Anti-Windup Design for Synchronous Machines

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L2S CentraleSupélec

International Workshop on Robust LPV Control Techniques and Anti-Windup Design

Toulouse, April 17th, 2018

Outline

1. *Motivation*: Use of electric machines, PI control
2. *Problem statement*: Improve transients of speed control of synchronous machines.
3. *Proposed solution*: Static Anti-windup.
4. *Example*: Application to a PMSM.

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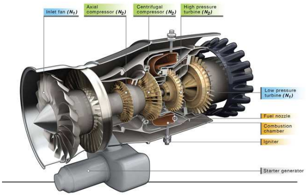
Electric Motors

Use of electric motors

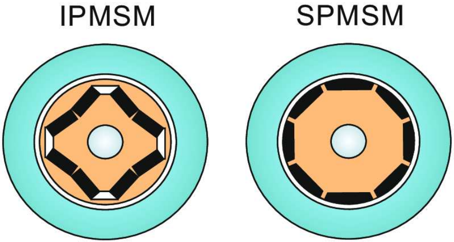


Anti-windup for electric motors:

Saqib, Rehan, Iqbal, and Hong. *Static Antiwindup Design for Nonlinear Parameter Varying Systems With Application to DC Motor Speed Control Under Nonlinearities and Load Variations*, IEEECST 18;
March and Turner. *Anti-Windup Compensator Designs for Nonsalient Permanent-Magnet Synchronous Motor Speed Regulators*, IEEETIA 09;
Sepulchre, Devos, Jadot, and Malrait. *Antiwindup Design for Induction Motor Control in the Field Weakening Domain*, IEEECST 13;

Permanent Magnet Synchronous Motors

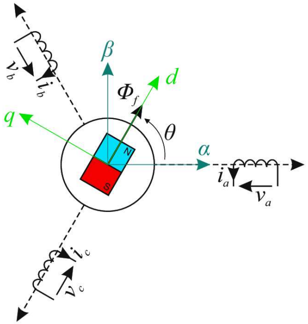
Internal vs Surface



Salient vs Non-Salient

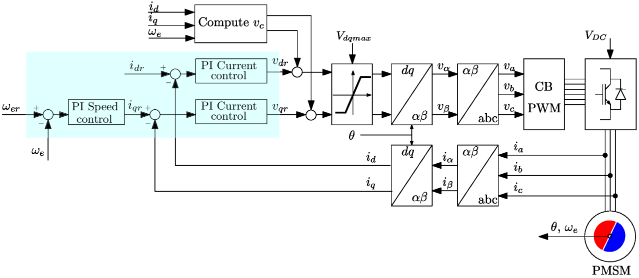
Permanent Magnet Synchronous Motors

Choice of coordinates: Park and Clarke transformation



Permanent Magnet Synchronous Motors

Architecture for speed control



Park Transformation

Choice of coordinates, in the rotor we have

$$\begin{cases} \frac{di_d}{dt} = -\frac{R_s}{L_d} i_d + \frac{L_q}{L_d}\omega_e i_q + \frac{1}{L_d} u_1 \\ \frac{di_q}{dt} = -\frac{R_s}{L_q} i_q - \frac{L_d}{L_q}\omega_e i_d - \frac{\psi_f}{L_q}\omega_e + \frac{1}{L_q} u_2 \\ \frac{d\omega_e}{dt} = \frac{N_p}{J}\gamma(i_d, i_q) - \frac{f}{J}\omega_e - \frac{N_p}{J}\gamma_L \end{cases}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in \mathcal{U}$$

$$\gamma(i_d, i_q) = \frac{3}{2} N_p(\psi_f + (L_d - L_q) i_d) i_q$$

i_q direct current, i_q quadrature current, ω_e electrical speed.
ψ_f, rotor flux, R_s, stator resistance, L_d, L_q direct and quadrature inductances.
J the moment of inertia of the rotor, N_p the number of pairs of poles, f viscous friction coefficient.

