Design of a *Real-Time LPV MPC* Scheme for Semi-Active Suspension Control of a Full Vehicle

Marcelo MENEZES MORATO
Olivier SENAME
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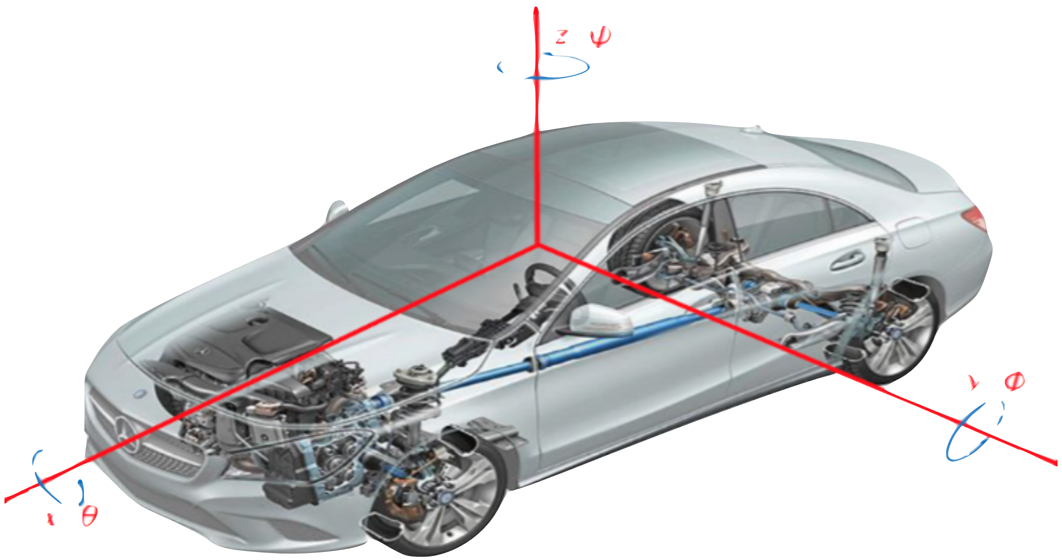
Research's Framework

- Master Project: *UFSC* & *ENSE*^{3}
- *PERSYVAL* **LPV4FTC** Project
- Internship at ***gipsa-lab***
- *Goal:* Development of Linear Parameter Varying Approaches as

  Advanced Control Techniques for Vehicle Suspension Systems

Outline

- Introduction & Motivation
- Problem Statement & Objectives
- System Model & Constraints
- Observer Design + Experimental Validation
- Optimal Solution
- Sub-Optimal Practical Solution
- Conclusions



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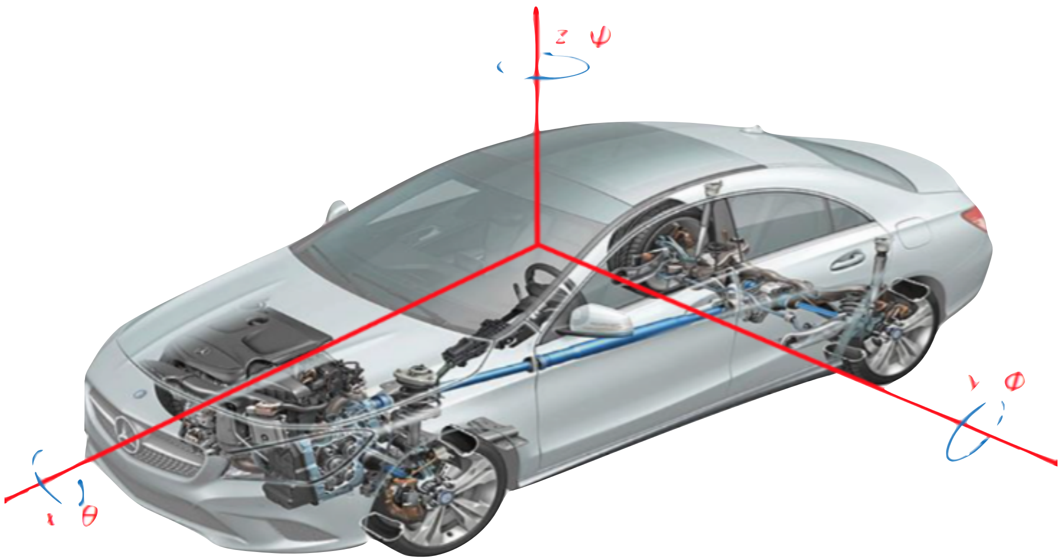

Introduction & Motivation

Modern Cars: More Safety and More Comfort!

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- Controlled Suspensions: Improve Comfort + Handling

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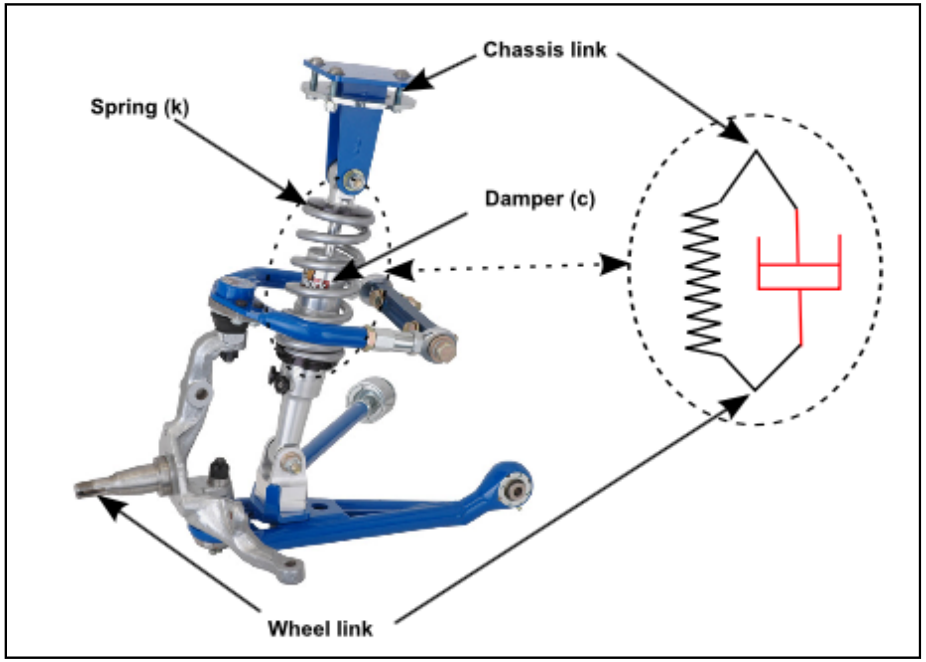

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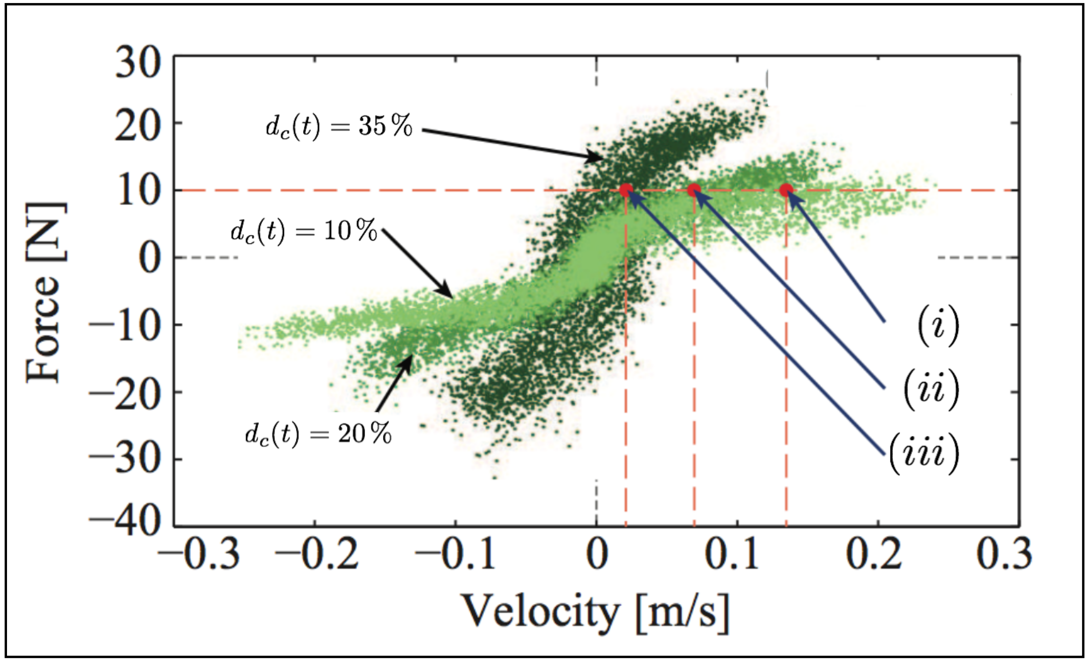

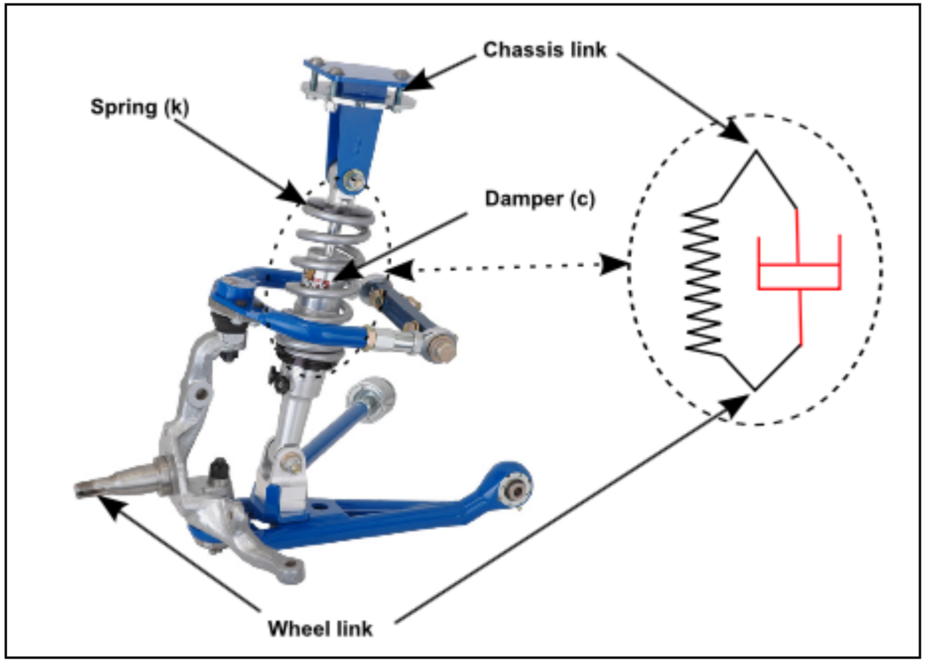

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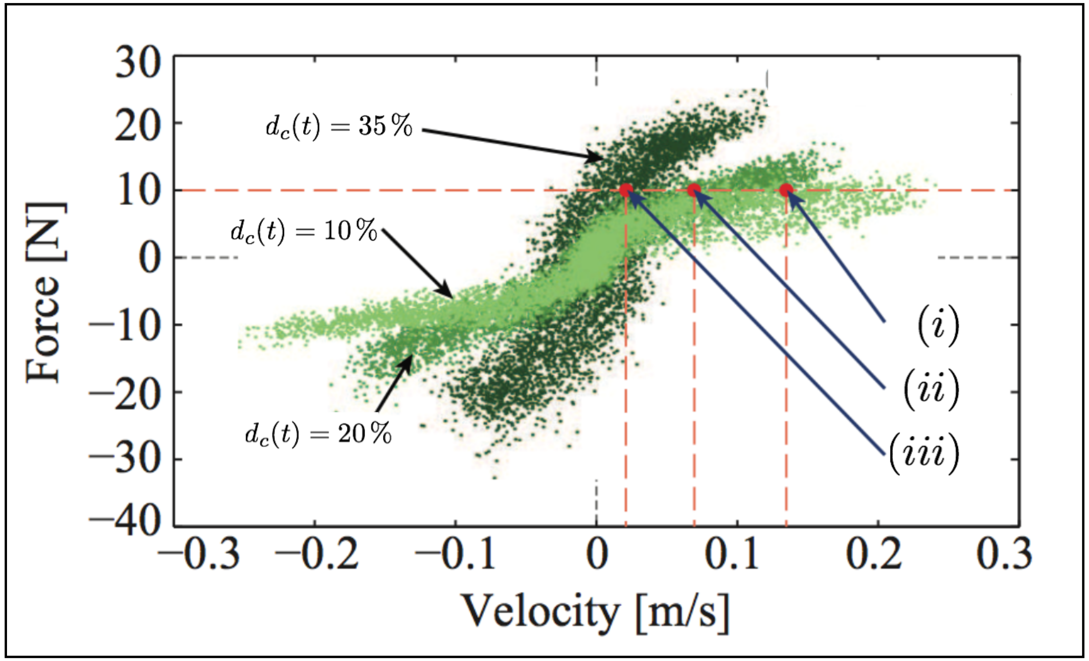
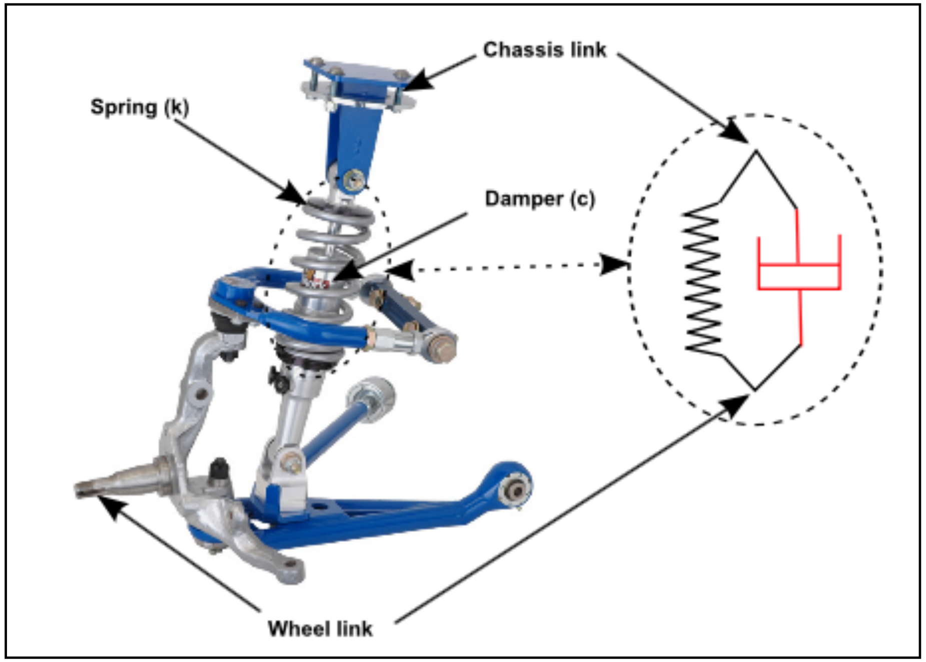
Semi-Active Suspension System

- High performances achieved
- Moderate Costs
- Problem: **Dissipativity Constraints**





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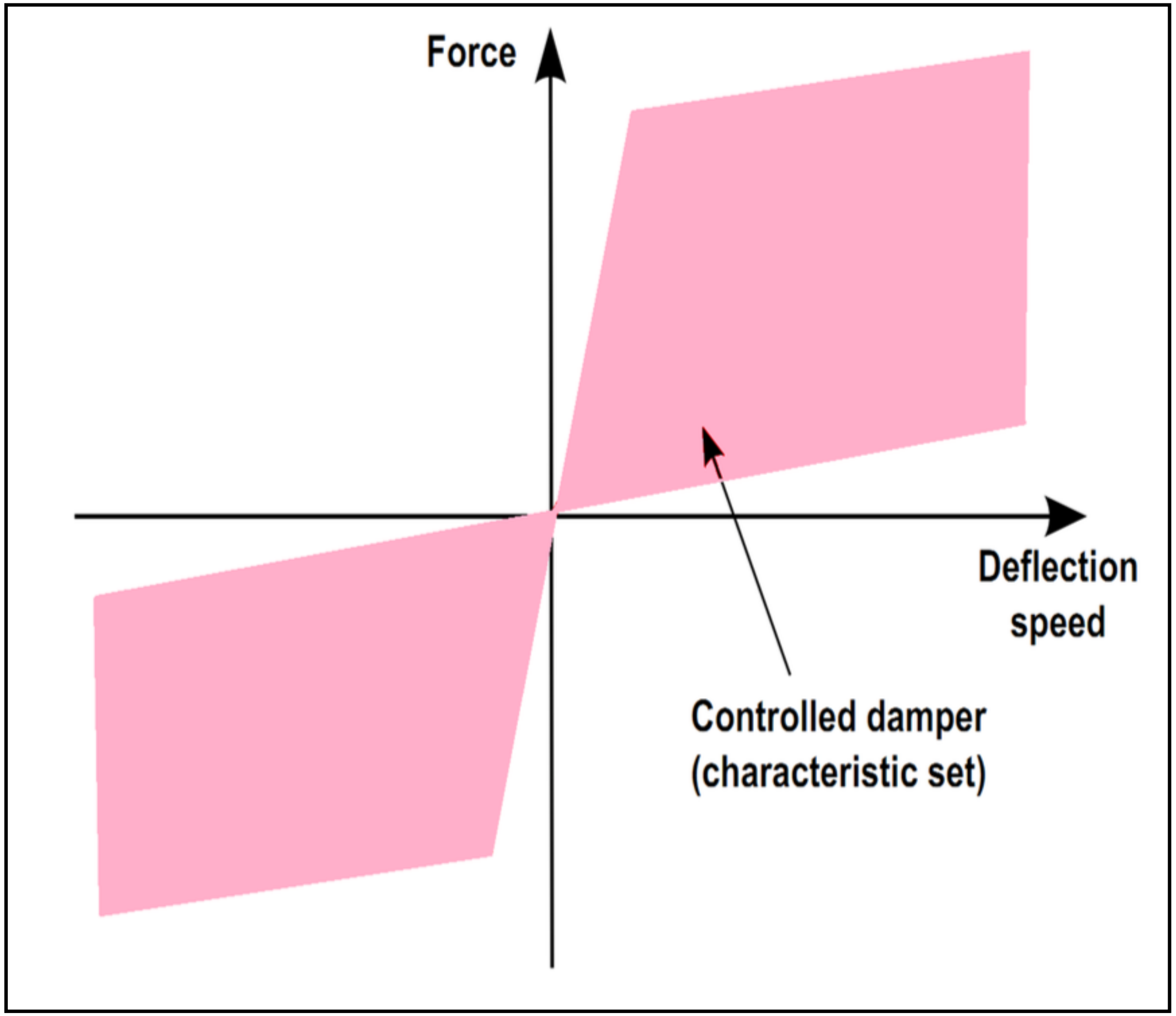

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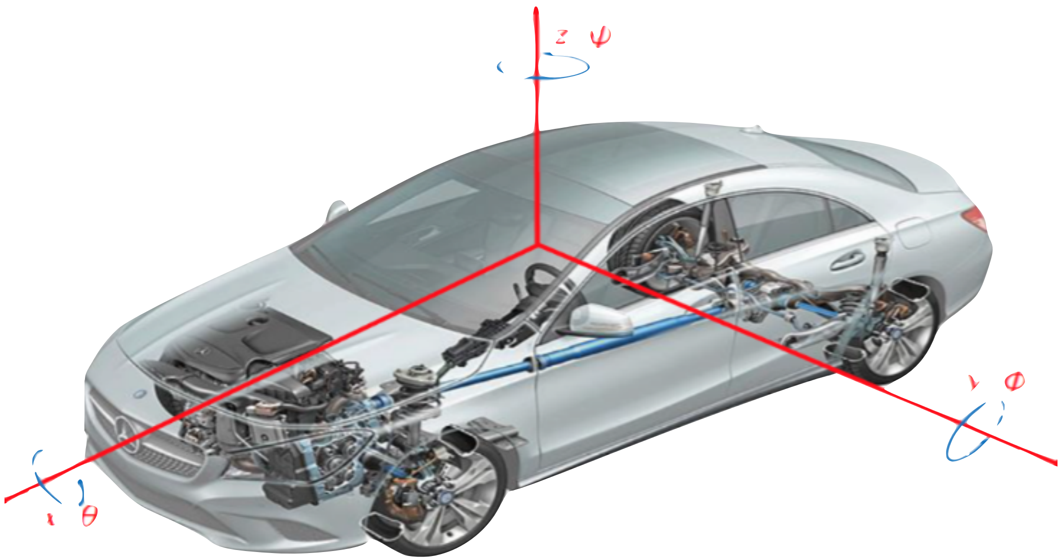

Problem Statement & Objectives

Control of **Semi-Active**
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  [Karnopp, D. (1974)]
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How to handle dissipativity constraints of dampers?

