Contrôle et analyse de stabilité de systèmes de dimension infinie
Approches directes et indirectes par l'interpolation de Loewner

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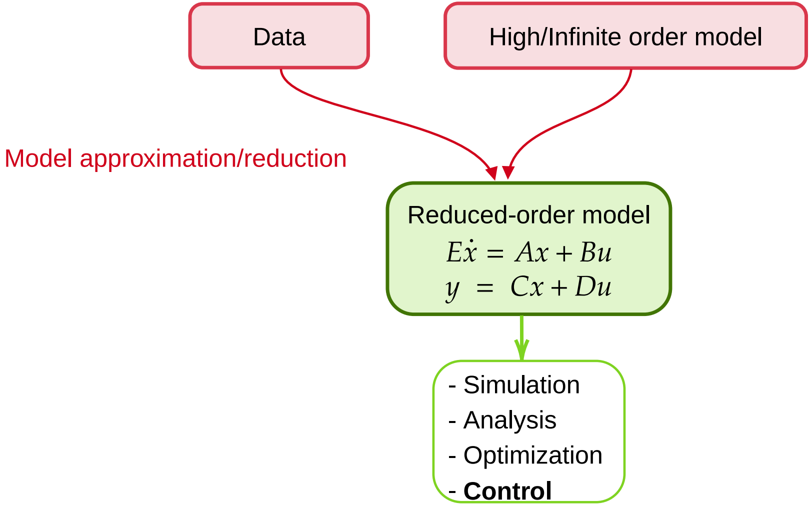
Journees Nationales d'Automatique de la SAGIP

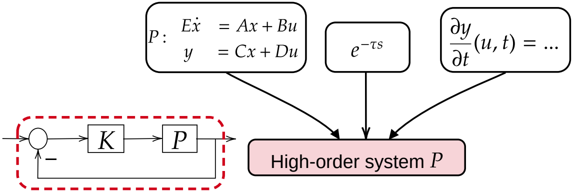
26/11/2020

Overview of the presentation

Motivations: why use model reduction?



Control of high-order systems



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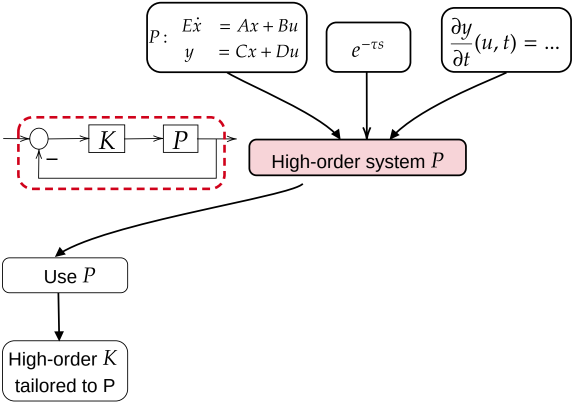
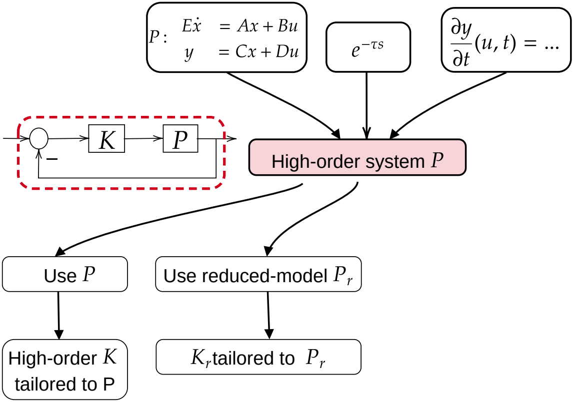


