Anti-windup for Model Reference Adaptive Control Systems

Matthew Turner
University of Leicester, UK

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Colleague

Dr. Jorge Sofrony
Nacional University
Colombia

Rebranding

New University of Leicester logo!

Sad Emoji?

Leicester crest?

Rebranding

Old Group:

Control Group

Rebranding

Old Group:

Control Group

New Group:

Computational Engineering and Control Group

Or:

Rebranding

Old Group:

Control Group

New Group:

Computational Engineering and Control Group

Or:

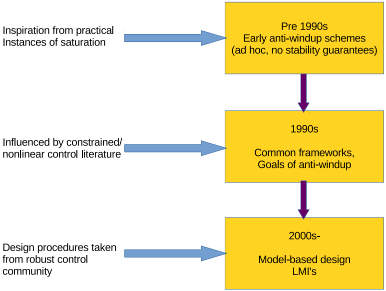
The Group Formerly Known As Control

Disclaimer

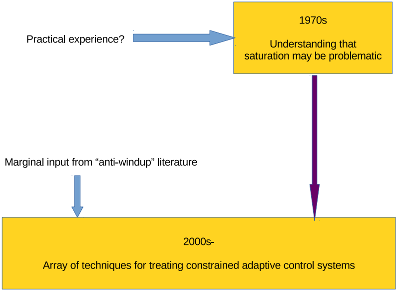
- Not an expert in adaptive control
- MRAC is not a perfect method
- Encouraging MRAC results from Wise and Lavretsky

Historical perspective

Anti-windup

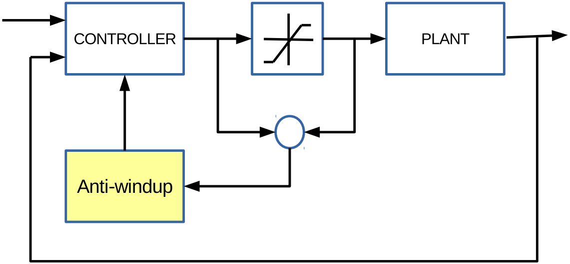


Constrained adaptive control



Architectural perspectives

Standard, agreed form of architecture for linear control laws:



Not much agreed at all with adaptive "anti-windup"....

Exceptions

Some systematic approaches to anti-windup for adaptive control exist:

- Kahveci, Ioannou *et al*: Anti-windup for indirect adaptive control schemes. Computationally intensive. Quite well developed. Questionnable stability guarantees.
- Tregouet, Arzelier, Peaucelle, Pittet, and Zaccarian. Adaptive law used to de-saturate when gains become too high (sort of saturation avoidance). Works on satellite.
- E. Johnson *et al*. “Hedging” schemes. Quite complicated. A lot of assumptions. Seems to work on examples.
- Lavretsky and Hovakimyan. *Positive μ modification*. Actually can be written as an anti-windup scheme. Inspiration for work here.

Plant and Assumptions

Plant:

$$\dot{x} = Ax + B\lambda \mathrm{sat}(u)$$

Assumptions:

- $A \in \mathbb{R}^{n \times n}$ unknown, *but* Hurwitz
- $B \in \mathbb{R}^{n}$ known
- $\lambda \in \mathbb{R}$ unknown, but positive
- State x available for feedback

Implication:

A Hurwitz and saturated input imply <u>state is bounded</u>

Typical MRAC algorithm

Reference Model:

$$\dot{x}_r = A_m x_r + B_m r$$

State-feedback:

$$u = \hat{K}_x(t)' x + \hat{K}_r(t)' r$$

Adaptation ($e = x - x_r$):

$$\begin{cases} \dot{\hat{K}}_x & = & -\Gamma_x x(e' PB) & \Gamma_x > 0 \\ \dot{\hat{K}}_r & = & -\Gamma_r r(e' PB) & \Gamma_r > 0 \end{cases}$$

Ensures asymptotic tracking if there exists matrices K_x^*, K_r^*, $P > 0$ such that

$$A_m = A + B\lambda K_x^* \quad B_m = B\lambda K_r^* \quad A_m' P + PA_m < 0$$

...and no saturation!