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Natural approach (process + constraints)
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Not-So-Rich Literature:

- Considering Quarter-Car Models:
  [Canale, M. (2006)]
- Clipped Analytical MPC:
  [Giorgetti, N. (2006)]
- MPC for Full Car, Road Preview
  [Sawodny, O. (2014)]
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Limited Sampling Period
for Real-Time Applications
—> 5 *ms*

Problem Statement & Objectives

Control Objectives:

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- **Use a Full Car 7-DOF Vehicle Model**
- Take into account the Dissipativity Constraints of all 4 Semi-Active Dampers
- Compute MPC Law in less then *5 ms* !
- Prediction of States and Road Disturbances ? —> Extended Observer

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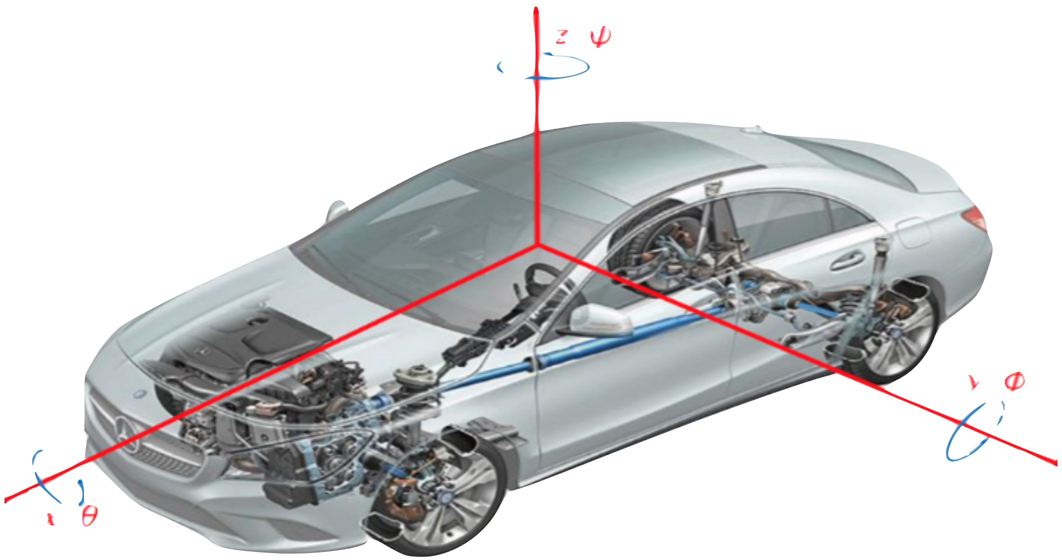
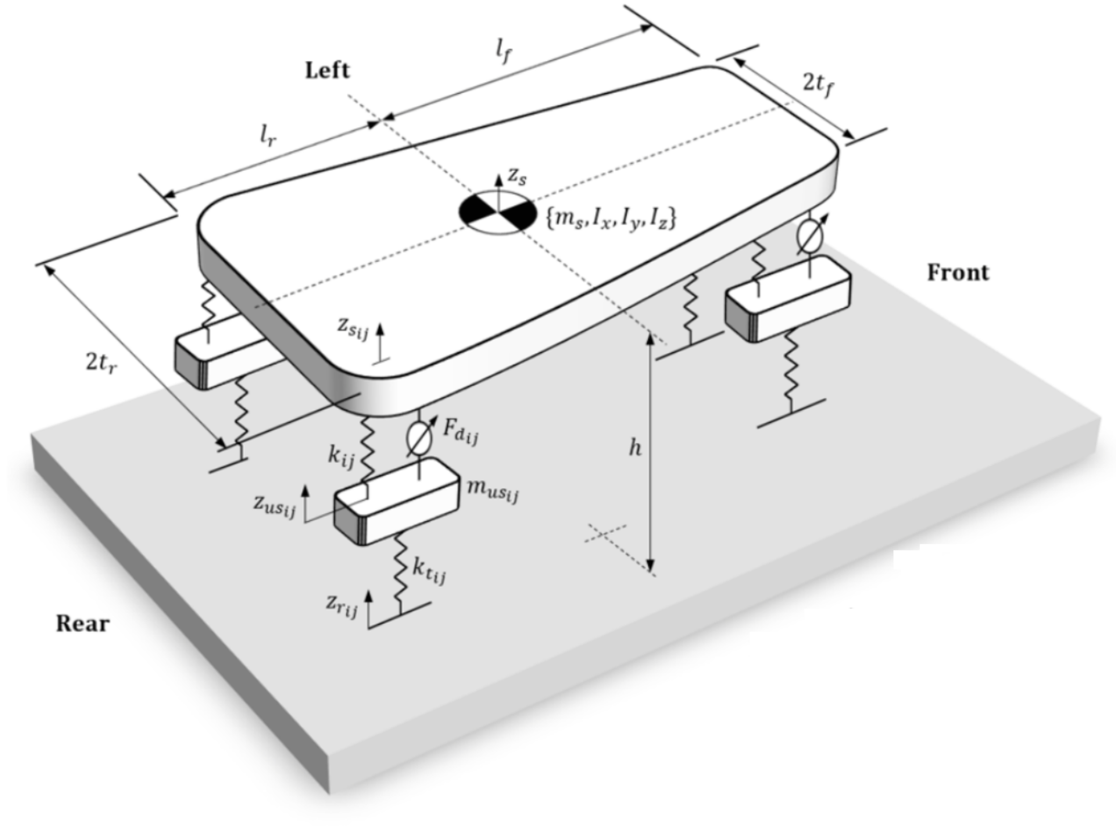


System Model & Constraints

Full Vertical Suspension System Model



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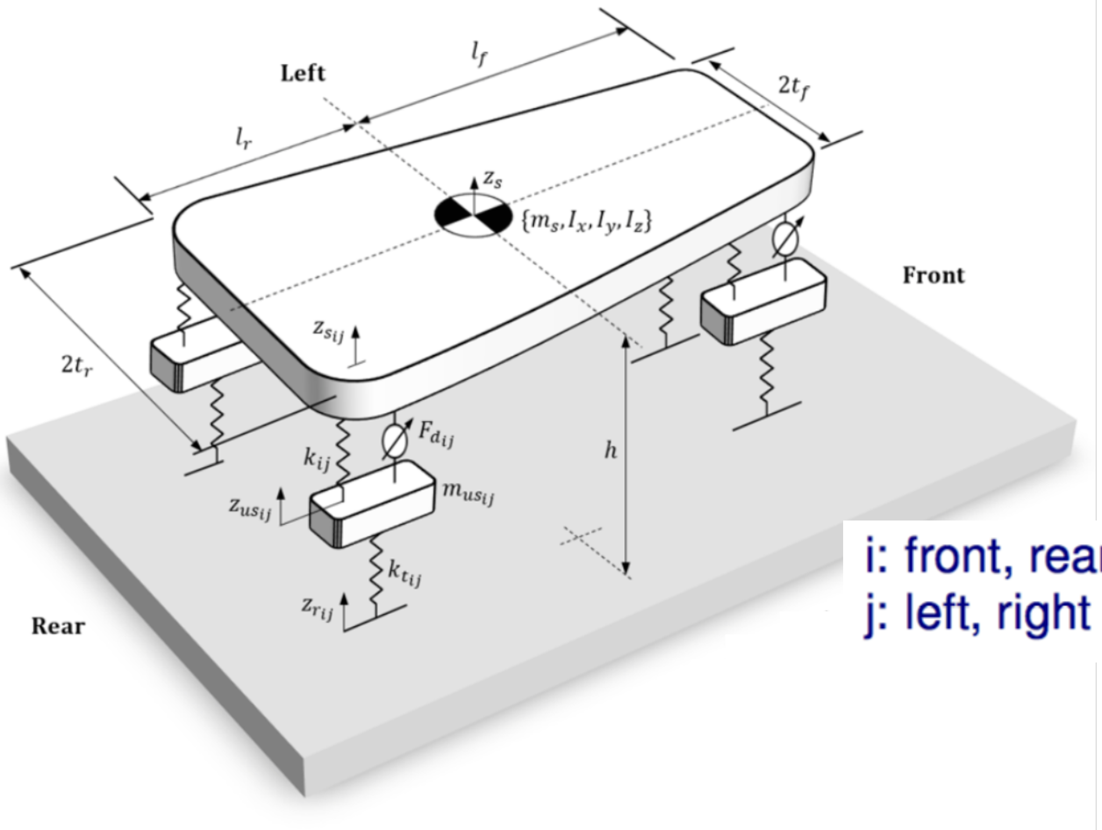

i: front, rear
j: left, right

Dynamical Equations of Motion

$$m_s.\ddot{z}_s = -F_{sfl} - F_{sfr} - F_{srl} - F_{srr}$$

$$I_x.\ddot{\theta} = (-F_{sfr} + F_{sfl}).t_f + (F_{srl} - F_{srr}).t_r$$

$$I_y.\ddot{\phi} = (F_{srr} + F_{srl}).l_r - (F_{sfr} + F_{sfl}).l_f$$

$$m_{us_{ij}}.\ddot{z_{us_{ij}}} = F_{s_{ij}} - F_{tz_{ij}}$$

System Model & Constraints

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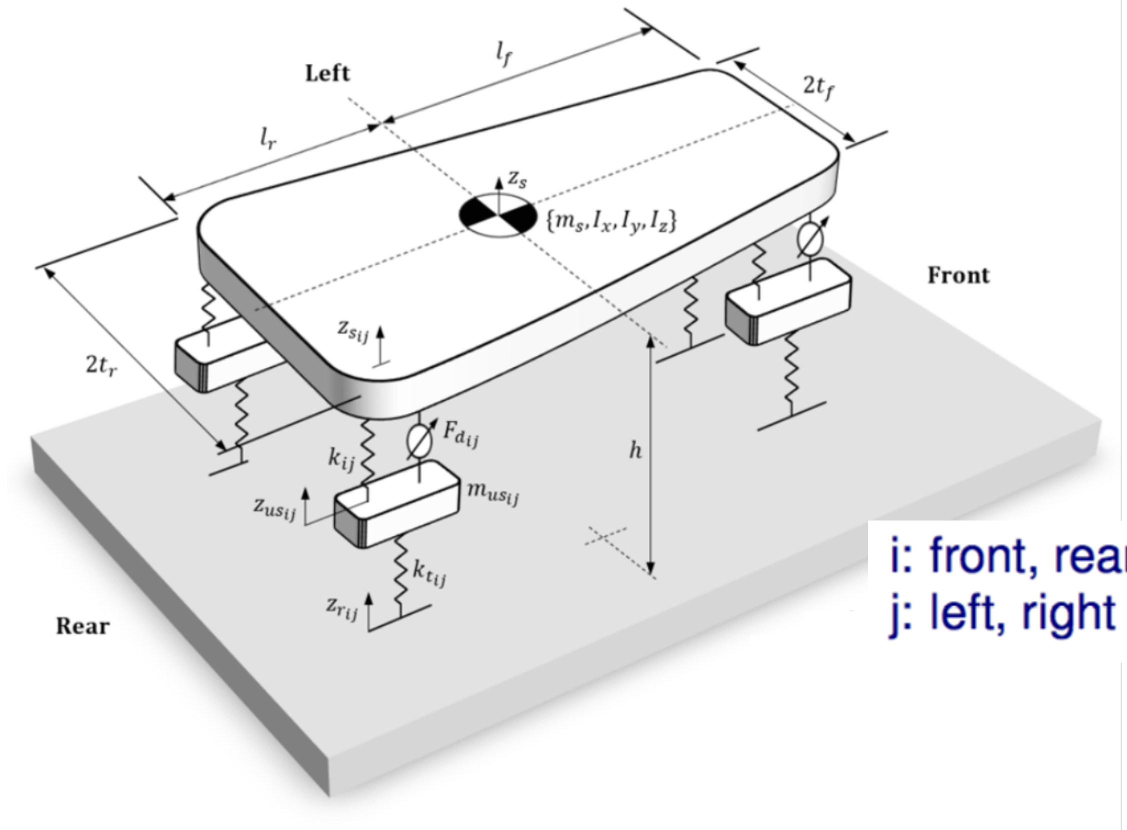

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Tire's Forces

$$F_{tz_{ij}} = k_{t_{ij}}.(z_{us_{ij}} - z_{r_{ij}})$$

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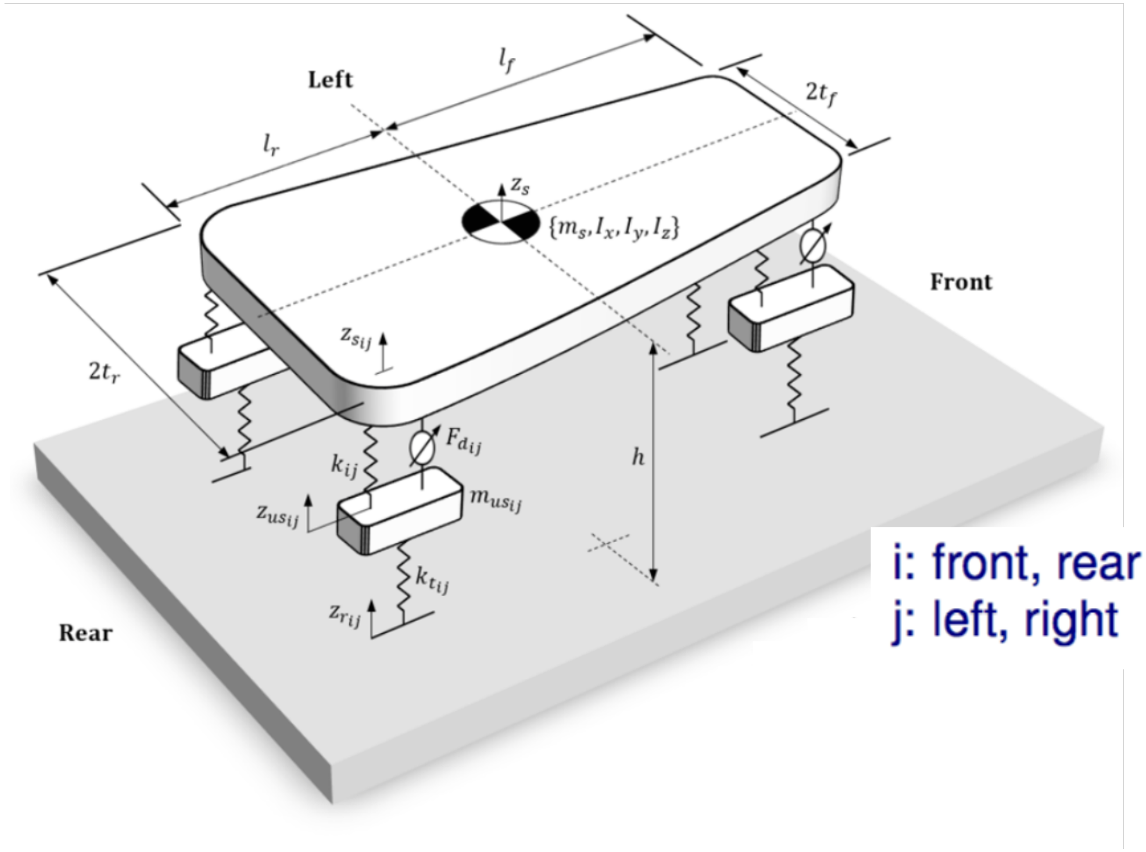

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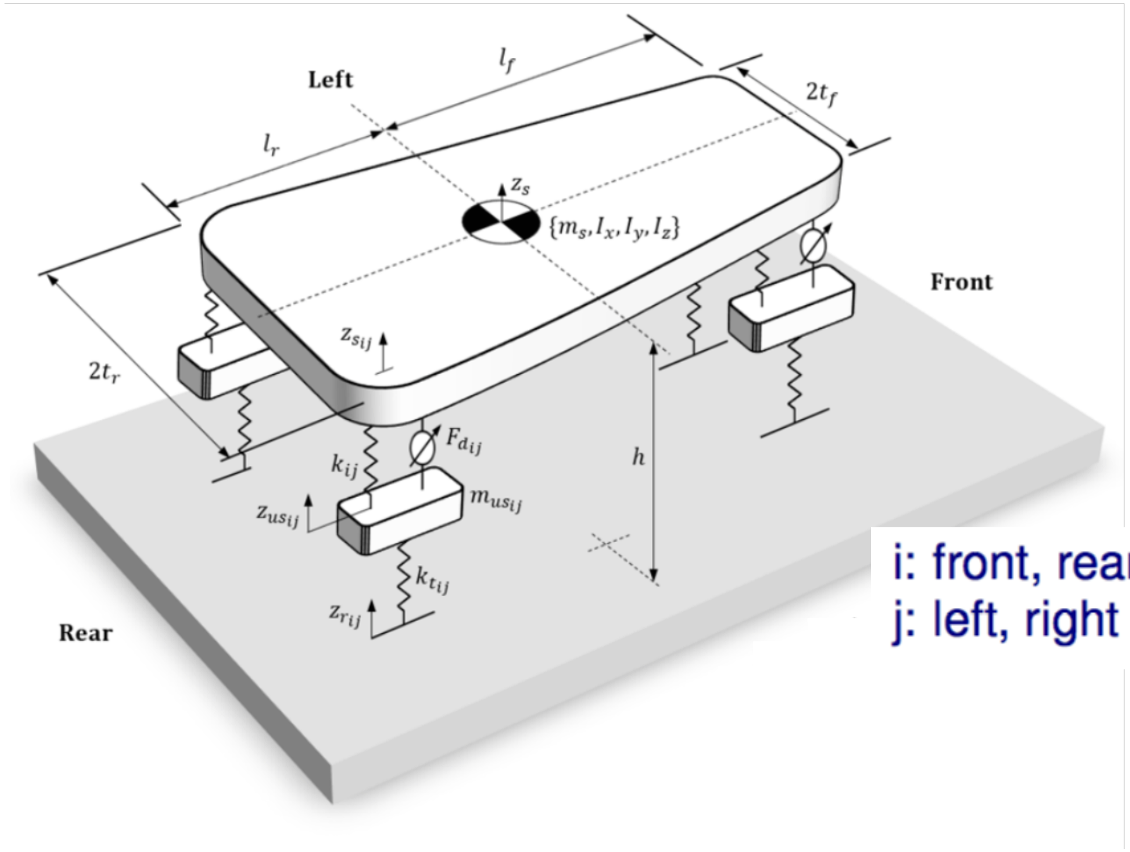
Semi-Active Suspension Forces

$$F_{s_{ij}} = k_{ij}.(z_{s_{ij}} - z_{us_{ij}}) + F_{d_{ij}}$$

System Model & Constraints

Full Vertical Suspension System Model



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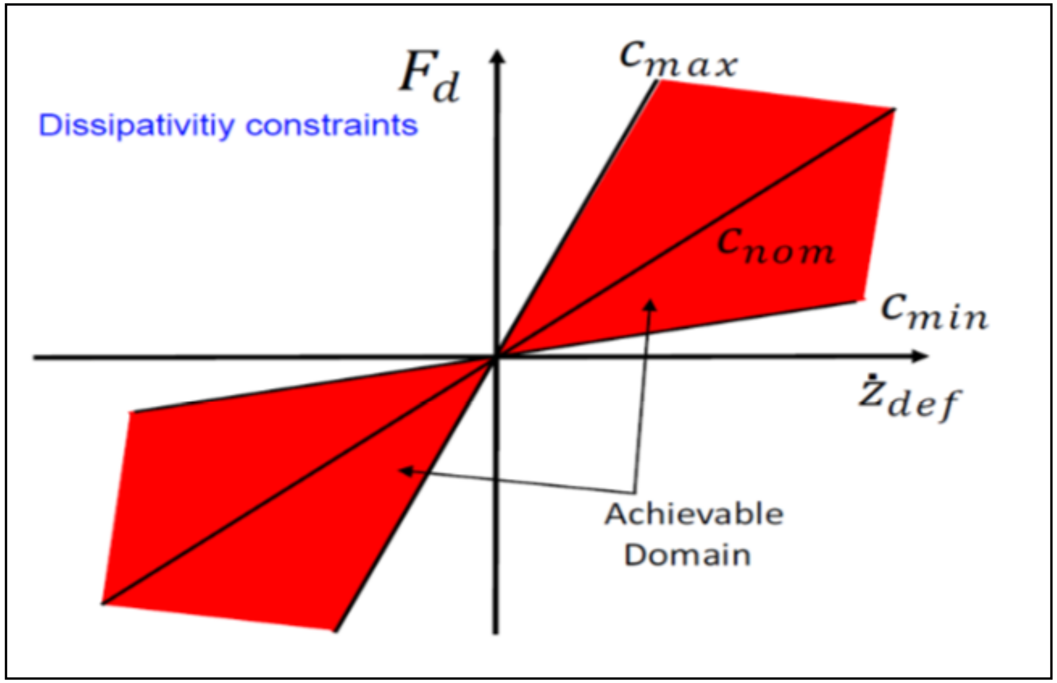
Linearization on Small Angles

$$z_{s_{fl}} = z_s - l_f.(\phi) + t_f.(\theta)$$
$$z_{s_{fr}} = z_s - l_f.(\phi) - t_f.(\theta)$$
$$z_{s_{rl}} = z_s + l_r.(\phi) + t_r.(\theta)$$
$$z_{s_{rr}} = z_s + l_r.(\phi) - t_r.(\theta)$$

