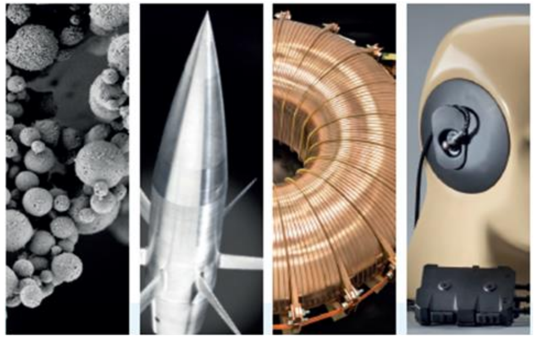

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Quasi-LPV Modeling and Control of Projectile Pitch Dynamics through State Transformation Technique

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A joint initiative of

Outline

1 Projectile Nonlinear Dynamics
- Project Overview
- Nonlinear Pitch Dynamics
- Control Deflections

2 State Transformation Approach
- Standard Model Derivation
- Augmented Model
- Quasi-LPV Models Architecture

3 Models Validations and Comparison
- Simulation Conditions
- Results Comparison: graphical
- Results Comparison: statistical

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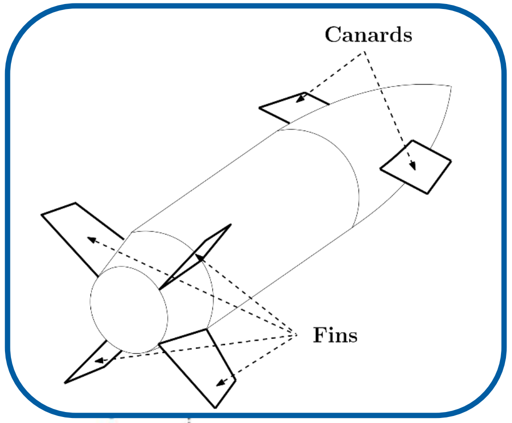
Development of guided munitions:

- Unguided munitions suffer from on-target dispersion.
- Employment of aerodynamic control surfaces (nose-mounted canards and/or tail fins).
- Stability design approaches:
  - spin-stabilized architecture [1-3].
  - fin-stabilized architecture [4-6].

Limitations of spin-stabilization….

- High spin rate generates nonlinear couplings [7].
- Operating range depends on the firing gun capability.

Long-Range Guided Projectile (LRGP):



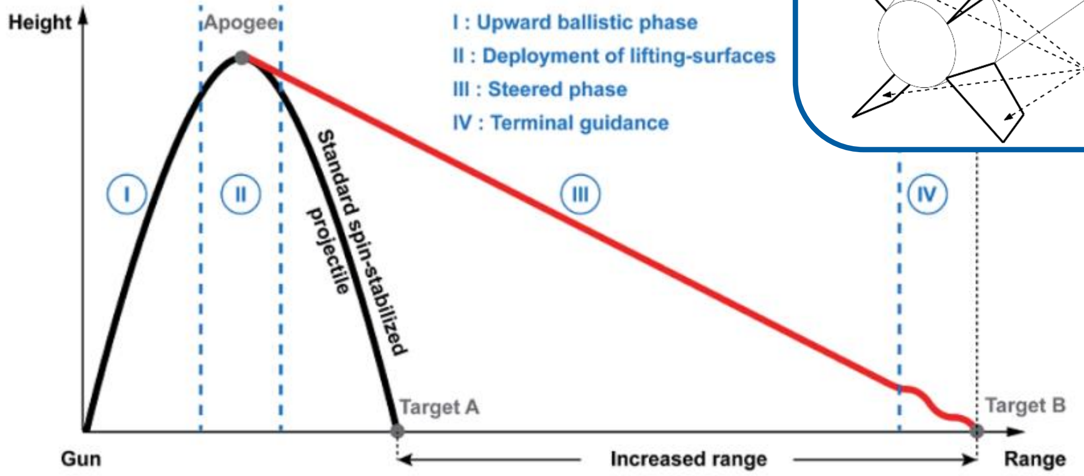


Figure 1. LRGP concept and flight strategy.

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Study of a new *Long-Range Guided Projectile* (LRGP) concept.

Development of a full 6DOF nonlinear guided simulator environment.

Derivation of a reliable Linear Parameter Varying (LPV) model of the projectile.

Design of a LPV-based Bank-To-Turn autopilot coupled with a gliding oriented guidance law.

Linear Parameter Varying formulation:

- Nontrivial transformation process.
- Standard approach in aerospace: linearization-based models [1-2].
- Alternative methods:
  - Function Substitution [3-4];
  - Velocity-based [5];
  - State Transformation [6-7].
- State Transformation: no approximations are involved in the process.