MRAC with anti-windup (Lavretsky's scheme in disguise)

Modifed Reference Model (£):

$$\dot{x}_m = A_m x_m + B_m r - B(1 + \mu(x, x_m))\hat{K}_u(t)' \mathrm{Dz}(u)$$

State-feedback ($):

$$u = \hat{K}_x(t)' x + \hat{K}_r(t)' r - \mu(x, x_m)\mathrm{Dz}(u) \quad \mu(.,.) : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$$

Adaptation ($e = x - x_m$) (€):

$$\left\{ \begin{array}{lcll} \dot{\hat{K}}_x & = & -\Gamma_x x(e'PB) & \Gamma_x > 0 \\ \dot{\hat{K}}_r & = & -\Gamma_r r(e'PB) & \Gamma_r > 0 \\ \dot{\hat{K}}_u & = & -\Gamma_u(1 + \mu(x, x_m))\mathrm{Dz}(u)(e'PB) & \Gamma_u > 0 \end{array} \right.$$

When no saturation occurs $\mathrm{Dz}(u) \equiv 0$, standard MRAC is recovered.

Intermediate Result

Tracking result 1 (essentially main result of Lavretsky)

The MRAC algorithm (£), ($), (€) ensures

- All closed loop signals are bounded ($\mathcal{L}_\infty$ stability)
- Asymptotic tracking

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (x(t) - x_m(t)) = 0$$

But is this what we want?

MRAC reference model:

$$\dot{x}_m = A_m x_m + B_m r - B(1+\mu(x,x_m))\hat{K}_u(t)'\mathrm{Dz}(u)$$

Ideal reference model:

$$\dot{x}_r = A_m x_r + B_m r$$

More satisfactory result

Tracking result 2

The MRAC algorithm (£), ($), (€) ensures

- All closed loop signals are bounded ($\mathcal{L}_\infty$ stability)
- Asymptotic tracking

$$\lim_{t\to\infty} e_m(t) = \lim_{t\to\infty} (x_m(t) - x_r(t)) = 0$$

if

1. $\mathrm{Dz}(u^*) \in \mathcal{L}_2$ $\qquad u^* := K_x^* x(t) + K_r^* r(t)$ (ideal nominal control signal)
2. $\Delta u \in \mathcal{L}_2$ $\qquad \Delta u := \Delta K_x(t)' x(t) + \Delta K_r(t)' r(t)$

where

$$\Delta K_x' = \hat{K}_x' - K_x^* \qquad \Delta K_r' = \hat{K}_r' - K_r^*$$

MRAC with anti-windup terms achieves tracking under anti-windup-like conditions

Critique

- $\mathrm{Dz}(u^*) \in \mathcal{L}_2$ mirrors typical linear anti-windup results
- Only two tuning parameters: $\mu(.,.)$ and $\Gamma_u > 0$

- $\Delta u \in \mathcal{L}_2$ essentially means that adaptive gains have to converge to their ideal values:

$$K_x(t) \to K_x^* \quad K_r(t) \to K_r^*$$

Perhaps not always possible....

A modified result

Define:

$$\lim_{t\to\infty} \hat{K}_x(t) := K_{x,ss} \qquad \lim_{t\to\infty} \Delta K_x(t) := \Delta K_{x,ss}$$

$$\lim_{t\to\infty} \hat{K}_r(t) := K_{r,ss} \qquad \lim_{t\to\infty} \Delta K_r(t) := \Delta K_{r,ss}$$