System Model & Constraints

Damper Dissipativity Constraints



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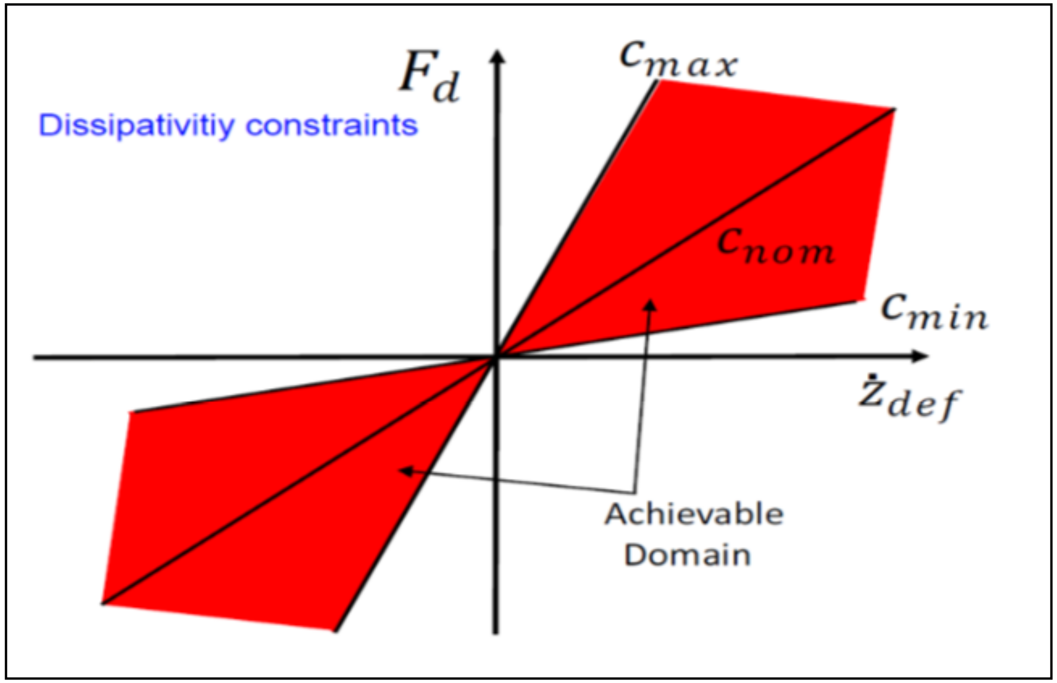
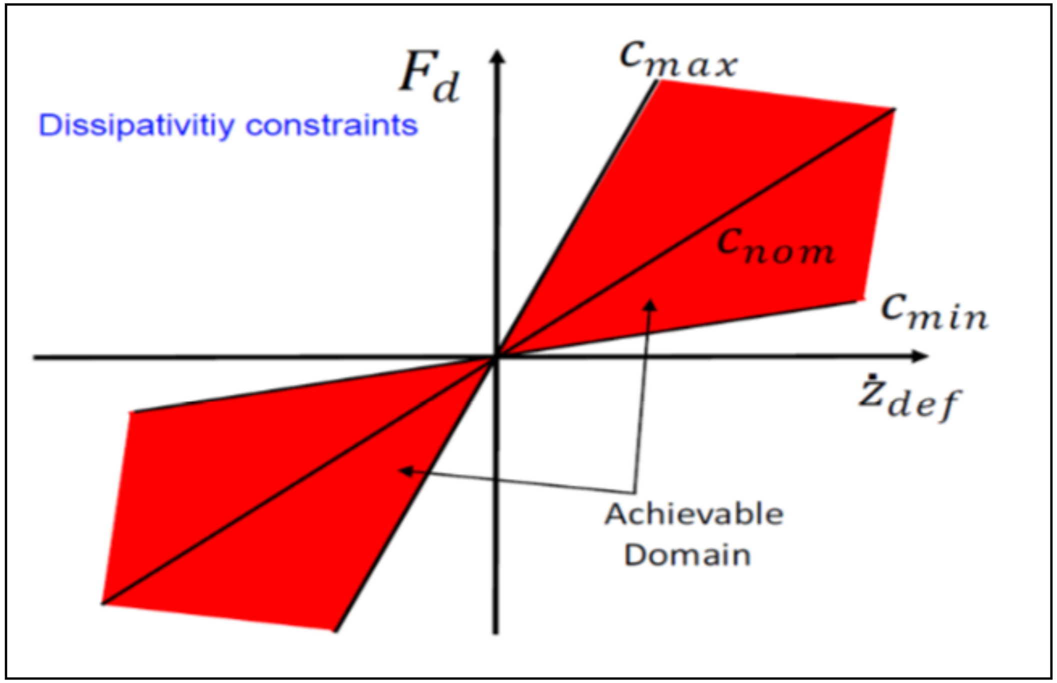


Damper Dissipativity Constraints

$$F_{d_{ij}} = c_{ij}(\cdot).\dot{z}_{def_{ij}}$$

System Model & Constraints



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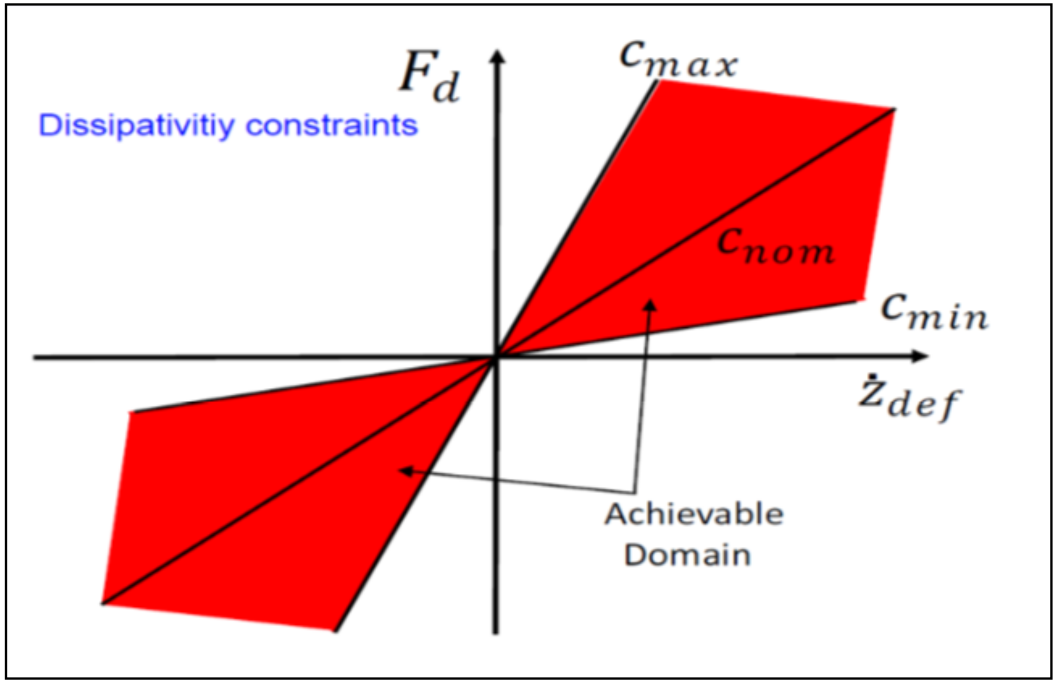

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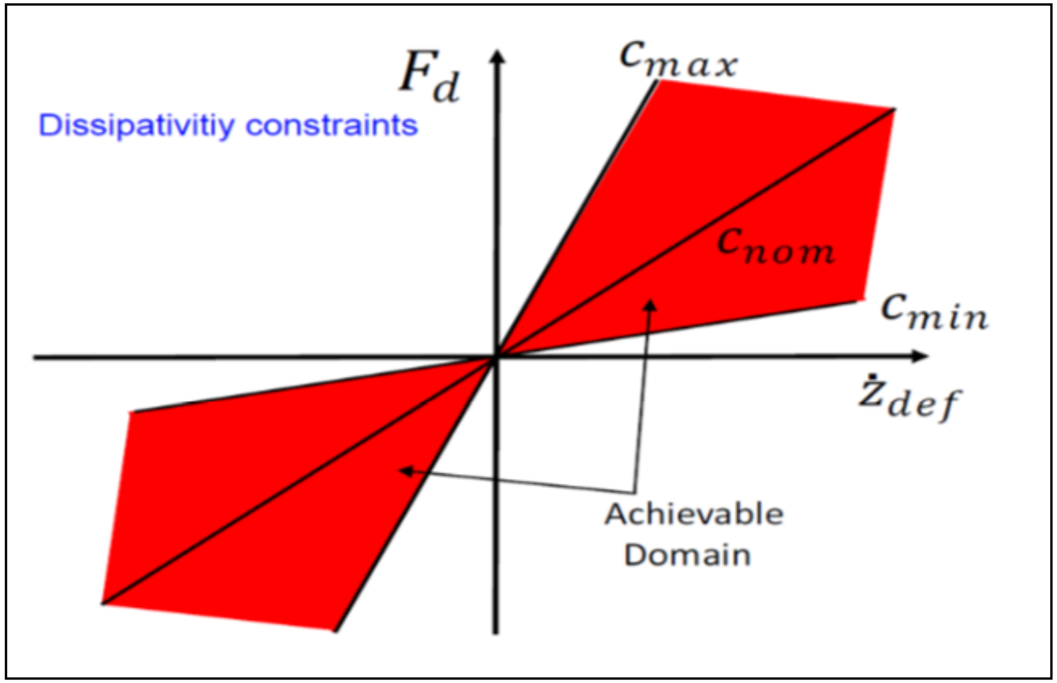

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System Model & Constraints

State-Space Representation

$$\sum_{Full\,Veh.}^{T_s} := \left\{ \begin{array}{ccccccc} x[k+1] & = & A_d.x[k] & + & B_{1d}.w[k] & + & B_{2d}.u[k] \\ y[k] & = & C_d.x[k] & + & D_{1d}.w[k] & + & D_{2d}.u[k] \end{array} \right\}$$

$$x = \left[\begin{array}{cccccccccccccc} z_s & \theta & \phi & z_{us_{fl}} & z_{us_{fr}} & z_{us_{rl}} & z_{us_{rr}} & \dot{z_s} & \dot{\theta} & \dot{\phi} & \dot{z_{us_{fl}}} & \dot{z_{us_{fr}}} & \dot{z_{us_{rl}}} & \dot{z_{us_{rr}}} \end{array} \right]$$

$$u = \left[\begin{array}{cccc} u_{fl} & u_{fr} & u_{rl} & u_{rr} \end{array} \right]$$

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$T_s = 5\ ms$

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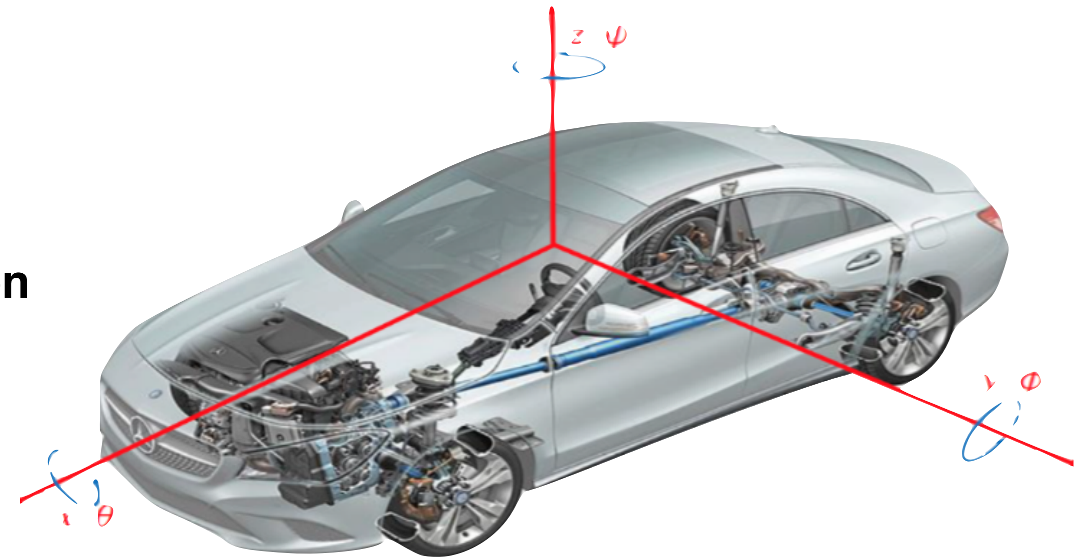

Observer Design + Experimental Validation

- **MPC —> Need for State and Disturbance Knowledge**
- Future Disturbance Estimation —> Enhance *CL* Performance
- Extended $\boldsymbol{H_2}$ Observer
- Trade-Off: Convergence Speed *vs* Noise Attenuation

Observer Design + Experimental Validation

Disturbance Preview:
[Sawodny, O. (2014)]

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$$w[k+n] = w[k]$$

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Pole Placement

$$\mathcal{C}(\mu, \varrho)$$

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H_2 Extended Observer Design

$$\sum_{Aug.Sys.}^{T_s} := \left\{ \begin{array}{lcl} \begin{bmatrix} x[k+1] \\ w[k+1] \end{bmatrix} & = & \overbrace{\begin{bmatrix} A_d & B_{1d} \\ 0 & \mathbb{I} \end{bmatrix}}^{A_{obs}} . \begin{bmatrix} x[k] \\ w[k] \end{bmatrix} + \overbrace{\begin{bmatrix} B_{2d} \\ 0 \end{bmatrix}}^{B_{obs}} .u[k] \\ y[k] & = & \underbrace{\begin{bmatrix} C_d & D_{1d} \end{bmatrix}}_{C_{obs}} . \begin{bmatrix} x[k] \\ w[k] \end{bmatrix} + D_{2d}.u[k] \end{array} \right\}$$

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Observer Design + Experimental Validation

H_2 Extended Observer Design

- Problem Definition:

$$
\begin{aligned}
||T_{e\nu}(z)||_2 \leq \gamma \quad \text{under} \quad e[k=0] = 0 \\
\lim_{k\to\infty} e[k] \to 0 \quad \text{for} \quad \nu[k] \equiv 0 \\
e[k+1] = (A_{obs} - L.C_{obs}).e[k] - L.F_u \nu[k]
\end{aligned}
$$

Observer Design + Experimental Validation

H_2 Extended Observer Design

- Problem Definition:

$$||T_{e\nu}(z)||_2 \leq \gamma \quad \text{under} \quad e[k=0]=0$$

$$\lim_{k\to\infty} e[k] \to 0 \quad \text{for} \quad \nu[k] \equiv 0$$

$$e[k+1] = (A_{obs} - L.C_{obs}).e[k] - L.F_u\nu[k]$$

H_2 Norm —> Impulse to Energy Gain!
Noise —> Estimation Error

Observer Design + Experimental Validation

H_2 Extended Observer Design

- Problem Solution:

$$\begin{bmatrix} P & P(\frac{A_{obs}-\mu.\mathbb{I}}{\varrho}) - Y.\frac{C_{obs}}{\varrho} & -Y \\ \star & P & 0 \\ \star & \star & \mathbb{I} \end{bmatrix} > 0,$$

$$\begin{bmatrix} R & \mathbb{I} & 0 \\ \star & P & 0 \\ \star & \star & \mathbb{I} \end{bmatrix} > 0,$$

$$Trace(R) < \gamma$$

Observer Design + Experimental Validation

H_2 Extended Observer Design

- Problem Solution:

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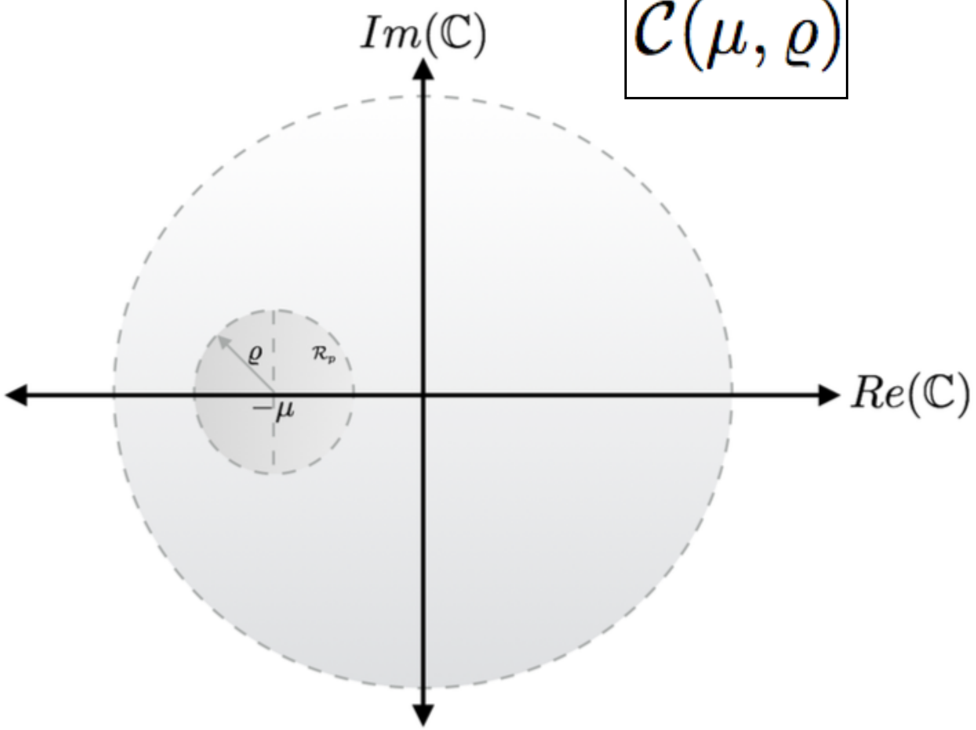
Pole Placement