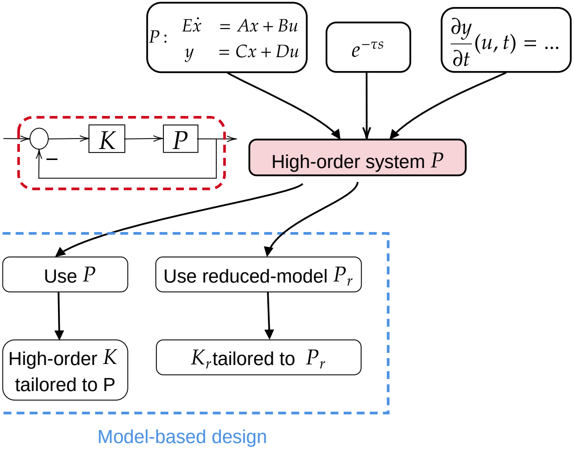
Foias, C., Özbay, H., Tannenbaum, A. (1996). *Robust control of infinite dimensional systems: frequency domain methods*. Springer.

Control of high-order systems



Morris, K., Levine, W. S. (2010). Control of systems governed by partial differential equations. The control theory handbook.

Control of high-order systems



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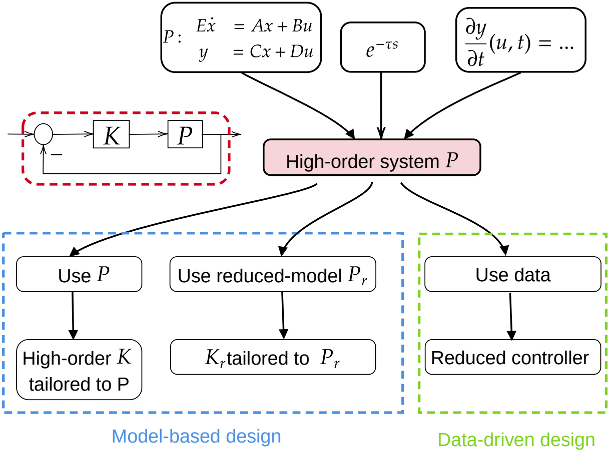
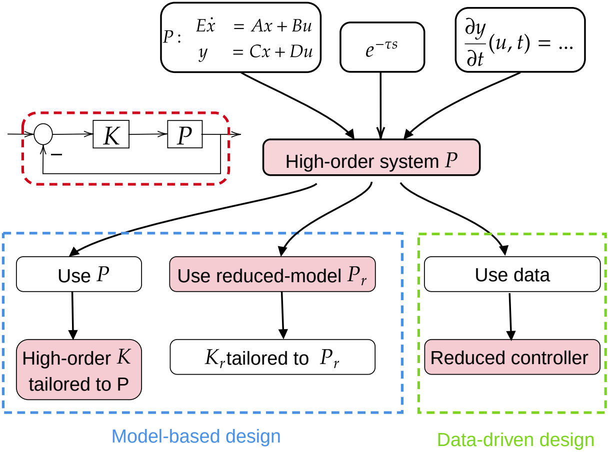


Kergus, P (2020), *Data-driven control of infinite dimensional systems: Application to a continuous crystallizer*, submitted to IEEE Control System Letters.

Control of high-order systems

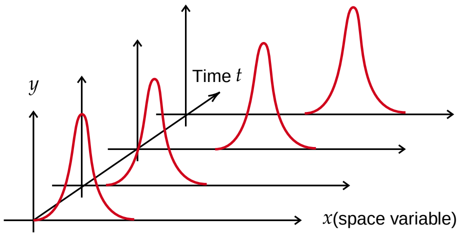


Model reduction is everywhere

Antoulas, A. C. (2005). *Approximation of large-scale dynamical systems*. SIAM.

