Parameter dependent models for mechanical systems in closed loop kinematic chains

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Outline

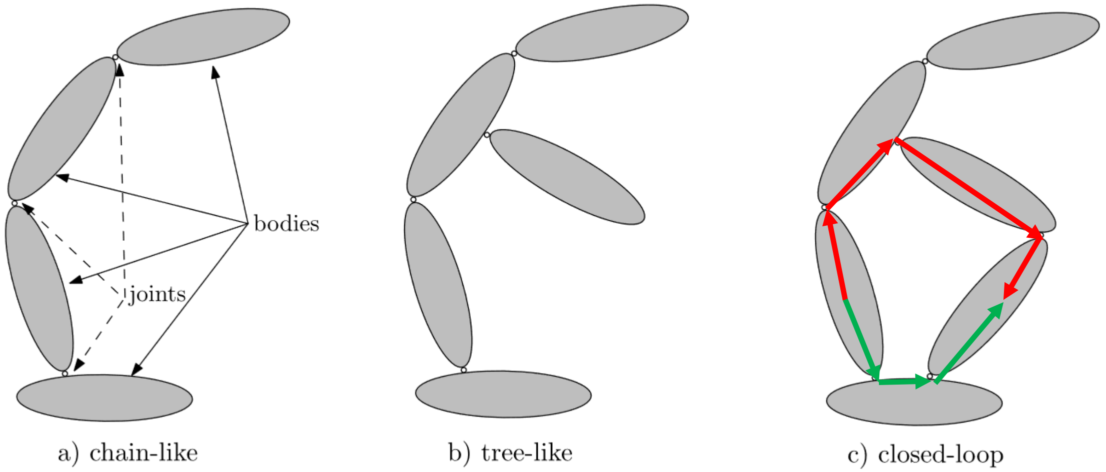
Objectives:

- Presents some insight on the **modelling of closed loop kinematics chains**
- Develop a linear model of closed loop kinematics multibody system **parameterized according to the geometric configuration of the mechanism** (i.e. position of the end-effector or configuration of the joints) in the Linear Fractional Transformation (LFT) setting
- Model such complex systems in a **sub-structured** way (by defining and assembling elementary blocks)

Main Challenge:

- the mechanical system with closed loop kinematic chains needs to **always satisfy a set of non-linear equations defining the loop closure constraints.**

Introduction (Closed loop kinematics)



Multibody systems (image credit: [1])

A **kinematic chain** is an assembly of links that is connected by joints.

When every link in a kinematic chain is connected to other links by at least two distinct paths, then it is called a **closed-loop chain**.

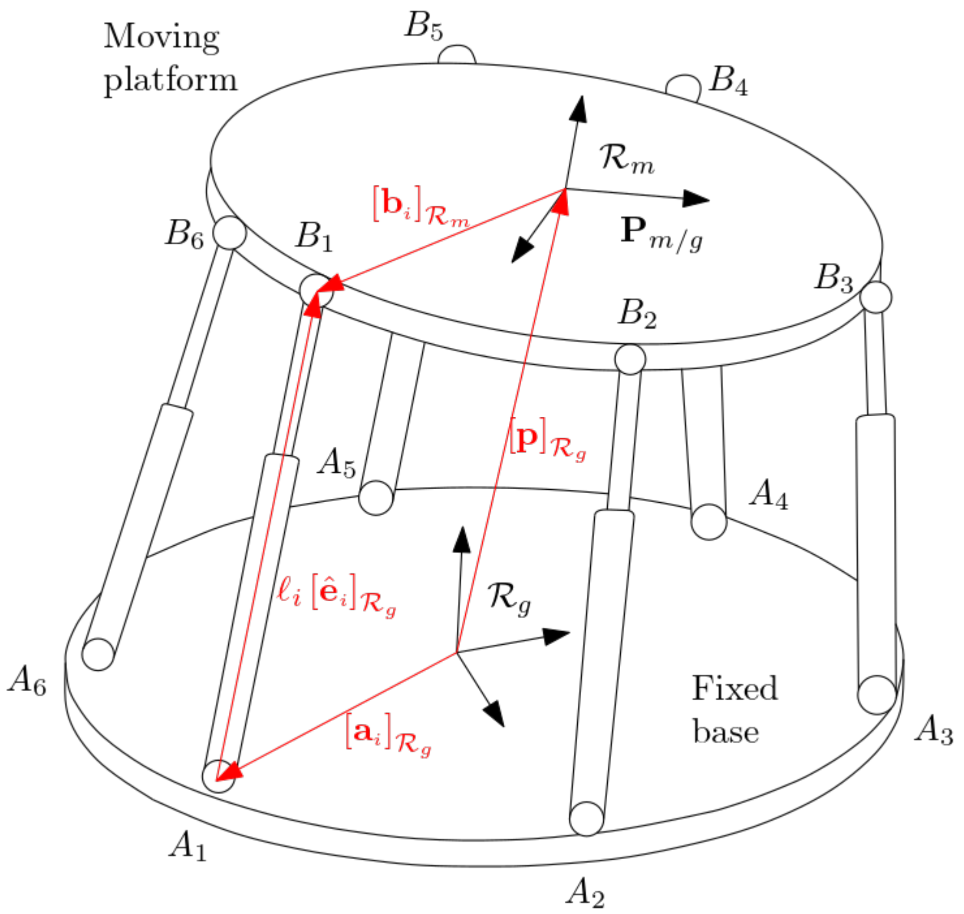
For closed kinematic chain systems, the number of rigid and independent DoFs is reduced due to **the loop closure constraints:**

$\boldsymbol{g}(\boldsymbol{p}) = 0$ (Set of holonomic constraints) $\boldsymbol{p}(t)$: position coordinates of all bodies depending on time t.

The classical Lagrange approach using the Lagrange multipliers λ leads to an augmented system of Differential-Algebraic Equations (DAEs):

$$\begin{pmatrix} \boldsymbol{M}(\boldsymbol{p}) & \boldsymbol{G}^{\mathrm{T}}(\boldsymbol{p}) \\ \boldsymbol{G}(\boldsymbol{p}) & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{p}} \\ \lambda \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}(\boldsymbol{p}, \dot{\boldsymbol{p}}, t) \\ -\boldsymbol{z}(\boldsymbol{p}, \dot{\boldsymbol{p}}) \end{pmatrix}$$

Further details reported in [2]



Stewart-Gough Platform schematic representation

The **Stewart Gough Platform (SGP)** is composed of:

- a **moving platform** (or end-effector) generally housing sensitive instruments
- a **main base** (fixed or attached to an another systems i.e. spacecraft)
- **six active (or hybrid) legs** used to control the pose of the moving platform.

$$\begin{aligned} \ell_i\,[\hat{\mathbf{e}}_i]_{\mathcal{R}_g} &= [\mathbf{p}]_{\mathcal{R}_g} + [\mathbf{b}_i]_{\mathcal{R}_g} - [\mathbf{a}_i]_{\mathcal{R}_g} \\ &= [\mathbf{p}]_{\mathcal{R}_g} + \mathbf{P}_{m/g}\,[\mathbf{b}_i]_{\mathcal{R}_m} - [\mathbf{a}_i]_{\mathcal{R}_g} \end{aligned} \qquad i = 1, 2, \ldots, 6$$

Forward Kinematics $\quad \ell_i \rightarrow \left\{[\mathbf{p}]_{\mathcal{R}_g},\, \mathbf{P}_{m/g}\right\}$

Inverse Kinematics $\quad \left\{[\mathbf{p}]_{\mathcal{R}_g},\, \mathbf{P}_{m/g}\right\} \rightarrow \ell_i$

Introduction (SGP Space Applications)

Hexapod used as **high accuracy pointing system**, developed to support ISS external payloads.

