ON MODELING & ROBUST LPV/$\mathcal{H}_\infty$ BASED OBSERVATION OF FUEL SLOSH DYNAMICS

Application to spacecraft attitude control

Workshop on Robust Control & Convex Optimization - Toulouse, March 2019

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Introduction to Sloshing in Spacecraft

Key Elements [1/2]

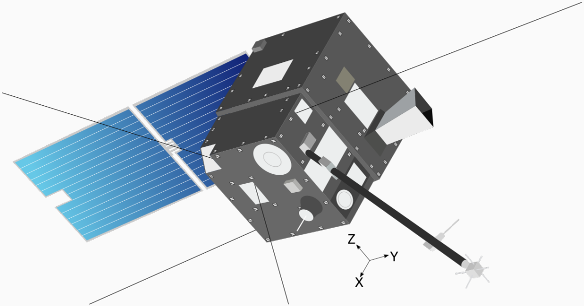
- Sloshing : liquid free surface movement inside tanks or containers[1]
- Low frequency and badly damped phenomenon
- Spacecraft carry lifespan-defining mass of liquid propellant
  e.g. 4% (DEMETER, 2004) to 38% (DAWN, 2007 & Astra 2A, 1998) of launch mass
- Coupled fluid-structure dynamics $\rightarrow$ disruptive forces and torques
- Alteration of spacecraft pointing accuracy
- Compromises system perf. and stability $\rightarrow$ more complex controller design[2]
- Can lead to severe consequences : NEAR[3] (1998), ATS-5 (1969), Falcon 1 (2007)

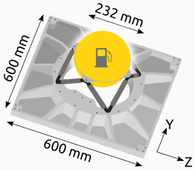
[1] R. A. Ibrahim, *Liquid sloshing dynamics: theory and applications*. Cambridge University Press, 2005.
[2] P. Mason and S. Starin, “The effects of propellant slosh dynamics on the SDO,” in *AIAA GNC*, 2011, p. 6731.
[3] E. J. Hoffman *et al.*, “The NEAR rendezvous burn anomaly of december 1998,” *Johns Hopkins Univ.*, 1999.

- Space application $\rightarrow$ surface tension effects has to be considered
- Microgravity conditions are difficult to reproduce in laboratories
  e.g. 0G flights or drop towers (short duration $\sim$ 20 s)
- Very complex analytical descriptions $\rightarrow$ Computational Fluid Dynamics
- In-situ experiments: Sloshsat-FLEVO (ESA), Spheres (NASA) and Fluidics (ESA)[4]
- Flight data have been used to adjust and validate CFD models
  e.g. `DIVA` (IMFT)[5] and `COMFLO` (University of Groningen)[6]

[4] J. Mignot *et al.*, "Fluid dynamics in space experiment," , IAC, 2017.
[5] M. Lepilliez *et al.*, "On two-phase flow solvers in irregular domains with contact line," *Journal of Computational Physics*, vol. 321, pp. 1217–1251, 2016.
[6] A. E. Veldman *et al.*, "The numerical simulation of liquid sloshing on board spacecraft," *Journal of Computational Physics*, vol. 224, no. 1, pp. 82–99, 2007.

Illustration : μ-satellite DEMETER (CNES)





Usual methods and their drawbacks [1/2]

- Baffles and bladders in propellant tanks[7]
  - ⊕ Increases sloshing frequency and reduces its amplitude
  - ⊖ Heavier satellite and more expensive mission
- Time margins between aggressive maneuvers to let propellant settle down
  - ⊕ Avoid propellant over-excitation
  - ⊖ Reduces mission availability
- Smoothed angular velocity references profiles
  - ⊕ Reduces propellant excitation
  - ⊖ Whole satellite agility may no longer be exploited

[7] F. T. Dodge, "Engineering study of flexible baffles for slosh suppression (nasa cr-1880)," Tech. Rep., 1971.