$$\mathcal{C}(\mu, \varrho)$$

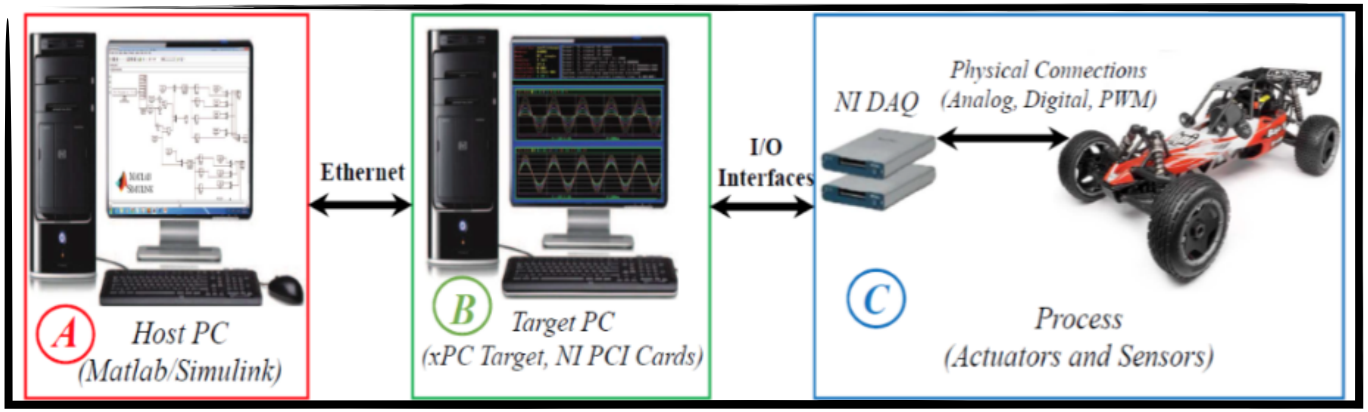


Observer Design + Experimental Validation

Experimental Test-bench:
INOVE Soben-Car

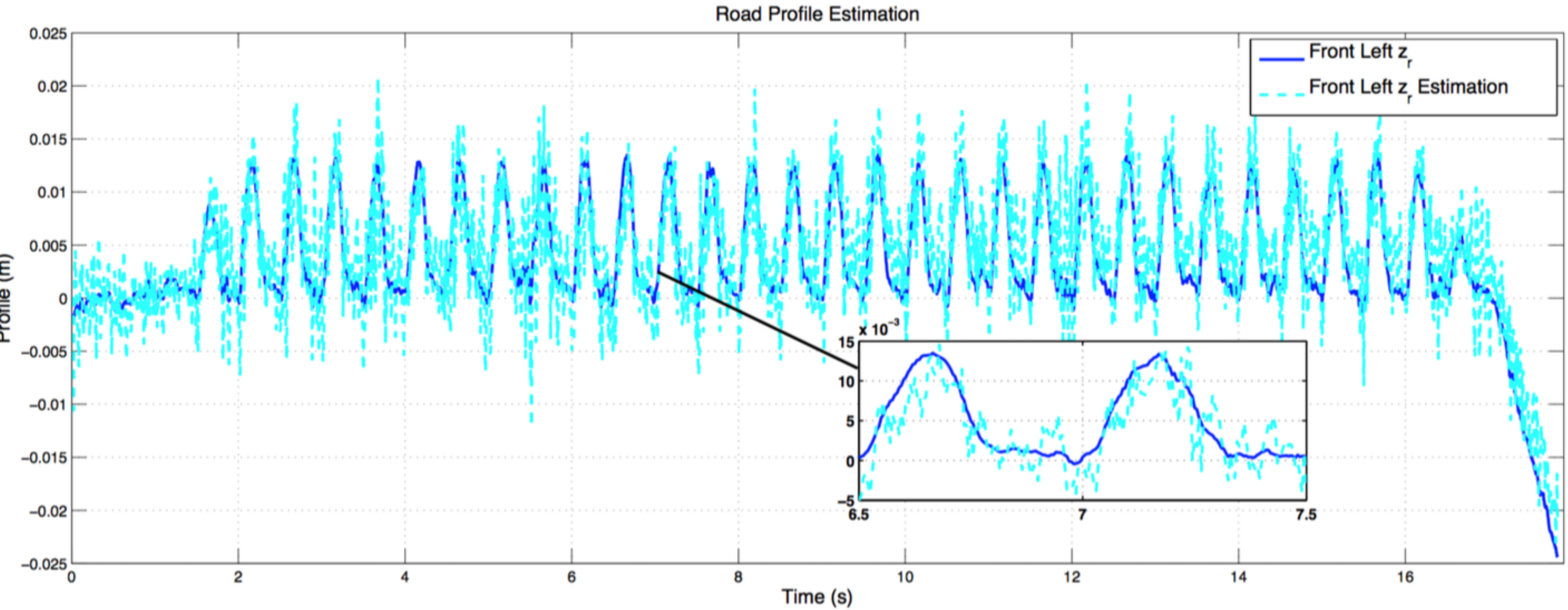
Observer Design + Experimental Validation

Experimental Test-bench:
INOVE Soben-Car



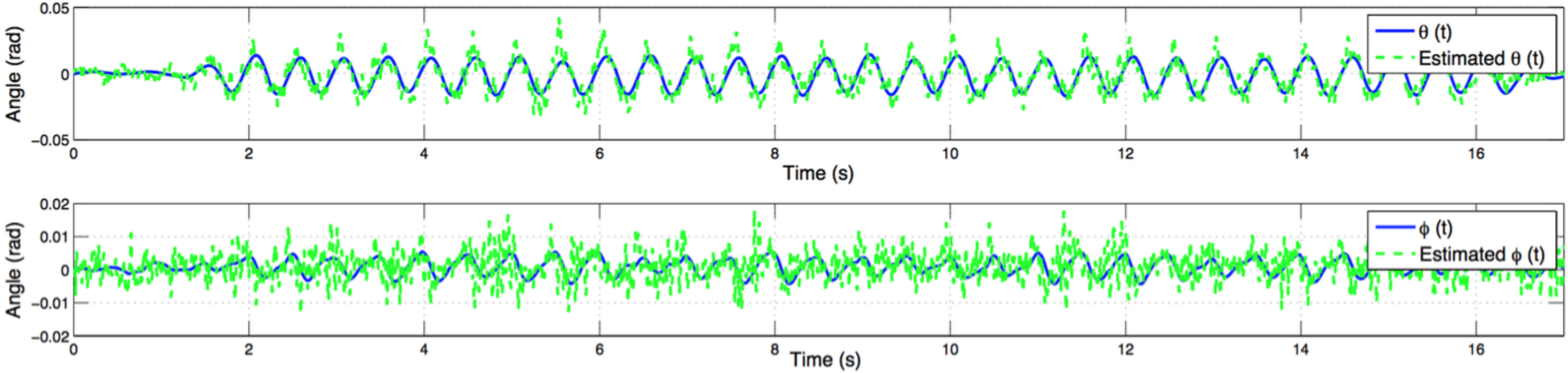
Observer Design + Experimental Validation

Validation Results:
Road Estimation



Observer Design + Experimental Validation

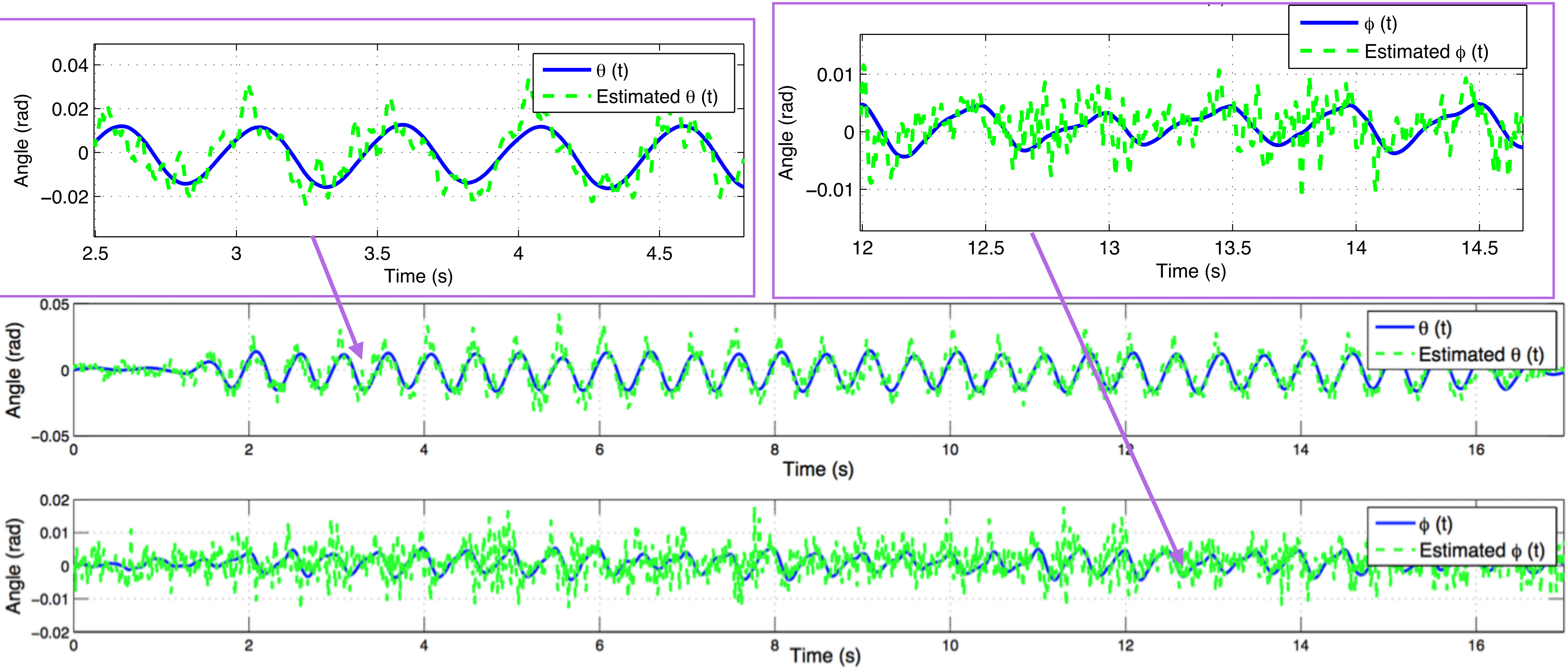
Validation Results:
Roll & Pitch Angles



Observer Design + Experimental Validation

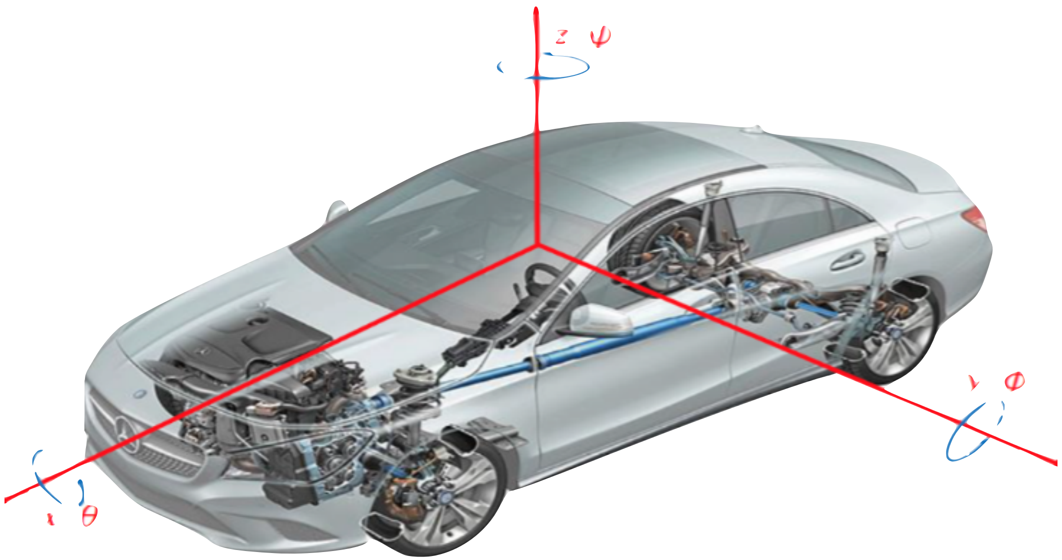
Validation Results:

Roll & Pitch Angles

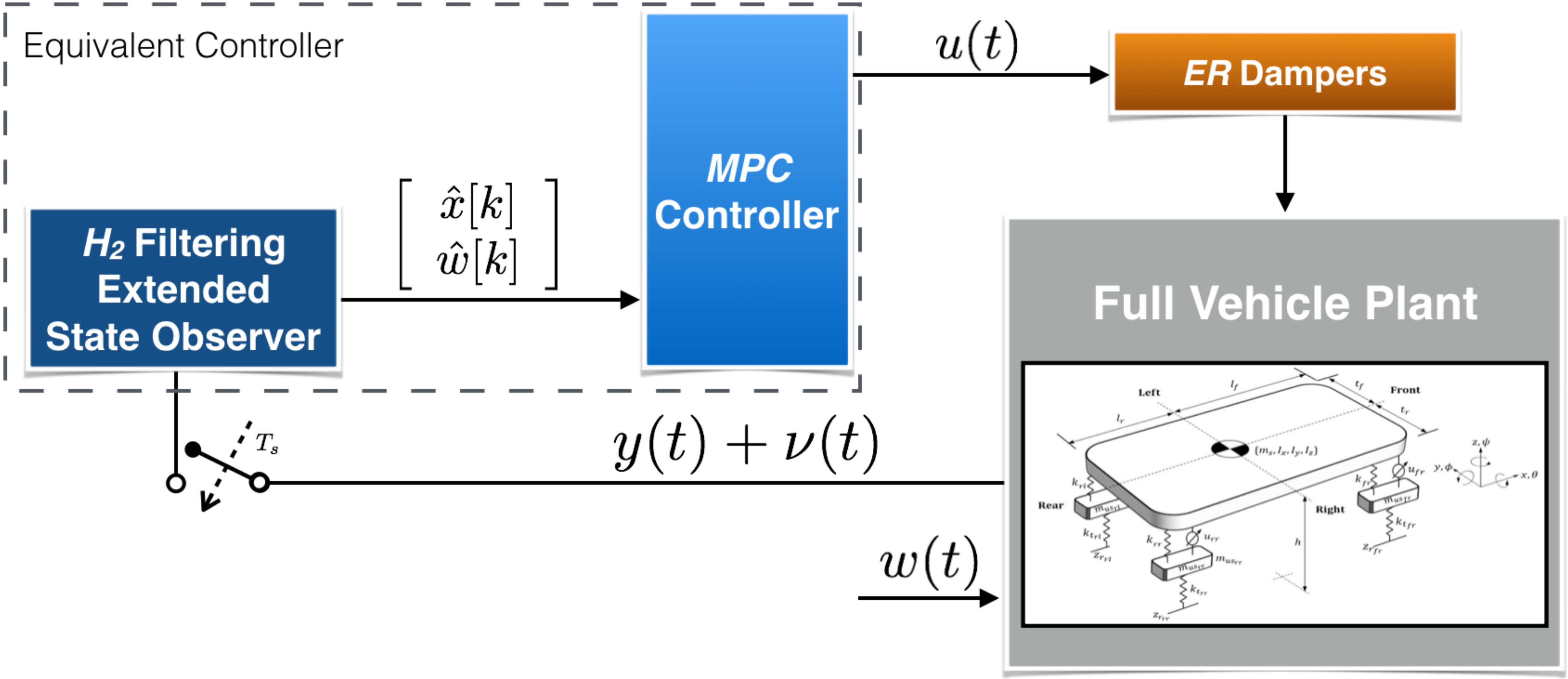


Outline

- Introduction & Motivation
- Problem Statement & Objectives
- System Model & Constraints
- Observer Design + Experimental Validation
- **Optimal Solution**
- Sub-Optimal Practical Solution
- Conclusions



Optimal Solution



- **MPC —> Optimization of Performance Indexes**
- Cost Function
- Computational Time Constraints —> Faster Approaches

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$$COMFORT: J_c = \int_0^T \ddot{z}_s^2(t)dt \leftrightarrow \ Body\ acceleration\ \ddot{z}_s$$

$$HANDLING: J_h = \int_0^T \theta^2(t)dt \leftrightarrow \ Body\ roll\ angle\ \theta$$

Optimal Solution

Proposed Optimal **MPC** Design

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+ Chassis Displacement

Optimal Solution

Proposed Optimal **MPC** Design

- MPC —> Optimization of Performance Indexes
- **Cost Function**
- Computational Time Constraints —> Faster Approaches

$$J(U, x[k], w, N_p, N_c) = \sum_{j=1}^{N_p} [(\xi_1)(\frac{\ddot{z}_s[k+j|k]}{\ddot{z}_s^{\max}})^2 + \xi_2(\frac{\theta[k+j|k]}{\theta^{\max}})^2] + \sum_{j=1}^{N_p} [\xi_3.(\frac{z_s[k+j|k]}{z_s^{\max}})^2] + \sum_{j=0}^{N_c-1} u^T[k+j|k].Q_u.u[k+j|k]$$

Optimal Solution

Proposed Optimal **MPC** Design

- MPC —> Optimization of Performance Indexes
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Computational Time $>> T_s$

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Proposed Optimal **MPC** Design

[Nguyen, M. Q. (2016)]

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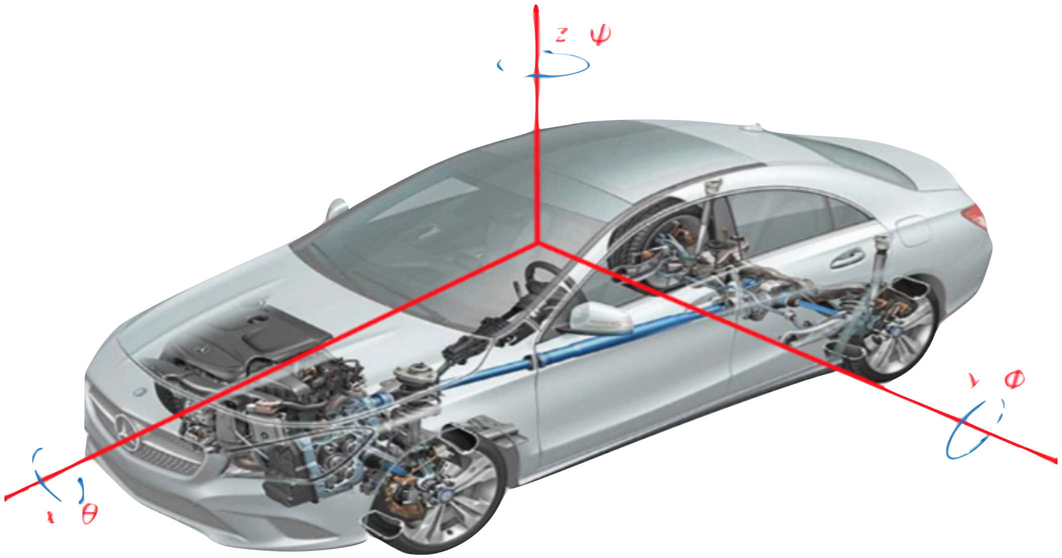

Sub-Optimal Pratical Solution

- ***LPV* Representation**
- **New Control Inputs**
- Linear Constraints

Sub-Optimal Pratical Solution

$$F_{d_{ij}} = c_{nom_{ij}} . \dot{z}_{def_{ij}} + \underbrace{\Delta c_{ij} . \dot{z}_{def_{ij}}}_{u_{ij}}$$

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Sub-Optimal Pratical Solution

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Control

- ***LPV* Representation**
- **New Control Inputs**
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Sub-Optimal Pratical Solution

$$F_{d_{ij}} = c_{nom_{ij}} . \dot{z}_{def_{ij}} + \underbrace{\Delta c_{ij} . \dot{z}_{def_{ij}}}_{u_{ij}}$$

Control

Scheduling Parameter

- ***LPV* Representation**
- **New Control Inputs**
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Sub-Optimal Pratical Solution

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- **New Control Inputs**
- Linear Constraints

$$\text{Control Inputs} = \begin{bmatrix} \Delta c_{fl} & \Delta c_{fr} & \Delta c_{rl} & \Delta c_{rr} \end{bmatrix}^T$$

$$\rho = \begin{bmatrix} (\dot{z_{s_{fl}}} - \dot{z_{us_{fl}}}) & (\dot{z_{s_{fr}}} - \dot{z_{us_{fr}}}) & (\dot{z_{s_{rl}}} - \dot{z_{us_{rl}}}) & (\dot{z_{s_{rr}}} - \dot{z_{us_{rr}}}) \end{bmatrix}^T$$

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$$B_{2d}^{\text{LPV}} = B_{2d}.\text{diag}(\dot{z_{def_{ij}}}[k])$$
$$D_{2d}^{\text{LPV}} = D_{2d}.\text{diag}(\dot{z_{def_{ij}}}[k])$$

$$\sum_{Full}^{T_s} := \left\{ \begin{array}{ccccccc} x[k+1] & = & A_d.x[k] & + & B_{1d}.w[k] & + & B_{2d}(\rho).\Delta c[k] \\ y[k] & = & C_d.x[k] & + & D_{1d}.w[k] & + & D_{2d}(\rho).\Delta c[k] \end{array} \right\}$$