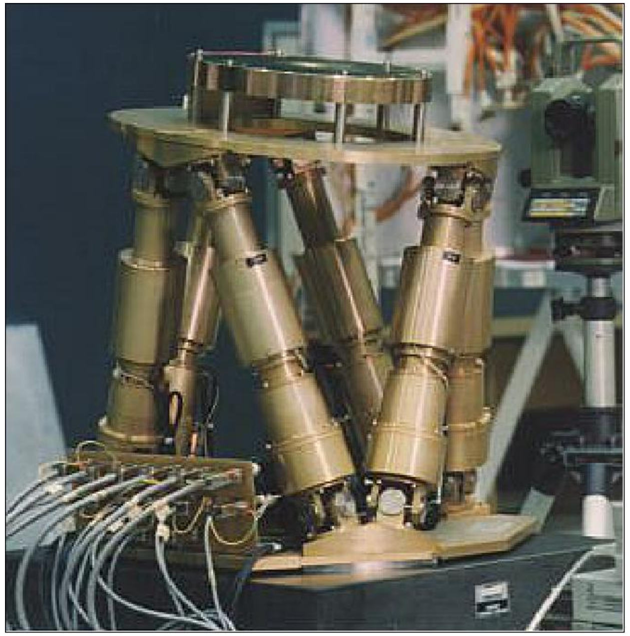
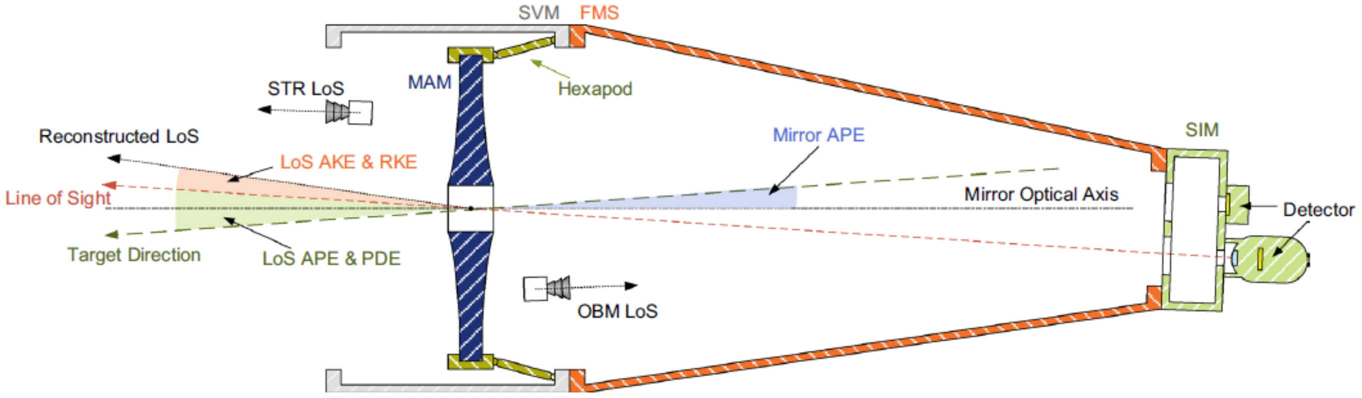
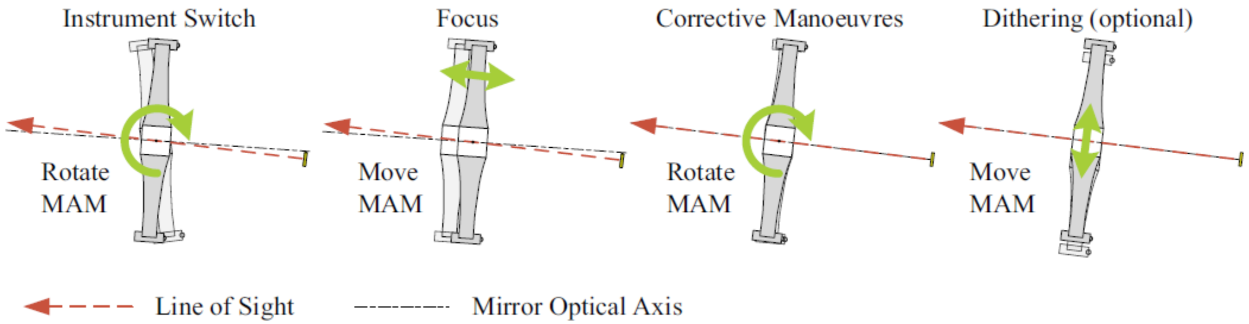


Photo of the Hexapod development model (image credit: Thales Alenia Space-Italy)

Athena X-ray Observatory (launch foreseen in 2028)



Pointing geometry of the S/C and Mirror assembly module (image credit: [3])



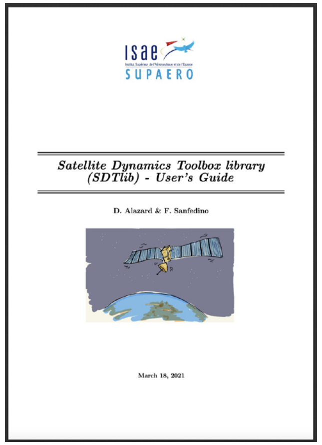
SGP manoeuvres (image credit: [3])

STDlib & Theoretical Framework

The Satellite Dynamics Toolbox library (STDlib) [4-7] is a MATLAB/SIMULINK library developed to linear model of multibody space systems presenting the following features:

- Model complex space systems in a **sub-structured** way
- Include **all possible parametric/complex uncertainties** (with minimal repetitions)
- Include **all possible varying parameters** (with minimal repetitions)
- Plug different flexible sub-structures for preliminary design
- Cop with the existing **Robust Control tools**