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[1] Theodoulis, S., Morel, Y., Wernert, P., and Tzes, A. (2010). LPV modeling of guided projectiles for terminal guidance.
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[7] Carter, L.H. and Shamma, J.S. (1996). Gain-scheduled bank-to-turn autopilot design using linear parameter varying transformations.

Flight Dynamics

$$
\begin{cases}
\dot{\alpha} = \left(\dfrac{1}{mV\cos\beta}\right)\left(-X\sin\alpha + Z\cos\alpha + mg(\sin\alpha\sin\vartheta + \cos\alpha\cos\vartheta\cos\varphi)\right) \\
\qquad + q - p\tan\beta\cos\alpha - r\tan\beta\sin\alpha \\
\dot{q} = \dfrac{1}{I_{yy}}\left(M - (I_{xx} - I_{zz})pr\right)
\end{cases}
$$

Equation 1. Projectile pitch dynamics.

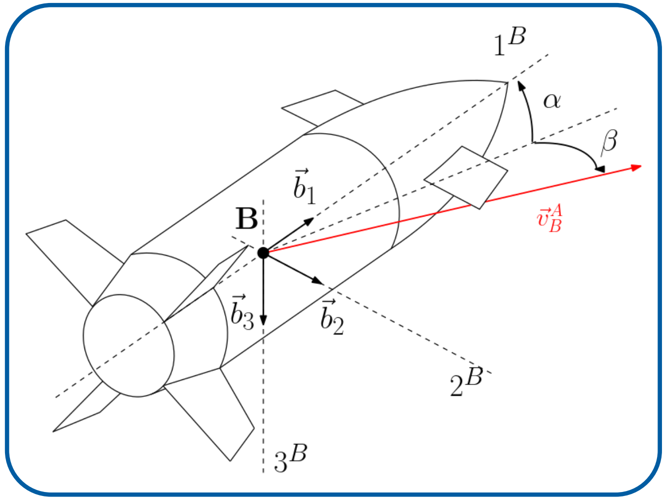

Figure 2. Body reference frame and coordinates.

Aerodynamics

$$
\begin{cases}
X = \bar{q}S\left(C_{X_{\alpha 0}}(\mathcal{M}) + C_{X_{\alpha 2}}(\mathcal{M})\sin^2\alpha + C_{X_{\alpha 4}}(\mathcal{M})\sin^4\alpha + C_{X_{\delta 0}}(\mathcal{M}) + C_{X_{\delta 2}}(\mathcal{M})\sin^2\delta_{\mathrm{eff}}\right) \\
Z = \bar{q}S\left(C_{Z_{\alpha 1}}(\mathcal{M})\sin\alpha + C_{Z_{\delta 1}}(\mathcal{M})\sin\delta_q + C_{Z_{\delta 3}}(\mathcal{M})\sin^3\delta_q\right) \\
M = \bar{q}dS\left(C_{m_{\alpha 1}}(\mathcal{M})\sin\alpha + C_{m_{\alpha 3}}(\mathcal{M})\sin^3\alpha + C_{m_{\alpha 5}}(\mathcal{M})\sin^5\alpha + C_{m_{\delta 1}}(\mathcal{M})\sin\delta_q + C_{m_{\delta 3}}(\mathcal{M})\sin^3\delta_q\right)
\end{cases}
$$

Equation 2. Projectile aerodynamic forces and moments.

Control Deflections

- Local canards deflections (δ_r, δ_l) are combined into virtual roll and pitch deflections (δ_p, δ_q).

Allocation Matrix

$$\delta_p = \frac{-\delta_r + \delta_l}{2} \qquad \delta_q = \frac{\delta_r + \delta_l}{2} \quad \Rightarrow \quad \begin{bmatrix} \delta_p \\ \delta_q \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_l \end{bmatrix}$$

Equation 3. Control allocator relation.

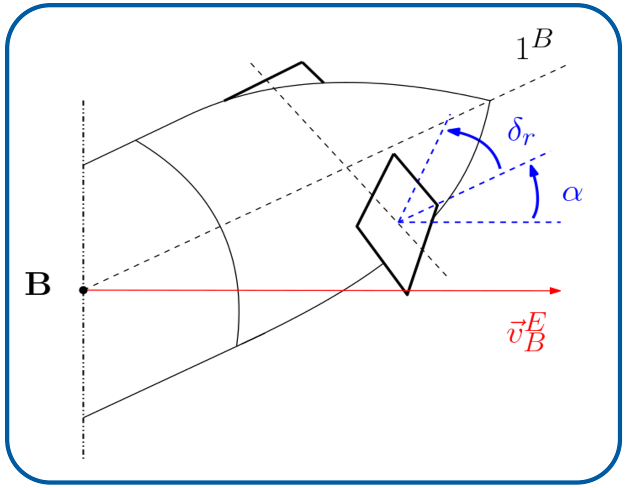

Figure 3. Canards local deflection angles.