Vas. *Sensorless Vector and Direct Torque Control*, Oxford 98;
Grellet and Clerc. *Actionneurs Electriques, Principes, Modèles, Commande*, Eyrolles 97;

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$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in \mathbb{R}^2$$

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Speed control

For speed control, a strategy consists in taking

$$\gamma(i_d, i_q) = \frac{3}{2} N_p(\psi_f + (L_d - L_q) i_d) i_q$$

as the *input* for the speed control. A reference torque γ_r is obtained with

$$(i_{dr}, i_{qr}) = (0, K_\psi \gamma_r).$$

where $K_\psi = \left(\frac{3}{2} N_p \psi_f\right)^{-1}$.

PID for speed control

The control inputs (v_d, v_q) generated (using (i_{dr}, i_{qr}))

$$\begin{cases} \dot{e}_d &= -i_d + i_{dr} \\ \dot{e}_q &= -i_q + i_{qr} \\ \dot{e}_{\omega_e} &= -\omega_e + \omega_{er} \\ \gamma_r &= \frac{K_i}{T_{ii}} e_{\omega_e} - K_i \omega_e + K_i \omega_{er} \\ v_{dr} &= \frac{K_d}{T_{id}} e_d - K_d i_d + K_d i_{dr} \\ v_{qr} &= \frac{K_q}{T_{iq}} e_q - K_q i_q + K_q i_{qr}. \end{cases}$$

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Non-linear compensation

Remove the nonlinear terms with

$$v_c = \begin{bmatrix} -L_q \omega_e i_q \\ L_d \omega_e i_d + \psi_f \omega_e \end{bmatrix}$$

and applying in $v = v_r + v_c$.

Non-saturated system

The closed-loop

$$\dot{\xi} = A\xi + f(\xi) + B_\omega \omega_{er} + B_\gamma \gamma_L$$

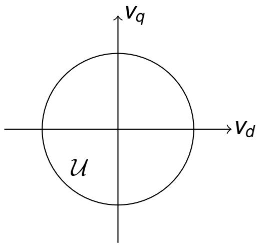
with $\xi = [i_d \;\; i_q \;\; \omega_e \;\; e_d \;\; e_q \;\; e_{\omega_e}]^T$

$$A = \left[\begin{array}{ccc|ccc} \frac{-R_s - K_d}{L_d} & 0 & 0 & \frac{K_d}{T_{id}L_d} & 0 & 0 \\ 0 & \frac{-R_s - K_q}{L_q} & \frac{-\psi_f - K_q K_\psi K_i}{L_q} & 0 & \frac{K_q}{L_q T_{iq}} & \frac{K_q K_\psi K_i}{L_q T_{ii}} \\ 0 & \frac{3}{2}\frac{N_p^2}{J}\psi_f & -\frac{f}{J} & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -K_\psi K_i & 0 & 0 & \frac{K_\psi K_i}{T_{ii}} \\ 0 & 0 & -1 & 0 & 0 & 0 \end{array}\right]; \; f(\xi) = \begin{bmatrix} 0 \\ 0 \\ \frac{3}{2}\frac{N_p^2}{J}(L_d - L_q) i_d i_q \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B_\omega = \begin{bmatrix} 0 \\ \frac{K_q K_\psi K_i}{L_q} \\ 0 \\ 0 \\ K_\psi K_i \\ 1 \end{bmatrix}; \; B_\gamma = \begin{bmatrix} 0 \\ 0 \\ -\frac{N_p}{J} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Input Constraints

The actual constraint set is $\mathcal{U} = \{ u \in \mathbb{R}^2 \mid \|u\|_2 \leq u_{\max} \}$



Define a mapping $\sigma : \mathbb{R}^2 \to \mathcal{U}$ such that

$$u = \sigma(v), \quad v = \begin{bmatrix} v_d \\ v_q \end{bmatrix}.$$

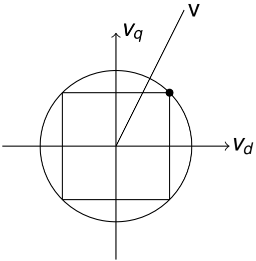
Standard saturation

Let us denote

$$\sigma_A : \mathbb{R}^2 \to \left\{ u \in \mathbb{R}^2 \mid \|u\|_\infty \leq \frac{\sqrt{2}}{2} u_{\max} \right\} \subset \mathcal{U}$$

the standard decentralized saturation with a fixed saturation level.