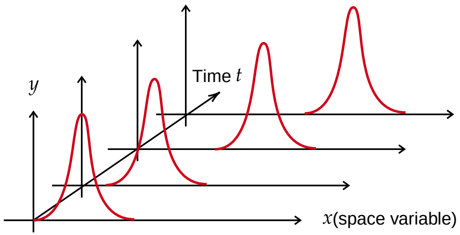
MOR-based control of infinite dimensional systems

Illustrative example



$$
\begin{aligned}
\frac{\partial \tilde{y}(x,t)}{\partial x} + 2x\frac{\partial \tilde{y}(x,t)}{\partial t} &= 0 && \text{(transport equation)}\\
\tilde{y}(x,0) &= 0 && \text{(initial condition)}\\
\tilde{y}(0,t) &= \frac{1}{\sqrt{t}}\tilde{u}_f(0,t) && \text{(control input)}\\
\frac{\omega_0^2}{s^2+m\omega_0 s+\omega_0^2}u(0,s) &= u_f(0,s) && \text{(controller bandwidth)},
\end{aligned}
$$

Illustrative example



$$\boxed{y(x,s) = \frac{\sqrt{\pi}}{\sqrt{s}} e^{-x^2 s} \frac{\omega_0^2}{s^2 + m\omega_0 s + \omega_0^2} u(s) = H(x,s)u(0,s)}$$

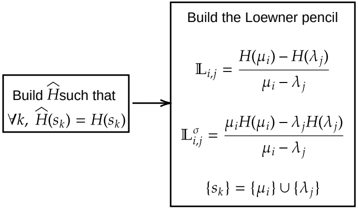
$$x = 1.9592$$

The Loewner framework

Build $\widehat{H}$ such that
$\forall k,\ \widehat{H}(s_k) = H(s_k)$

A tutorial introduction to the Loewner framework for model reduction, Antoulas, A. C., Lefteriu, S., Ionita, A. C., Benner, P., Cohen, A. (2017), *Model Reduction and Approximation: Theory and Algorithms*.

The Loewner framework



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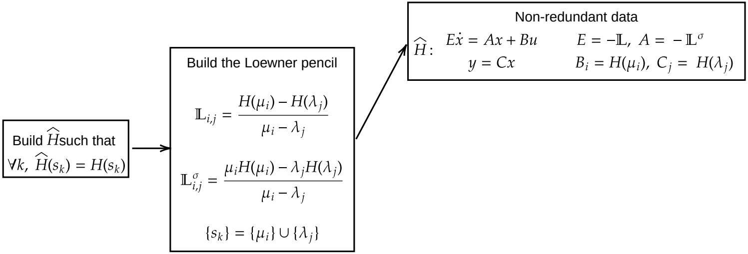

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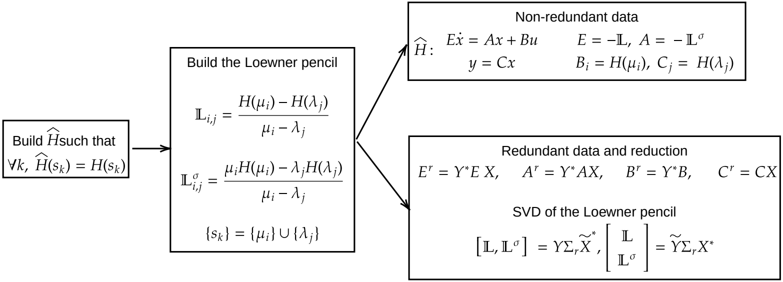

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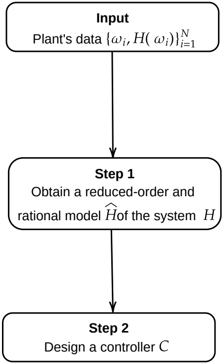
Model-based approach



