - \psi_{des} \end{aligned}$$



- Sliding surface

$$S_i = \dot{e}_i + \alpha_i e_i, \quad \alpha_i > 0$$

- Lyapunov function

$$V_i = \frac{1}{2} S_i^2$$

State-space form

- System

$$\dot{X} = f(X, U, d)$$

- State

$$X = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & \phi & \theta & \psi & p & q & r \end{bmatrix}^T$$

- Control

$$U = \begin{bmatrix} U_z \\ U_\theta \\ U_\phi \\ U_\psi \end{bmatrix} = \begin{bmatrix} K_f & K_f & K_f & K_f \\ K_f lc_j & K_f lc_j & K_f lc_j & K_f lc_j \\ -K_f ls_j & -K_f ls_j & -K_f ls_j & -K_f ls_j \\ K_m & -K_m & K_m & -K_m \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

where $K_f = \rho A R^2 C_{T\,stat}$, $K_m = \rho A R^3 \big(\frac{\sigma C_{D0}}{8} + \lambda_{stat} \sigma a (\frac{\theta_0}{6} - \frac{\lambda_{stat}}{4})\big)$.
Control inputs are proportional to the terms with ω_j^2. The other terms dependent linearly on ω_j and wind velocities are considered as disturbances. Since we do not know in advance the wind perturbations, then we cannot use these terms in controls.

Disturbances

- Upper-bound of the control equation from Jensen's inequality

$$\sum_{j=1}^{4} |\omega_j| \leq K\sqrt{|U_z|}, \qquad K = \frac{2}{\sqrt{K_f}}$$

- Disturbance upper-bounds after substitutions

$$|d_x| \leq \bar{K}_D \left(|X| + D_x\right) \sqrt{|U_z|}$$

$$|d_y| \leq \bar{K}_D \left(|X| + D_y\right) \sqrt{|U_z|}$$

$$|d_z| \leq \bar{K}_z \left(|X| + D_z\right) \sqrt{|U_z|}$$

$$|d_\phi| \leq \tilde{K}_\phi \left(f_{\phi 1}\left(X\right) + D_{\phi 1}\right) \sqrt{|U_z|} + \bar{K}_\phi \left(f_{\phi 2}\left(X\right) + D_{\phi 2}\right)$$

$$|d_\theta| \leq \tilde{K}_\theta \left(f_{\theta 1}\left(X\right) + D_{\theta 1}\right) \sqrt{|U_z|} + \bar{K}_\theta \left(f_{\theta 2}\left(X\right) + D_{\theta 2}\right)$$

$$|d_\psi| \leq \tilde{K}_\psi \left(f_{\psi 1}\left(X\right) + D_{\psi 1}\right) \sqrt{|U_z|} + \bar{K}_\psi \left(f_{\psi 2}\left(X\right) + D_{\psi 2}\right)$$

Altitude control

Steps to design the altitude control:

- System in compact form

$$\ddot{z} = g - (\cos\phi\cos\theta)\,\frac{1}{m}(U_z + d_z)$$

- Error between reference signal and state value

$$e_z = z - z_{des}$$

- Derivative of the sliding surface

$$\dot{S}_z = \ddot{z} + \alpha_z\dot{z} = g - \frac{\cos\theta\cos\phi}{m}(U_z + d_z) + \alpha_z\dot{z}$$

- Control equation

$$U_z = \frac{m}{\cos\theta\cos\phi}\,(g - \tilde{u}_z + \alpha_z\dot{z}) \leq \frac{m}{\gamma}(|g + \alpha_z\dot{z}| + |\tilde{u}_z|)$$