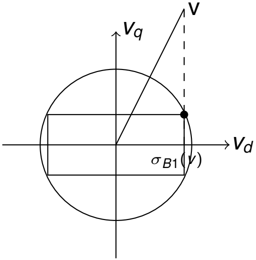
$$\sigma_A(v) = \begin{bmatrix} sign(v_d)\max(|v_d|, \frac{\sqrt{2}}{2}u_{\max}) \\ sign(v_q)\max(|v_q|, \frac{\sqrt{2}}{2}u_{\max}) \end{bmatrix}$$



Alternative saturation

Let us denote $\sigma_B : \mathbb{R}^2 \rightarrow \mathcal{U}$ a mapping that gives priority to the first input

$$\sigma_B(v) = \left[\begin{array}{c} sign(v_d)\max(|v_d|, u_{\max}) \\ sign(v_q)\max(|v_q|, \sqrt{u_{\max}^2 - \sigma_{B1}^2(v)}) \end{array} \right]$$

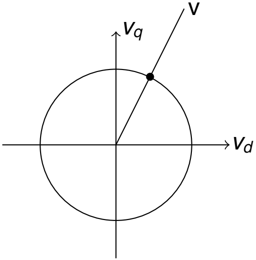


Introducing a *variable* saturation level for the second input

Directionality preserving saturation

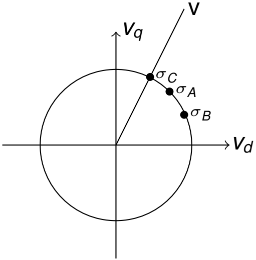
Let us denote $\sigma_C : \mathbb{R}^2 \to \mathcal{U}$ the directionality preserving map

$$\sigma_C(v) = \frac{u_{\max}}{\max(u_{\max}, \|v\|_2)} v$$



Sector conditions for directionality preserving

For the same value v different inputs



How to incorporate these functions for the analysis/synthesis?

$\longrightarrow$ Sector conditions

Sector conditions for directionality preserving

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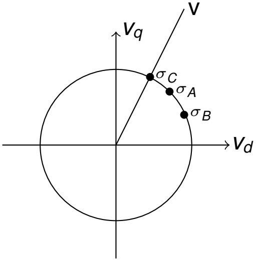
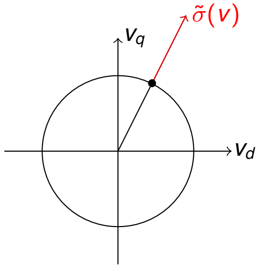


How to incorporate these functions for the analysis/synthesis?
$\longrightarrow$ Sector conditions

Sector condition

Consider the directionality preserving non-linearity $\sigma(v) = \sigma_C(v)$ and define $\tilde{\sigma}(v) := v - \sigma(v)$. We have

$$
\begin{aligned}
\tilde{\sigma}(v) &= v - \sigma(v) \\
&= v - \frac{u_{\max}}{\max(u_{\max}, \|v\|_2)} v \\
&= \underbrace{\left(1 - \frac{u_{\max}}{\max(u_{\max}, \|v\|_2)}\right)}_{\in[0,1]} v
\end{aligned}
$$



Sector condition

Lemma 1.1

For any $T \in \mathbb{S}^m_{\geq 0}$ *the inequality*

$$\tilde{\sigma}^T(u)T(\tilde{\sigma}(u) - u) \leq 0.$$

holds for all $u \in \mathbb{R}^m$*.*

Proof.

Since $\tilde{\sigma}(u) = (1 - \beta(u))u$ and $\beta(u) \in [0, 1]$, we have

$$\begin{aligned} \tilde{\sigma}^T(u)T(\tilde{\sigma}(u) - u) &= (1 - \beta(u))u^T T(-\beta(u))u \\ &= (1 - \beta(u))(-\beta(u))u^T Tu \\ &\leq 0. \end{aligned}$$

□

State constraints, current limitation

Also need to prevent high peak currents, that is

- total peak current $\leq \|i_{peak}\|_2$
- total steady state current $\leq \|i_{ss}\|_2$

These are *state constraints* involving the total current $\|i\|_2^2 = i_d^2 + i_q^2$.
$\longrightarrow$ Possible solution: bound the reference signal i_{dr} and i_{qr}.