- The longitudinal control is the nonlinear combination, δ_{eff}:

$$\delta_{\text{eff}} = \sqrt{{\delta_p}^2 + {\delta_q}^2}$$

Equation 4. Longitudinal control contribution.

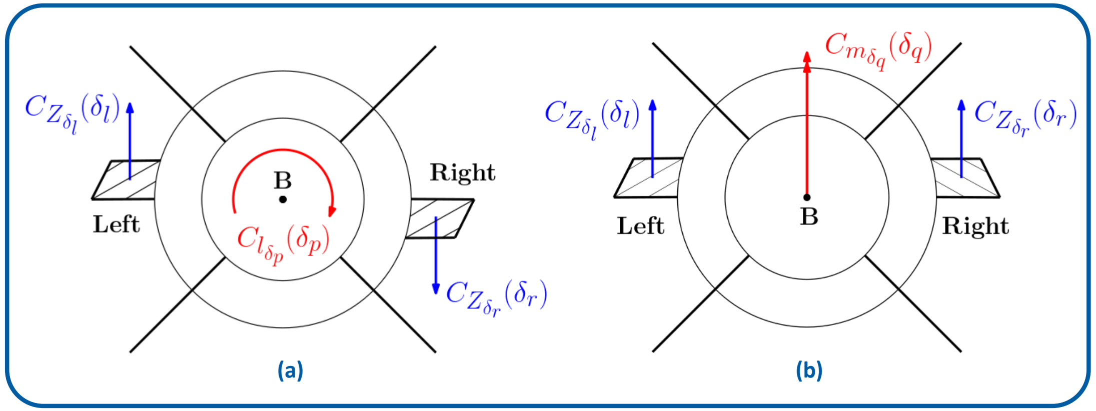

Figure 4. Virtual deflections: (a) virtual roll; (b) virtual pitch.

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State transformation aimed to remove all the nonlinearities present in the model, that do not depend on the scheduling variables [1-2].

- Consider an output dependent system:

$$\begin{bmatrix}\dot{z}\\ \dot{w}\end{bmatrix} = \begin{bmatrix}f_1(\rho)\\ f_2(\rho)\end{bmatrix} + \begin{bmatrix}A_{11}(\rho) & A_{12}(\rho)\\ A_{21}(\rho) & A_{22}(\rho)\end{bmatrix}\begin{bmatrix}z\\ w\end{bmatrix} + \begin{bmatrix}B_1(\rho)\\ B_2(\rho)\end{bmatrix}u$$
$$y = z \qquad (1)$$

- State: $x(t) = [z(t)\;\; w(t)],\; z(t) \in \mathbb{R}^{n_z}, w(t) \in \mathbb{R}^{n_w}$,
- Input: $u(t) \in \mathbb{R}^{n_u}$,
- Nonlinearities: $f_1(\rho), f_2(\rho)$,
- Scheduling vector: $\rho(t) = [z(t)\;\; \Omega(t)]$.

- Assuming:
  - $f_1(\rho), f_2(\rho)$ only functions of the measured output $z(t)$,
  - trim functions $w_{eq}(\rho)$, $u_{eq}(\rho)$ are continuously differentiable,
  - $n_z = n_u$,

then impose the trimming:

$$\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}f_1(\rho)\\ f_2(\rho)\end{bmatrix} + \begin{bmatrix}A_{11}(\rho) & A_{12}(\rho)\\ A_{21}(\rho) & A_{22}(\rho)\end{bmatrix}\begin{bmatrix}z\\ w_{eq}(\rho)\end{bmatrix} + \begin{bmatrix}B_1(\rho)\\ B_2(\rho)\end{bmatrix}u_{eq}(\rho) \qquad (2)$$

⇒ Trimming functions: $w_{eq}(\rho)$, $u_{eq}(\rho)$

References

[1] Leith, D.J. and Leithead, W.E. (2000). Survey of gain-scheduling analysis and design.
[2] Shamma, J.S. and Cloutier, J.R. (1993). Gain-scheduled missile autopilot design using linear parameter varying transformations.

- By subtracting (1)-(2) and introducing the state derivative, $\dot{w}_{eq} = \frac{dw_{eq}}{dt} = \frac{dw_{eq}}{d\rho}\dot{\rho}$

$$\begin{bmatrix} \dot{z} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & A_{12}(\rho) \\ 0 & \tilde{A}_{22}(\rho) \end{bmatrix} \begin{bmatrix} z \\ \xi \end{bmatrix} + \begin{bmatrix} B_1(\rho) \\ \tilde{B}_2(\rho) \end{bmatrix} \nu + \begin{bmatrix} 0 \\ E(\rho) \end{bmatrix} \dot{\Omega}$$

Equation 5. Quasi-LPV State Transformation-based model.