Sub-Optimal Pratical Solution

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Affine on ρ

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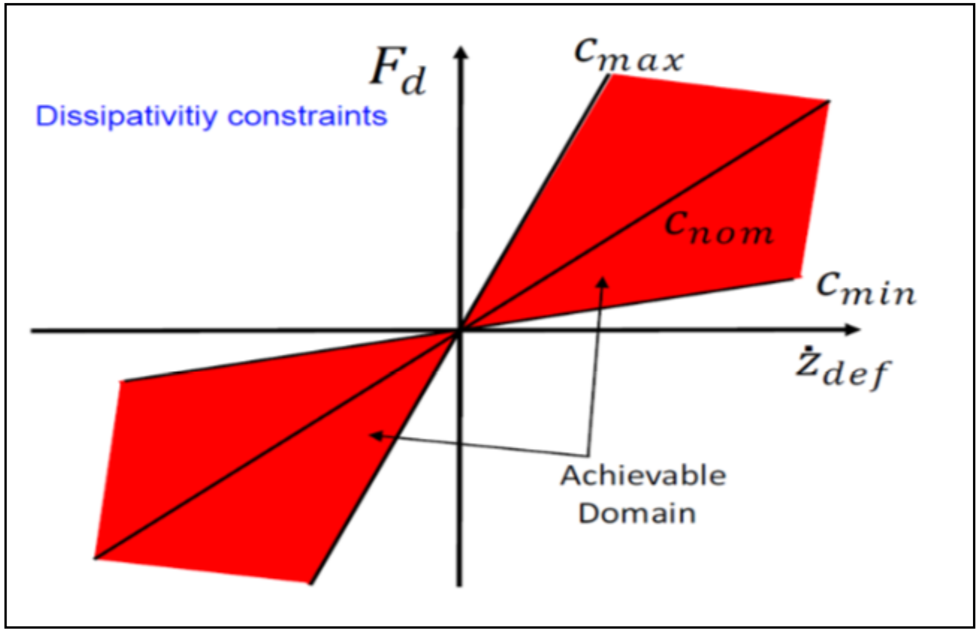
Sub-Optimal Pratical Solution

- *LPV* Representation
- New Control Inputs
- **Linear Constraints**

$$\underline{\Delta c} \leq \Delta c[k] \leq \overline{\Delta c}$$

$$\underline{x} \leq x[k] \leq \overline{x}$$



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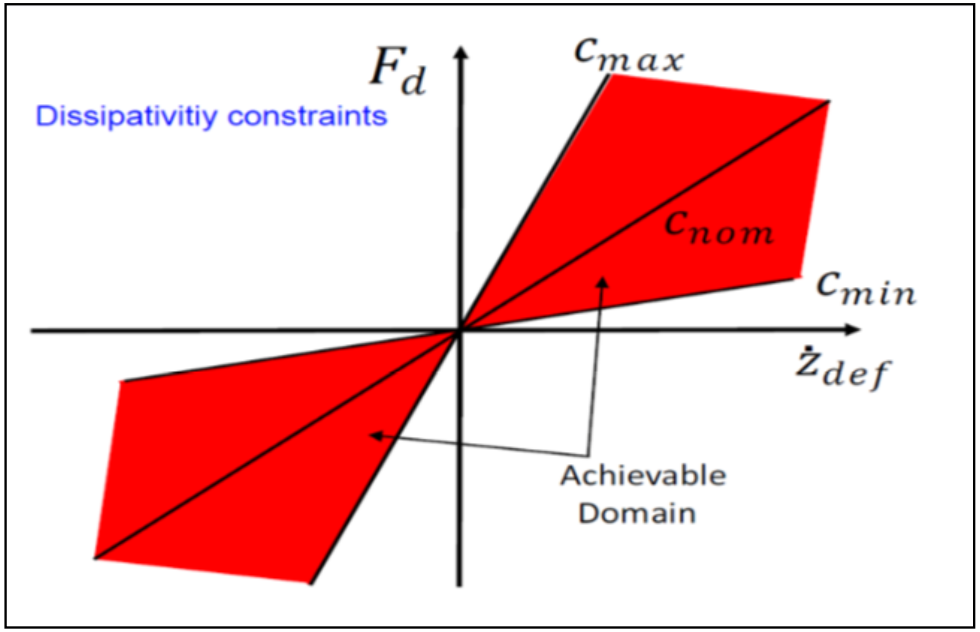
Sub-Optimal Pratical Solution

- *LPV* Representation
- New Control Inputs
- **Linear Constraints**

[Nguyen, M. Q. (2016)]
Mixed Integer Constraints



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Sub-Optimal Pratical Solution

Computation of LPV MPC Law

- LPV Matrices *fixed* through prediction horizon
- The **FMPC** Method, Proposed by **[Boyd, 2008]**

Sub-Optimal Pratical Solution

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Sub-Optimal Pratical Solution

- LPV Matrices *fixed* through prediction horizon
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    - **LPV —> fixed at instant *k***
    - Primal-Barrier Interior-Point Method
    - Infeasible Start Newton Method
    - Warm Start Techniques

Sub-Optimal Pratical Solution

Computation of LPV MPC Law

- LPV Matrices *fixed* through prediction horizon
- The **FMPC** Method, Proposed by **[Boyd, 2008]**
    - LPV —> fixed at instant *k*
    - **Primal-Barrier Interior-Point Method**
    - Infeasible Start Newton Method
    - Warm Start Techniques

Approximate J, Primal-Barrier Term instead of linear inequality constraints

Sub-Optimal Pratical Solution

Computation of LPV MPC Law

- LPV Matrices *fixed* through prediction horizon
- The **FMPC** Method, Proposed by **[Boyd, 2008]**
    - LPV —> fixed at instant k
    - Primal-Barrier Interior-Point Method
    - **Infeasible Start Newton Method**
    - Warm Start Techniques

Solving the approximate QP with infeasible Start Newton Method, use of dual variables, primal and dual residual search

<u>Sub-Optimal Pratical Solution</u>

- LPV Matrices *fixed* through prediction horizon
- The **FMPC** Method, Proposed by **[Boyd, 2008]**
  - LPV —> fixed at instant *k*
  - Primal-Barrier Interior-Point Method
  - Infeasible Start Newton Method
  - **Warm Start Techniques**

Last array of control steps *U* shifted (z^{-1}) as start for next computation

Sub-Optimal Pratical Solution

Simulation Results:

Objective:
Control all 4 Semi-Active *ER* Dampers
Provide Suitable Trade-Off Handling vs Comfort Performances
Abide to all Dissipativity Constraints
Computational Time < 5 *ms*

Sub-Optimal Pratical Solution

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Control all 4 Semi-Active *ER* Dampers
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Simulation Scenario:
Straight Road, Constant Speed
Frontal and Lateral Bumps
Comparison with Analytical Clipped MPC, [Giorgetti, N. (2006)]

Sub-Optimal Pratical Solution

Simulation Results:

Objective:

Control all 4 Semi-Active *ER* Dampers

Provide Suitable Trade-Off Handling vs Comfort Performances

Abide to all Dissipativity Constraints

Computational Time < 5 *ms*

Simulation Scenario: $N_c = N_p = 10$

Straight Road, Constant Speed

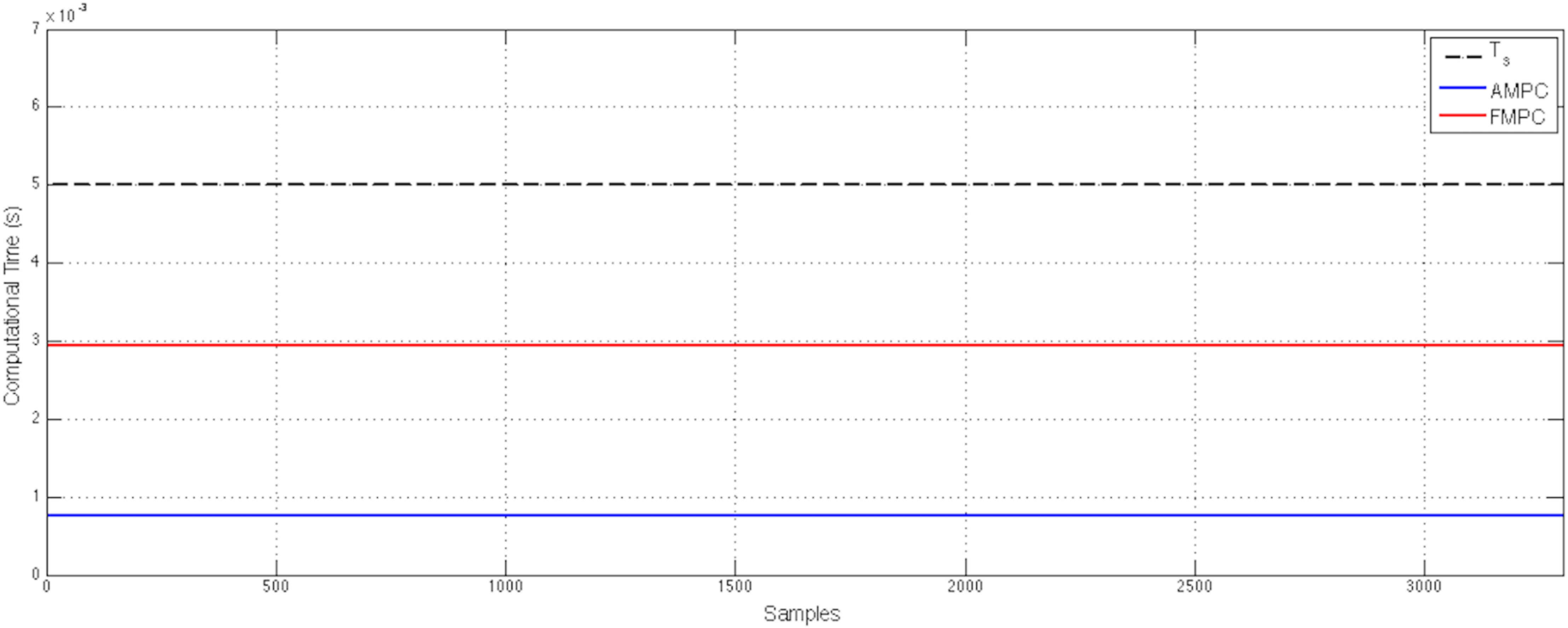
Frontal and Lateral Bumps

Comparison with Analytical Clipped MPC, [Giorgetti, N. (2006)]

Sub-Optimal Pratical Solution

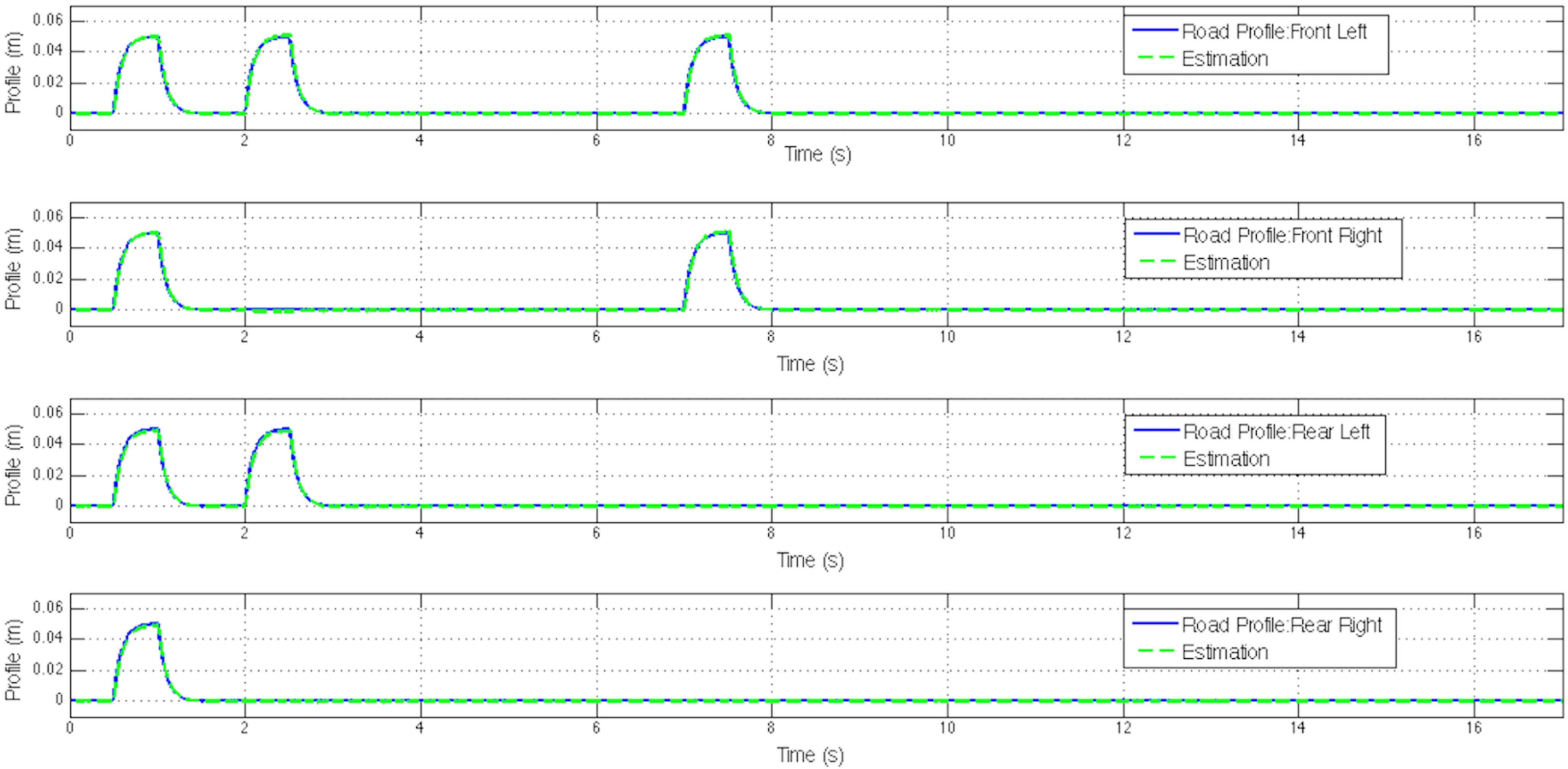
Simulation Results:

Computation of Control Law



Sub-Optimal Pratical Solution

Simulation Results:
Road Profile + Estimation



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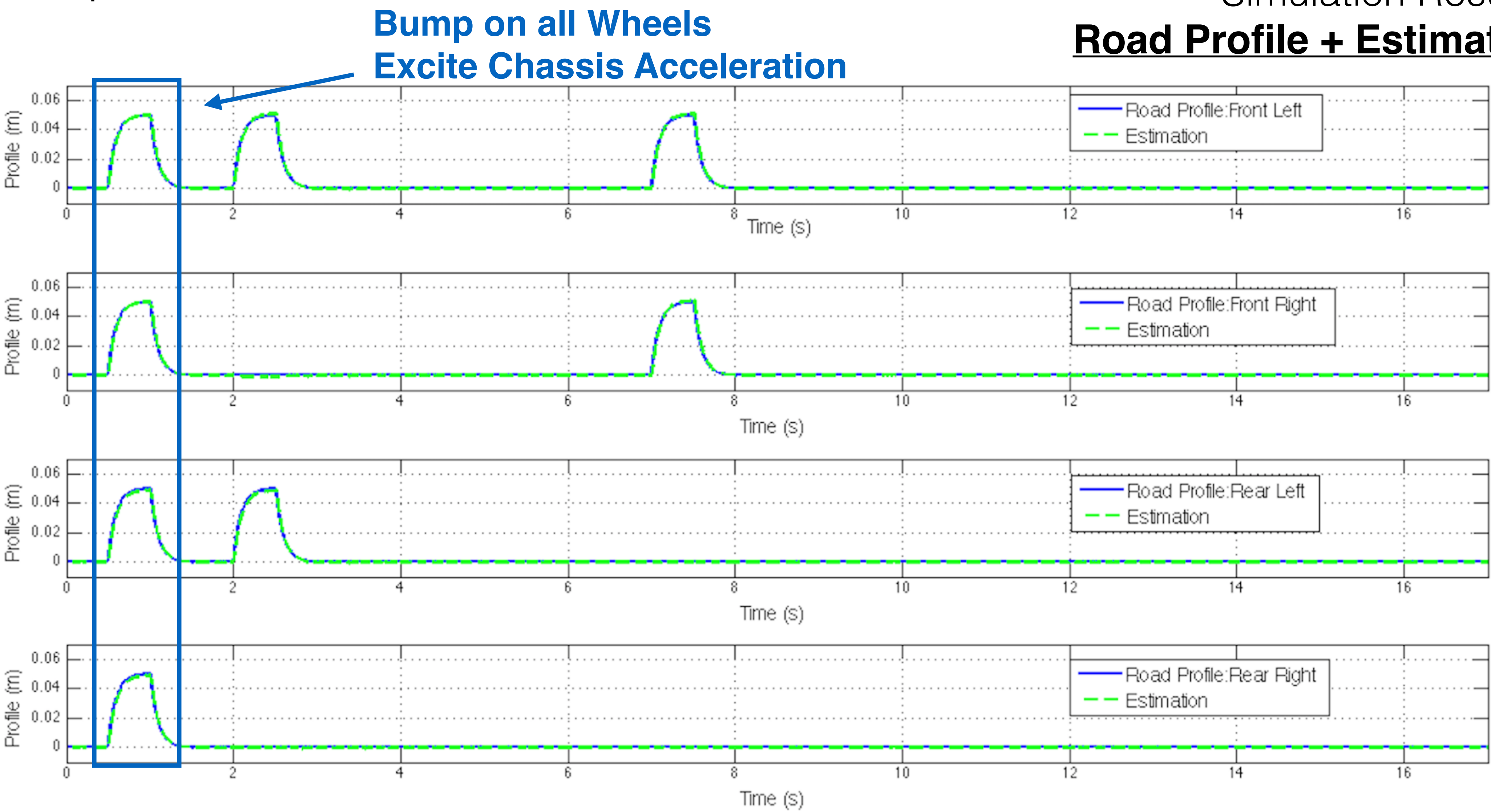

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Simulation Results:

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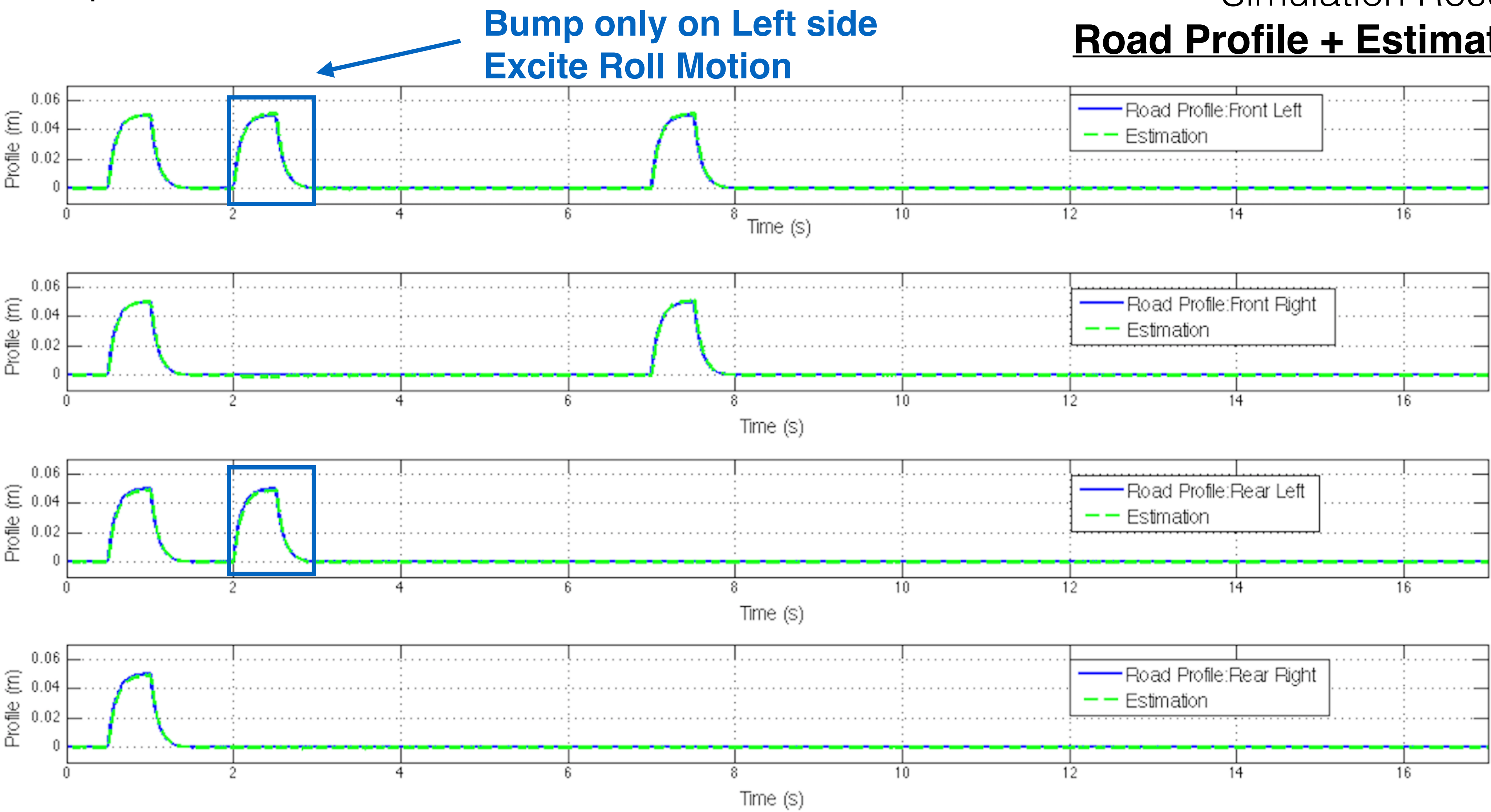
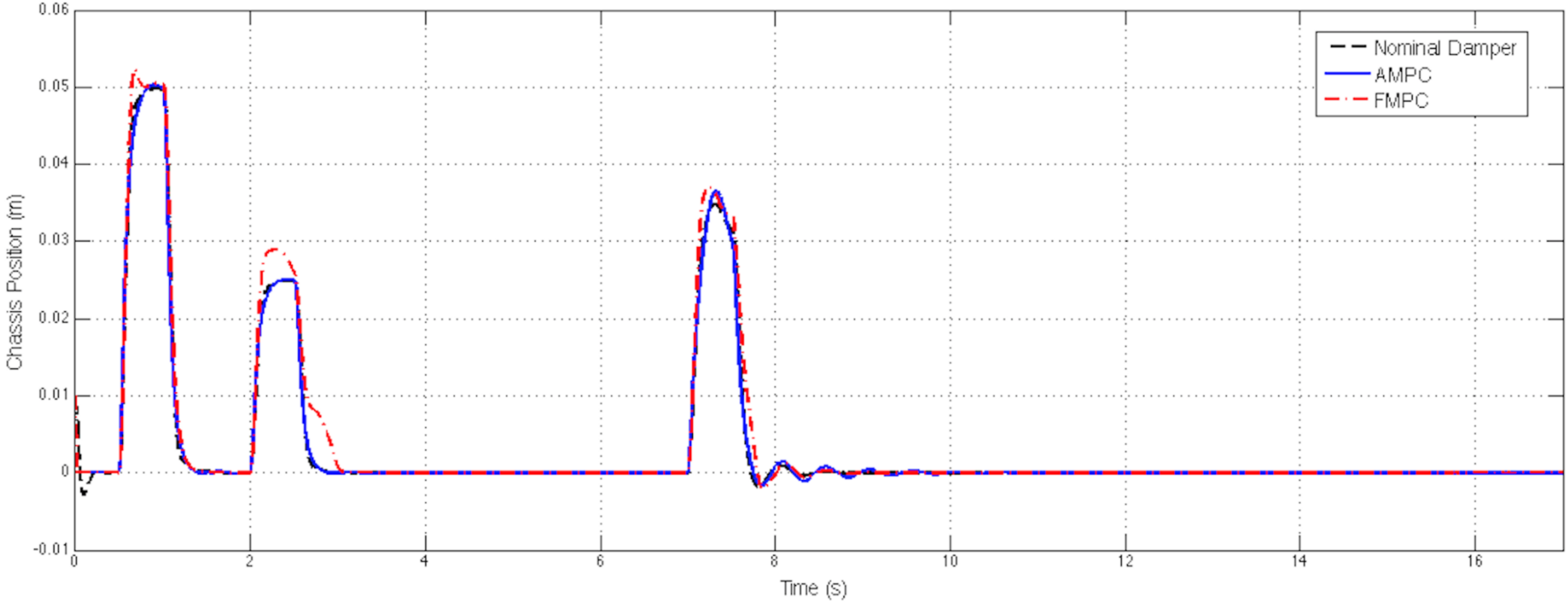


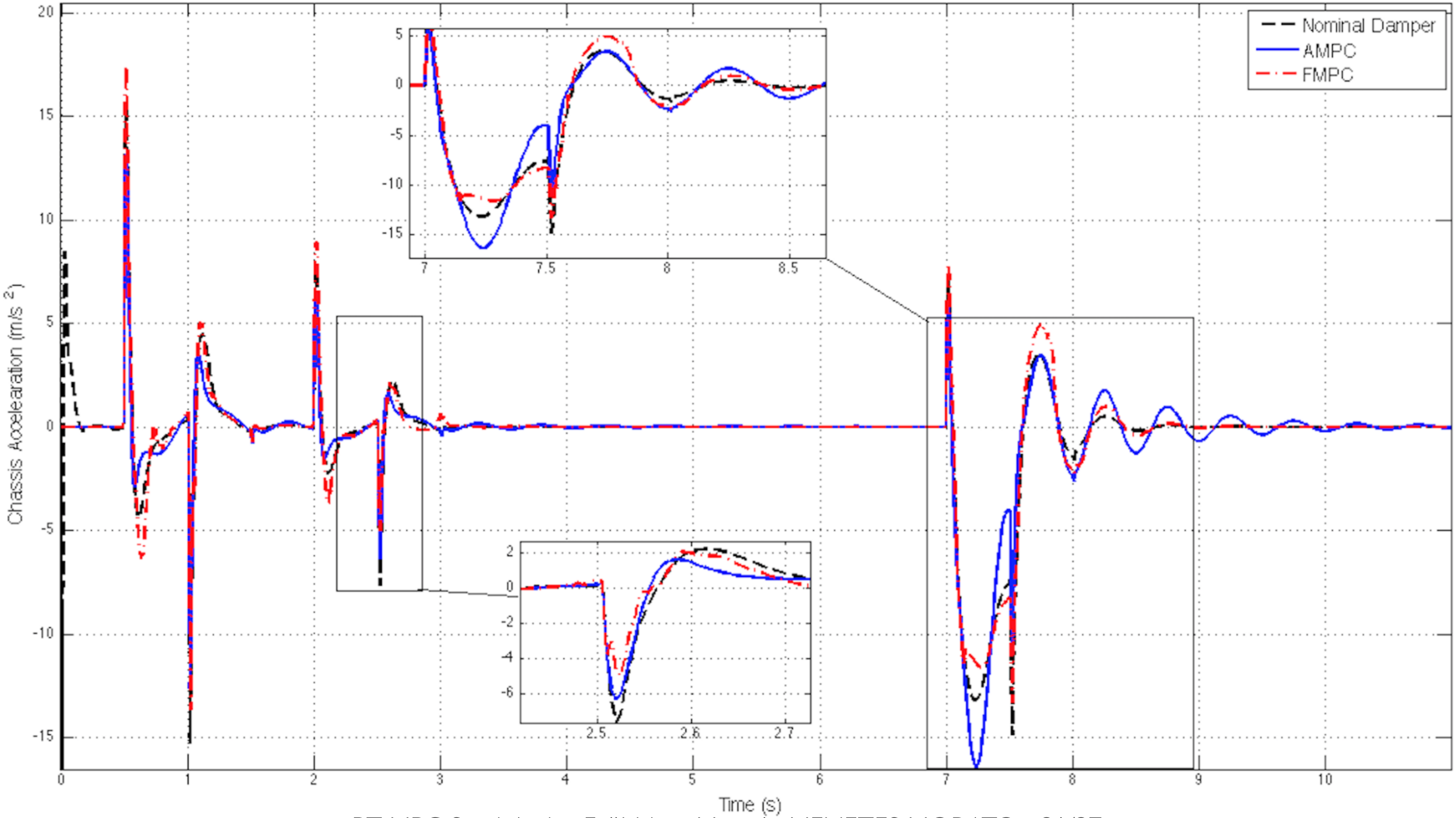
Sub-Optimal Pratical Solution

Simulation Results:
Chassis Displacement



Sub-Optimal Pratical Solution

Simulation Results:
Chassis Acceleration



Sub-Optimal Pratical Solution

Simulation Results:
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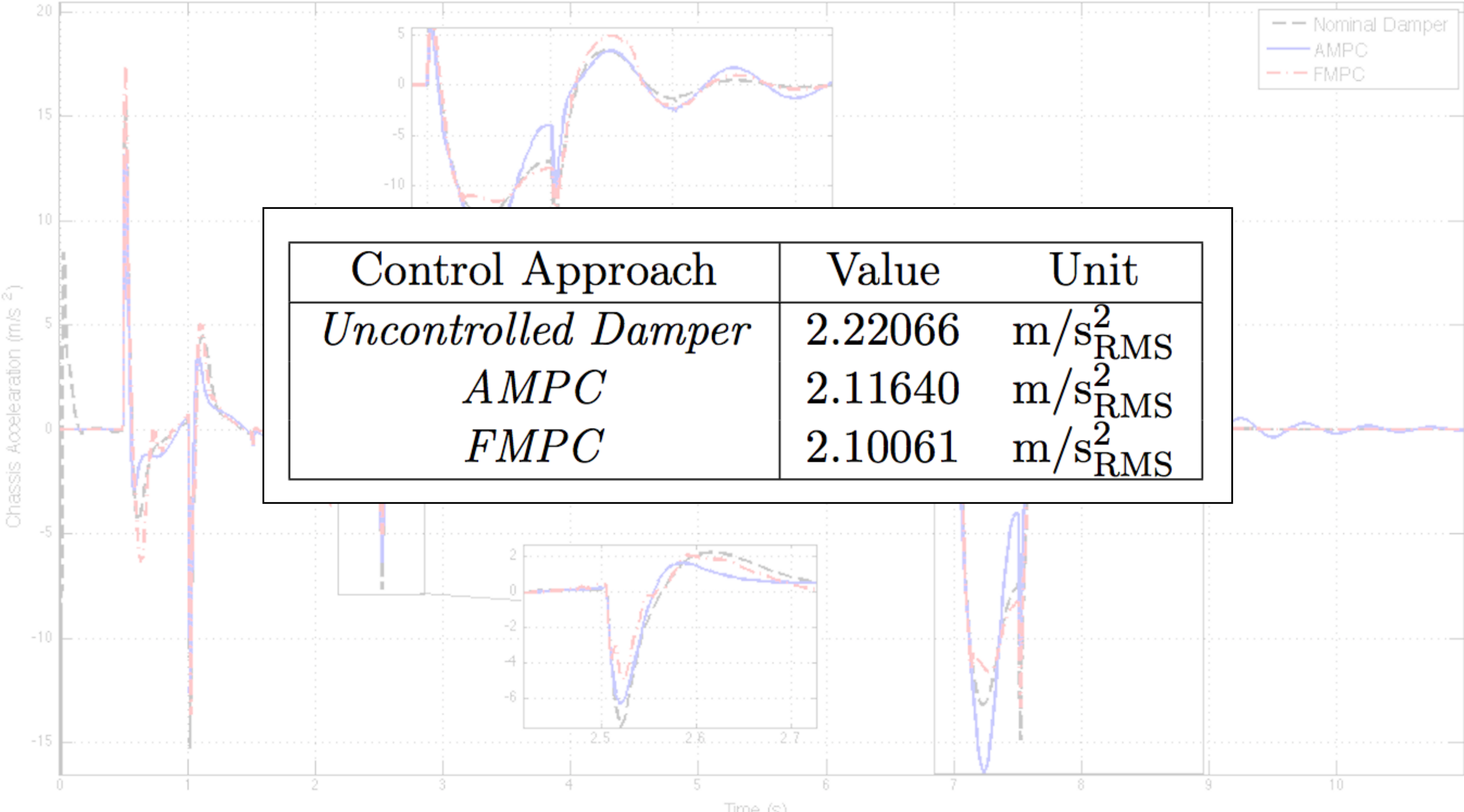
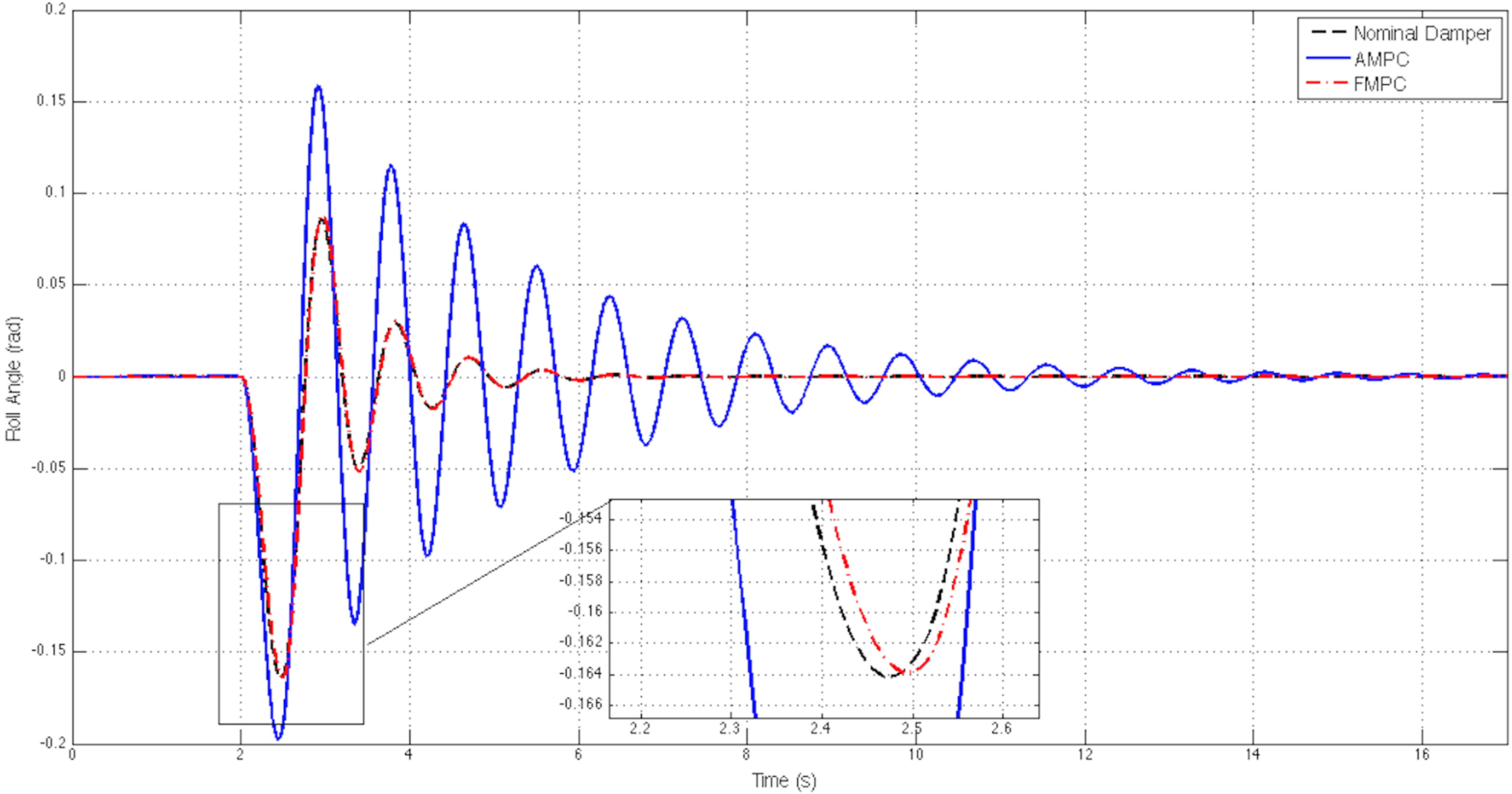


Control Approach	Value	Unit
Uncontrolled Damper	2.22066	m/s^2_{RMS}
AMPC	2.11640	m/s^2_{RMS}
FMPC	2.10061	m/s^2_{RMS}