- Notch filters[8]
  - ➕ Mitigates sloshing influence
  - ➖ Reduces satellites bandwidth (sloshing frequencies are uncertain)
- Linear Time Invariant Models (will be further detailed later)
  - ➕ Suitable for model-based control
  - ➖ Valid only for specific cases and small amplitude motion
- Infinite-Dimensional Models[9]
  - ➕ More representative
  - ➖ Unsuitable for 2D/3D coupled motion in microgravity

[8] A. Preumont, *Vibration control of active structures*. Springer, 1997, vol. 2.
[9] F. L. Cardoso-Ribeiro, D. Matignon, and V. Pommier-Budinger, “Control design for a coupled fluid-structure system with piezoelectric actuators,” *Proceedings of the 3rd CEAS EuroGNC*, pp. 13–15, 2015.

Problem Statement and Proposed Solution

- Problem Statement
  - Always more stringent attitude pointing accuracy and stability requirements
  - Need for very effective Attitude Control Systems
- Proposed Solution
  - Development of a new model of propellant sloshing torque
  - Observer design to enhance attitude control by compensating torque

From CFD to LPV models

Equivalent Mechanical Models

Approximation of the liquid with a mechanical system[10]
e.g. spring-mass, pendulum, free-mass or mass constrained on a surface

- ⊕ Successfully used for decades, for launchers and satellites[11]
- ⊕ Can be addressed like flexible modes[12]
- ⊕ Model-based controller design[13]
- ⊖ Based on linearized fluid dynamics models
- ⊖ Often valid only for axisymmetric problems with small amplitude motion
- ⊖ Not dependent on inertial forces acting on the fluid during attitude maneuver

[10] H. N. Abramson *et al.*, "The dynamic behavior of liquids in moving containers, with applications to space vehicle technology (nasa-sp-106)," Tech. Rep., 1966.
[11] P. J. Enright and E. C. Wong, "Propellant slosh models for the cassini spacecraft," Jet Propulsion Laboratory, Caltech, Tech. Rep., 1994.
[12] L. Mazzini, *Flexible Spacecraft Dynamics, Control and Guidance*. Springer, 2015.
[13] J. R. Hervas and M. Reyhanoglu, "Control of a spacecraft with time-varying propellant slosh parameters," in *Control, Automation and Systems (ICCAS), 2012 12th International Conference on*, IEEE, 2012, pp. 1621–1626.

Example : **IMFT** study for several bang-off maneuvers (square shape acc. profile)[14]

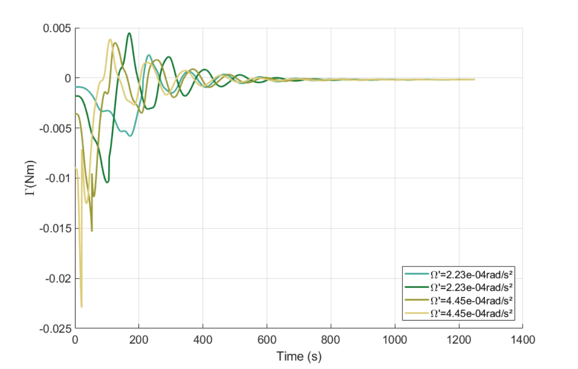


Figure 1: Torque Γ_Z along the Z-axis for a $4.72 \times 10^{-2} rad/s$ steady-state velocity

System : spherical tank, diameter - $0.585\ m$, filling ratio - 50%, lever arm - $0.4\ m$

[14] M. Lepilliez, "Simulation numérique des ballotements d'ergols dans les réservoirs de satellites en microgravité et à faible nombre de bond," PhD thesis, Université Paul Sabatier-Toulouse III, 2015.

Parameters affecting sloshing

Tank filling ratio
Thrusters saved for orbital maneuvers $\rightarrow$ constant filling ratio

Gravity vector w.r.t. the spacecraft, linked to the attitude $\boldsymbol{\theta}$
Gravity effects can be neglected (microgravity)

Liquid properties, e.g. density, viscosity, surface tension
Propellant properties do not change

Tank geometry and position inside the spacecraft
Rigid tank with fixed position

Angular speed $\boldsymbol{\Omega}$ and acceleration $\dot{\boldsymbol{\Omega}}$
Linked to inertial forces acting on the fluid during attitude maneuvers