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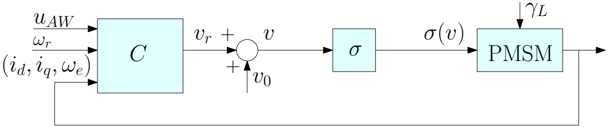
Speed Control Architecture

A block diagram of the control loop of the linearized system including the saturation is given below



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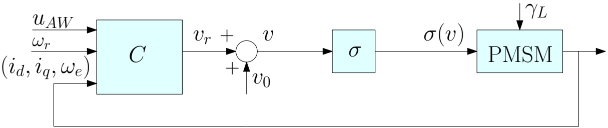

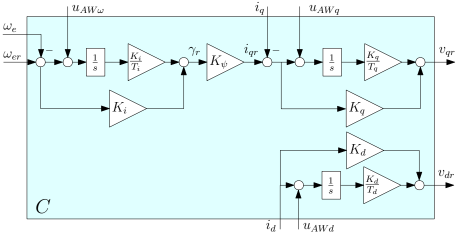

Speed Control Architecture

And the control inputs (v_d, v_q) are obtained from

$$\begin{cases} \dot{e}_d &= -i_d + v_{AWd} \\ \dot{e}_q &= -i_q + K_\psi \gamma_r + v_{AWq} \\ \dot{e}_{\omega_e} &= -\omega_e + \omega_{er} + v_{AW\omega} \\ \gamma_r &= \frac{K_i}{T_{ii}} e_{\omega_e} - K_i \omega_e + K_i \omega_{er} \\ v_d &= \frac{K_d}{T_{id}} e_d - K_d i_d + K_d i_{dr} \\ v_q &= \frac{K_q}{T_{iq}} e_q - K_q i_q + K_q i_{qr}. \end{cases}$$

Speed control with anti-windup input

Using $\tilde{\sigma}(v) := v - \sigma(v)$ the closed-loop system becomes

$$\dot{\xi} = A\xi - B\tilde{\sigma}(v) + B_{AW}v_{AW} + f(\xi) + B_{\omega}\omega_{er} + B_{\gamma}\gamma_L$$

$$B = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};\ B_{AW} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\ v_{AW} = \begin{bmatrix} v_{AWd} \\ v_{AWq} \\ v_{AW\omega} \end{bmatrix}.$$

Compute a *static Anti-Windup* $u_{AW} = K_{AW}\tilde{\sigma}(v)$.
The LMI based method used here can be found in

Zaccarian and Teel. *Modern Anti-Windup Synthesis - Control Augmentation for Actuator Saturation*, Princetown Series in Applied Mathematics 2011(Section 4.3.1);
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Static Anti-Windup

Consider the system

$$\begin{cases} \dot{\xi} &= A_{cl}\xi + (B_{cl,q} + B_{cl,AW}K_{AW})\tilde{\sigma}(v) + B_{cl,w}w \\ v &= C_{cl}\xi + D_{cl,u}\tilde{\sigma}(v) + D_{cl,w}w \\ z &= C_{cl,z}\xi + D_{cl,z}\tilde{\sigma}(v) + D_{cl,z}w \end{cases}$$

Theorem 2.1

If there exist $Q \in \mathbb{S}^n_{>0}$, $S \in \mathbb{R}^{m\times n}$, $T \in \mathbb{D}^m_{>0}$, and a scalar $\gamma > 0$ such that

$$He \begin{pmatrix} A_{cl}Q & B_{cl,q}T + B_{cl,AW}S & B_{cl,w} & 0_{n\times p} \\ C_{cl,u}Q & (D_{cl,u} - I_m)T & D_{cl,uw} & 0_{m\times p} \\ 0_{m_w\times n} & 0_{m_w\times m} & -\frac{1}{2}\gamma I_{m_w} & 0_{m_w\times p} \\ C_{cl,z}Q & D_{cl,z}T & D_{cl,zw} & -\frac{1}{2}\gamma I_p \end{pmatrix} < 0$$

Then $K_{AW} = ST^{-1}$ guarantees $\frac{\|w\|_2}{\|z\|_2} < \gamma$ for the closed-loop system.