Where:

$$\xi \coloneqq w(t) - w_{eq}(\rho(t))$$

$$\nu \coloneqq u(t) - u_{eq}(\rho(t))$$

$$\tilde{A}_{22}(\rho) \coloneqq A_{22}(\rho) - \frac{dw_{eq}}{dz} A_{12}(\rho)$$

$$\tilde{B}_2(\rho) \coloneqq B_2(\rho) - \frac{dw_{eq}}{dz} B_1(\rho)$$

$$E(\rho) \coloneqq \frac{dw_{eq}}{d\Omega}$$

Considerations:

- $E(\rho)$ dynamics of the exogenous variables is assumed as a disturbance to be rejected, neglected in the model [1].
- Input ν depends on the selected equilibrium condition u_{eq}.

Theoretical solution:

- Imposition of $u_{eq}(\rho) = 0$ ⇒ High limitation on the feasible trim map.

References

[1] Balas, G.J. (2002). Linear, parameter-varying control and its application to aerospace systems.

- Augment an integrator at the plant input, $u(t) = \int \sigma(t)dt$:

$$\begin{bmatrix} \dot{z} \\ \dot{\xi} \\ \dot{\nu} \end{bmatrix} = \begin{bmatrix} 0 & A_{12}(\rho) & B_1(\rho) \\ 0 & \tilde{A}_{22}(\rho) & \tilde{B}_2(\rho) \\ 0 & \tilde{A}_{32}(\rho) & \tilde{B}_3(\rho) \end{bmatrix} \begin{bmatrix} z \\ \xi \\ \nu \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \sigma$$

Equation 6. Quasi-LPV integrator-augmented model.

Where:

$$\tilde{A}_{32}(\rho) := -\frac{du_{eq}}{dz} A_{12}(\rho)$$

$$\tilde{B}_3(\rho) := -\frac{du_{eq}}{dz} B_1(\rho)$$

Considerations:

- Input is uniformly zero at every equilibrium point.
- Motivated by the intention of designing a controller with pure integral action.

Advantages:

✓ Exact transformation between the original nonlinear system and the obtained quasi-LPV model.

Standard quasi-LPV

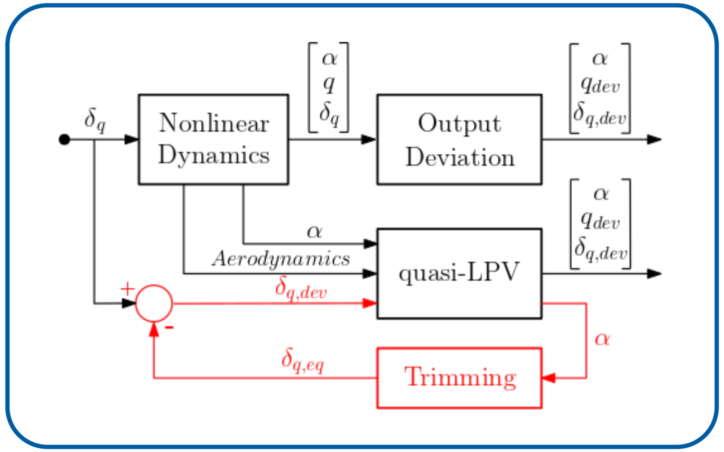

Figure 5. Quasi-LPV model: simulation architecture.

❖ Feedback loop updating the input at the current trim point.

$$\begin{bmatrix}\dot{z}\\ \dot{\xi}\end{bmatrix} = \begin{bmatrix}\dot{\alpha}\\ \dot{q}_{\text{dev}}\end{bmatrix} = \begin{bmatrix}0 & A_{12}(\rho)\\ 0 & \tilde{A}_{22}(\rho)\end{bmatrix}\begin{bmatrix}\alpha\\ q_{\text{dev}}\end{bmatrix} + \begin{bmatrix}B_1(\rho)\\ \tilde{B}_2(\rho)\end{bmatrix}\delta_{q,\text{dev}}$$

$$q_{\text{dev}} = q - q_{\text{eq}} \qquad \delta_{q,\text{dev}} = \delta_q - \delta_{q,\text{eq}}$$

$$\rho(t) = [z(t)\ \ \Omega(t)] = [\alpha(t)\ h(t)\ V(t)]$$

Equation 7. Quasi-LPV standard pitch dynamics model.