The output of STDlib is a model $\mathbf{M}^{\mathcal{S}}\left(\mathrm{s}, \boldsymbol{\theta}_{ref}, \boldsymbol{\Delta}\right)$ of a multibody system $\mathcal{S}$ parametrized according to the:


isae SUPAERO

Satellite Dynamics Toolbox library (SDTlib) - User's Guide

D. Alazard & F. Sanfedino

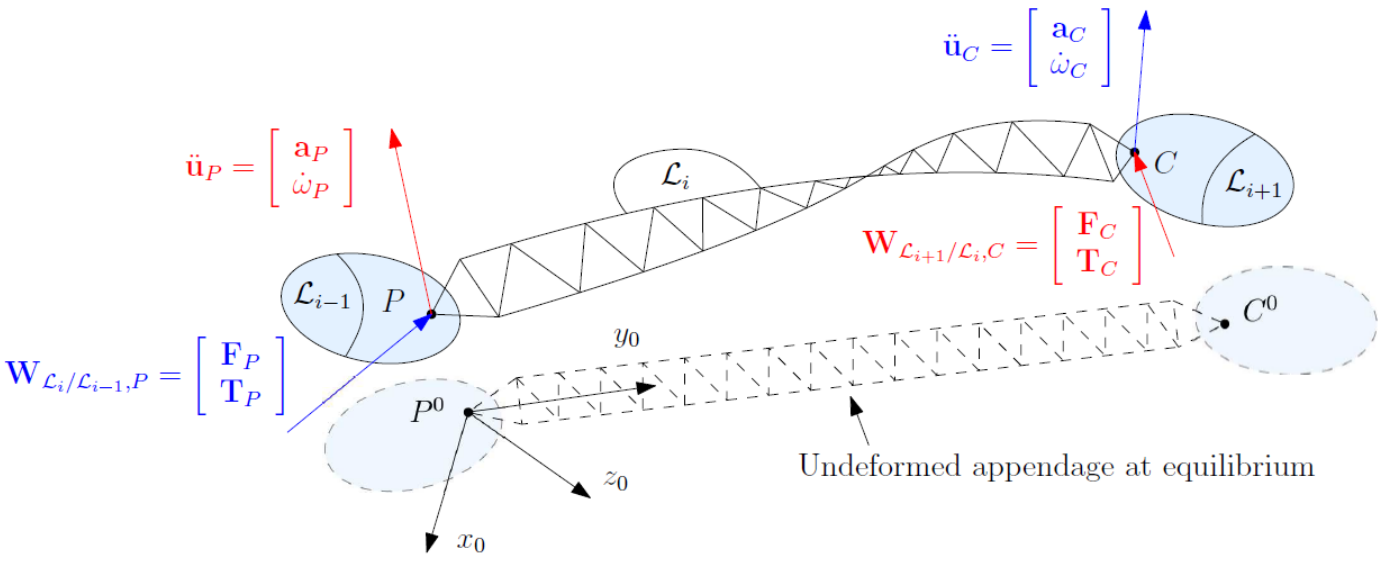
March 18, 2021

- Geometric configuration $\boldsymbol{\theta}_{ref}$ →
- Mechanical parametric configuration $\boldsymbol{\Delta}$

For closed loop kinematics chains the parametrization according to the geometric configuration can be done only if an **analytical solution (or an approximation) of the loop closure equations** is found.

User's Guide:
https://nextcloud.isae.fr/index.php/s/oPQjcytZMxL27a5#pdfviewer

STDlib & Theoretical Framework



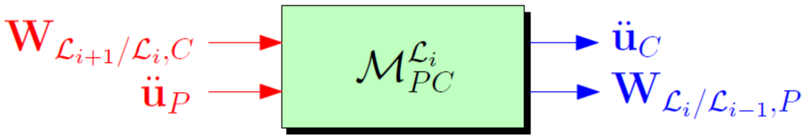
Two-Input Two-Output Port (TITOP) model of a flexible appendage [6]

TITOP approach allows to model different kind of substructures and mechanical systems:

- Analytical models for elemental bodies (beams, plates, multi-port rigid bodies, etc.)
- Interface with complex FEM models directly retrieved from PATRAN/NASTRAN model [8]
- Analytical models of mechanisms (RW, SADM, Revolute joints, etc.) [9]

Two input ports:

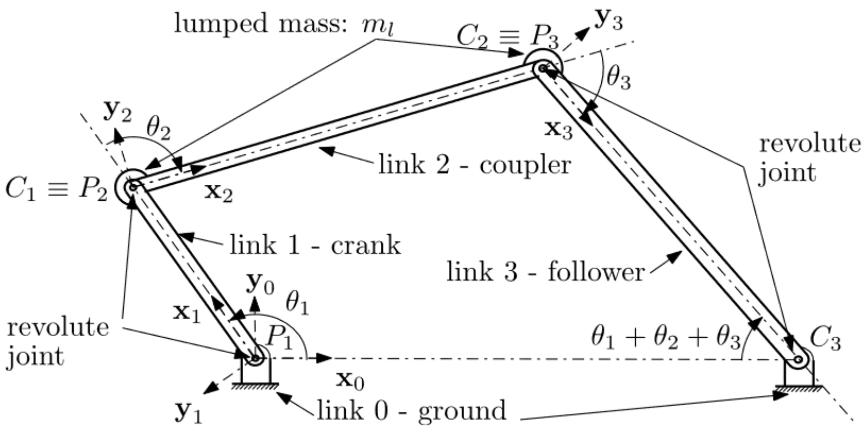
- Force/Torques applied by $\mathcal{L}_{i+1}$ to $\mathcal{L}_i$ at point C
- Acceleration at point P



Two output ports:

- Acceleration at point C
- Force/Torques applied by $\mathcal{L}_i$ to $\mathcal{L}_{i-1}$ at point P

Solving the loop closure (An Example)



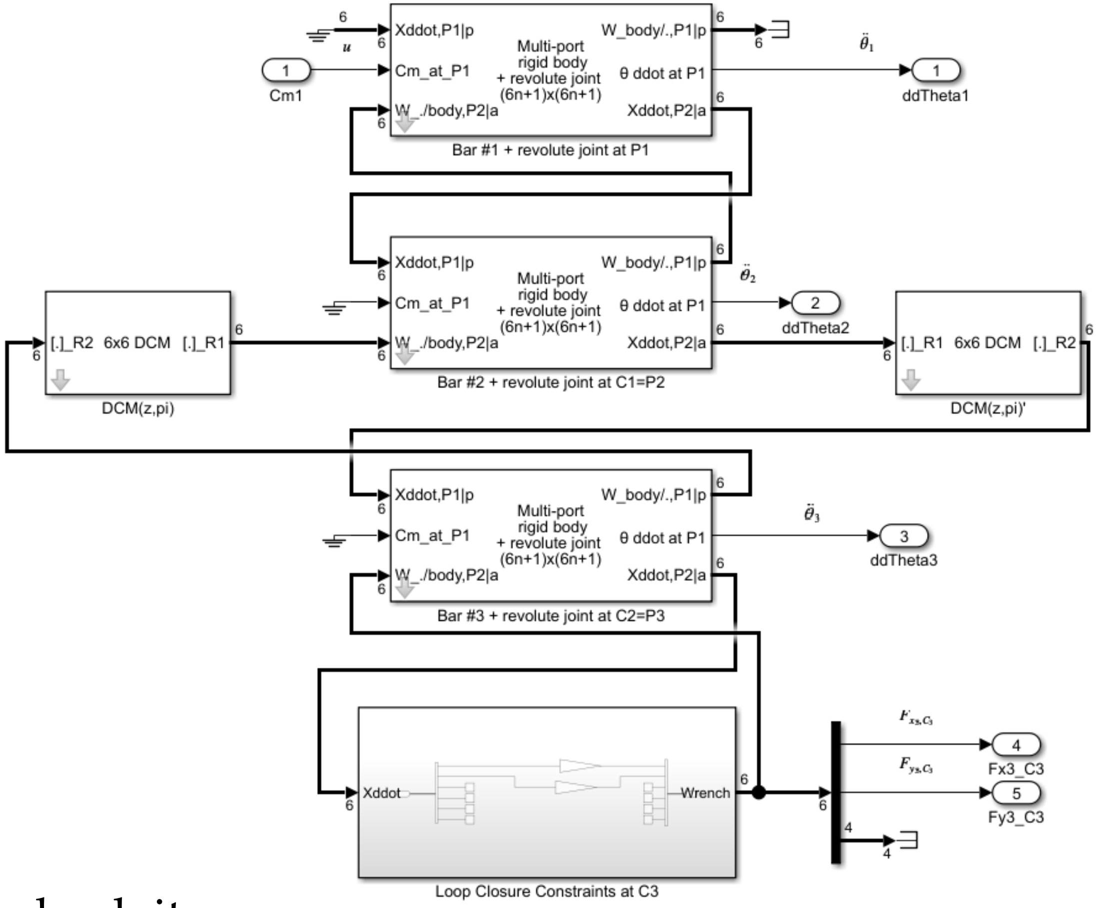
4 bar mechanism (image credit: [1])

$$l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3) = l_0$$
$$l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) = 0$$