Experimental results

We applied the above to a *non-salient machine* with $R_s = 0.95\Omega$, $L_d = L_q = 13.6mH$, $\phi_f = 0.284\,Wb$, and $u_{\max} = 34\,V$. $N_p = 4$, $J = 3.2 \times 10^{-3} kgm^2$, $f = 0.0001\,Nms^{-1}$.
PI gains $K_q = K_d = 34\ T_{iq} = T_{id} = 0.0143$ and $K_i = 0.2011\ T_{ii} = 0.0796$.

With

$$w = \gamma_L$$

and

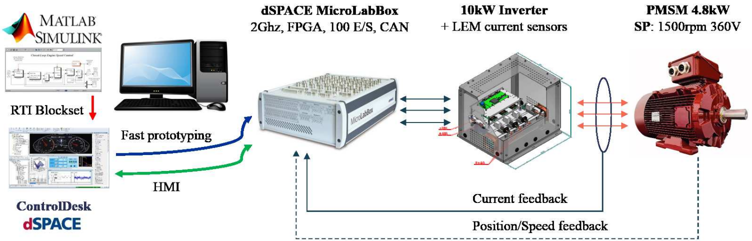
$$z = \omega_e - \omega_{er},$$

we obtain

$$K_{AW} = \begin{bmatrix} -1.3408 & 0.000 \\ 0.0006 & -1.0563 \\ -0.0012 & -2.3856 \end{bmatrix}.$$

Experimental results

Experimental setup



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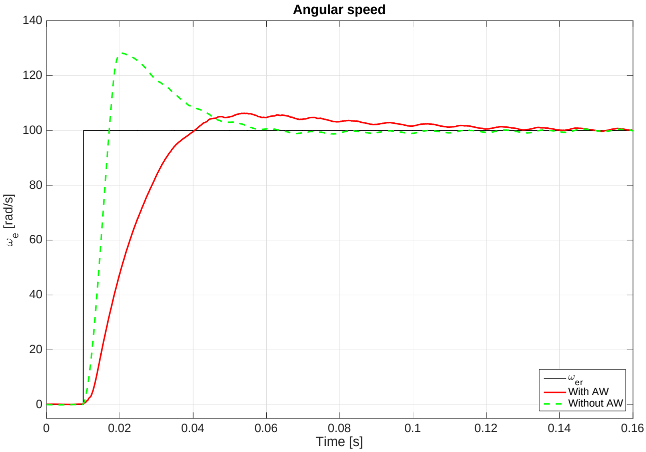

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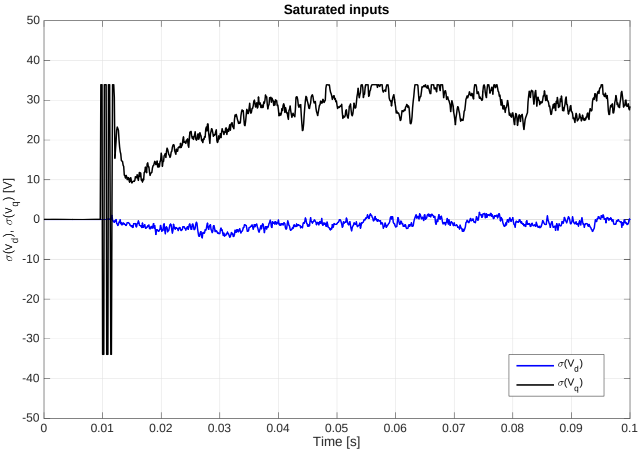