- Derivative of Lyapunov function

$$\dot{V} = S_z\dot{S}_z \leq S_z\tilde{u}_z + |S_z||d_z\frac{1}{m}| = S_z\tilde{u}_z + |S_z|\frac{1}{m}\bar{K}_z\,(|X| + D_z)\,\sqrt{|U_z|}$$

$$\begin{aligned}\dot{V} \leq & S_z \tilde{u}_z + |S_z| \left(\varrho(X) + \nu(X) \sqrt{|\tilde{u}_z|} \right) \\ \varrho(X) = & \frac{1}{m} \sqrt{\frac{m}{\gamma}} \bar{K}_z \left(|X| + D_z \right) \sqrt{|g + \alpha_z \dot{z}|} \\ \nu(X) = & \frac{1}{m} \sqrt{\frac{m}{\gamma}} \bar{K}_z \left(|X| + D_z \right)\end{aligned}$$

- Auxiliary control

$$\begin{aligned}\tilde{u}_z = & -\beta(X) \operatorname{sign}(S_z) \\ \beta(X) = & \frac{1}{2} \left(\nu(X)^2 + 2\varrho(X) + \nu(X) \sqrt{\nu^2(X) + 4\varrho(X)} \right) + \delta\end{aligned}$$

- Finite time stability proved

$$\dot{V} < -\sqrt{2}\delta\sqrt{V}$$

Attitude control

- Attitude is equivalent to a control of linear acceleration so it leads to stabilizing the linear speed.

Steps to design the roll control:

- System in compact form

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\frac{I_{yy} - I_{zz}}{I_{xx}} + \frac{1}{I_{xx}}(U_\phi + d_\phi)$$

- Error between reference signal and state value

$$e_\phi = \phi - \phi_{des}$$

- Derivative of the sliding surface

$$\dot{S}_\phi = \ddot{\phi} + \alpha_\phi\dot{\phi} = \dot{\theta}\dot{\psi}\frac{I_{yy} - I_{zz}}{I_{xx}} + \frac{1}{I_{xx}}(U_\phi + d_\phi) + \alpha_\phi\dot{\phi}$$

- Control equation

$$U_\phi = I_{xx}\left(-\dot{\theta}\dot{\psi}\frac{I_{yy} - I_{zz}}{I_{xx}} + \tilde{u}_\phi - \alpha_\phi\dot{\phi}\right)$$

Attitude control

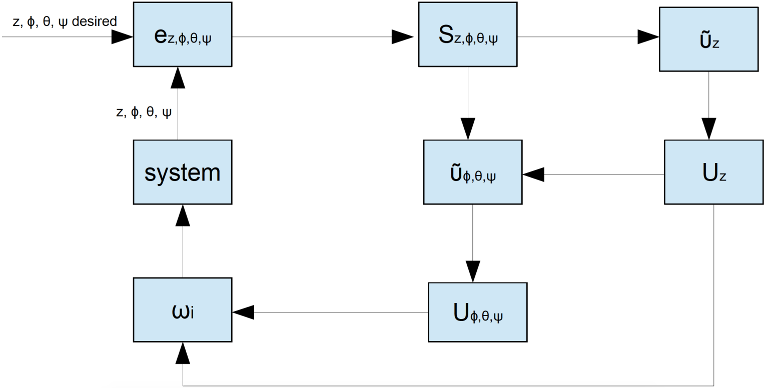
- Derivative of Lyapunov function

$$\begin{aligned}\dot{V} =& S_\phi \dot{S}_\phi \leq S_\phi \tilde{u}_\phi + |S_\phi||d_\phi \frac{1}{I_{xx}}| \\ &\leq S_\phi \tilde{u}_\phi + \frac{|S_\phi|}{I_{xx}}\left(\tilde{K}_\phi\left(f_{\phi 1}(X) + D_{\phi 1}\right)\sqrt{|U_z|} + \bar{K}_\phi\left(f_{\phi 2}(X) + D_{\phi 2}\right)\right)\end{aligned}$$

- Auxiliary control

$$\begin{aligned}\tilde{u}_\phi =& -\frac{1}{I_{xx}}\mathrm{sign} S_\phi\left(\tilde{K}_\phi\left(f_{\phi 1}\left(X\right) + D_{\phi 1}\right)\sqrt{|U_z|}\right. \\ &\left. + \bar{K}_\phi\left(f_{\phi 2}\left(X\right) + D_{\phi 2}\right)\right)\end{aligned}$$