In the case where $l_1 = l_3$ and $l_0 = l_4$ then:

$$\theta_1 = \theta, \quad \theta_2 = -\theta, \quad \theta_3 = \theta - \pi$$

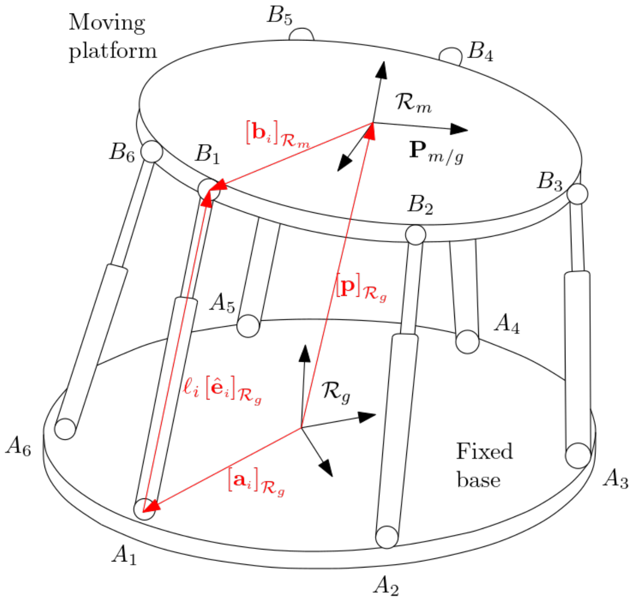
Thus, in this particular case where the closure constraints are solved, it is possible to build the dynamic model of this mechanism **fully parameterized** according to θ .



4 bar mechanism with rigid bars built in STDlib (image credit: [6])

Solving the loop closure for the SGP

$$\begin{aligned}\ell_i \left[\hat{\mathbf{e}}_i\right]_{\mathcal{R}_g} &= [\mathbf{p}]_{\mathcal{R}_g} + [\mathbf{b}_i]_{\mathcal{R}_g} - [\mathbf{a}_i]_{\mathcal{R}_g} \\ &= [\mathbf{p}]_{\mathcal{R}_g} + \mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m} - [\mathbf{a}_i]_{\mathcal{R}_g}\end{aligned} \qquad i = 1, 2, \ldots, 6$$



Tackling the Forward Kinematic (FK) problem $\ell_i \rightarrow \left\{ [\mathbf{p}]_{\mathcal{R}_g},\ \mathbf{P}_{m/g} \right\}$

- The **FK problem** is highly nonlinear and is extremely difficult to solve (is cast into 12 nonlinear equations with 12 unknowns)
- Multiple solutions are derived from the problem (Schemes are further needed to find a unique actual pose of the platform among all the possible solutions)
- Several **analytical closed-form FK solution** for the SGP where proposed in the years all based on different hypothesis

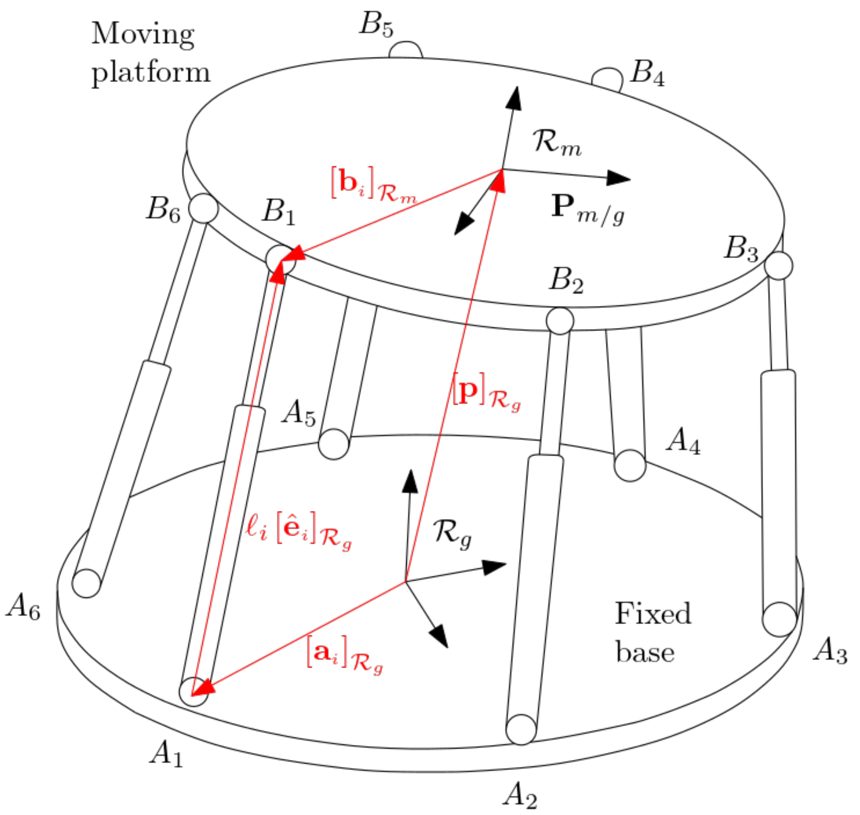
P. Ji. and H.Wu [10]: 8 solutions based on planar bases and similar hexagons hypothesis

J. Yang and Z. J. Geng. [11]: 8 solutions (very similar hypothesis to [10])

F. Wen and C. Liang.[12]: 40 solutions (nearly general 6–6 SGP)

Tackling the Inverse Kinematics problem $\left\{ [\mathbf{p}]_{\mathcal{R}_g}, \mathbf{P}_{m/g} \right\} \to \ell_i$

$$\begin{aligned} \ell_i \, [\hat{\mathbf{e}}_i]_{\mathcal{R}_g} &= [\mathbf{p}]_{\mathcal{R}_g} + [\mathbf{b}_i]_{\mathcal{R}_g} - [\mathbf{a}_i]_{\mathcal{R}_g} \\ &= [\mathbf{p}]_{\mathcal{R}_g} + \mathbf{P}_{m/g} \, [\mathbf{b}_i]_{\mathcal{R}_m} - [\mathbf{a}_i]_{\mathcal{R}_g} \end{aligned} \qquad i = 1, 2, \ldots, 6$$



To obtain the length of each actuator and eliminate $[\hat{\mathbf{e}}_i]_{\mathcal{R}_g}$ it is sufficient to dot multiply each side by itself:

$$\ell_i = \Big[[\mathbf{p}]^T_{\mathcal{R}_g} [\mathbf{p}]_{\mathcal{R}_g} + [\mathbf{b}]^T_{\mathcal{R}_m} [\mathbf{b}]_{\mathcal{R}_m} + [\mathbf{a}]^T_{\mathcal{R}_g} [\mathbf{a}]_{\mathcal{R}_g} - 2 \, [\mathbf{p}]^T_{\mathcal{R}_g} [\mathbf{a}]_{\mathcal{R}_g} + 2 \, [\mathbf{p}]^T_{\mathcal{R}_g} \left[\mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m} \right] - 2 \left[\mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m} \right]^T [\mathbf{a}]_{\mathcal{R}_g} \Big]^{1/2}$$

One unique solution to the limb length exists if the position and orientation of the moving platform lie in the feasible workspace of the manipulator.