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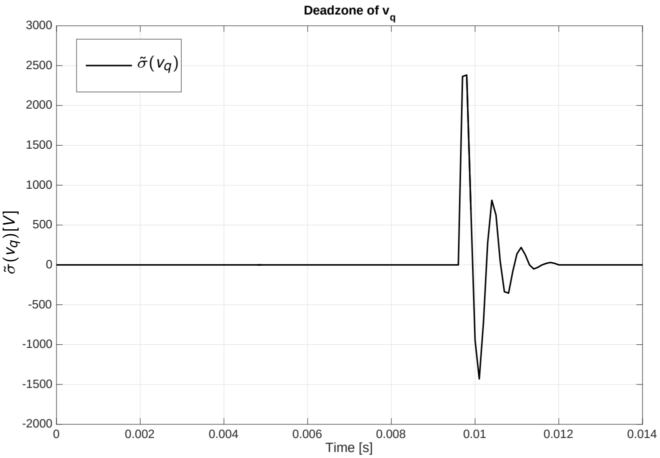

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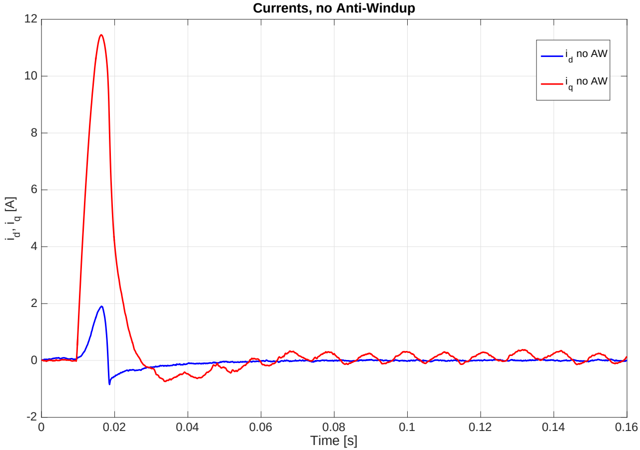

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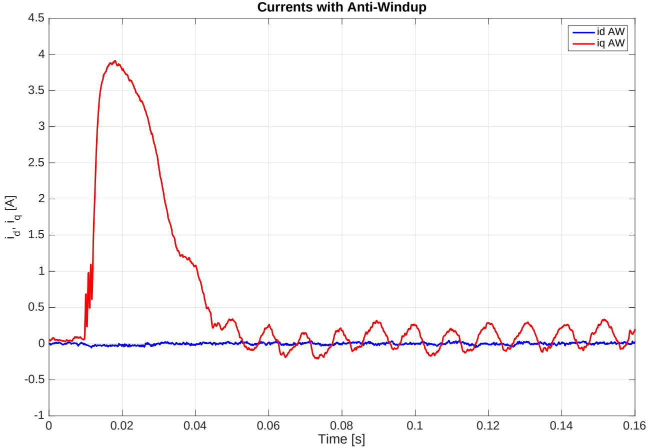

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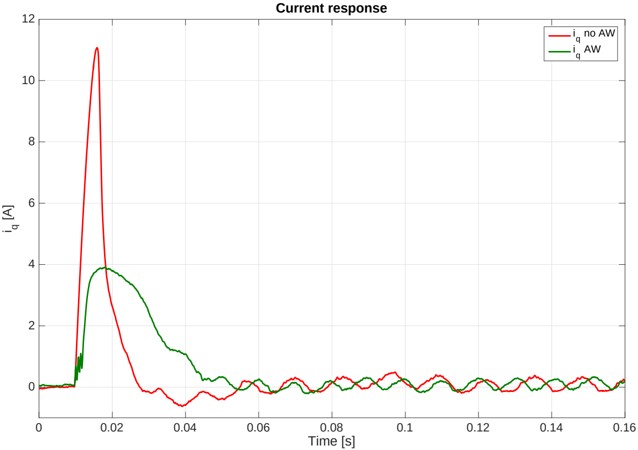