Model-based approach

Input
Plant's data $\{\omega_i, H(\omega_i)\}_{i=1}^{N}$

↓

Step 1
Obtain a reduced-order and rational model $\widehat{H}$ of the system H

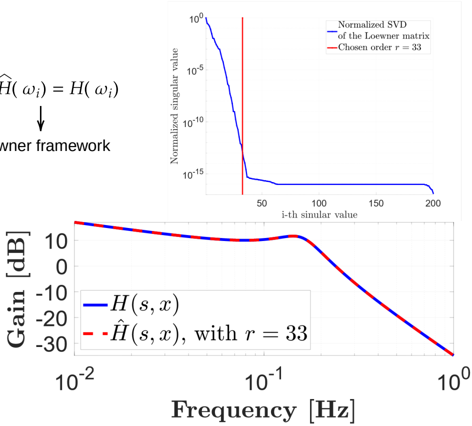
↓

Step 2
Design a controller C

$\forall i,\ \widehat{H}(\omega_i) = H(\omega_i)$

↓

Loewner framework



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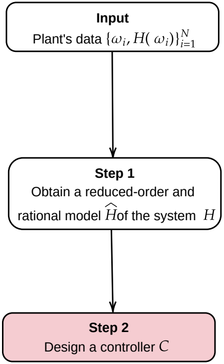
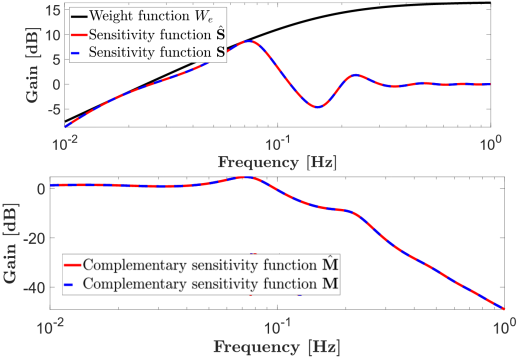
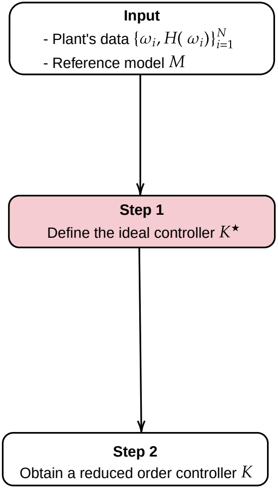


$\mathcal{H}_\infty$ structured synthesis (**hinfstruct**)

$$C(s) = k_p + \frac{k_i}{s} \qquad k_p = 0.191 \qquad k_i = 0.0252$$

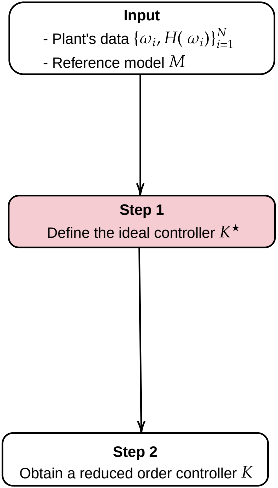


Data-driven approach



$$K^\star = H^{-1}M(1-M)^{-1}$$

Data-driven approach



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Risk of instabily compensation in the open-loop

$$H(0) = \infty \quad \Rightarrow \quad M(0) = 1$$
$$H(\infty) = 0 \quad\quad\quad M(\infty) = 0$$

Data-driven approach

Input
- Plant's data $\{\omega_i, H(\omega_i)\}_{i=1}^{N}$
- Reference model M

↓

Step 1
Define the ideal controller $K^\star$

↓

Step 2
Obtain a reduced order controller K

$$K^\star = H^{-1}M(1-M)^{-1}$$

Risk of instabily compensation in the open-loop

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$$M_1(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2s}{\omega_0} + 1} \qquad M_2 = \frac{\widehat{HC}}{1 + \widehat{HC}}$$

$\omega_0 = 0.5 rad/s$

Data-driven approach

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Loewner framework

$$K_1(s) = \frac{0.1347\, s + 0.009259}{s + 0.001303} \qquad K_2(s) = \frac{0.1914\, s + 0.02517}{s + 1.526 \cdot 10^{-5}} \approx C(s)$$