$$\Delta u_{ss}(t) := \Delta \hat{K}_{x,ss} x(t) + \Delta \hat{K}_{r,ss} r(t)$$

Tracking result 3

The MRAC algorithm (£), ($), (€) ensures

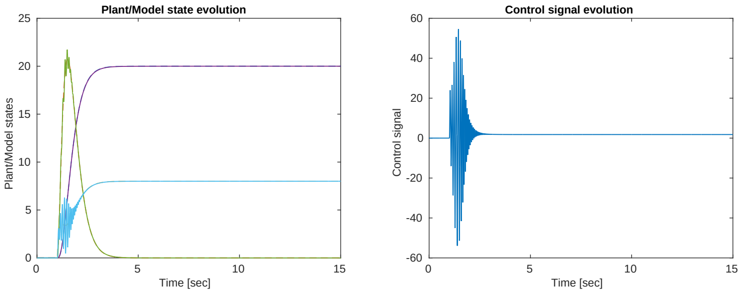
- All closed loop signals are bounded ($\mathcal{L}_\infty$ stability)
- Asymptotic tracking

$$\lim_{t\to\infty} e_m(t) = \lim_{t\to\infty} (x_m(t) - x_r(t)) = 0$$

if

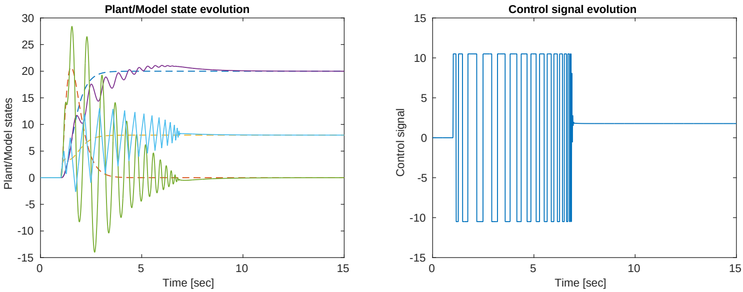
1. $\mathrm{Dz}(u^* + \Delta u_{ss}) \in \mathcal{L}_2$ (nominal, but not necessarily ideal control signal)
2. $\Delta u - \Delta u_{ss} \in \mathcal{L}_2$ (gains must converge to steady state values)

Hydraulic Actuator - no saturation



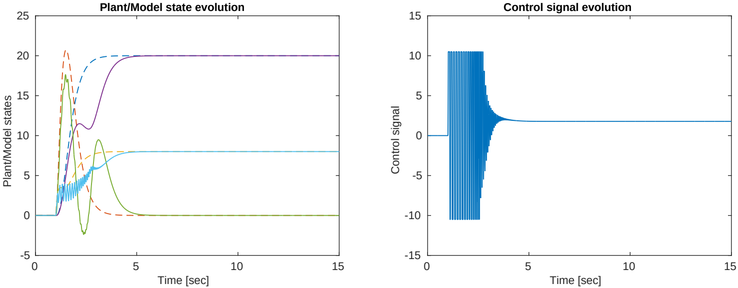
Response of adaptive control system without input saturation: left, plant/model state evolution; right, control signal

Hydraulic Actuator - no saturation



Response of adaptive control system with input saturation: left, plant/model state evolution; right, control signal

Hydraulic Actuator - saturation, anti-windup



Response of adaptive control system with input saturation and anti-windup: left, plant/model state evolution; right, control signal

Conclusion

Positive μ modification exhibits an anti-windup-like structure

- It also provides anti-windup-like solutions
- It requires minimal tuning (Γ_u, $\mu(.,.)$)
- It seems to be effective if MRAC itself is effective
- Can be extended to rate-limited actuators (some technical difficulties)

It has some problems:

- Some fundamental difficulties for open-loop unstable plants (A is unknown and if unknown how can a region of attraction be estimated?)
- Incorporation into an *incremental* adaptive scheme may make it more useful in practice

References

A E. Lavretsky and N. Hovakimyan. Stable adaptation in the presence of input constraints. *Systems and Control Letters*, 2007.

B M.C. Turner. Positive μ modification as an anti-windup mechanism. *Systems and Control Letters*, 2017

C M.C. Turner, J. Sofrony. Preliminary results on anti-windup for MRAC systems with rate-limits. Submitted to CDC, 2018.