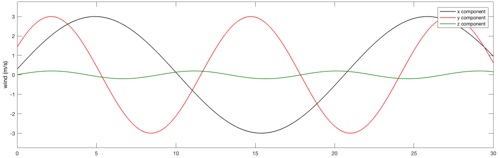
Summary scheme



Simulations

Simulation data and constraints:

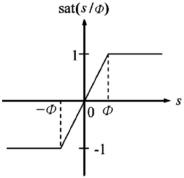
- wind signal

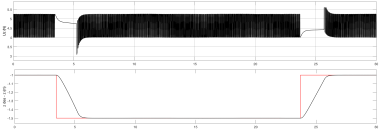


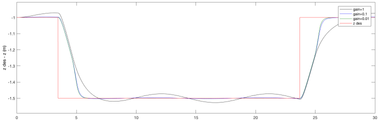
- mass UAV 0.47Kg
- max rotor speed 400rad/s
- max thrust rotors 5.6N
- rotors dynamics $\frac{1}{1+0.03s}$

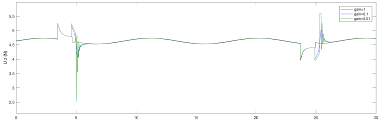
- For a sufficiently small ϕ, if for a sign function all trajectories converge to an equilibrium, then with a saturation all trajectories converge in a compact set around that equilibrium.
- Saturation function:

$$\text{sat}_\phi(x) = \begin{cases} \text{sign}(x) & \text{if} \quad |x| > 1 \\ \arctan\left(\frac{1}{\phi}x\right) & \text{otherwise} \end{cases}$$

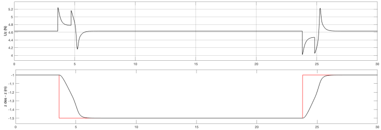




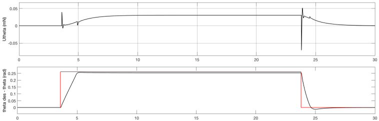




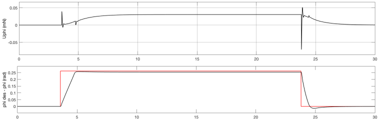
- System response to desired input with no wind disturbances



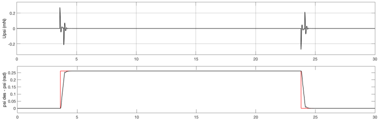
(i) z correction and altitude control



(j) pitch correction and control

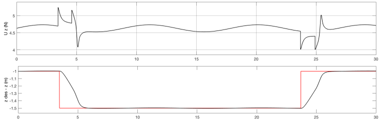


(k) roll correction and control

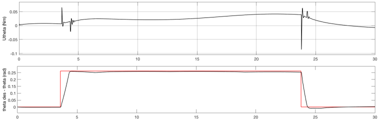


(l) yaw correction and control

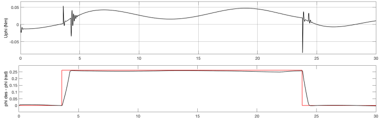
- Robustness of the proposed control under wind disturbances and convergence



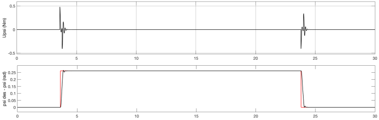
(m) z correction and altitude control



(n) pitch correction and control



(o) roll correction and control



(p) yaw correction and control

Summarizing:

- Choice of UAV physical model influenced by wind disturbance;
- SMC applied to UAV problems (attitude and altitude) with simplified coefficients equations;
- Robustness of the proposed SMC method with respect to:
  - wind disturbances;
  - uncertainty of identified model parameters;
  - unmodeled rotor dynamics;
- Further work in trajectory considering also x_{des},y_{des},$\dot{x}_{des}$,$\dot{y}_{des}$.

Thank you for your attention!