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Stability analysis and stabilization of jump LPV systems with piecewise differentiable parameters using continuous and sampled-data controllers

Corentin Briat

Department of Biosystems Science and Engineering, ETH-Zürich

Outline

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LPV systems

LPV systems

LPV systems are generically represented as

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t), \; x(0) = x_0 \tag{1}$$

where

- x and u are the state of the system and the control input
- $\rho(t) \in \mathcal{P}$, $\mathcal{P} \subset \mathbb{R}^N$ compact, is the value of the parameter vector at time t
- The matrix-valued functions $A(\cdot)$ and $B(\cdot)$ are "nice enough", i.e. continuous on $\mathcal{P}$

Rationale

- Can be used to approximate nonlinear systems [Sha88, BPB04]
- Can be used to model a wide variety of real-world processes [MS12, HW15, Bri15a]
- Convenient framework for the design gain-scheduled controllers [RS00]

Quadratic stability [SGC97]

Definition

The LPV system

$$\begin{array}{rcl} \dot{x}(t) & = & A(\rho(t))x(t) \\ x(0) & = & x_0 \end{array} \quad (2)$$

is said to be quadratically stable if $V(x) = x^T P x$ *is a Lyapunov function for the system.*

Theorem

The LPV system (2) *is quadratically stable if and only if there exists a matrix* $P \in \mathbb{S}^n_{\succ 0}$ *such that the LMI*

$$A(\theta)^T P + PA(\theta) \prec 0 \quad (3)$$

holds for all $\theta \in \mathcal{P}$.

Remarks

- All the possible trajectories $\rho : \mathbb{R}_{\geq 0} \mapsto \mathcal{P}$ are (implicitly) considered (together with the assumption of existence of solutions)
- Semi-infinite dimensional LMI problem (can be checked using various methods)

Robust stability [Wu95]

Definition

The LPV system

$$\begin{array}{rcl}\dot{x}(t) & = & A(\rho(t))x(t) \\ x(0) & = & x_0\end{array} \quad (4)$$

with $\rho(t) \in \mathcal{P}$ *and* $\dot{\rho}(t) \in \mathcal{D}$*, for some given compact sets* $\mathcal{P}, \mathcal{D} \subset \mathbb{R}^N$*, is said to be robustly stable if* $V(x,\rho) = x^T P(\rho)x$ *is a Lyapunov function for the system.*

Theorem

The LPV system (4) *is robustly stable if and only if there exists a differentiable matrix-valued function* $P : \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the LMI*

$$\sum_{i=1}^{N} \theta_i' \partial_{\theta_i} P(\theta) + A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0 \quad (5)$$

holds for all $\theta \in \mathcal{P}$ *and all* $\theta' \in \mathcal{D}$*.*

Remarks

- Trajectories of the parameters are continuously differentiable (can be relaxed)
- Infinite-dimensional LMI problem (can be approximately checked)

Summary

Some remarks

- Two main classes of parameter trajectories associated with two main stability concepts
- But these classes are very far apart!
- Parameter trajectories are defined in a very loose/restrictive way
- The accuracy of the tools developed for periodic, switched and Markov jump systems stems from the fact that they are tailor-made
- In the end, LPV systems suffer from a very vague description which may prevent the development of accurate tools

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- Two main classes of parameter trajectories associated with two main stability concepts
- But these classes are very far apart!
- Parameter trajectories are defined in a very loose/restrictive way
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Issues

- What if we consider piecewise differentiable parameters?
- Robust stability not applicable and quadratic stability too conservative
- So, we need something else!

LPV systems with piecewise differentiable parameters

Class of parameters

- Piecewise differentiable with aperiodic discontinuities

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Stability results

- Stability condition using hybrid systems method $\rightarrow$ minimum dwell-time condition
- Connections with quadratic and robust stability
- Example

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Stabilization results

- Continuous-time controllers
- Sampled-data controllers [TGW02, RMG12, GMFP15]
- Examples