Augmented quasi-LPV

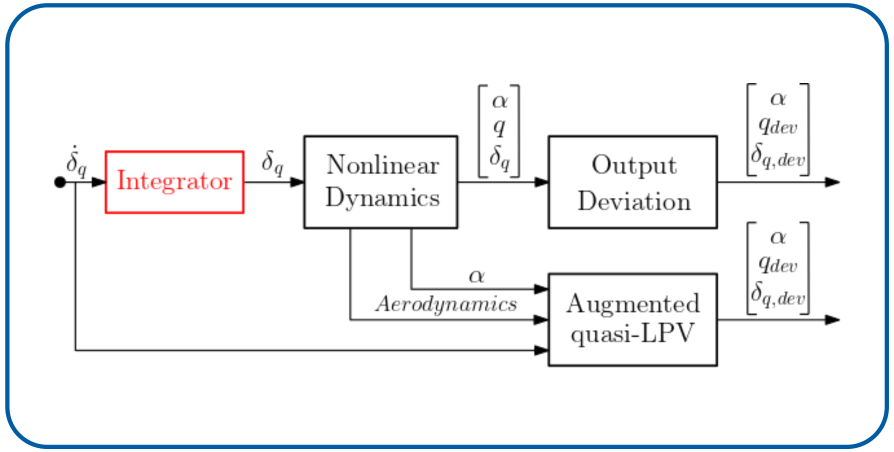

Figure 6. Augmented quasi-LPV model: simulation architecture.

❖ Integrator at the input of the NL model for compensation.

$$\begin{bmatrix}\dot{z}\\ \dot{\xi}\\ \dot{\nu}\end{bmatrix} = \begin{bmatrix}\dot{\alpha}\\ \dot{q}_{\text{dev}}\\ \dot{\delta}_{q,\text{dev}}\end{bmatrix} = \begin{bmatrix}0 & A_{12}(\rho) & B_1(\rho)\\ 0 & \tilde{A}_{22}(\rho) & \tilde{B}_2(\rho)\\ 0 & \tilde{A}_{32}(\rho) & \tilde{B}_3(\rho)\end{bmatrix}\begin{bmatrix}\alpha\\ q_{\text{dev}}\\ \delta_{q,\text{dev}}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ I\end{bmatrix}\sigma$$

$$\delta_{q,\text{dev}} = \int \sigma$$

Equation 8. Quasi-LPV Augmented pitch dynamics model.

- The pitch nonlinear dynamics is not input affine. (nonlinear with respect to δ_{eff} and δ_q.)

Not feasible for State Transformation formulation.

Adopted solution:

- Assume a narrower canards deflection range $\delta_{\text{r}}, \delta_{\text{l}}$ to assure linear response.
- Fit the reduced CFD dataset with a linear regression models:

$$C_{Z_\delta} = C_{Z_\delta}(\mathcal{M}) \sin \delta_q$$
$$C_{m_\delta} = C_{m_\delta}(\mathcal{M}) \sin \delta_q$$

- Neglect the nonlinear longitudinal term δ_{eff} .

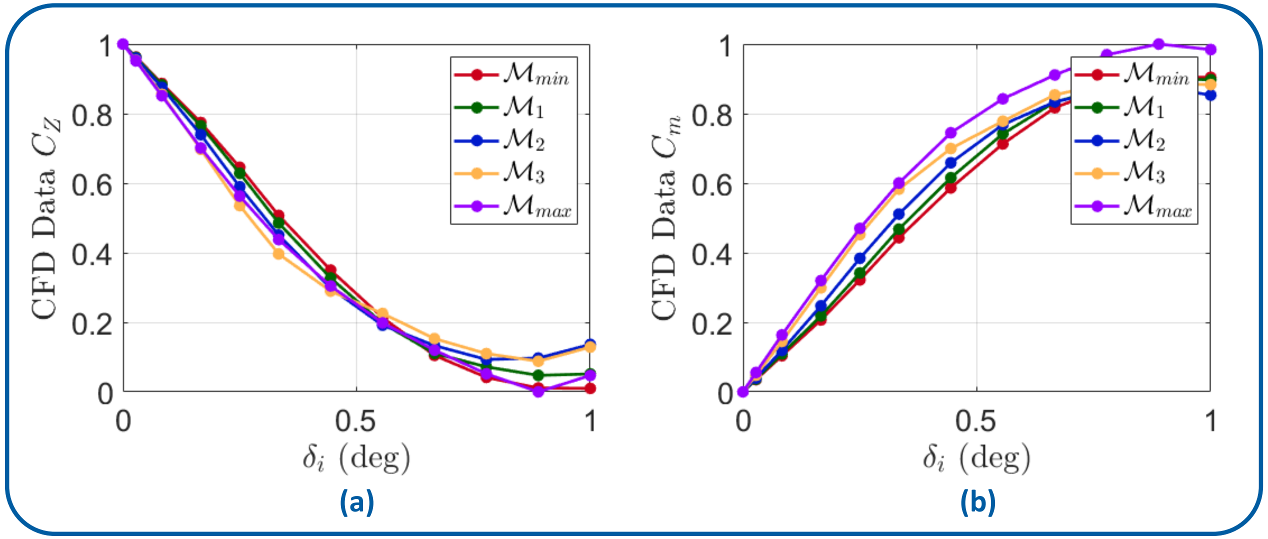

Figure 4. CFD aerodynamic data: (a) vertical force; (b) pitching moment.