Solving the loop closure for the SGP

$$\ell_i = \Big[[\mathbf{p}]_{\mathcal{R}_g}^T [\mathbf{p}]_{\mathcal{R}_g} + [\mathbf{b}]_{\mathcal{R}_m}^T [\mathbf{b}]_{\mathcal{R}_m} + [\mathbf{a}]_{\mathcal{R}_g}^T [\mathbf{a}]_{\mathcal{R}_g} - 2\,[\mathbf{p}]_{\mathcal{R}_g}^T [\mathbf{a}]_{\mathcal{R}_g} + 2\,[\mathbf{p}]_{\mathcal{R}_g}^T \left[\mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m}\right] - 2\left[\mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m}\right]^T [\mathbf{a}]_{\mathcal{R}_g} \Big]^{1/2} \qquad i = 1, 2, \ldots, 6$$

In order to derive a model parametrization for the SGP system in the **Linear Fractional Transformation (LFT) form a rational approximation of the loop closure equations** is needed.

A **linear approximation** is selected

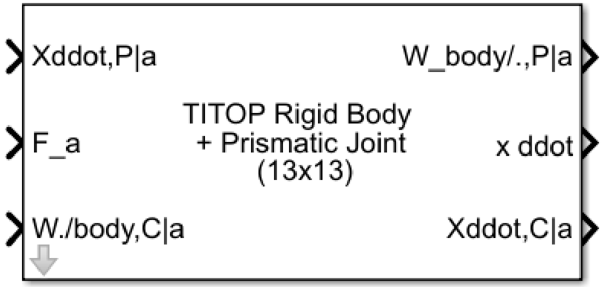
- Provides a sufficient approximation in the domain of interest of the platform variables
- Provides the least number of repetition of the parameters
  - **24 for the rotations** (8 for each of the 3 angles to define the pose of the moving platform wrt the base)
  - **12 for the translation** (4 for each axis)

The **direction of the leg** is then given by:

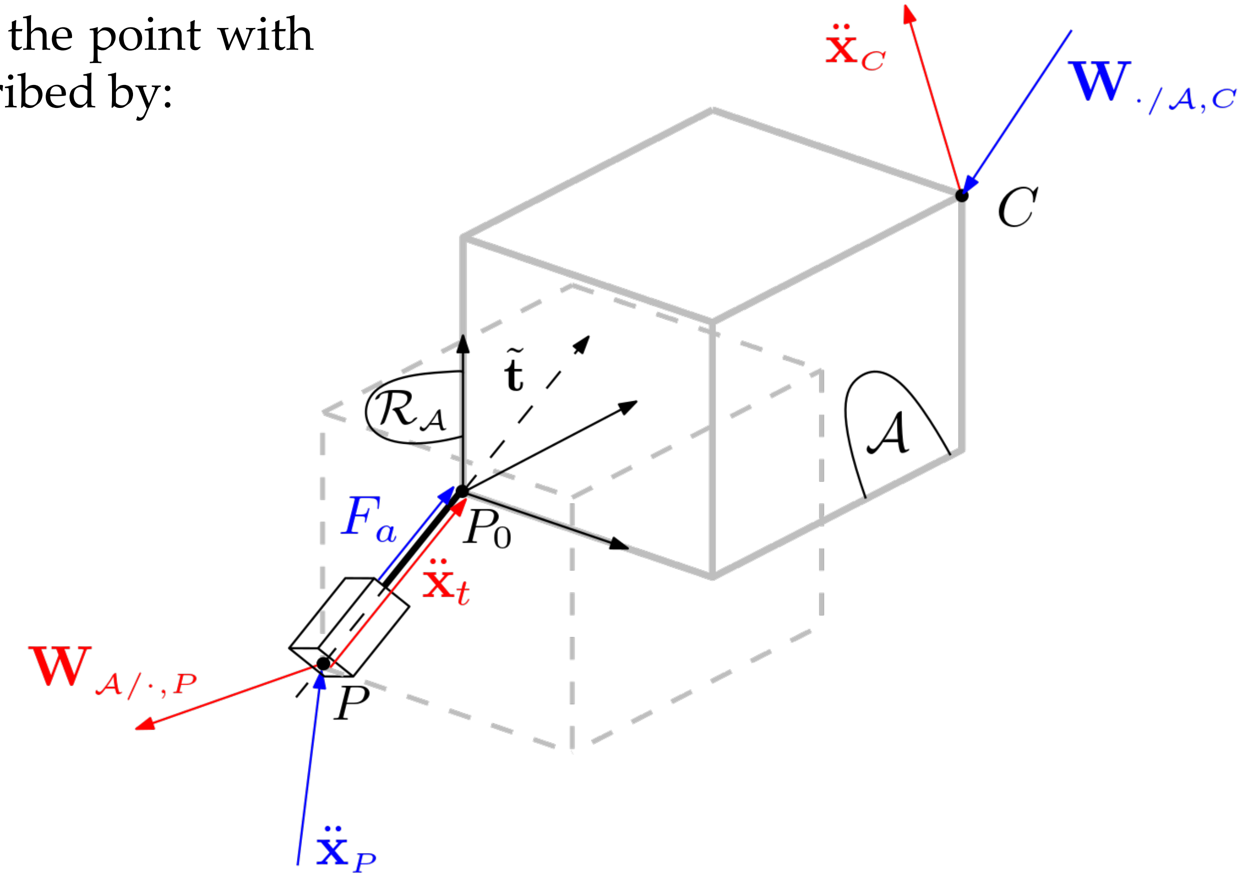
$$[\hat{\mathbf{e}}_i]_{\mathcal{R}_g} = \frac{[\mathbf{p}]_{\mathcal{R}_g} + \mathbf{P}_{m/g} [\mathbf{b}_i]_{\mathcal{R}_m} - [\mathbf{a}_i]_{\mathcal{R}_g}}{\ell_i} \qquad i = 1, 2, \ldots, 6$$

The dynamics of the rigid body connected to body at the point with a prismatic joint is a (6 + 6 + 1) × (6 + 6 + 1) model described by:

$$\begin{bmatrix} \mathbf{W}_{\mathcal{A}/\cdot,P} \\ \ddot{\mathbf{x}}_t \\ \ddot{\mathbf{x}}_C \end{bmatrix} = \left[\mathcal{M}^{\mathcal{A}}_{P,C}\right]_{\mathcal{R}_\mathcal{A}} \begin{bmatrix} \ddot{\mathbf{x}}_P \\ F_a \\ \mathbf{W}_{\cdot/\mathcal{A},C} \end{bmatrix}$$