Outline

Preliminaries

Let us consider the LPV system

$$\dot{x}(t) = A(\rho(t))x(t),\ x(0) = x_0 \tag{6}$$

with parameter trajectories ρ in $\mathscr{P}_{\geqslant \bar{T}}$ where

$$\mathscr{P}_{\geqslant \bar{T}} := \left\{ \rho : \mathbb{R}_{\geq 0} \mapsto \mathcal{P} \left| \begin{array}{c} \dot{\rho}(t) \in \mathcal{Q}(\rho(t)), t \in [t_k, t_{k+1}) \\ T_k \geq \bar{T},\ \rho(t_k) \neq \rho(t_k^+) \in \mathcal{P}, k \in \mathbb{Z}_{\geq 0} \end{array} \right. \right\} \tag{7}$$

where $\rho(t_k^+) := \lim_{s \downarrow t_k} \rho(s)$, $t_0 = 0$ (no jump at t_0), $T_k := t_{k+1} - t_k$, $\bar{T} > 0$,

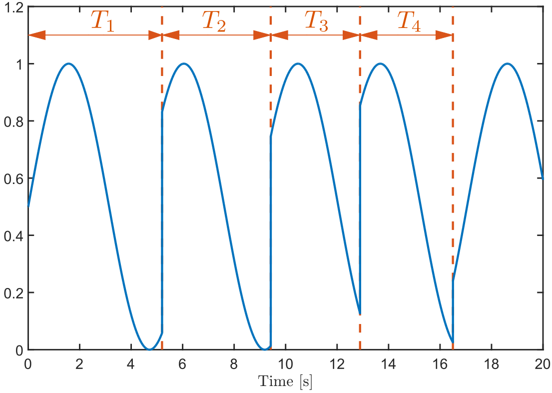
$$\begin{array}{rcl} \mathcal{P} & =: & \mathcal{P}_1 \times \ldots \times \mathcal{P}_N,\ \mathcal{P}_i := [\underline{\rho}_i,\ \bar{\rho}_i],\ \underline{\rho}_i \leq \bar{\rho}_i,\ i = 1, \ldots, N \\ \mathcal{D} & =: & \mathcal{D}_1 \times \ldots \times \mathcal{D}_N,\ \mathcal{D}_i := [\underline{\nu}_i,\ \bar{\nu}_i],\ \underline{\nu}_i \leq \bar{\nu}_i,\ i = 1, \ldots, N \end{array}$$

and $\mathcal{Q}(\rho) = \mathcal{Q}_1(\rho) \times \ldots \times \mathcal{Q}_N(\rho)$ with

$$\mathcal{Q}_i(\rho) := \begin{cases} \mathcal{D}_i & \text{if } \rho_i \in (\underline{\rho}_i, \bar{\rho}_i), \\ \mathcal{D}_i \cap \mathbb{R}_{\geq 0} & \text{if } \rho_i = \underline{\rho}_i, \\ \mathcal{D}_i \cap \mathbb{R}_{\leq 0} & \text{if } \rho_i = \bar{\rho}_i. \end{cases} \tag{8}$$

Illustration

- Minimum dwell-time $\bar{T} = 3.3$
- Discontinuities separated by at least $\bar{T} = 3.3$ seconds



System reformulation

- The key idea is to reformulate the system in a way that will allow us to capture the both the dynamics of the system and the dynamics of the parameters.

System reformulation

- The key idea is to reformulate the system in a way that will allow us to capture the both the dynamics of the system and the dynamics of the parameters.
- Hence, we propose the following hybrid system formulation [GST12]

$$\left\{\begin{array}{l}\left\{\begin{array}{rcl}\dot{x}(t) & = & A(\rho(t))x(t)\\ \dot{\rho}(t) & \in & \mathcal{Q}(\rho(t))\\ \dot{\tau}(t) & = & 1\\ \dot{T}(t) & = & 0\end{array}\right| \begin{array}{c}\text{if } (x(t),\rho(t),\tau(t),T(t))\in C\\ (\text{eq. } \tau(t)<T(t))\end{array}\right\}\\ \left\{\begin{array}{rcl}x(t^+) & = & x(t)\\ \rho(t^+) & \in & \mathcal{P}\\ \tau(t^+) & = & 0\\ T(t^+) & \in & [\bar{T},\infty)\end{array}\right| \begin{array}{c}\text{if } (x(t),\rho(t),\tau(t),T(t))\in D\\ (\text{eq. } \tau(t)=T(t))\end{array}\right\}\end{array} \tag{9}$$