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- The selected scheduling parameter:

$$\rho(t) = [z(t)\ \ \Omega(t)] = [\alpha(t)\ h(t)\ V(t)],$$

$\alpha \in [10, 16]$ (deg),

$V \in [31.60, 316.05]$ (m/s).

Trimmed flight conditions:

- Altitude assumed constant, $h = 6029$ (m).
- Trimming analysis to define the initial equilibrium conditions, q_{eq} and $\delta_{q,\mathrm{eq}}$.
- Selection criteria:
  - Lower trimmed input value (allows larger input command in simulations);
  - Stability characterization of the projectile.

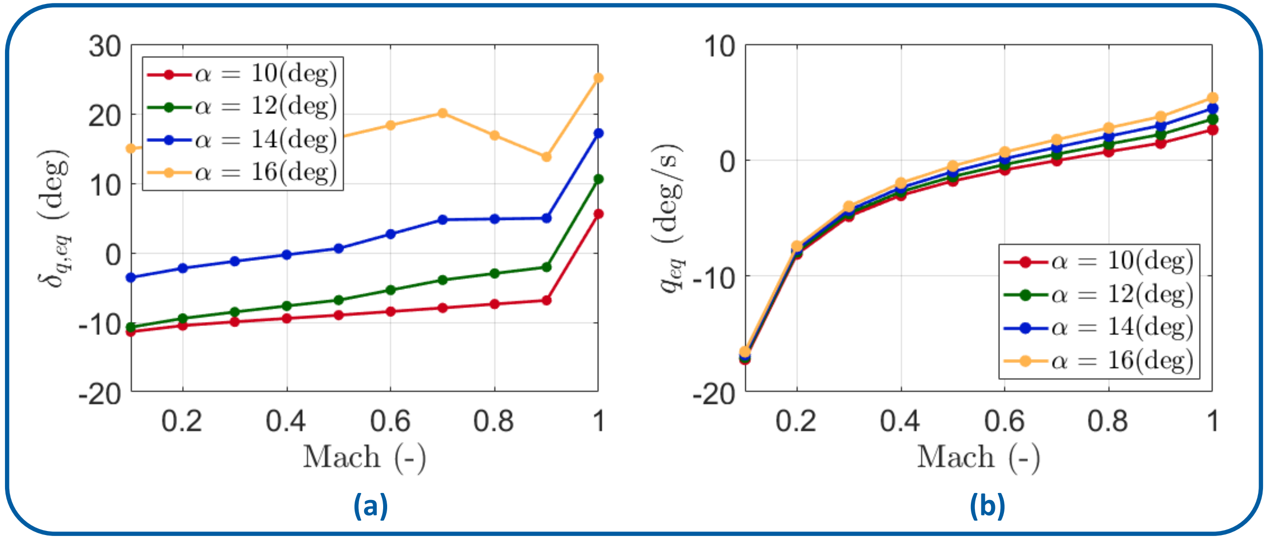


Figure 7. Trimming analysis: (a) control deflection; (b) pitch rate.

α (deg)	V (m/s)	h (m)	q_{eq} (deg/s)	$\delta_{q,\mathrm{eq}}$ (deg)
14	158.026	6029	-1	0.6

Table 1. Selected trimming conditions.

Simulation parameters:

- Deflection commands [1]:
  - $\delta_{q_1} = 20\ \text{(deg)}, \quad \text{at } t_1 = 5\ \text{(s)},$
  - $\delta_{q_2} = -15\ \text{(deg)}, \text{at } t_2 = 30\ \text{(s)}.$

Considerations:

- $\mathrm{NL_{sim}}$ and q-LPV curves are perfectly overlapped for both the models (exact transformation).
- Mismatch between NL and q-LPV curves due to the simplified aerodynamic model.
- Larger oscillation for the quasi-LPV model due to the destabilizing effect of the inner feedback loop.