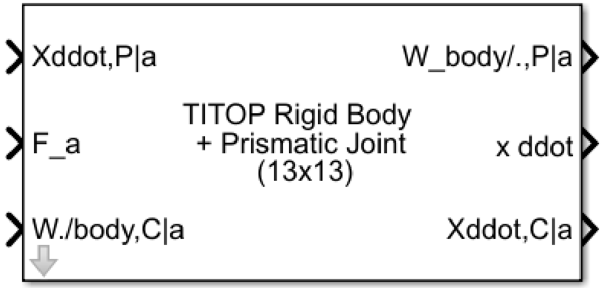


Simulink subsystem for the TITOP rigid body + Prismatic joint

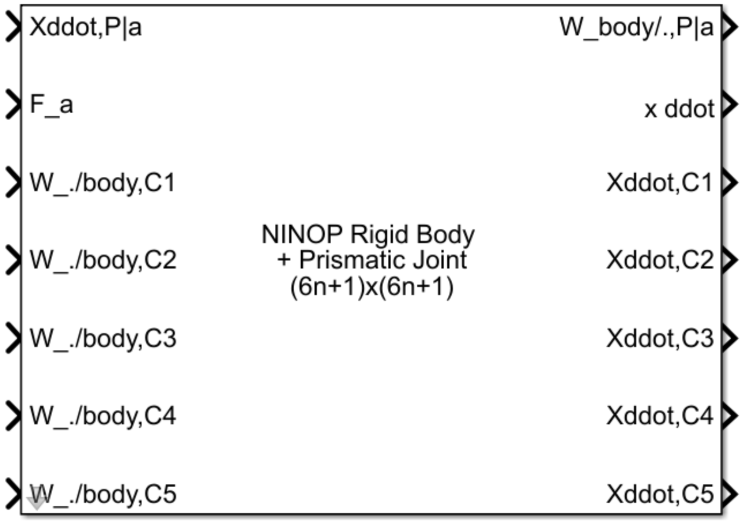


Schematic representation of a rigid body A with a prismatic joint at point P0

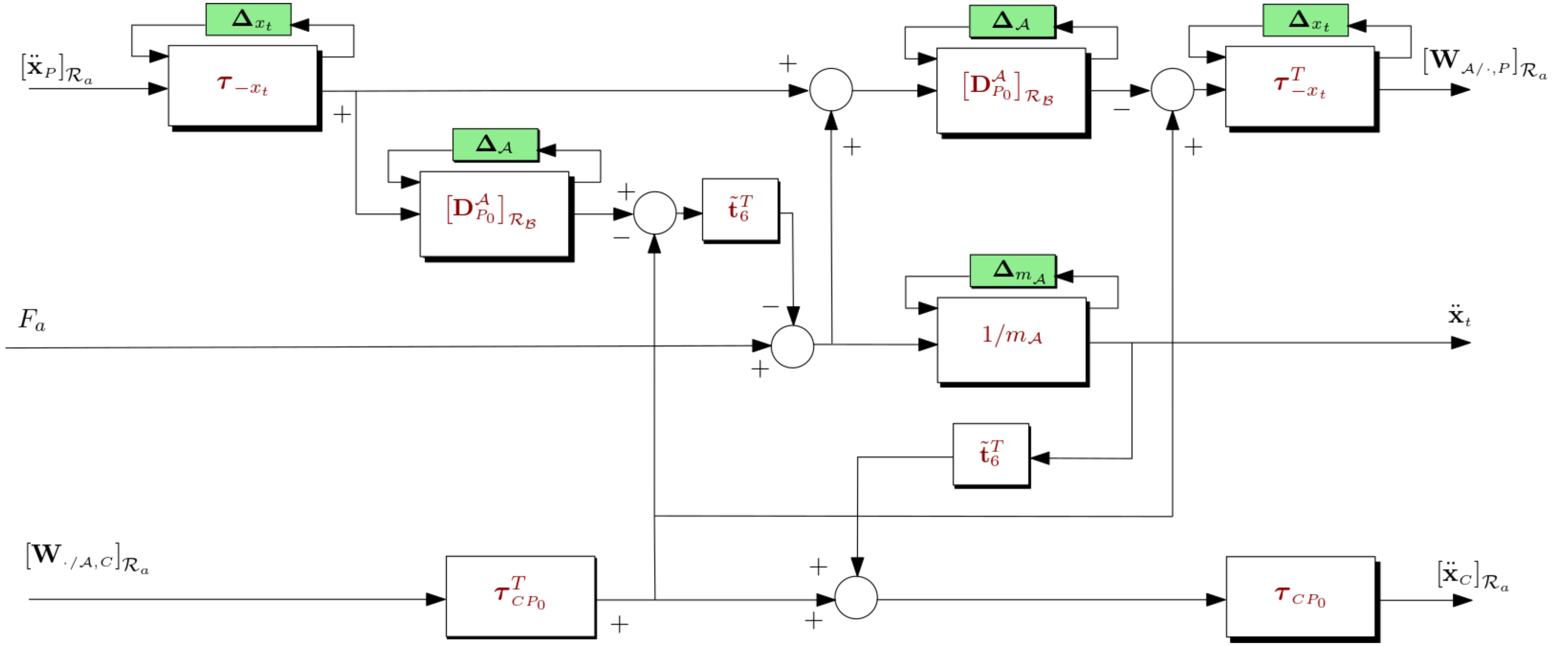
TITOP rigid body + Revolute joint



Simulink subsystem for the TITOP rigid body + Prismatic joint



Simulink subsystem for the NINOP rigid body + Prismatic joint



Block diagram representation of the TITOP rigid body + Prismatic joint

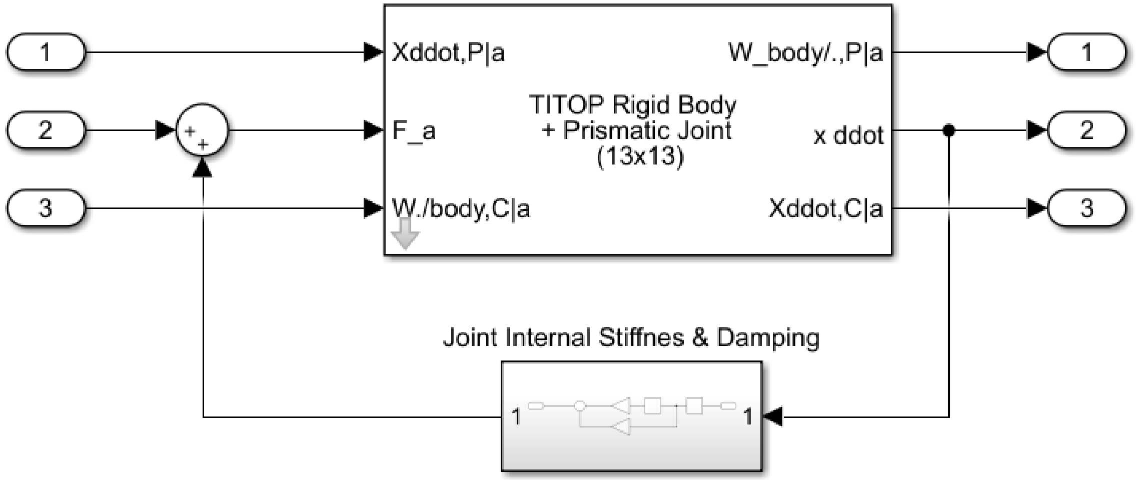
$$\boldsymbol{\Delta}_{x_t} = \delta_{x_t} \mathbf{I}_2$$