where

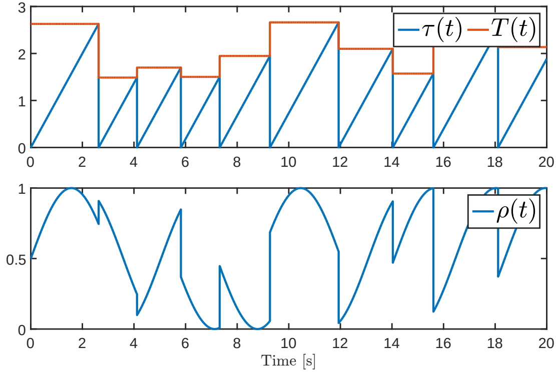
$$\begin{array}{rcl}C & = & \mathbb{R}^n\times\mathcal{P}\times E_<,\\ D & = & \mathbb{R}^n\times\mathcal{P}\times E_=\\ E_\square & = & \{\varphi\in\mathbb{R}_{\geq 0}\times[\bar{T},\infty):\varphi_1\square\varphi_2\},\ \square\in\{<,=\}\end{array} \tag{10}$$

and

$$(x(0),\rho(0),\tau(0),T(0))\in\mathbb{R}^n\times\mathcal{P}\times\{0\}\times[\bar{T},\infty). \tag{11}$$

Illustration

- Let the t_k's be the time instants for which $\tau(t_k) = T(t_k)$
- We consider a parameter trajectory given by $\rho(t) = (1 + \sin(t + \phi(t)))/2$ where $\phi(t) = \phi_k$, $t \in [t_k, t_{k+1})$ and the ϕ_k's are uniform random variables taking values in $[0, 2\pi]$
- At each t_k, a new value for ϕ_k is drawn, which introduces a discontinuity in the parameter trajectory



Main result

Theorem (Minimum dwell-time)

Let $\bar{T} \in \mathbb{R}_{>0}$ be given and assume that there exist a bounded continuously differentiable matrix-valued function $S : [0, \bar{T}] \times \mathcal{P} \mapsto \mathbb{S}^n_{\succ 0}$ and a scalar $\varepsilon > 0$ such that the conditions

$$\partial_\tau S(\tau, \theta) + \sum_{i=1}^{N} \partial_{\rho_i} S(\tau, \theta)\mu_i + \mathrm{Sym}[S(\tau, \theta)A(\theta)] + \varepsilon I \preceq 0 \quad (12)$$

$$\sum_{i=1}^{N} \partial_{\rho_i} S(\bar{T}, \theta)\mu_i + \mathrm{Sym}[S(\bar{T}, \theta)A(\theta)] + \varepsilon I \preceq 0 \quad (13)$$

and

$$S(0, \theta) - S(\bar{T}, \eta) \preceq 0 \quad (14)$$

hold for all $\theta, \eta \in \mathcal{P}$, $\mu \in \mathcal{D}$ and all $\tau \in [0, \bar{T}]$. Then, the LPV system (6) with parameter trajectories in $\mathscr{P}_{\geqslant \bar{T}}$ is asymptotically stable.

- For a square matrix M, we define $\mathrm{Sym}[M] = M + M^T$

Proof

Let us consider the system

$$\begin{array}{rcl} \dot{\chi}(t) & \in & F(\chi(t)) \text{ if } \chi(t) \in C \\ \chi(t^+) & \in & G(\chi(t)) \text{ if } \chi(t) \in D \end{array} \quad (15)$$

where $\chi(t) \in \mathbb{R}^d$, $C \subset \mathbb{R}^d$ is open, $D \subset \mathbb{R}^d$ is compact and $G(D) \subset C$. The flow map and the jump map are the set-valued maps $F : C \rightrightarrows \mathbb{R}^n$ and $G : D \rightrightarrows C$, respectively. We also assume for simplicity that the solutions are complete. We then have the following stability result:

Theorem (Persistent flowing [GST12])

Let $\mathcal{A} \subset \mathbb{R}^d$ be closed. Assume that there exist a function $V : \bar{C} \cup D \mapsto \mathbb{R}$ that is continuously differentiable on an open set containing $\bar{C}$ (i.e. the closure of C), functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and a continuous positive definite function α_3 such that

(a) $\alpha_1(|\chi|_\mathcal{A}) \le V(x) \le \alpha_2(|\chi|_\mathcal{A})$ *for all* $\chi \in \bar{C} \cup D$;

(b) $\langle \nabla V(\chi), f \rangle \le -\alpha_3(|\chi|_\mathcal{A})$ *for all* $\chi \in C$ *and* $f \in F(\chi)$;

(c) $V(g) - V(\chi) \le 0$ *for all* $\chi \in D$ *and* $g \in G(\chi)$.