Nonlinear Model of Propellant Sloshing Torque [1/2]

- We will consider a satellite *bang-off-bang* attitude maneuver around a single axis
- Our reasoning can be generalized to any maneuver given appropriate CFD data
- Model sloshing disruptive torque instead of propellant behavior
- Sloshing torque Γ_F as the output of a nonlinear 2nd order system with varying frequency ω and damping ratio ϵ:

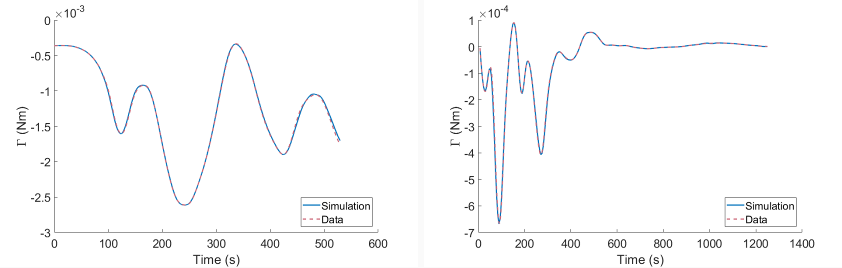
$$\begin{aligned} \ddot{\Gamma}_F + C_s(\Omega,\dot{\Omega})\dot{\Gamma}_F + K_s(\Omega,\dot{\Omega})\Gamma_F &= -A_s(\Omega,\dot{\Omega})\Omega - B_s(\Omega,\dot{\Omega})\dot{\Omega} && (1)\\ C_s(\Omega,\dot{\Omega}) &= 2\xi(\Omega,\dot{\Omega})\omega(\Omega,\dot{\Omega}) && (2)\\ K_s(\Omega,\dot{\Omega}) &= \omega(\Omega,\dot{\Omega})^2 && (3) \end{aligned}$$

- Generalization/abstraction of Equivalent Mechanical Models
- Nonlinearity results from the dependence of A_S, B_S, C_S and K_S to $(\Omega,\dot{\Omega})$

Nonlinear Model of Propellant Sloshing Torque [2/2]

A_S, B_S, C_S and K_S can be identified by using CFD results :

- Definition of N small time intervals
- On each interval Ω and $\dot{\Omega}$ are assumed constant
- On each interval the nonlinear model becomes Linear Time Invariant
- ω and ϵ can be bounded by analyzing CFD results
- Use a Constrained Least Squares method (Matlab^{TM} `lsqlin` routine)
- Result : sets $\{C_{s_i}, K_{s_i}, A_{s_i}, B_{s_i}\}_{i \leq N}$ associated to $\{\Omega_i, \dot{\Omega}_i\}_{i \leq N}$
- Note that better results (relative error $\leq \ 10\%$) are obtained by proceeding on two different submodels, one for each side of the acceleration discontinuity



(a) submodel *before the discontinuity* **(b)** submodel *after the discontinuity*

Figure 2: Identification results examples

- Sloshing state-space representation :

$$\underbrace{\begin{pmatrix} \dot{\Gamma}_F \\ \ddot{\Gamma}_F \end{pmatrix}}_{\dot{x}_F} = \underbrace{\begin{pmatrix} 0 & 1 \\ -K_S & -C_S \end{pmatrix}}_{A_F(K_S,\, C_S)} \underbrace{\begin{pmatrix} \Gamma_F \\ \dot{\Gamma}_F \end{pmatrix}}_{x_F} + \underbrace{\begin{pmatrix} 0 & 0 \\ -A_S & -B_S \end{pmatrix}}_{B_F(A_S,\, B_S)} \begin{pmatrix} \Omega \\ \dot{\Omega} \end{pmatrix} \tag{4}$$

$$\Gamma_F = \begin{pmatrix} 0 & 1 \end{pmatrix} x_F \tag{5}$$

- Uncertainties arise from numerical simulation, identification and modeling errors
- Poorly known uncertainties $\rightarrow$ useless to develop accurate model (e.g. LFT based)
- Bounded disturbance w such that $||w||_2 \leq \overline{w}$ is introduced :

$$\dot{x}_F = A_F(K_S, C_S)x_F + B_F(A_S, B_S) \begin{pmatrix} \Omega \\ \dot{\Omega} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w \tag{6}$$