Standard quasi-LPV | Augmented quasi-LPV

Angle-of-Attack α

Pitch Rate q_{dev}

Pitch Deflection $\delta_{q,\mathrm{dev}}$

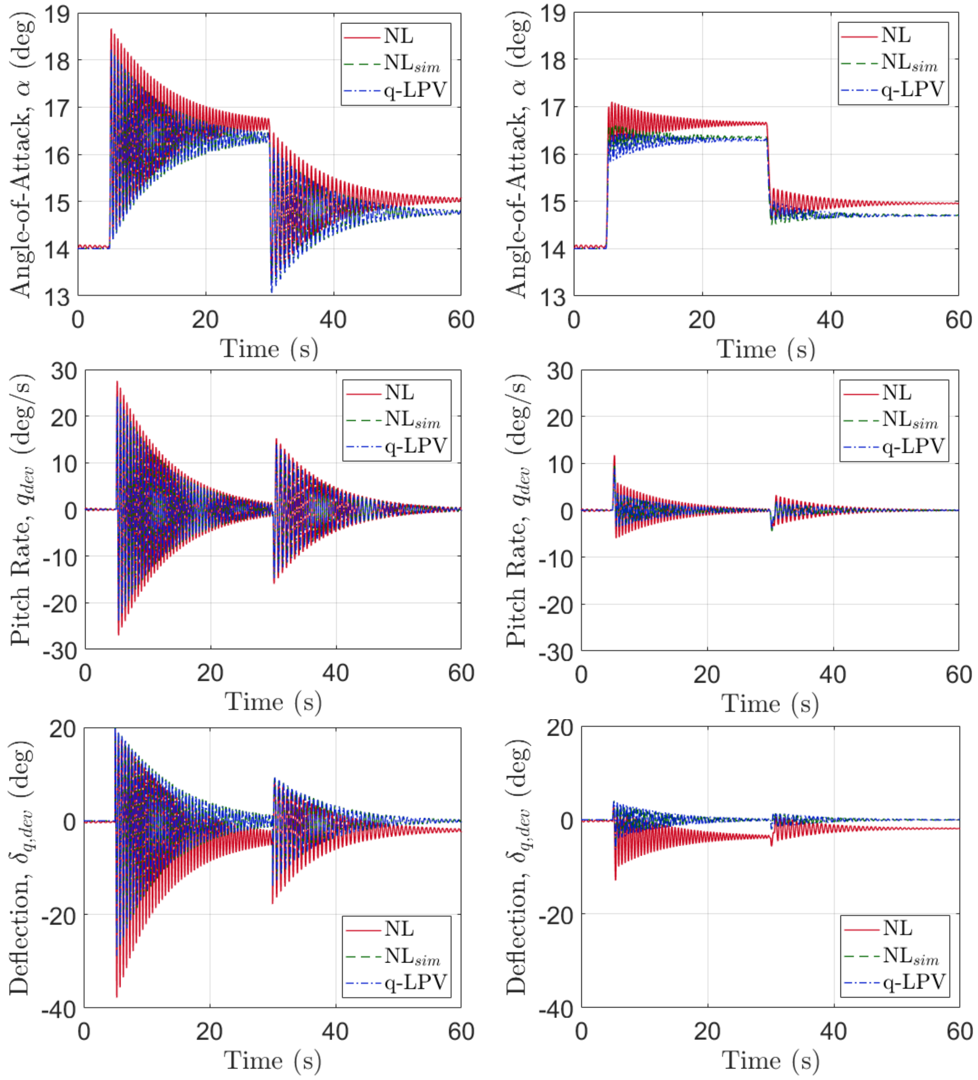


Figure 8. Simulation results comparison.

[1] Modelled as pulses for the Augmented quasi-LPV model.

RMSE evaluated between the three models and normalized by the original nonlinear system values.

Considerations:

- Perfect correspondence between the $\mathrm{NL} - \mathrm{NL_{sim}}$ and $\mathrm{NL} - \mathrm{qLPV}$.