Assume further that for each $r > 0$, there exists a $\gamma_r \in \mathcal{K}_\infty$ and an $N_r \ge 0$ such that for every solution ϕ to the system (15), *we have that $|\phi(0,0)|_\mathcal{A} \in (0, r]$, $(t, j) \in \operatorname{dom} \phi$, $t + j \ge T$ imply $t \ge \gamma_r(T) - N_r$, then $\mathcal{A}$ is uniformly globally asymptotically stable for the system* (15).

Proof

- Assume that the full trajectory of $T(t)$ is known.
- This is possible since $T(t)$ is independent of the other components of the state of the system (9).
- Then, there exists a $T_{max} < \infty$ such that $\bar{T} \leq T(t) \leq T_{max}$ for all $t \geq 0$.

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- Define then the set $\mathcal{A} = \{0\} \times \mathcal{P} \times ((E_< \cup E_=) \cap [0, T_{max}]^2)$
- Note that the LPV system (6) with parameter trajectories in $\mathscr{P}_{\geqslant \bar{T}}$ is asymptotically stable if and only if the set $\mathcal{A}$ is asymptotically stable for the system (9).

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- Note that the LPV system (6) with parameter trajectories in $\mathscr{P}_{\geqslant \bar{T}}$ is asymptotically stable if and only if the set $\mathcal{A}$ is asymptotically stable for the system (9).
- To prove the stability of this set, let us consider the Lyapunov function

$$V(x,\tau,\rho) = \begin{cases} x^T S(\tau,\rho)x & \text{if } \tau \leq \bar{T}, \\ x^T S(\bar{T},\rho)x & \text{if } \tau > \bar{T}. \end{cases} \quad (16)$$

  where $S(\tau,\rho) \succ 0$ for all $\tau \in [0, \bar{T}_{max}]$ and all $\rho \in \mathcal{P}$.
- Applying then the conditions of Theorem 6 yields the result.

Connection with quadratic and robust stability

Theorem (Quadratic stability)

When $\bar{T} \to 0$ *in the minimum dwell-time theorem, then we recover the quadratic stability condition*

$$A(\theta)^T P + PA(\theta) \prec 0, \; \theta \in \mathcal{P}. \tag{17}$$

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Theorem (Robust stability)

When $\bar{T} \to \infty$, then we recover the robust stability condition

$$\sum_{i=1}^{N} \partial_{\rho_i} P(\theta)\mu_i + A(\theta)^T P(\theta) + P(\theta)A(\theta) \prec 0, \ \theta \in \mathcal{P}, \ \mu \in \mathcal{D}. \tag{18}$$

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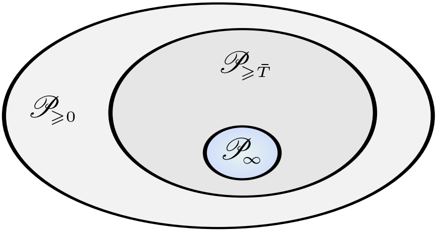

Computational aspects [Par00, PAV+13]

- We say that a symmetric polynomial matrix $M(\theta)$, $\theta \in \mathbb{R}^N$, is an SOS matrix if there exists a matrix $Q(\theta)$ such that $M(\theta) = Q(\theta)^T Q(\theta)$. An SOS matrix is positive semidefinite for all $\theta \in \mathbb{R}^N$. Checking whether $M(\theta)$ is an SOS matrix can be cast as an SDP [Par00]
- Now assume that we would like to prove that a matrix $M(\theta)$ is positive semidefinite for all $\theta \in \mathcal{P}$ where $\mathcal{P}$ is defined as

$$\mathcal{P} := \left\{\theta \in \mathbb{R}^N : g_i(\theta) \geq 0, i = 1, \ldots, b\right\}, \ g_i\text{'s are polynomials.} \tag{19}$$

- This is true if we can find SOS matrices $\Gamma_i(\theta)$, $i = 1, \ldots, b$, such that the matrix

$$M(\theta) - \sum_{i=1}^{b} \Gamma_i(\theta) g_i(\theta) \ \text{ is an SOS matrix.} \tag{20}$$

- If the above condition holds, then

$$M(\theta) \succeq \sum_{i=1}^{b} \Gamma_i(\theta) g_i(\theta) \tag{21}$$

  where the right-hand side is positive semidefinite for all $\theta \in \mathcal{P}$.
- The package SOSTOOLS [PAV+13] can be used to formalize and check SOS conditions

Example 1

System

- Let us consider the system [XSF97]

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2-\rho & -1 \end{bmatrix} x \tag{22}$$

  where $\rho(t) \in \mathcal{P} = [0, \bar{\rho}]$, $\bar{\rho} > 0$.
- It is known [XSF97] that this system is quadratically stable if and only if $\bar{\rho} \leq 3.828$
- This bound can be improved in the case of piecewise constant parameters provided that discontinuities do not occur too often [Bri15b].