Steps for non-linear analysis

Due to the nonlinear terms, the *compensation* is lost when the system saturates. For the non-salient case we have

$$\begin{cases} \dot{\xi} &= A_{cl}\xi + (B_{cl,q} + B_{cl,AW}K_{AW})\tilde{\sigma}(v) + B_{cl,w}w \\ v &= C_{cl}\xi + v_c(\xi) + D_{cl,u}\tilde{\sigma}(v) + D_{cl,w}w \\ z &= C_{cl,z}\xi + D_{cl,z}\tilde{\sigma}(v) + D_{cl,z}w \end{cases}$$

not

$$\begin{cases} \dot{\xi} &= A_{cl}\xi + (B_{cl,q} + B_{cl,AW}K_{AW})\tilde{\sigma}(v) + B_{cl,w}w \\ v &= C_{cl}\xi + D_{cl,u}\tilde{\sigma}(v) + D_{cl,w}w \\ z &= C_{cl,z}\xi + D_{cl,z}\tilde{\sigma}(v) + D_{cl,z}w \end{cases}$$

$$v_c(\xi) = \begin{bmatrix} -L_q\omega_e i_q \\ L_d\omega_e i_d \end{bmatrix}$$

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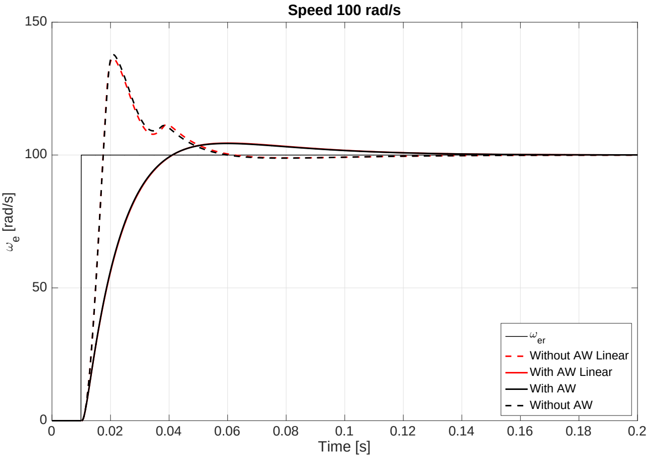
$$v_c(\xi) = \begin{bmatrix} -L_q\omega_e i_q \\ L_d\omega_e i_d \end{bmatrix}$$

Steps for non-linear analysis

Due to the nonlinear terms, the *compensation* is lost when the system saturates. We have



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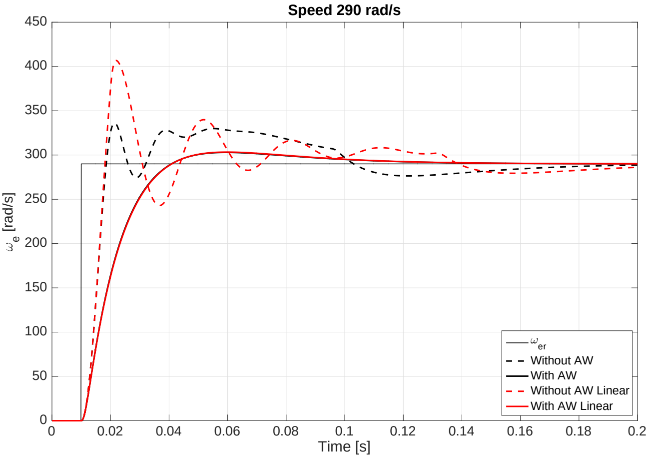

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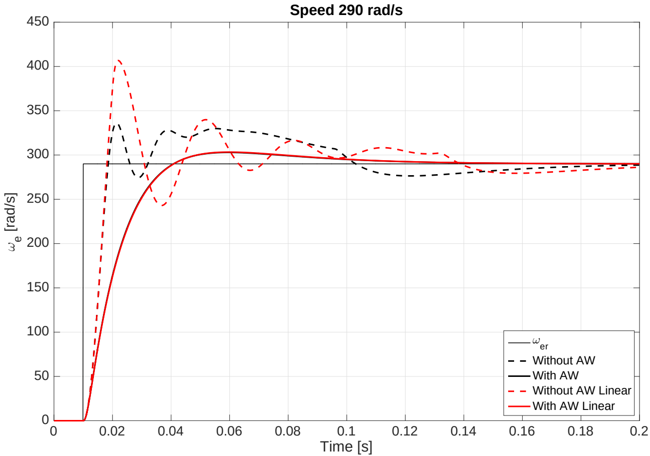