LPV Model of Propellant Sloshing Torque [2/4]

- Single-axis dynamics of an actuated satellite :

$$\dot{x}_{SAT} = A_{SAT}x_{SAT} + B_{SAT}(\Gamma_F + \Gamma_P + \Gamma_C) \quad (7)$$

$$\theta = C_\theta x_{SAT} \quad (8)$$

Γ_P is a non-sloshing disturbing torque, Γ_C is the control torque

- To also estimated Γ_P the state vector is extended and $\dot{\Gamma}_P = 0$ is considered
- Further analysis of the identif. results highlights a link between parameters :

$$B_S = \alpha_{AB}A_S + \beta_{AB} \quad (9)$$

$$C_S = \alpha_{KC}K_S + \beta_{KC} \quad (10)$$

- Combining equations → uncertain LPV model of the liquid-filled satellite :

$$\dot{x} = A(\alpha(t))x + B_u\Gamma_C + \underbrace{[0\ 1\ 0\ \ldots\ 0]^T}_{B_w} w \quad (11)$$

$$\begin{pmatrix} \theta \\ \Gamma_D \end{pmatrix} = \begin{bmatrix} C_m \\ C_z \end{bmatrix} x \quad (12)$$

where :

$$\begin{aligned} \alpha(t) &= (\alpha_A(t), \alpha_K(t)) \\ &= (A_S[\Omega(t), \dot{\Omega}(t)], K_S[\Omega(t), \dot{\Omega}(t)]) \quad (13) \\ \Gamma_D &= \Gamma_F + \Gamma_P \quad (14) \\ x &= [x_F\ x_{SAT}\ \Gamma_P]^T \quad (15) \end{aligned}$$

- Filtering effect of the low-pass actuators → param. variations only in the A matrix

- Using α as parameter, instead of $(\Omega, \dot{\Omega})$, has the following advantages :
  - $A(\alpha)$ is a linear function of α (simplifies observer design and stab. analysis)
  - A_S, B_S, C_S and K_S do not need to be explicitly written as functions of $(\Omega, \dot{\Omega})$
- Reactions wheels limitation :
  - Bounded control torque capacity
  - Restricted variations of $(\Omega(t), \dot{\Omega}(t))$
- This permits to characterize a narrowed definition domain for A_S and K_S
- $\alpha(t)$ takes its values in a polytope $\mathcal{P}$ of 9 vertices $\mathcal{P}_i,\ i \in \{1, 2, \ldots, 9\}$, i.e.

$$\alpha(t) \ \in \ \mathcal{P} \ := \ Co\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_9\} \tag{16}$$

$\mathcal{H}_\infty$-based Observer Design

Problem Statement

- Aim is to enhance attitude control independently of any existing controller
- Decoupling of the satellite from sloshing dynamics obtained by canceling the disturbing torques estimate from the control input
- Solution : design of a reliable LPV observer
- Estimated torque has to be accurate in spite of model disturbances w
- Observer has to compensate the small delay induced by actuators dynamics
- Observer state-space representation :

$$\begin{aligned} \dot{\hat{x}} &= A(\alpha(t))\hat{x} + B_u\Gamma_C + L(\alpha(t))(\theta - \hat{\theta}) && (17) \\ &= \underbrace{(A(\alpha) - L(\alpha)C_m)}_{A_{Obs}}\hat{x} + \underbrace{[B_u \quad L(\alpha)]}_{B_{Obs}}[\Gamma_C \quad \theta]^T && (18) \\ \hat{\Gamma}_D &= C_z x + \underbrace{[0 \quad 0]}_{D_{Obs}}[\Gamma_C \quad \theta]^T && (19) \end{aligned}$$

where $\hat{x}$ and $\hat{\Gamma}_D$ are x and Γ_D estimates, $L(\alpha)$ is the observer gain

Characterization as a multi-model $\mathcal{H}_\infty$ design problem [1/4]