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Results

- We choose polynomials of order 4, which corresponds to an SDP with 2409 primal variables and 315 dual variables.
- Building this program takes 6.04 seconds while solving it takes 1.25 second.

Example 1

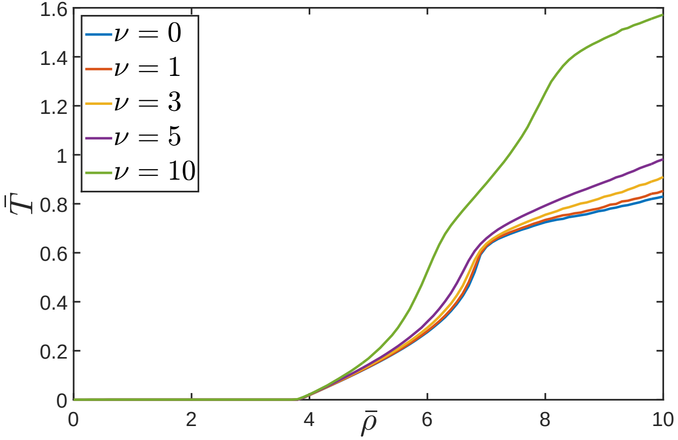

Figure: Evolution of the computed minimum upper-bound on the minimum stability-preserving minimum dwell-time with $|\dot{\rho}| \leq \nu$ using an SOS approach with polynomials of degree 4.

Example 2

- Let us consider the system [Wu95]

$$\dot{x} = \begin{bmatrix} 3/4 & 2 & \rho_1 & \rho_2 \\ 0 & 1/2 & -\rho_2 & \rho_1 \\ -3\upsilon\rho_1/4 & \upsilon\left(\rho_2 - 2\rho_1\right) & -\upsilon & 0 \\ -3\upsilon\rho_2/4 & \upsilon\left(\rho_1 - 2\rho_2\right) & 0 & -\upsilon \end{bmatrix} x \tag{23}$$

  where $\upsilon = 15/4$ and $\rho \in \mathcal{P} = \{z \in \mathbb{R}^2 : ||z||_2 = 1\}$. This system is not quadratically stable.

- We define $\rho_1(t) = \cos(\beta(t))$ and $\rho_2(t) = \sin(\beta(t))$ where $\beta(t)$ is piecewise differentiable.
- Differentiating these equalities yields $\dot{\rho}_1(t) = -\dot{\beta}(t)\rho_2(t)$ and $\dot{\rho}_2(t) = \dot{\beta}(t)\rho_1(t)$ where $\dot{\beta}(t) \in [-\nu, \nu]$, $\nu \geq 0$,

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Table: Evolution of the computed minimum upper-bound on the minimum dwell-time with $|\dot{\beta}| \leq \nu$ using an SOS approach with polynomials of degree d. The number of primal/dual variables of the semidefinite program and the preprocessing/solving time are also given.

	$\nu = 0$	$\nu = 0.1$	$\nu = 0.3$	$\nu = 0.5$	$\nu = 0.8$	$\nu = 0.9$	p/d vars.	time (sec)
$d = 2$	2.7282	2.9494	3.5578	4.6317	11.6859	26.1883	9820/1850	20/27
$d = 4$	1.7605	1.8881	2.2561	2.9466	6.4539	num. err.	43300/4620	212/935

Outline

Setup

System

- Let us consider the LPV system

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Control laws

- Continuous-time controllers

$$u(t) = \begin{cases} K(t - t_k, \rho(t_k))x(t), & t \in [t_k, t_k + \bar{T})\\ K(\bar{T}, \rho(t_k))x(t), & t \in [t_k + \bar{T}, t_{k+1}) \end{cases} \quad (24)$$

where $\{t_k\}_{k\in\mathbb{Z}_{>0}}$ is the sequence of time instants at which the parameter vector changes value.