4 total repetition of the parameter inside the TITOP rigid body + Prismatic joint

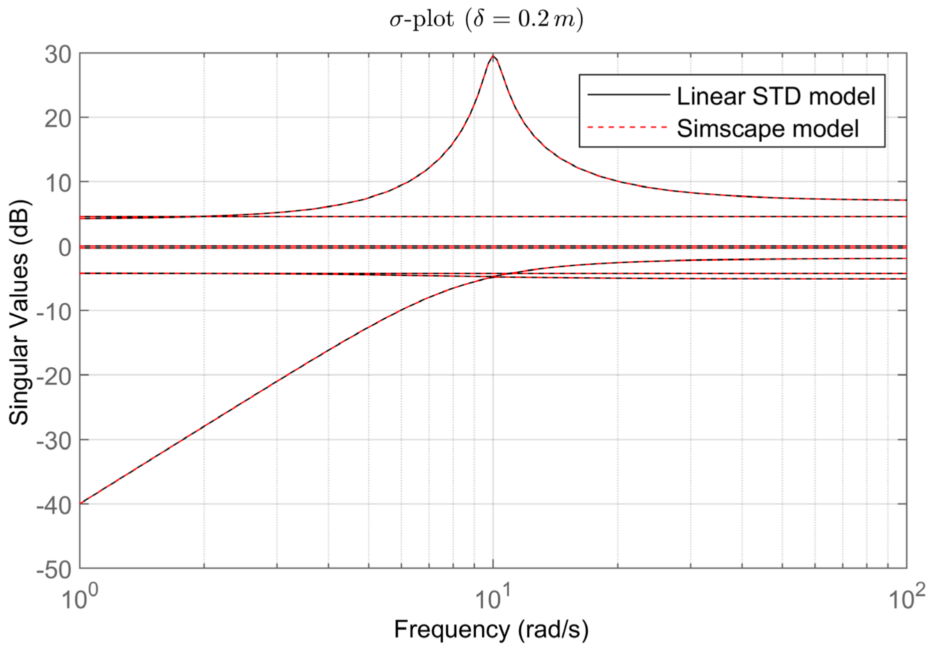
TITOP rigid body + Prismatic joint

The **TITOP rigid body + Prismatic joint** has been validated with a non-linear model built with SIMSCAPE Multibody and linearized around the equilibrium condition.

- For the nominal configuration (displacement = 0)
- For $\delta \neq 0$



Simulink system for the TITOP rigid body + prismatic joint and internal stiffness and damping

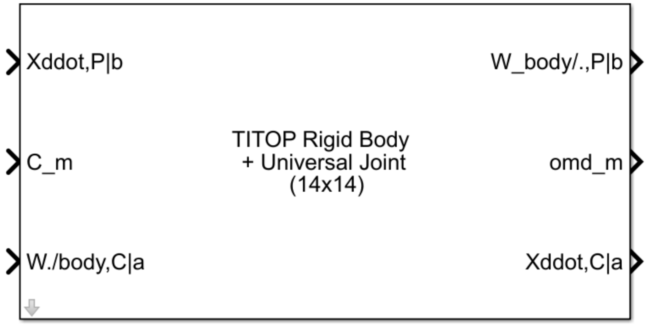


Sigma plot comparison between STD model and linearized SIMSCAPE model

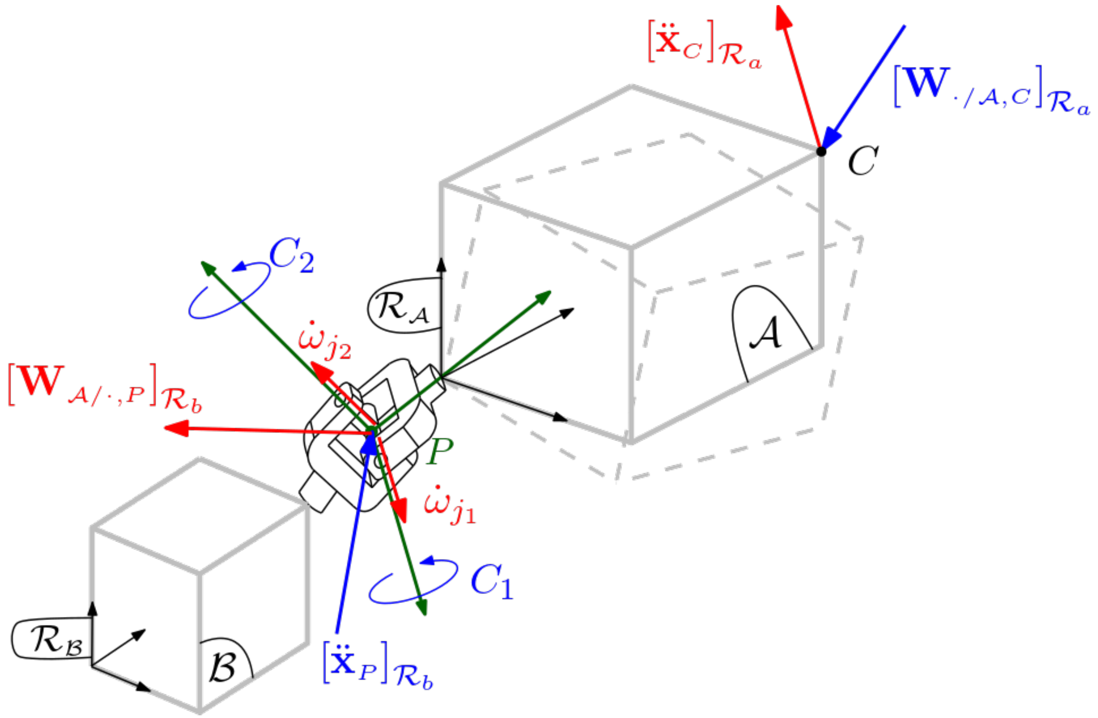
TITOP rigid body + Universal joint

The dynamics of the rigid body connected to body at the point with an universal joint is a (6 + 6 + 2) × (6 + 6 + 2) model described by:

$$\begin{bmatrix} [\mathbf{W}_{\mathcal{A}/\cdot,P}]_{\mathcal{R}_b} \\ \dot{\boldsymbol{\omega}}_j \\ [\ddot{\mathbf{x}}_C]_{\mathcal{R}_a} \end{bmatrix} = \left[\mathcal{M}^{\mathcal{A}}_{P,C}\right]_{\mathcal{R}_\mathcal{A}} \begin{bmatrix} [\ddot{\mathbf{x}}_P]_{\mathcal{R}_b} \\ \mathbf{C}_j \\ [\mathbf{W}_{\cdot/\mathcal{A},C}]_{\mathcal{R}_a} \end{bmatrix}$$