Data-driven approach

Input

- Plant's data $\{\omega_i, H(\omega_i)\}_{i=1}^{N}$
- Reference model M

Step 1

Define the ideal controller $K^\star$

Step 2

Obtain a reduced order controller K

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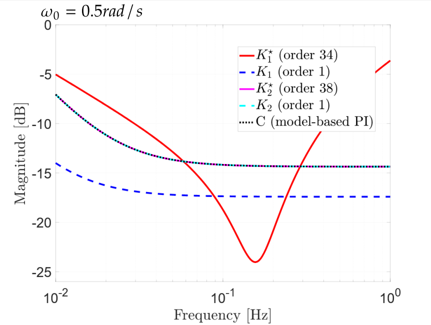
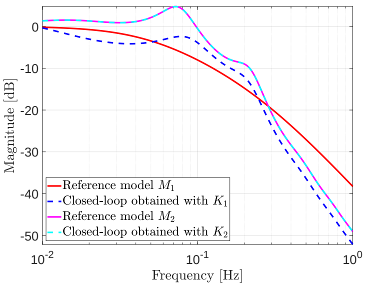
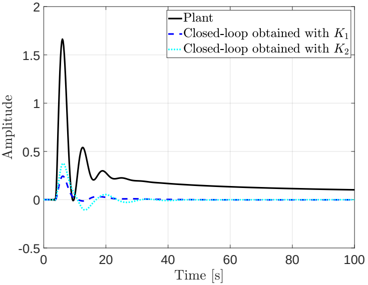


Model-based vs data-driven control

The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions

- Model based approach
- Data-driven approach

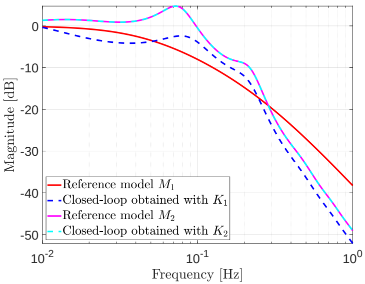


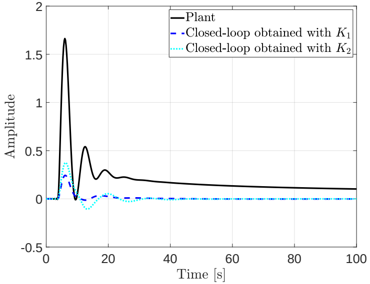


Model-based vs data-driven control

The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions

- Model based approach
  - \- more steps
- Data-driven approach
  - \+ direct control design
  - \- less flexible specifications

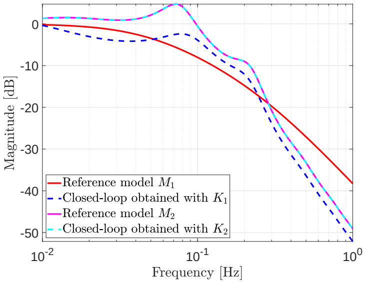


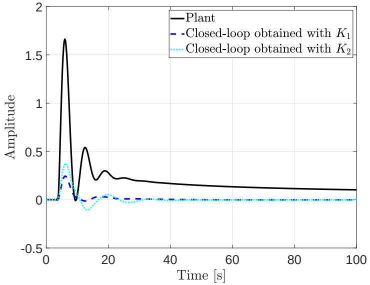


Model-based vs data-driven control

The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions

- Model based approach
  - \- more steps
  - +/- **guaranteed stability but for the reduced-order model $\hat{H}$**
- Data-driven approach
  - \+ direct control design
  - \- less flexible specifications
  - +/- **conservative data-driven stability test**

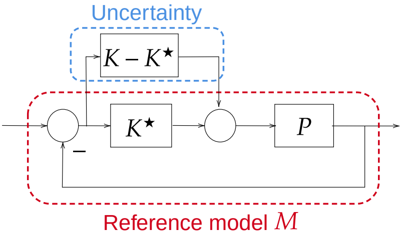




MOR-based stability analysis

Motivations

Does the controller stabilise the real system?