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  where $\{t_k\}_{k \in \mathbb{Z}_{>0}}$ is the sequence of time instants at which the parameter vector changes value.

- Sampled-data controllers

$$u(t_k + \tau) = K_1(\rho(t_k))x(t_k) + K_2(\rho(t_k))u(t_k), \;\; \tau \in (0, T_k], \;\; T_k \in [T_{min}, T_{max}] \quad (25)$$

  where $\{t_k\}_{k \in \mathbb{Z}_{>0}}$ is the sequence of time instants at which the control is updated.

Continuous state-feedback control - Minimum dwell-time

Theorem

Let $\bar{T} > 0$ *be given. Assume that there exist matrix-valued functions* $U : [0, \bar{T}] \times \mathcal{P} \to \mathbb{R}^{m \times n}$ *and* $\tilde{S} : [0, \bar{T}] \times \mathcal{P} \to \mathbb{S}^n_{\succ 0}$ *such that the conditions*

$$-\partial_\tau \tilde{S}(\tau, \theta) - \partial_\rho \tilde{S}(\tau, \theta)\nu + \mathrm{Sym}[A(\theta)\tilde{S}(\tau, \theta) + B(\theta)U(\tau, \theta)] \preceq 0 \tag{26}$$

$$\mathrm{Sym}[A(\theta)\tilde{S}(\bar{T}, \theta) + B(\theta)U(\bar{T}, \theta)] \prec 0, \tag{27}$$

and

$$\tilde{S}(\bar{T}, \eta) - \tilde{S}(0, \theta) \prec 0 \tag{28}$$

hold for all $\theta, \eta \in \mathcal{P}$ *and all* $\tau \in [0, \bar{T}]$.

Then the closed-loop LPV system is asymptotically stable for all $\rho \in \mathscr{P}_{\geqslant \bar{T}}$, *and a suitable controller gain is moreover given by*

$$K(\tau, \theta) = U(\tau, \theta)\tilde{S}(\tau, \theta)^{-1}. \tag{29}$$

Example

System

$$\dot{x} = \begin{bmatrix} 3-\theta & 1 \\ 1-\theta & 2+\theta \end{bmatrix} x + \begin{bmatrix} 1 \\ 1+\theta \end{bmatrix} u,\ \ \theta \in \mathcal{P} = [0,1],\ \ \mathcal{D} = [-\nu, \nu]. \tag{30}$$

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Proposition

No control law of the form $u = K(\theta)x$ *can quadratically stabilize the system* (30).

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Proposition

No control law of the form $u = K(\theta)x$ can quadratically stabilize the system (30)*.*

Proof

- Quadratically stabilizable if and only if the LMI (elimination lemma)

$$L(\theta) := B_\perp(\theta)[A(\theta)P + PA(\theta)^T]B_\perp(\theta)^T \prec 0$$

is feasible for all $\theta \in [0,1]$ where $B_\perp(\theta) = \begin{bmatrix} 1+\theta & -1 \end{bmatrix}$.

Example

System

$$\dot{x} = \begin{bmatrix} 3-\theta & 1 \\ 1-\theta & 2+\theta \end{bmatrix} x + \begin{bmatrix} 1 \\ 1+\theta \end{bmatrix} u,\ \theta \in \mathcal{P} = [0,1],\ \mathcal{D} = [-\nu, \nu]. \qquad (30)$$

Proposition

No control law of the form $u = K(\theta)x$ can quadratically stabilize the system (30).

Proof

- Quadratically stabilizable if and only if the LMI (elimination lemma)

  $$L(\theta) := B_\perp(\theta)[A(\theta)P + PA(\theta)^T]B_\perp(\theta)^T \prec 0$$

  is feasible for all $\theta \in [0,1]$ where $B_\perp(\theta) = \begin{bmatrix} 1+\theta & -1 \end{bmatrix}$.
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- But $f_1(p) < 0 \Leftrightarrow p \in (1,2)$ and $f_2(p) < 0 \Leftrightarrow p \in (2,4)$, a contradiction.

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- When $d = 2$ the number of primal/dual variables is 834/180 whereas, when $d = 3$, this number is 2414/315.
- When $d = 2$, it takes roughly 2.62sec; when $d = 3$, it takes around 6.31sec.