- Dynamics of the state error :

$$(\mathcal{S}) \begin{cases} \dot{\epsilon} = A_{Obs}x + B_w w, \ \epsilon = x - \hat{x} & (20) \\ z = C_z \epsilon & (21) \\ \quad = \Gamma_D - \hat{\Gamma}_D & (22) \end{cases}$$

- $A(\alpha)$ is a linear function of α :

$$A(\alpha) = A_0 + \alpha_A A_A + \alpha_K A_K \quad (23)$$

- Thus we propose to search a structured observer gain :

$$L(\alpha) = L_0 + \alpha_A L_A + \alpha_K L_K \quad (24)$$

- The system has then an affine LPV structure

Characterization as a multi-model $\mathcal{H}_\infty$ design problem [2/4]

- Recall : $\alpha(t) \in \mathcal{P} := Co\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_9\}$
- Affine LPV structure $\rightarrow$ a polytopic model can be easily deduced :

$$\alpha = \sum_{i=1}^{9} \beta_i \mathcal{P}_i,\ \beta_i \geq 0 \text{ and } \sum_{i=1}^{9} \beta_i = 1 \quad (25)$$

$$\mathcal{S}(\alpha) = \sum_{i=1}^{9} \beta_i \mathcal{S}(\mathcal{P}_i) \quad (26)$$

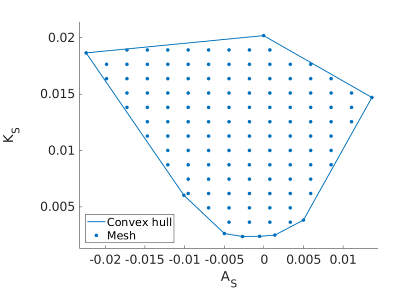


Figure 3: Vertices of the polytopic model

Characterization as a multi-model $\mathcal{H}_\infty$ design problem [3/4]

- Approach suitable to be addressed by a $\mathcal{H}_\infty$ multi-model robust design techniques on the 9 LTI models $(S_{i\leq 9})$ (LPV system frozen at the vertices $\mathcal{P}_{i\leq 9}$)[15]
- With `systune` Matlab^{TM} routine[16,17] it is possible :
  - to compute bounded gains L_0, L_A and L_K
  - to minimize the estimation error
  - to constrain the observer/error dynamics
- Remark: A resolution is also possible via extended LMI-based LPV techniques *to be proposed for LPVS 2019*

[15] J.-M. Biannic and P. Apkarian, "Missile autopilot design via a modified lpv synthesis technique," *Aerospace Science and Technology*, vol. 3, no. 3, pp. 153–160, 1999.
[16] P. Apkarian and D. Noll, "Nonsmooth $\mathcal{H}_\infty$ synthesis," *IEEE Trans. on Automatic Control*, vol. 51, no. 1, pp. 71–86, 2006.
[17] P. Apkarian, P. Gahinet, and C. Buhr, "Multi-model, multi-objective tuning of fixed-structure controllers," in *Proceedings of ECC 2014*, 2014, pp. 856–861.

Characterization as a multi-model $\mathcal{H}_\infty$ design problem [4/4]

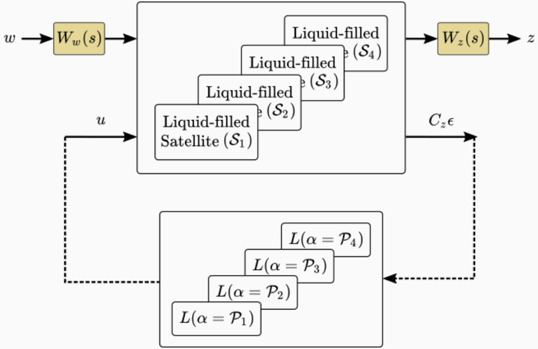


Figure 4: Design model block diagram

Implementation

- The following constraints have been defined for the $\mathcal{H}_\infty$ problem :
  - Minimum decay rate : 0.001 rad/s
  - Minimum damping ratio : 0.7
  - Maximum observer frequency : 5 rad/s
  - Absolute value of gains < 2
- Error signal is weighted by a low-pass transfer function $W_z(s)$ to minimize the steady-state estimated torque error :

$$W_z(s) = 2\frac{0.01}{s + 0.01} \tag{27}$$

- The model disturbance w is weighted by a constant filter $W_w(s) = 0.01$
- Actuators induced delays compensated by augmenting z with a derivative term :

$$z = (\Gamma_D - \hat{\Gamma}_D) + E(\dot{\Gamma}_F - \dot{\hat{\Gamma}}_F) \tag{28}$$

where the gain E is tuned according to the characteristics of the actuator.