Application of the small-gain theorem

The closed-loop is well-posed and internally stable for all stable $\Delta = K - K^\star$ such that $\|\Delta\|_\infty \leq \gamma^{-1}$ if and only if $\|(1 - \mathrm{M})\mathrm{P}\|_\infty < \gamma$

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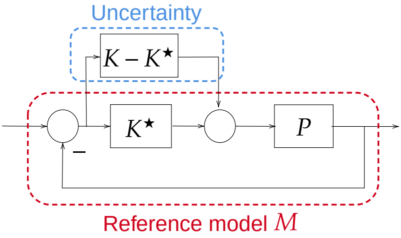

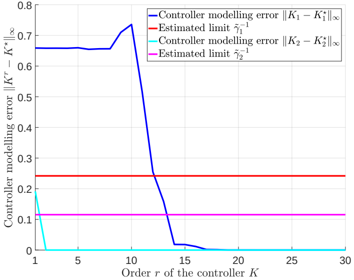

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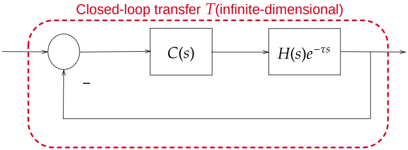
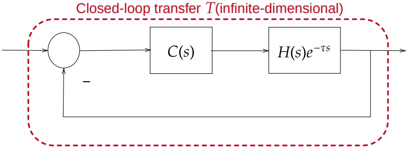
Loewner-based stability test



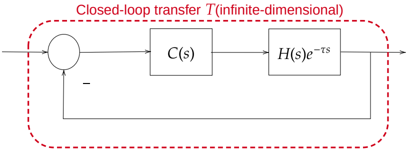
Interpolation-based infinite dimensional model control design and stability analysis, C. Poussot-Vassal, P. Kergus, P. Vuillemin, *chapter to appear.*

Loewner-based stability test



1. Compute samples $T(\jmath\omega_i) = \frac{C(\jmath\omega_i)H(\jmath\omega_i)e^{-\tau\jmath\omega_i}}{1+C(\jmath\omega_i)H(\jmath\omega_i)e^{-\tau\jmath\omega_i}}$

Loewner-based stability test



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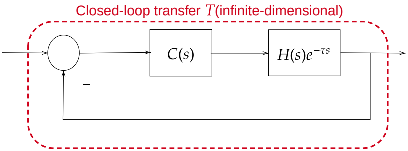
2. Obtain a minimal realisation $\hat{T}$ through the Loewner framework such that $\hat{T}(\jmath\omega_i) = T(\jmath\omega_i)$

Loewner-based stability test



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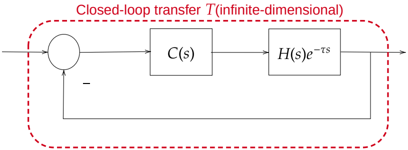
3. Compute $\hat{T}_s$

$$\hat{T}_s = \underset{T\in\mathbb{S}^+_{n,n_i,n_o}}{\arg\min} \|T - \hat{T}\|_\infty$$

On the closest stable descriptor system in the respective spaces $\mathcal{RH}_2$ and $\mathcal{RH}_\infty$, Köhler, M., Linear Algebra and its Applications, 2014.

Loewner-based stability test



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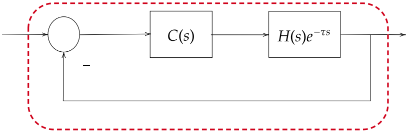
4. Compute the stability index as $S = \| \hat{T}_s - \hat{T} \|_\infty$

Loewner-based stability test



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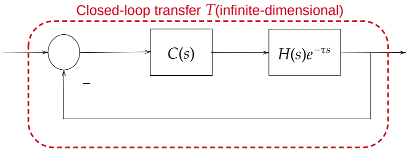
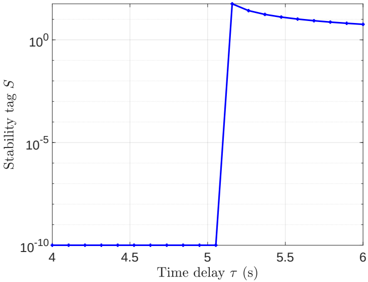
IF $S < \epsilon$ then T is stable