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$$\rho(t_k + \tau) = \frac{1 + \sin(2\nu(t_k + \tau) + \varphi_k)}{2}, \varphi_k \in \mathcal{U}(0, 2\pi), \tau \in (0, T_k], k \in \mathbb{Z}_{\geq 0}. \quad (31)$$

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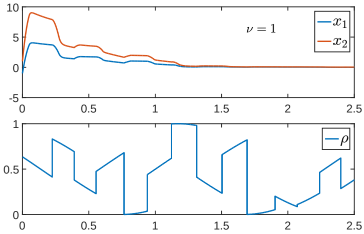

Outline

A preliminary stability result

We are interested here in deriving a stability result under a range dwell-time constraint for the sequence of jumping instants, that is, for all sequences of jumping instants in

$$\mathscr{T} := \left\{ \{t_k\}_{k\in\mathbb{Z}_{>0}} \;\middle|\; \begin{array}{c} t_{k+1} - t_k \in [T_{min}, T_{max}], \\ t_0 = 0,\ k \in \mathbb{Z}_{\geq 0} \end{array} \right\} \tag{32}$$

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for some $0 \leq T_{min} \leq T_{max} < \infty$. The corresponding hybrid system is given by

$$\begin{array}{l} \left\{ \left. \begin{array}{rcl} \dot{x}(t) & = & A(\rho(t))x(t) \\ \dot{\rho}(t) & \in & \mathcal{Q}(\rho(t)) \\ \dot{\tau}(t) & = & 1 \\ \dot{T}(t) & = & 0 \end{array} \right| \text{if } (x(t), \rho(t), \tau(t), T(t)) \in C \right\} \\ \left\{ \left. \begin{array}{rcl} x(t^+) & = & J(\rho(t))x(t) \\ \rho(t^+) & = & \rho(t) \\ \tau(t^+) & = & 0 \\ T(t^+) & \in & [T_{min}, T_{max}] \end{array} \right| \text{if } (x(t), \rho(t), \tau(t), T(t)) \in D \right\} \end{array} \tag{33}$$

where

$$\begin{array}{rcl} C & = & \mathbb{R}^n \times \mathcal{P} \times E_<, \\ D & = & \mathbb{R}^n \times \mathcal{P} \times E_=, \\ E_\square & = & \{\phi \in \mathbb{R}_{\geq 0} \times [T_{min}, T_{max}] : \phi_1 \square \phi_2\},\;\; \square \in \{<, =\}. \end{array} \tag{34}$$

General result

Theorem (Range dwell-time)

Let the scalars $0 < T_{min} \leq T_{max} < \infty$ be given and assume that there exist a bounded continuously differentiable matrix-valued function $S : [0, T_{max}] \times \mathcal{P} \mapsto \mathbb{S}^n_{\succ 0}$ and a scalar $\varepsilon > 0$ such that the conditions

$$-\partial_\tau S(\tilde{\tau}, \theta) + \partial_\rho S(\tilde{\tau}, \theta)\mu + \mathrm{Sym}[S(\tilde{\tau}, \theta)A(\theta)] \preceq 0 \qquad (35)$$

and

$$J(\theta)S(\sigma, \theta)J(\theta) - S(0, \theta) + \varepsilon I_n \preceq 0 \qquad (36)$$

hold for all $\theta \in \mathcal{P}$, all $\mu \in \mathcal{D}$, all $\tilde{\tau} \in [0, T_{max}]$ and all $\sigma \in [T_{min}, T_{max}]$.

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$$\mathscr{P}_\infty := \left\{ \rho : \mathbb{R}_{\geq 0} \mapsto \mathcal{P} \,\middle|\, \dot{\rho}(t) \in \mathcal{Q}(\rho(t)), t \geq 0 \right\}. \tag{37}$$

is asymptotically stable under the range dwell-time condition $[T_{min}, T_{max}]$*; i.e. for all sequences of jumping instants in* $\mathscr{T}$*.*

Problem formulation

- The sampled-data control input is assumed to be

$$u(t_k + \tau) = K_1(\rho(t_k))x(t_k) + K_2(\rho(t_k))u(t_k),\ \tau \in (0, T_k],\ T_k \in [T_{min}, T_{max}] \quad (38)$$

where $K_1(\cdot) \in \mathbb{R}^{m\times n}$ and $K_2(\cdot) \in \mathbb{R}^{m\times m}$ are the gains to be determined.