Sub-Optimal Pratical Solution

Simulation Results:
Roll Angle



Sub-Optimal Pratical Solution

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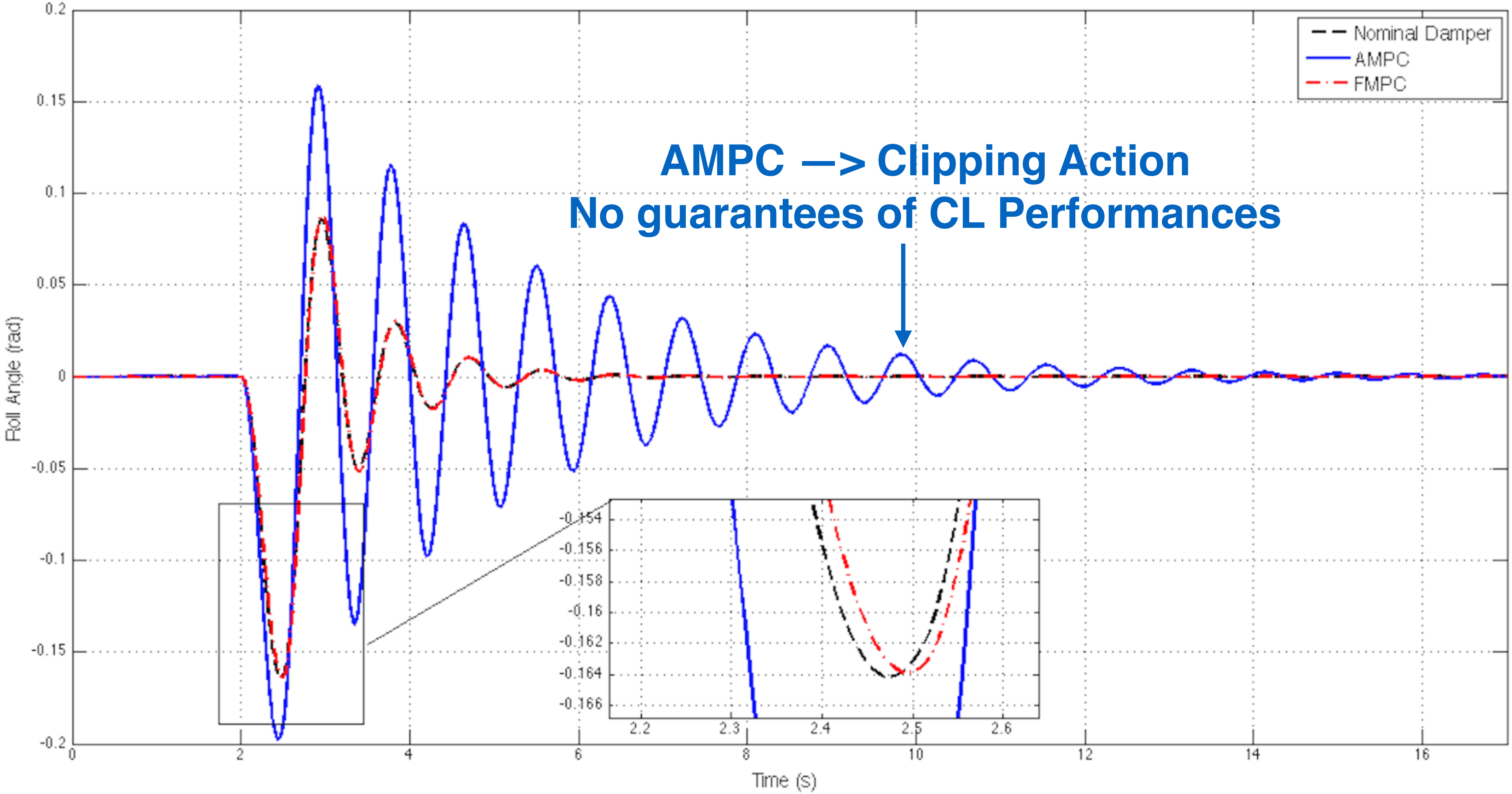
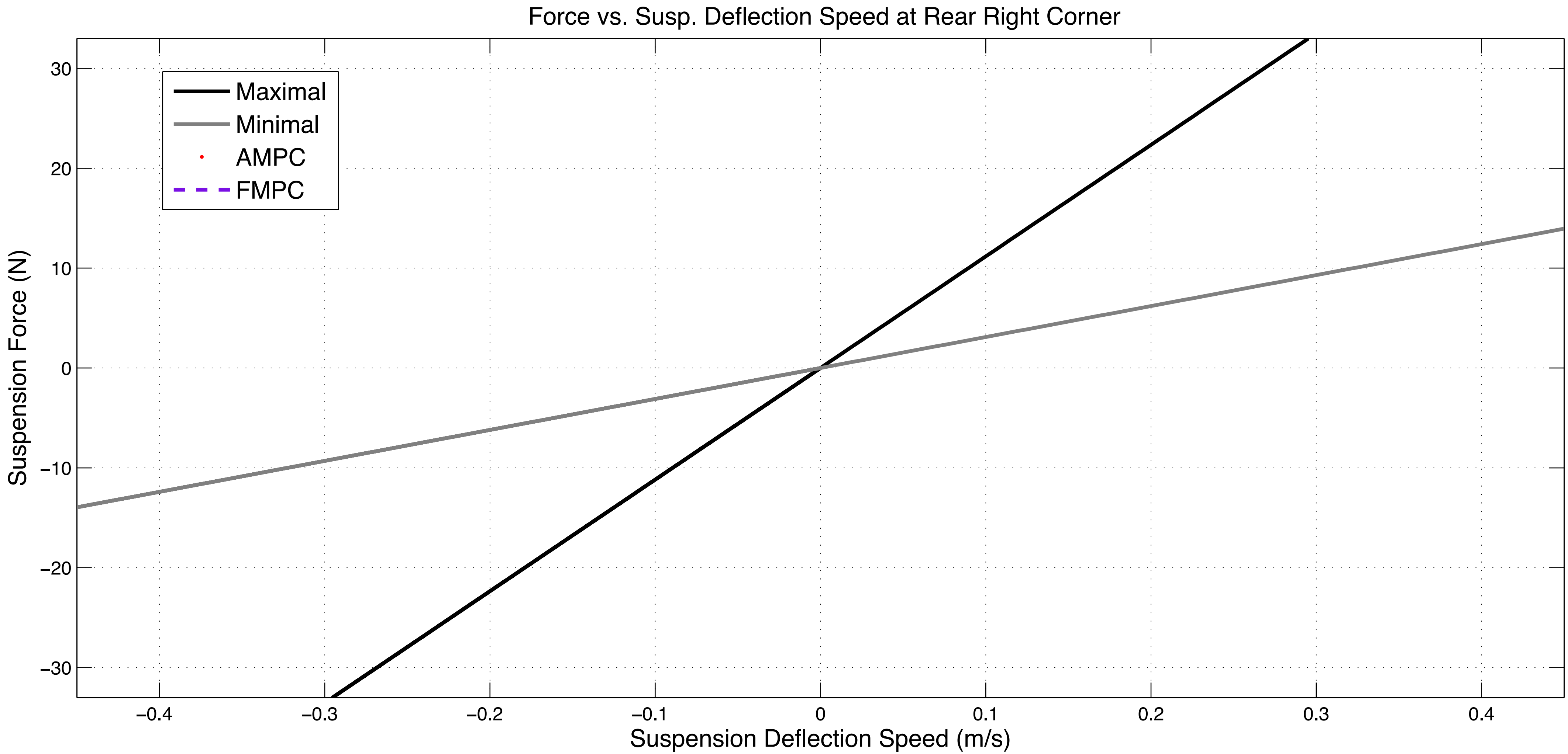


Sub-Optimal Pratical Solution

Simulation Results:
Semi-Active Damper Force



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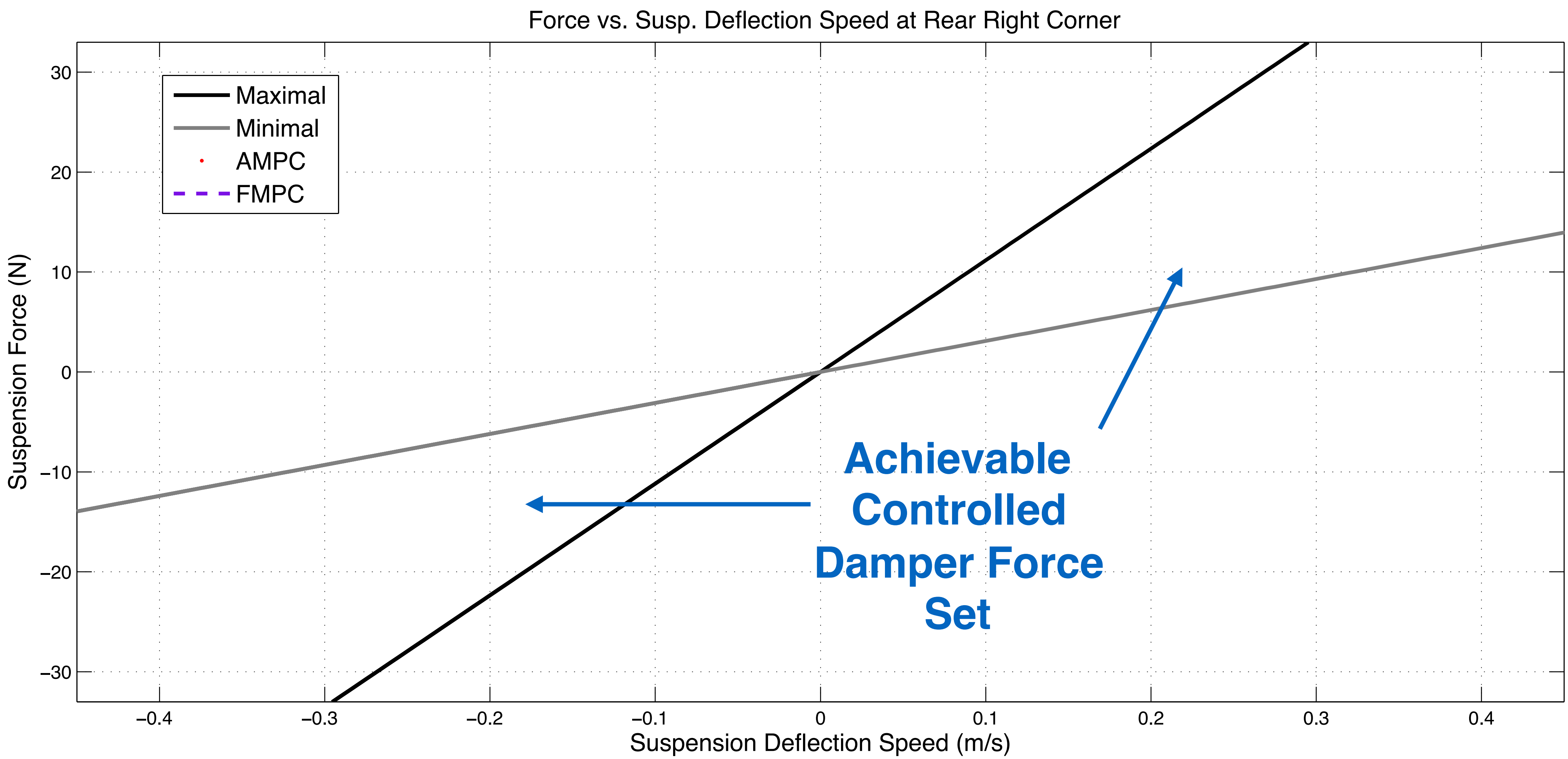

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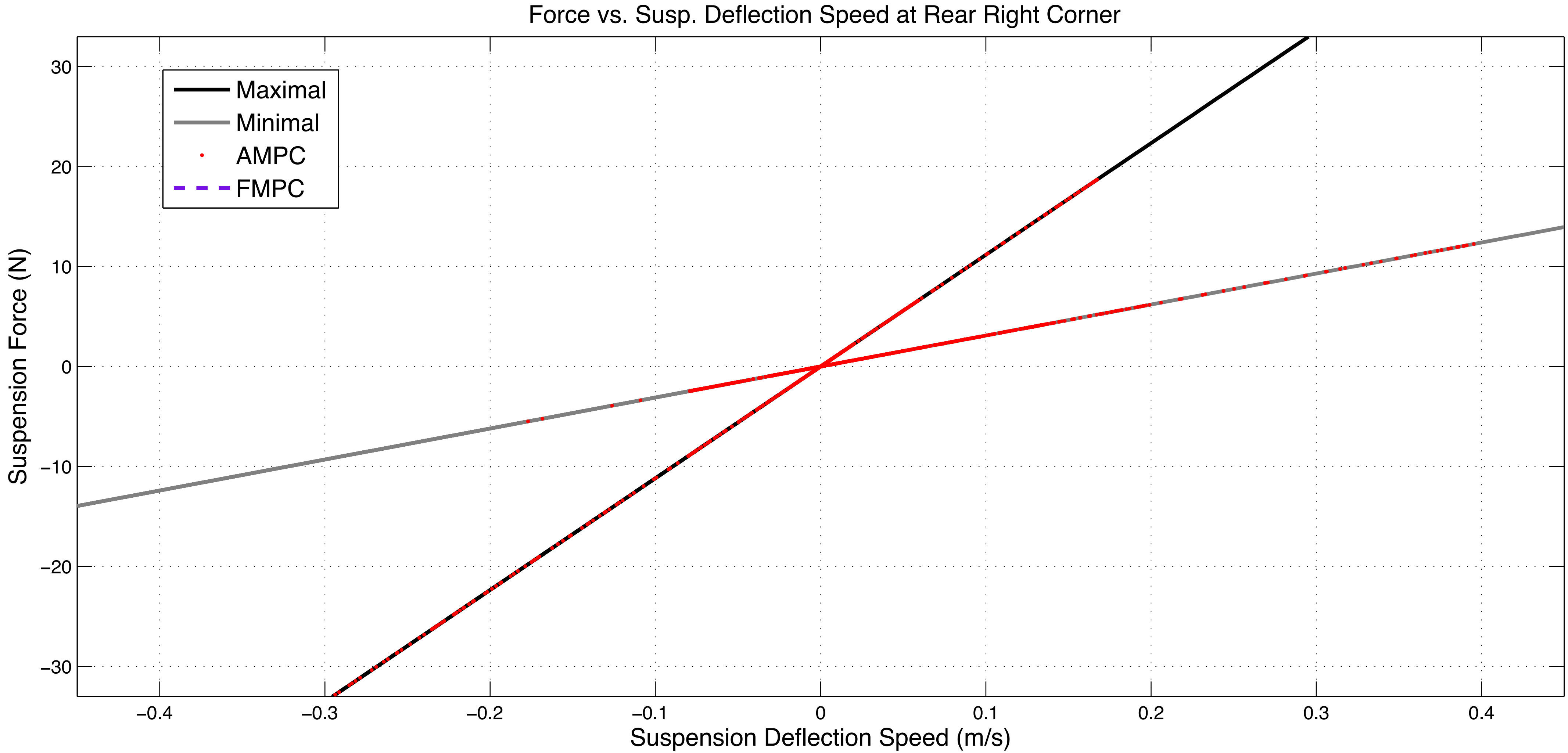

Sub-Optimal Pratical Solution

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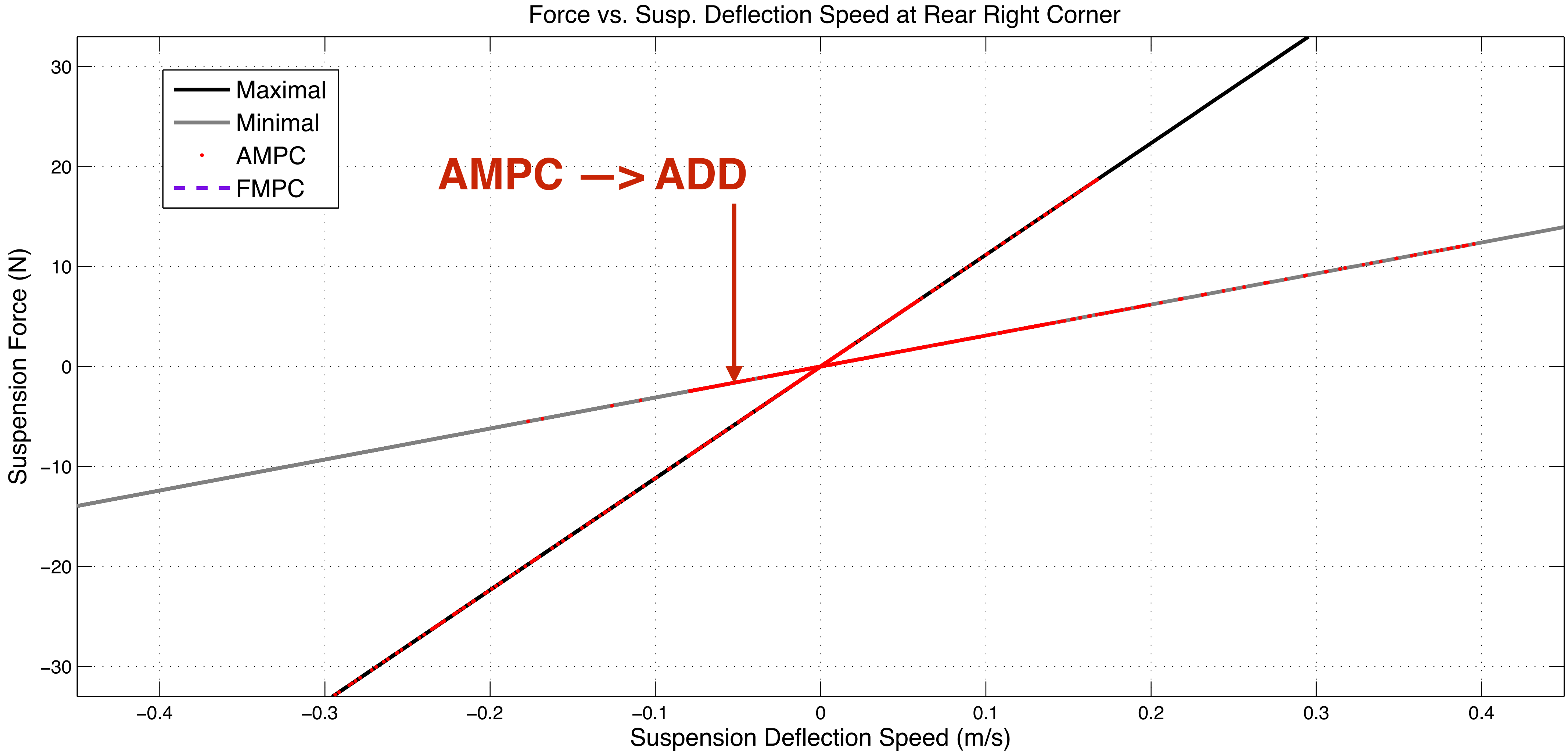

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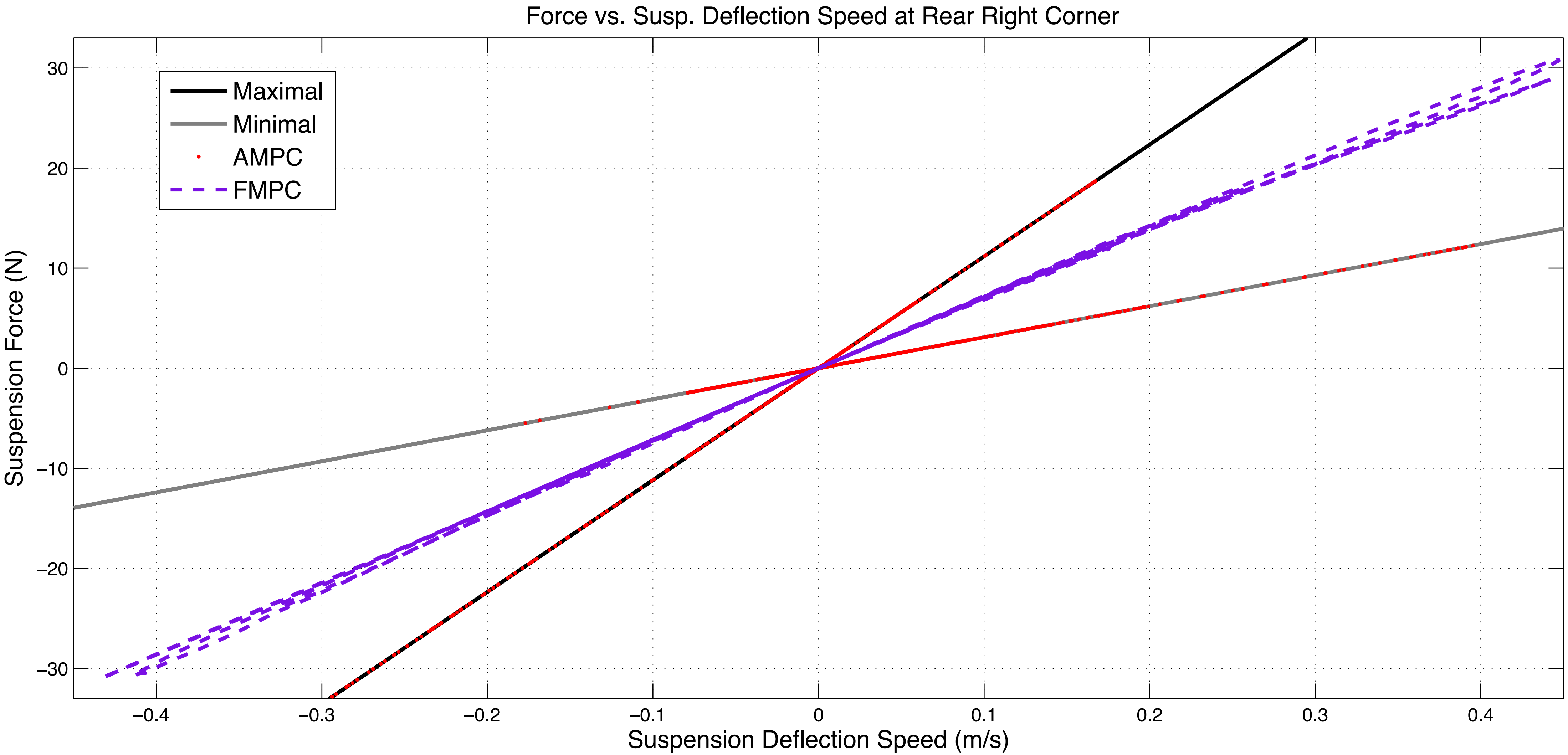