Illustration

System and requirements

- Required attitude control perf. inspired by DEMETER satellite bus benchmark[18] :
  - Pointing steady-state error < 0.04 deg
  - Pointing rate steady-state error < 0.1 deg/s
  - Angular momentum < 0.12 Nms
  - Control torque < 0.005 Nm
- Satellite inertia $I_z = 30$ kg.m^2
- Satellite controlled by a PD controller satisfying in the absence of sloshing :

$$\Gamma_C = 0.3553\delta_\theta + 6.2845\delta_\Omega \tag{29}$$

- The actuator is a reaction wheel modeled by the following transfer function :

$$RWS(s) = \frac{1.2s + 0.76}{s^2 + 2.4s + 0.76} \tag{30}$$

- To get faster responses, a guidance torque Γ_d is added in a feed-forward path

[18] C Pittet and D Arzelier, "Demeter: A benchmark for robust analysis and control of the attitude of flexible micro satellites," *IFAC Proceedings Vol.*, vol. 39, no. 9, pp. 661–666, 2006.

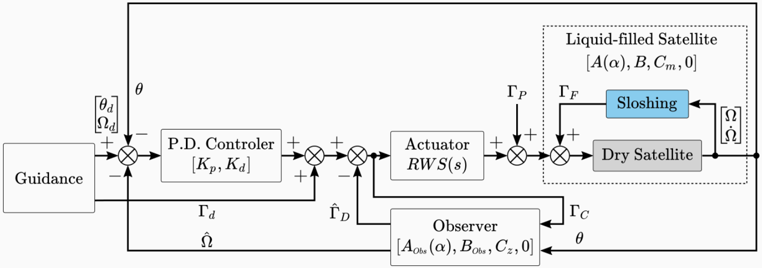
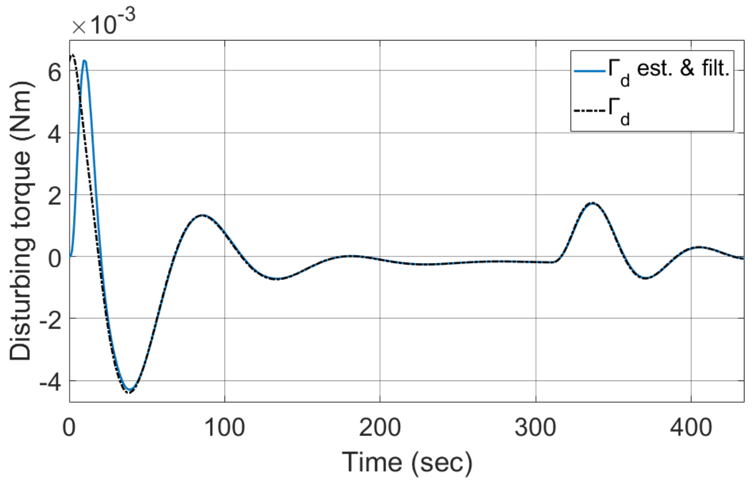