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- The hybrid system associated with the closed-loop system

$$\left.\begin{array}{l} \left\{\begin{array}{rcl} \dot{x}(t) &=& A(\rho(t))x(t) + B(\rho(t))u(t) \\ \dot{u}(t) &=& 0 \\ \dot{\rho}(t) &\in& \mathcal{Q}(\rho(t)) \\ \dot{\tau}(t) &=& 1 \\ \dot{T}(t) &=& 0 \end{array}\right| \text{if } (z(t),\rho(t),\tau(t),T(t)) \in C \right\} \\ \left\{\begin{array}{rcl} x(t^+) &=& J(\rho(t))x(t) \\ u(t^+) &=& K_1(\rho(t))x(t) + K_2(\rho(t))u(t) \\ \rho(t^+) &=& \rho(t) \\ \tau(t^+) &=& 0 \\ T(t^+) &\in& [T_{min}, T_{max}] \end{array}\right| \text{if } (z(t),\rho(t),\tau(t),T(t)) \in D \right\} \end{array} \quad (39)$$

  where $z = (x, u)$ and

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Main result

Define

$$\tilde{A}(\rho) := \begin{bmatrix} A(\rho) & B(\rho) \\ 0 & 0 \end{bmatrix}, \tilde{J}(\rho) := \begin{bmatrix} J(\rho) & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B} := \begin{bmatrix} 0 \\ I_m \end{bmatrix} \text{ and } \tilde{K}(\rho) := \begin{bmatrix} K_1(\rho) & K_2(\rho) \end{bmatrix}.$$

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Theorem

Let $\bar{T} \in \mathbb{R}_{>0}$ be given and assume that there exist a bounded continuously differentiable matrix-valued function $R : [0, T_{max}] \times \mathcal{P} \mapsto \mathbb{S}^{n+m}_{\succ 0}$, a matrix-valued function $U : \mathcal{P} \mapsto \mathbb{R}^{m \times (n+m)}$ and a scalar $\varepsilon > 0$ such that the conditions

$$\partial_\tau R(\tilde{\tau}, \theta) - \partial_\rho R(\tilde{\tau}, \theta)\mu + \mathrm{Sym}[\tilde{A}(\theta) R(\tilde{\tau}, \theta)] + \varepsilon I_n \preceq 0 \tag{41}$$

and

$$\begin{bmatrix} R(\sigma, \theta) & \tilde{J}(\theta) R(0, \theta) + \tilde{B} U(\theta) \\ \star & -R(0, \theta) \end{bmatrix} \preceq 0 \tag{42}$$

hold for all $\theta \in \mathcal{P}$, all $\mu \in \mathcal{D}^v$, all $\tilde{\tau} \in [0, T_{max}]$ and all $\sigma \in [T_{min}, T_{max}]$.

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hold for all $\theta \in \mathcal{P}$, all $\mu \in \mathcal{D}^v$, all $\tilde{\tau} \in [0, T_{max}]$ and all $\sigma \in [T_{min}, T_{max}]$. Then, the sampled-data LPV system with parameter trajectories in $\mathscr{P}_\infty$ is asymptotically stable under the range dwell-time condition $[T_{min}, T_{max}]$ (i.e. for all sequences of sampling instants in $\mathscr{T}$) with the controller gain $\tilde{K}(\theta) = U(\theta)R(\bar{T}, \theta)^{-1}$.

Example

- Let us consider now the system [GMFP15]

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 + 0.6\rho \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \ \mathcal{P} = [-1, 1], \ \mathcal{D} = [-\nu, \nu]. \tag{43}$$

- Choosing $d = 4$, we can show that, for both $\nu = 0.2$ and $\nu = 1$, we can find a controller that stabilizes the system for all $T_k \in [0.001, 1.3]$ in approximately 25sec
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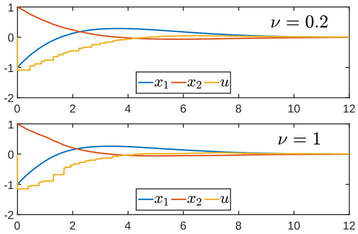

Conclusions

Concluding statements

- We can capture discontinuities in the parameters trajectories in a tractable way
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What else can be done ?

- Dissipativity analysis $\rightarrow$ IQC, multipliers, separators, scalings
- Performance analysis, e.g. L_2-performance
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An open question

Is it possible to obtain tractable conditions for the design a dynamic output feedback?

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Thanks everyone for your attention!
Any questions?