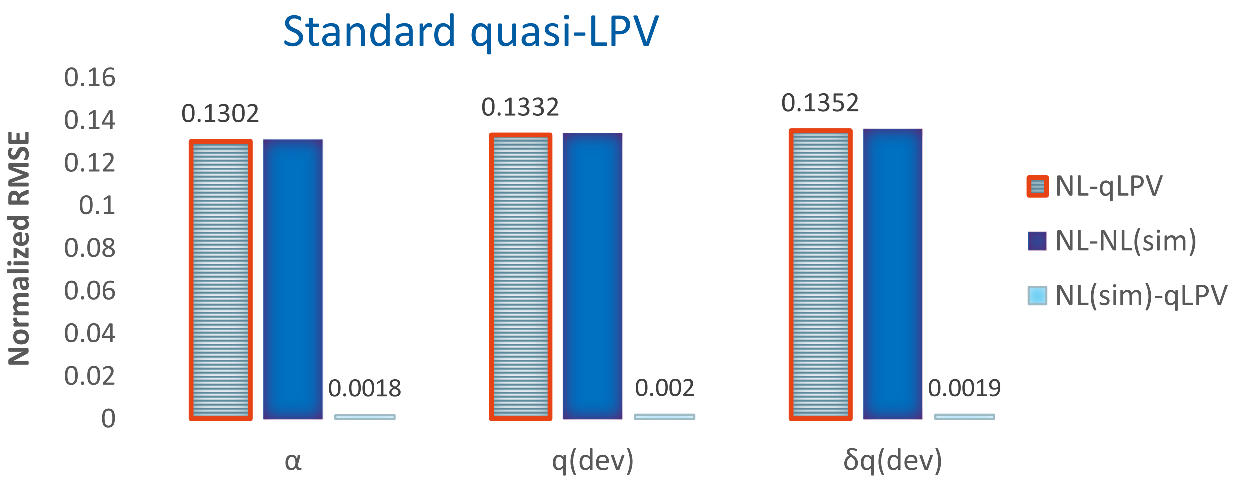


Figure 9. RMSE results related to the Standard quasi-LPV model.

Considerations:

- Larger $\mathrm{NL_{sim}} - \mathrm{qLPV}$ mismatch due to the different ways the integrator is implemented.
- Lower error between the original nonlinear system and the qLPV (higher accuracy).

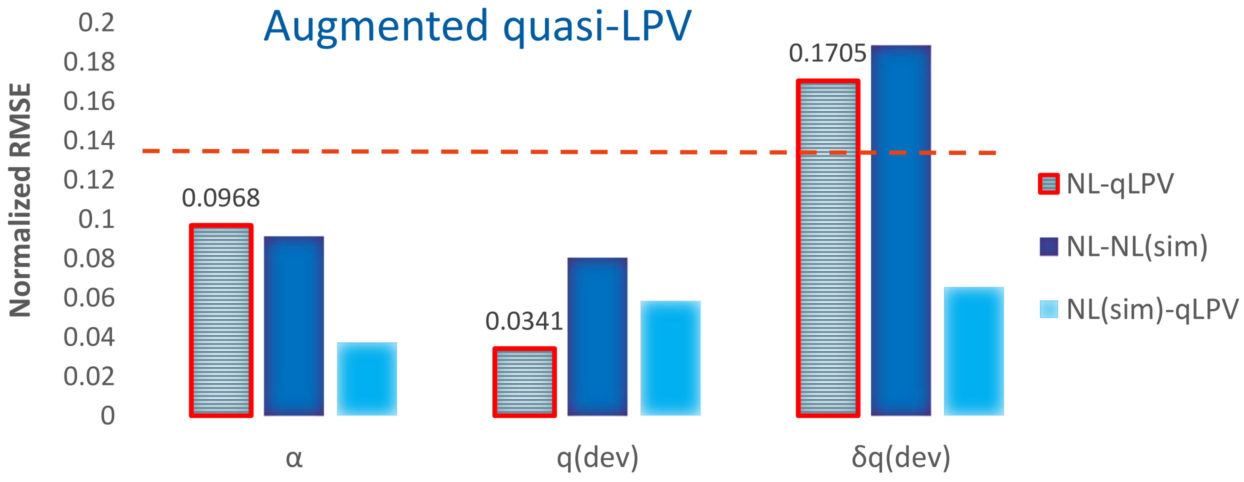


Figure 10. RMSE results related to the Augmented quasi-LPV model.

Conclusions

Results Overview:

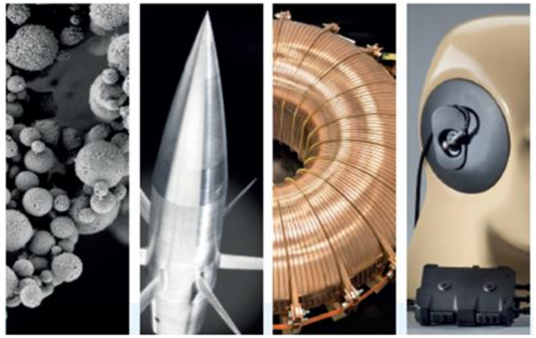
- Derivation of the pitch dynamics of the guided projectile.
- Investigation of the State Transformation formulation, and considerations on advantages and limitations.
- Development of two quasi-LPV models:
  - Standard formulation dependent on the trim point.
  - Augmented model with integrator at the input.
- Models comparison:
  - Simulation accuracy with respect to the original nonlinear dynamics.
  - Performance comparison (RMSE, graphical).

Future Works:

- Derivation of a quasi-LPV model of the roll-yaw projectile dynamics.
- Development of a LPV based Bank-To-Turn autopilot through polytopic design.
- Development of an appropriate guidance law for lift/drag maximization.
- State observer design for accurate Angle-of-Attack estimation to couple the control strategy with an appropriate gliding guidance law.

Thank you for your kind attention!

Any questions ?



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