Figure 5: Parameter-varying closed-loop model block diagram

Simulation results - Disturbing torque



Simulation results - Attitude

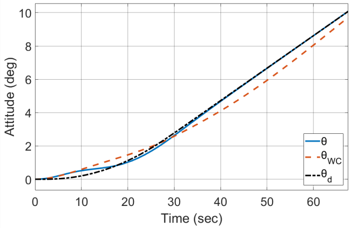


Figure 6: Start of the maneuver

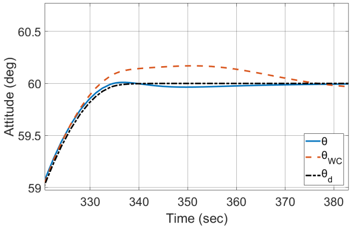


Figure 7: Reach of steady-state

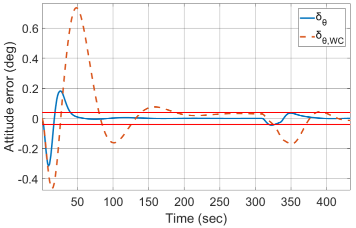


Figure 8: Error

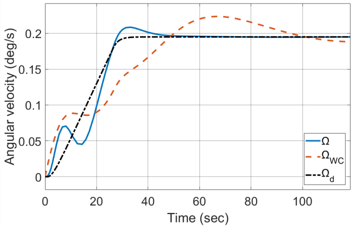


Figure 9: Start of the maneuver

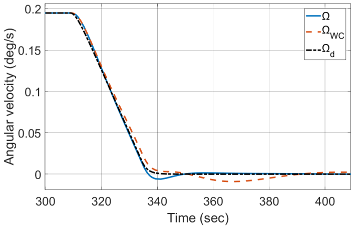


Figure 10: Reach of steady-state

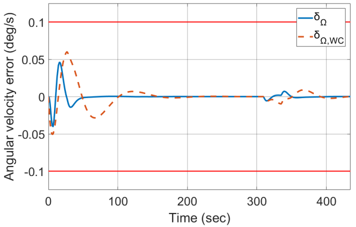


Figure 11: Error

Simulation results - Control torque and angular momentum requirements

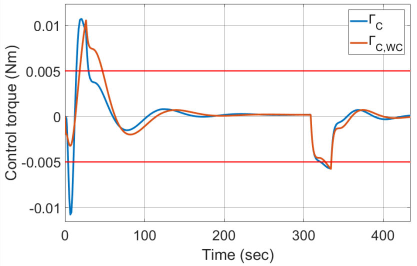


Figure 12: Control torque

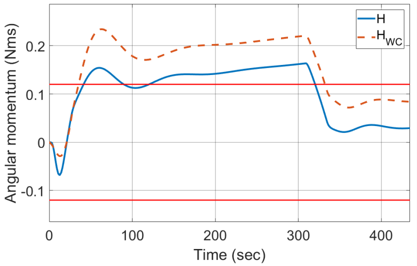


Figure 13: Angular momentum

Simulation results - Robustness to Parameters Errors $P \to P + \Delta_P$

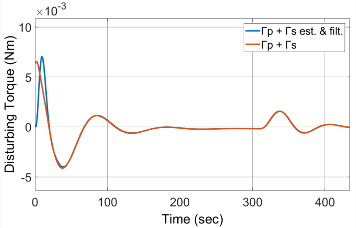


Figure 14: Est. Dist. Torque - $\Delta = 0\%$

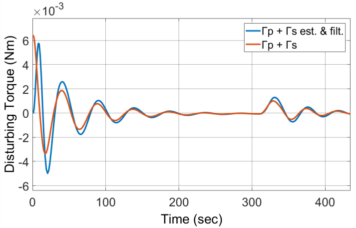


Figure 15: Est. Dist. Torque - $\Delta = 30\%$

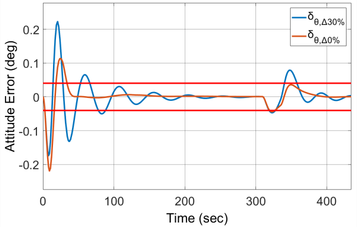


Figure 16: Attitude Error - Comparison

Observer and Closed-Loop Stability Analysis

Observer and Closed-Loop Stability Analysis

- No theoretical guarantee regarding time-varying stability
- Stability has then be checked *a posteriori*
- Achieved with quadratic and Parameter-Dependent Lyapunov functions (PDLF)