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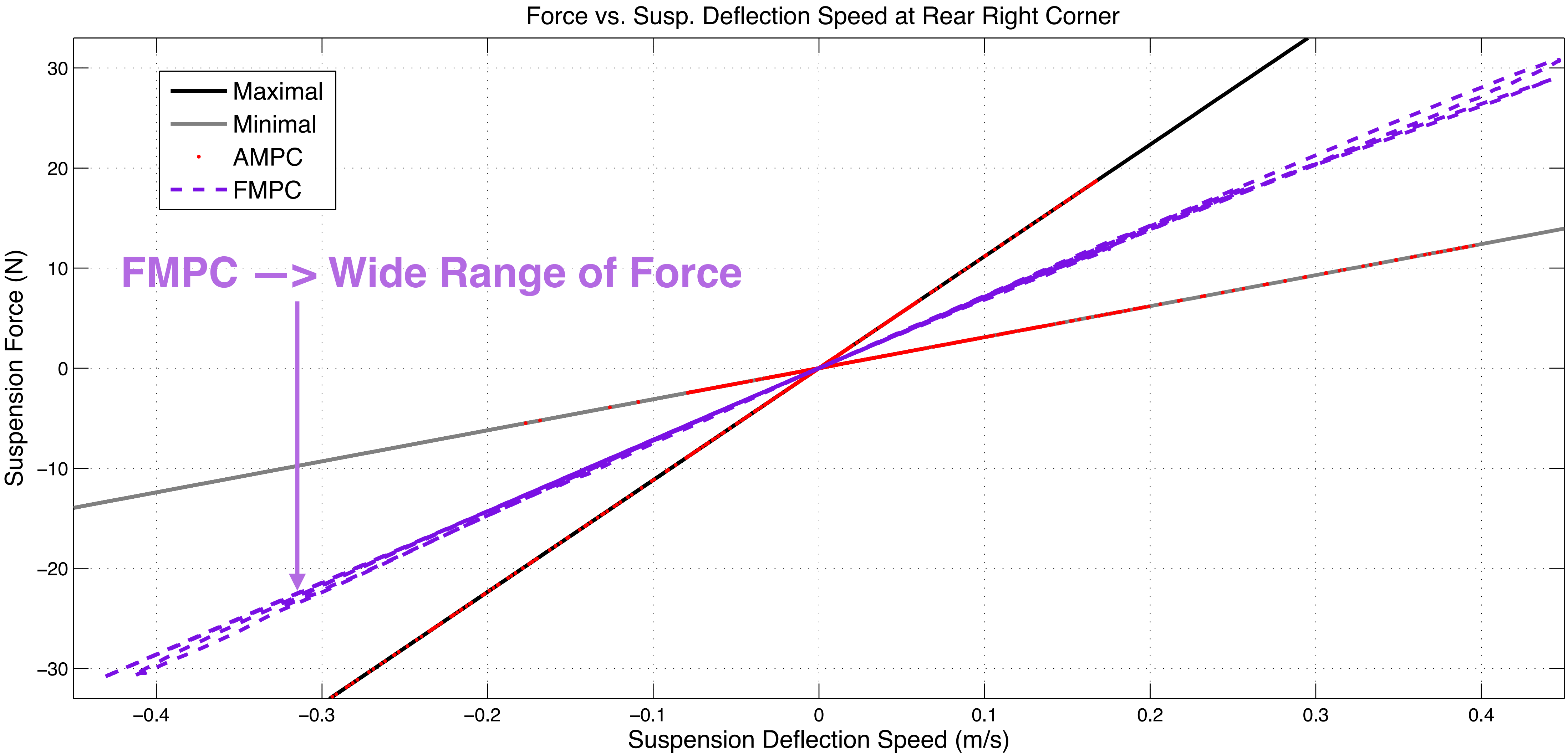

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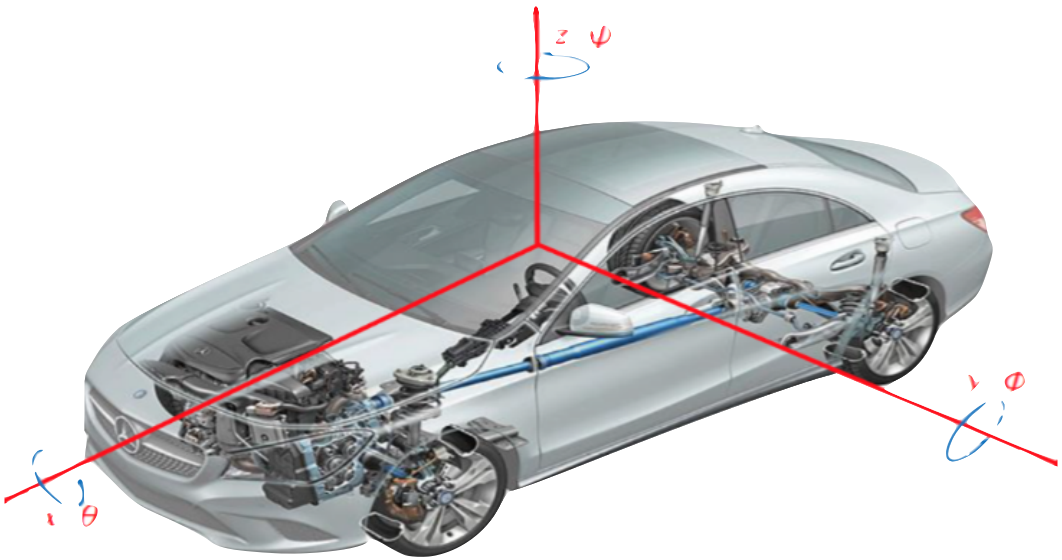

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- ***LPV FMPC* Technique —> Efficient Results!**

- Time Constraints Respected + Good Trade-Off Handling *vs* Comfort

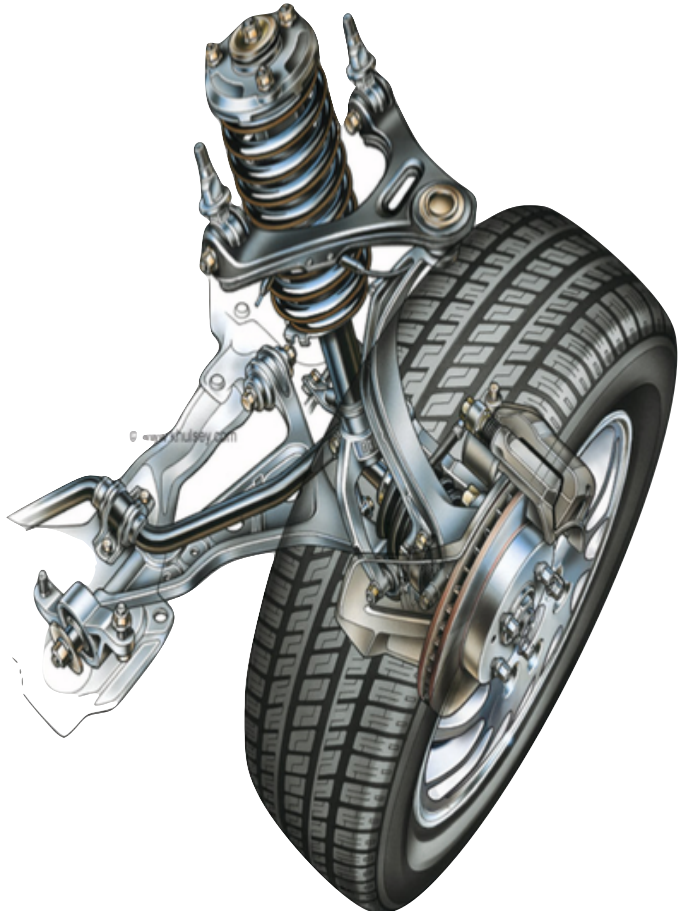
- AMPC —> Two State Control (Max Min)

- FMPC —> Wide Use of Damper Force

- Dissipativity Constraints of Dampers are Respected!

- Full 7-DOF Vehicle Semi-Active Suspension Control

- Good for Practical Implementation!



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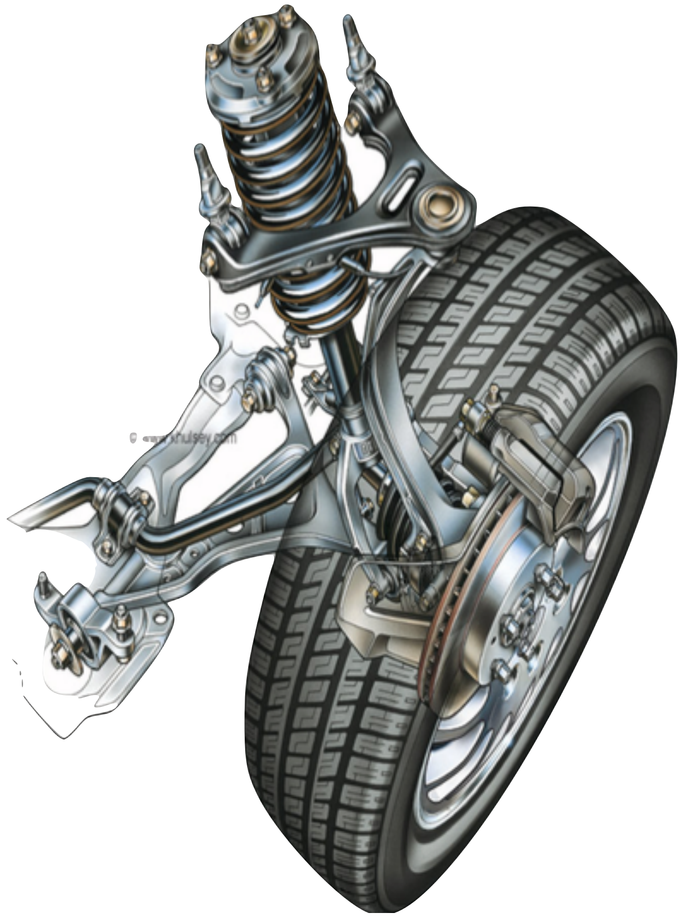

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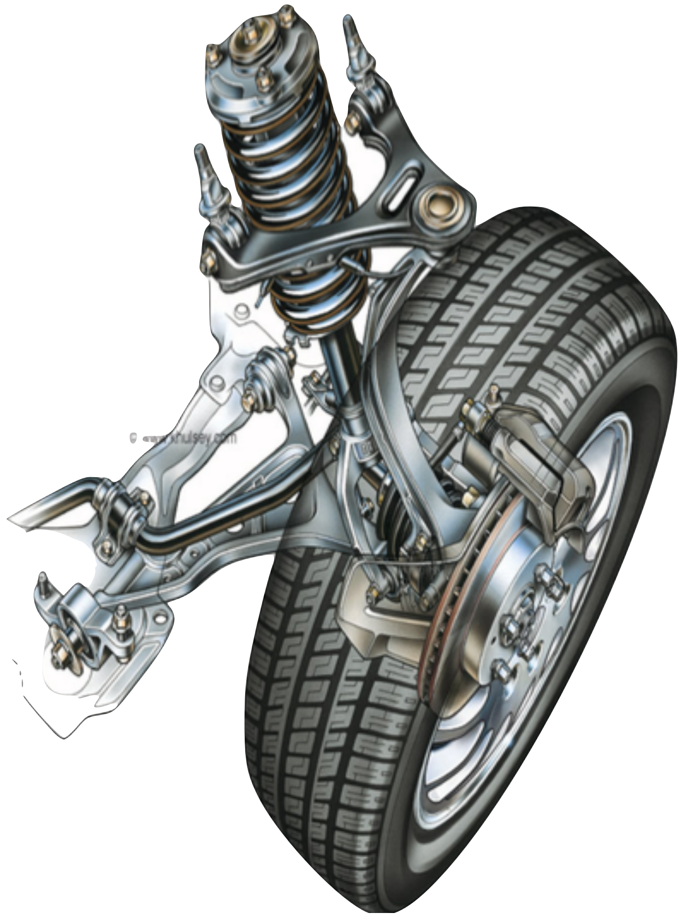

Conclusions

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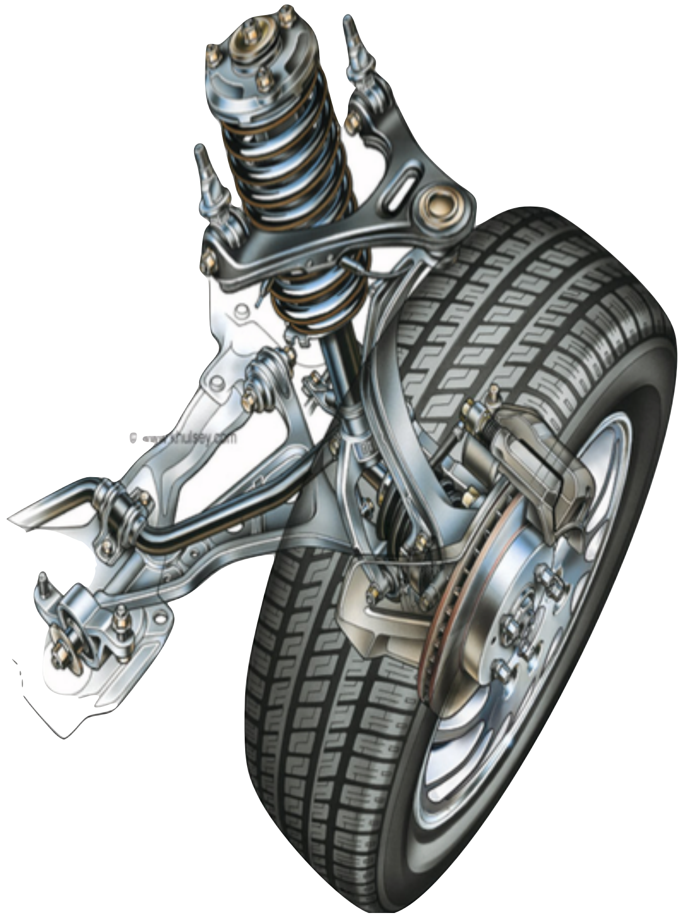

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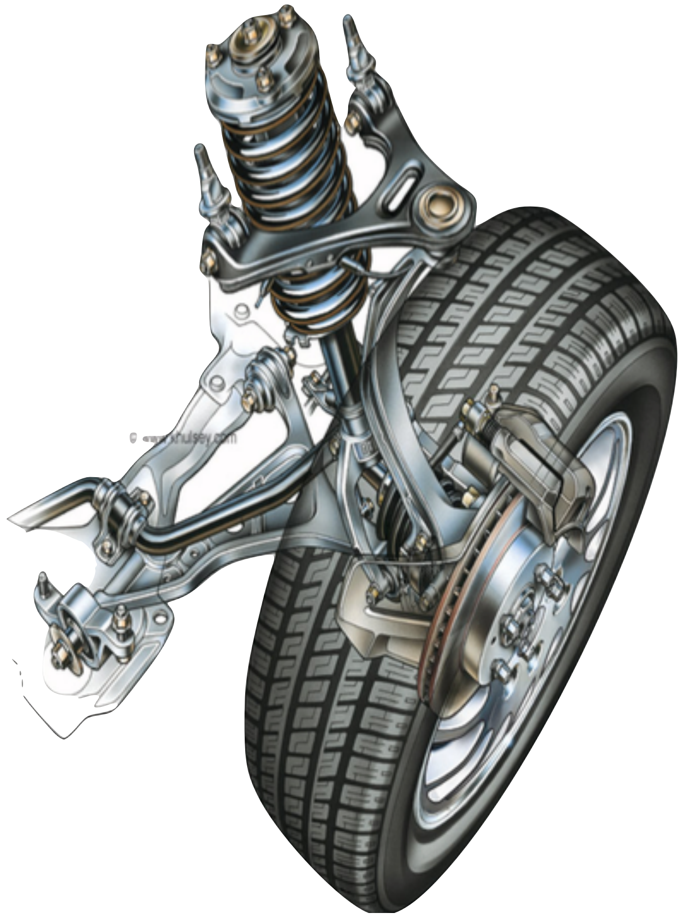

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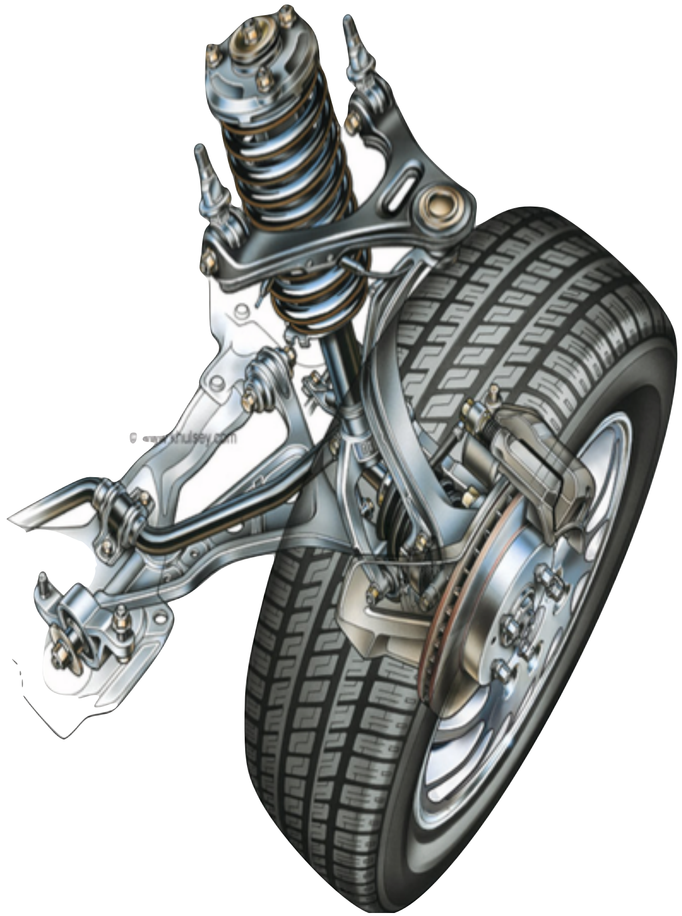
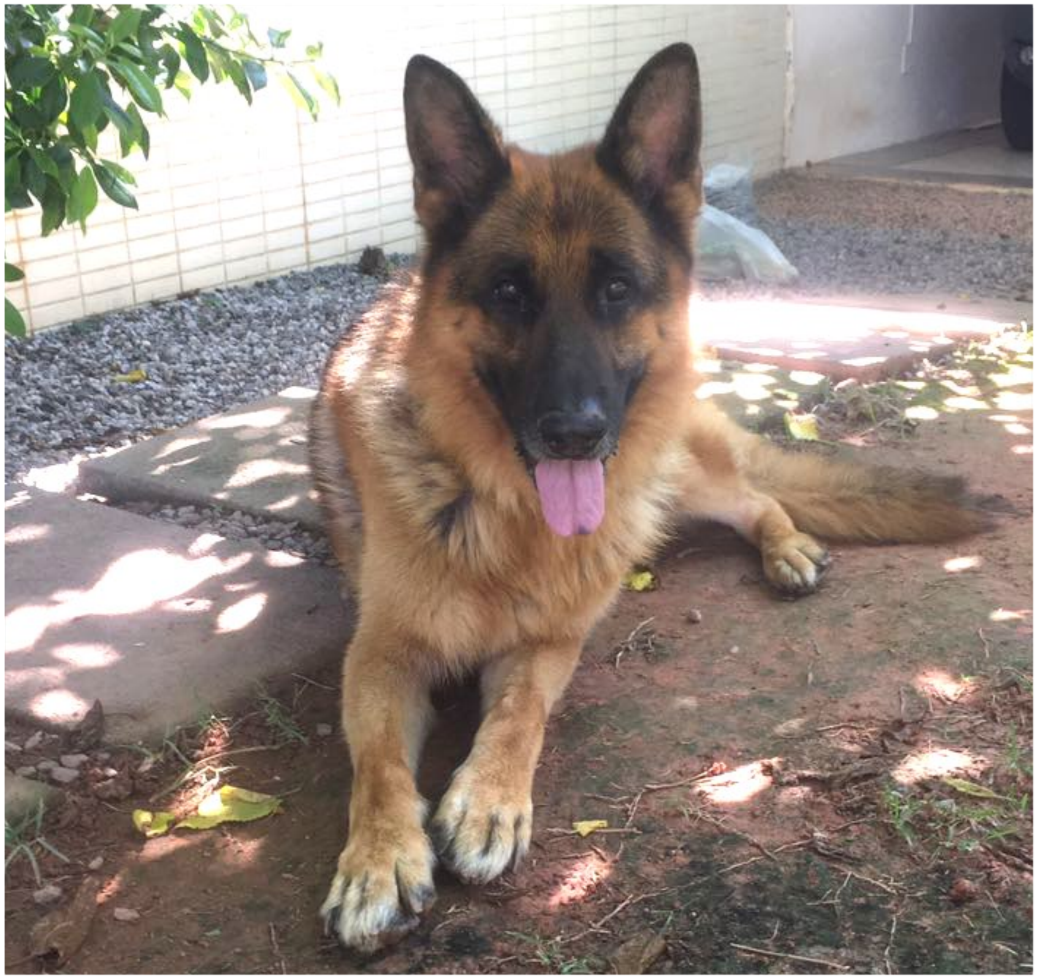


Conclusions

Future Work

- Application to real vehicle Test-Bench
- Compare with [Nguyen, M. Q. (2016)]
- Couple **FMPC** and Mixed-Integer Programming Techniques **?**

Merci!!!



Questions ?