ELSE $S > \epsilon$ then T is unstable

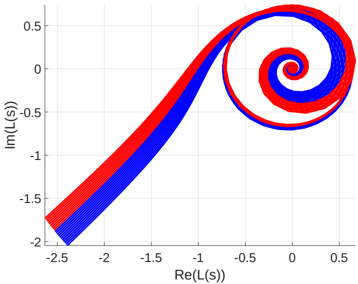
Results

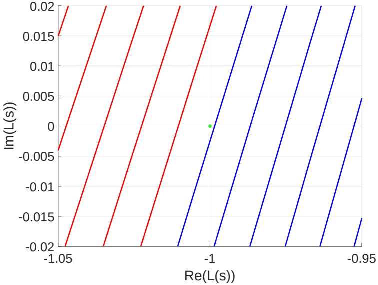




Stability tag as a function of the delay τ in the loop.

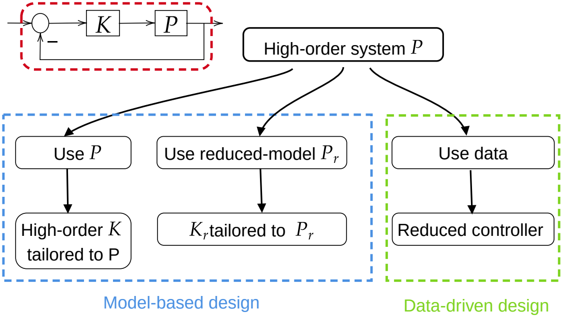
Results





Nyquist diagram for varying values of τ: $S < 10^{-10}$ (stable configuration) and $S > 10^{-10}$ (unstable configuration).

Conclusion



✓ The Loewner framework is a versatile and efficient tool for the control of high/infinite-order systems

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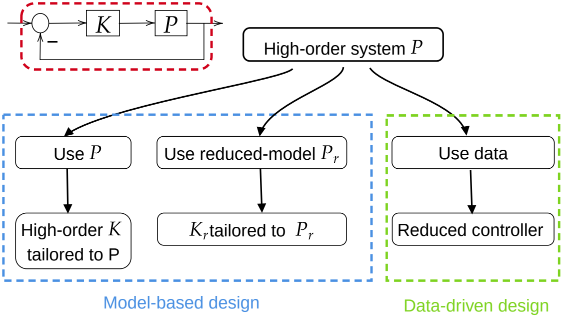

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- ✓ It provides a stability test when used with a projection on $\mathcal{RH}_\infty$

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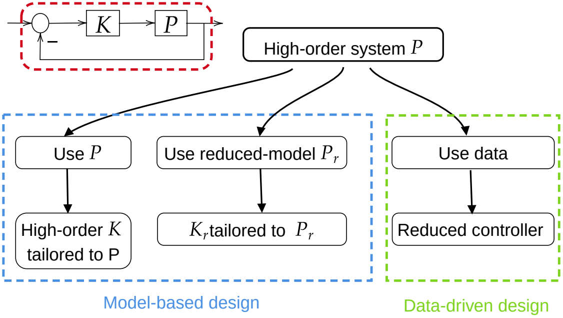

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- → Move toward robustness analysis

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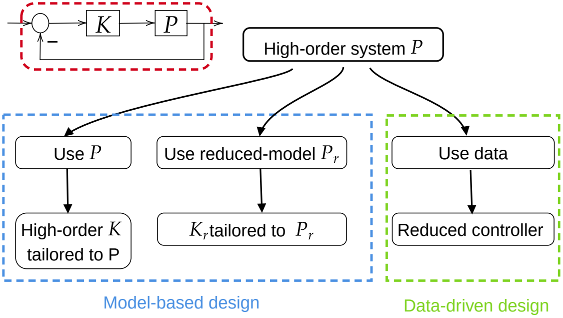

✓ The Loewner framework is a versatile and efficient tool for the control of high/infinite-order systems

✓ It provides a stability test when used with a projection on $\mathcal{RH}_\infty$

→ Move toward robustness analysis

→ Which frequencies to use? What about noise?