Observer Dynamics Parameter-Varying Stability Analysis

- Stability verified, independently of the rate of variation of the parameters, if a symmetric positive definite matrix $P_{Obs} > 0$ can be found such that:

$$A_{Obs}(\alpha)^T P_{Obs} + P_{Obs} A_{Obs}(\alpha) < 0,\ \forall \alpha \in \mathcal{P} \tag{31}$$

- Polytopic approach $\rightarrow$ condition reduces to 9 Linear Matrix Inequalities (LMI):

$$A_{Obs}(\mathcal{P}_i)^T P_{Obs} + P_{Obs} A_{Obs}(\mathcal{P}_i) < 0, \quad i = 1, \ldots, 9 \tag{32}$$

- The observer has 7 states $\rightarrow 7 \times 8/2 = 28$ decision variable
- Problem solved using the `feasp` Matlab$^T M$ LMI solver
- Observer is quadratically stable

- The closed-loop plant dynamics can be described by a matrix $A_C L(\alpha) \in \mathbb{R}^{13\times 13}$
- This matrix has the same properties as the observer A matrix, thus :

$$A_{CL}(\alpha) \in \mathcal{C}o\{A_{CL}(\mathcal{P}_1), \ldots, A_{CL}(\mathcal{P}_9)\} \tag{33}$$

- Quadratic stability is too conservative in this case and could not be established
- PDLF $P(\alpha)$ taking into account the parameters variation rate is needed :

$$P(\alpha) = P_0 + \alpha_A P_A + \alpha_K P_K + \alpha_A \alpha_K P_{AK} + \alpha_A^2 P_{A_2} + \alpha_K^2 P_{K_2} \tag{34}$$

- With $|\dot{\alpha}_A| < \rho_A$ and $|\dot{\alpha}_K| < \rho_K$, new stability conditions are obtained as, $\forall \alpha \in \mathcal{P}$:

$$\begin{aligned} A(\alpha)^T P(\alpha) + P(\alpha)A(\alpha) & \\ \pm \rho_A(P_A + \alpha_K P_{AK} + 2\alpha_A P_{A_2}) & \\ \pm \rho_K(P_K + \alpha_A P_{AK} + 2\alpha_K P_{K_2}) &< 0 \end{aligned} \tag{35}$$

$$P(\alpha) > 0 \tag{36}$$

- Both inequalities are nonlinear functions (second-order polynomial)
- A finite set of LMIs is obtained by searching $P_0, P_A, \ldots, P_{K_2}$ on a given grid
- *a posteriori* verif. that constraints are satisfied everywhere inside the polytope
- Test be performed by computing μ upper and lower bounds[19]
- A grid with 84 points has been considered :
  $5 \times 84 = 420$ LMIs and $6 \times 13 \times 14/2 = 546$ decision variables
- $\rho_A = 5.6 \times 10^{-4}$ and $\rho_K = 2.5 \times 10^{-3}$
- Solution has been found and validated with μ test in less than 5 min
- Cloosed-loop is asymptotically stable

[19] J.-M. Biannic, C. Roos, and C. Pittet, “Linear parameter varying analysis of switched controllers for attitude control systems,” *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 5, pp. 1561–1567, 2011.

Conclusion and Future Work

Conclusion and Future Work [1/2]

- New way to model sloshing disturbing torque as an LPV system
- Model successfully exploited to design an LPV torque observer
- Pert. compensation to enhance existing controller designed without sloshing
- Observer quadratic stability over the parametric domain
- Closed-loop asymptotic stability with PDLF
- Proposed for ACA 2019

- Study case corresponds to a tank larger than the one fitted to DEMETER satellite
- This tank is wall less and half-filled (worst case scenario)
- Despite such conditions our approach succeeded in reducing attitude error
- Likely the use of this approach could permit to reduce tank complexity and mass
- Control torque and angular momentum max. values are sometimes exceeded
- Future work: address this issue with *reference governors*[20] to adapt the reference
  Proposed for EUCASS 2019

[20] I. Kolmanovsky, E. Garone, and S. Di Cairano, "Reference and command governors: A tutorial on their theory and automotive applications," in *American Control Conference (ACC), 2014*, IEEE, 2014, pp. 226–241.

Thank you for your attention !

Questions and comments are welcomed !

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