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Consider the quadratic function

$$V_0(i_d, i_q, \omega_e) = \frac{1}{2} \begin{bmatrix} i_d \\ i_q \\ \omega_e \end{bmatrix}^T P_0(p_1) \begin{bmatrix} i_d \\ i_q \\ \omega_e \end{bmatrix}$$

with

$$P_0(p_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & p_1 & 0 \\ 0 & 0 & \left(\frac{3}{2}\frac{N_p^2}{J}(L_d - L_q)\right)^{-1}\left(\frac{L_q}{L_d} - \frac{L_d}{L_q}p_1\right) \end{bmatrix}$$

$P_0(p_1) > 0$ provided

$$0 < p_1 < \left(\frac{L_q}{L_d}\right)^2, \text{ if } L_d > L_q \quad \text{or} \quad p_1 > \left(\frac{L_q}{L_d}\right)^2, \text{ if } L_d < L_q.$$

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Energy-preserving nonlinearities

We have $\langle \nabla V_0, f(\xi) \rangle$

$$\left\langle \nabla V_0, \begin{bmatrix} \frac{L_q}{L_d}\omega_e i_q \\ -\frac{L_d}{L_q}\omega_e i_d \\ \frac{3}{2}\frac{N_p^2}{J}(L_d - L_q) i_d i_q \end{bmatrix} \right\rangle = \begin{bmatrix} i_d \\ i_q \\ \omega_e \end{bmatrix}^T P_0(p_1) \begin{bmatrix} \frac{L_q}{L_d}\omega_e i_q \\ -\frac{L_d}{L_q}\omega_e i_d \\ \frac{3}{2}\frac{N_p^2}{J}(L_d - L_q) i_d i_q \end{bmatrix} = 0$$

Take $P_c > 0$ and consider

$$V(x) = V_0(i_d, i_q, \omega_e) + \frac{1}{2} \begin{bmatrix} e_d \\ e_q \\ e_{\omega_e} \end{bmatrix}^T P_c \begin{bmatrix} e_d \\ e_q \\ e_{\omega_e} \end{bmatrix}$$

which, from the above gives $\dot{V}$ as a quadratic function... However with the above $\dot{V} \not< 0$.

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- Applied static Anti-Windup synthesis to a PMSM model.
- Defined non-linearities to exploit the input set.
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- Next steps include:
  - saturation of the reference current i_{qr},
  - non-linear analysis,
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Review

Thank you!

Local Sector Condition

The lemma below provides an inequality that holds only locally.

Lemma 4.1

For any $T \in \mathbb{S}^m_{\geq 0}$, and any scalar $\eta \in (0,1)$ the inequality

$$S(u,\eta) := \tilde{\sigma}^T(u) T(\tilde{\sigma}(u) - (1-\eta)u) \leq 0$$

holds for all $u \in \eta^{-1}\mathcal{U}$.

$$\text{maximize} \quad \beta \quad \text{subject to} \quad \beta \in [0,1], \;\; \beta u \in \mathcal{U}. \tag{3}$$

Proof.

We have $\eta^{-1}\mathcal{U} = \{u \in \mathbb{R}^m | \eta u \in \mathcal{U}\}$. Thus, from (3), the mapping β satisfies $\beta(u) \geq \eta$ for all $u \in \eta^{-1}\mathcal{U}$.
Hence we have

$$\begin{aligned} S(u,\eta) &= \tilde{\sigma}^T(u) T(\tilde{\sigma}^T(u) - (1-\eta)u) \\ &= (1-\beta(u))u^T T((1-\beta(u))u - (1-\eta)u) \\ &= (1-\beta(u))(\eta - \beta(u))u^T Tu. \end{aligned}$$

Since $(1-\beta(u)) \geq 0$ and $u^T Tu \geq 0 \; \forall u \in \mathbb{R}^m$ and $(\eta - \beta(u)) \leq 0 \; \forall u \in \eta^{-1}\mathcal{U}$, we have $S(u,\eta) \leq 0$ $\forall u \in \eta^{-1}\mathcal{U}$. □