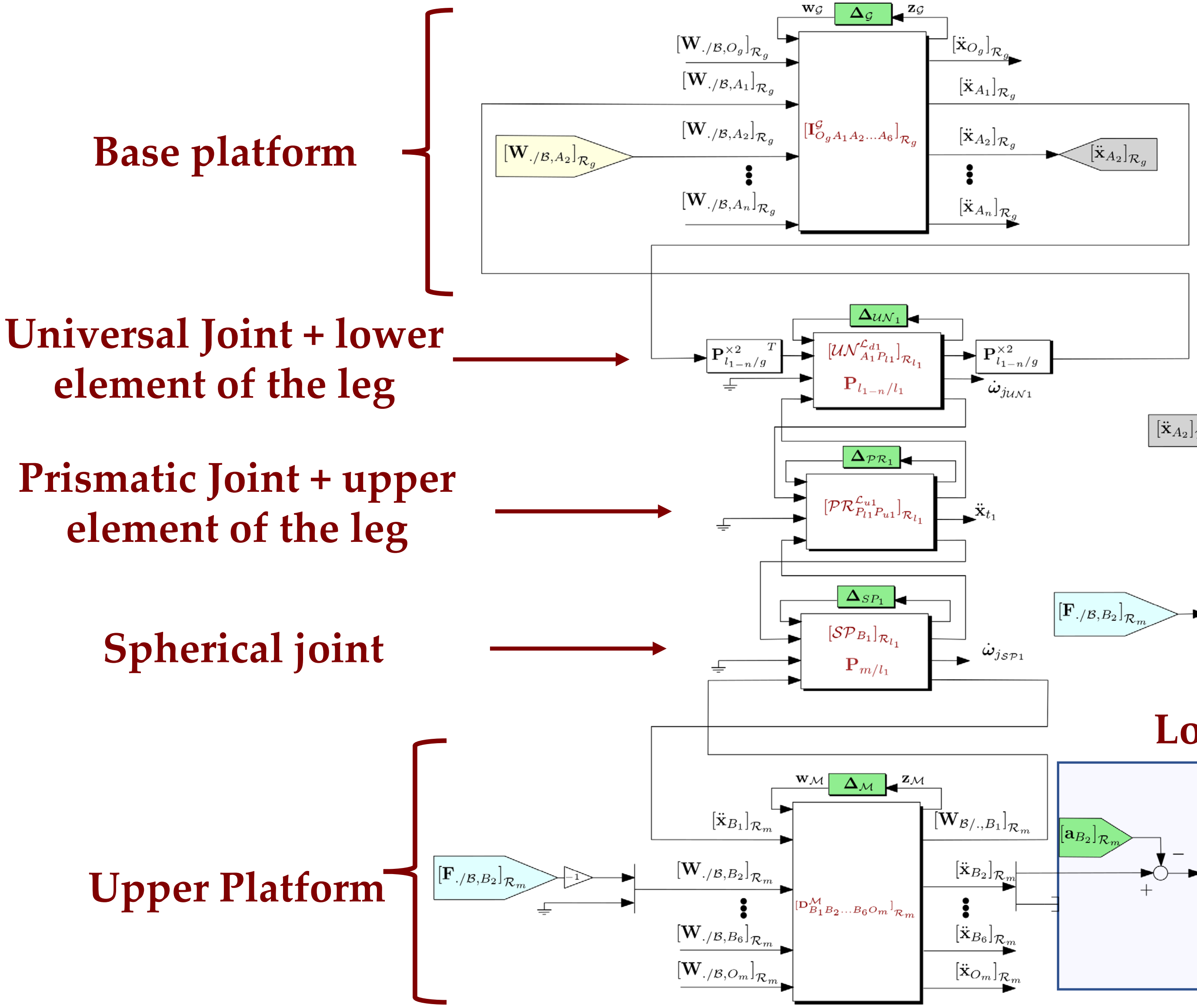
Simulink subsystem for the TITOP rigid body + Universal joint

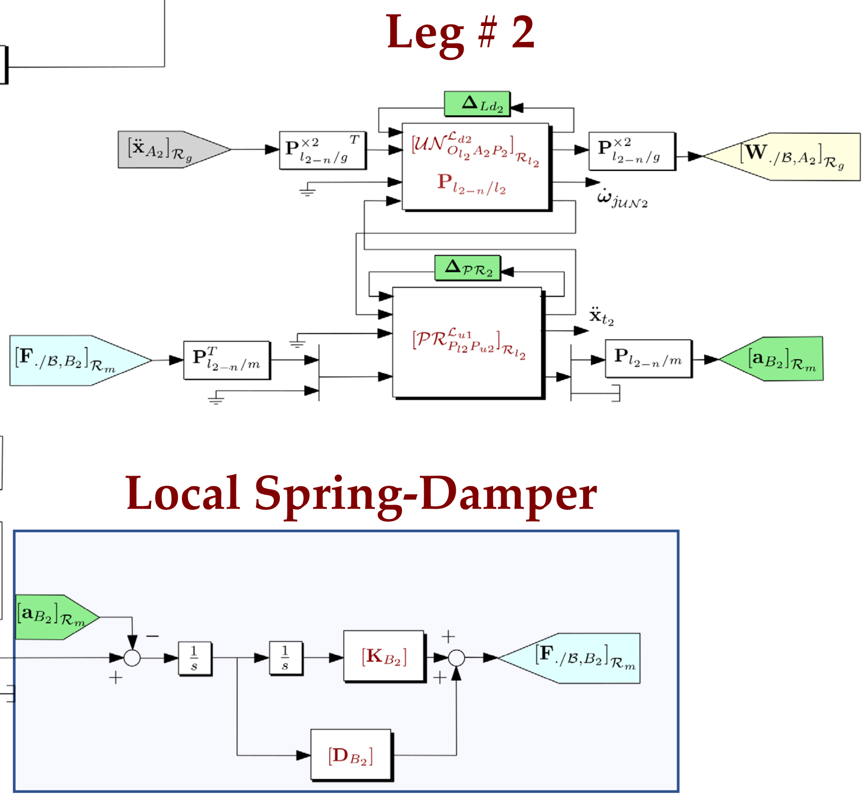


Schematic representation of a rigid body A connected to body B via an universal joint

Partial view of the Stewart-Gough platform

(Only Leg #1 and #2 are shown – The other legs are a repetition of Leg#2 and a Local Spring-Damper between the leg and the moving platform to impose the loop closure constraint)





Stewart-Gough Platform

3 Linear models can be obtained from the previous diagram:

1) **Fully parametrized:** It uses the second order Taylor expansion for the cosine function (not feasible to implement due to the huge number of repetition of the parameters)

2) **Small rotation model** (size of the Δ block still for the parametrization ~ 5000)

3) **Nominal joint rotations** (size of the Δ block for the parametrization ~ 720)

Reduced complexity and number of repetitions

The size can be further reduced considering that the motion of the platform does not involve simultaneously all 6 DoFs of the moving platform

Tip/tilt (roll-pitch rotations)
size of the Δ block: 384

Focusing (z-axis motion)
size of the Δ block: 48

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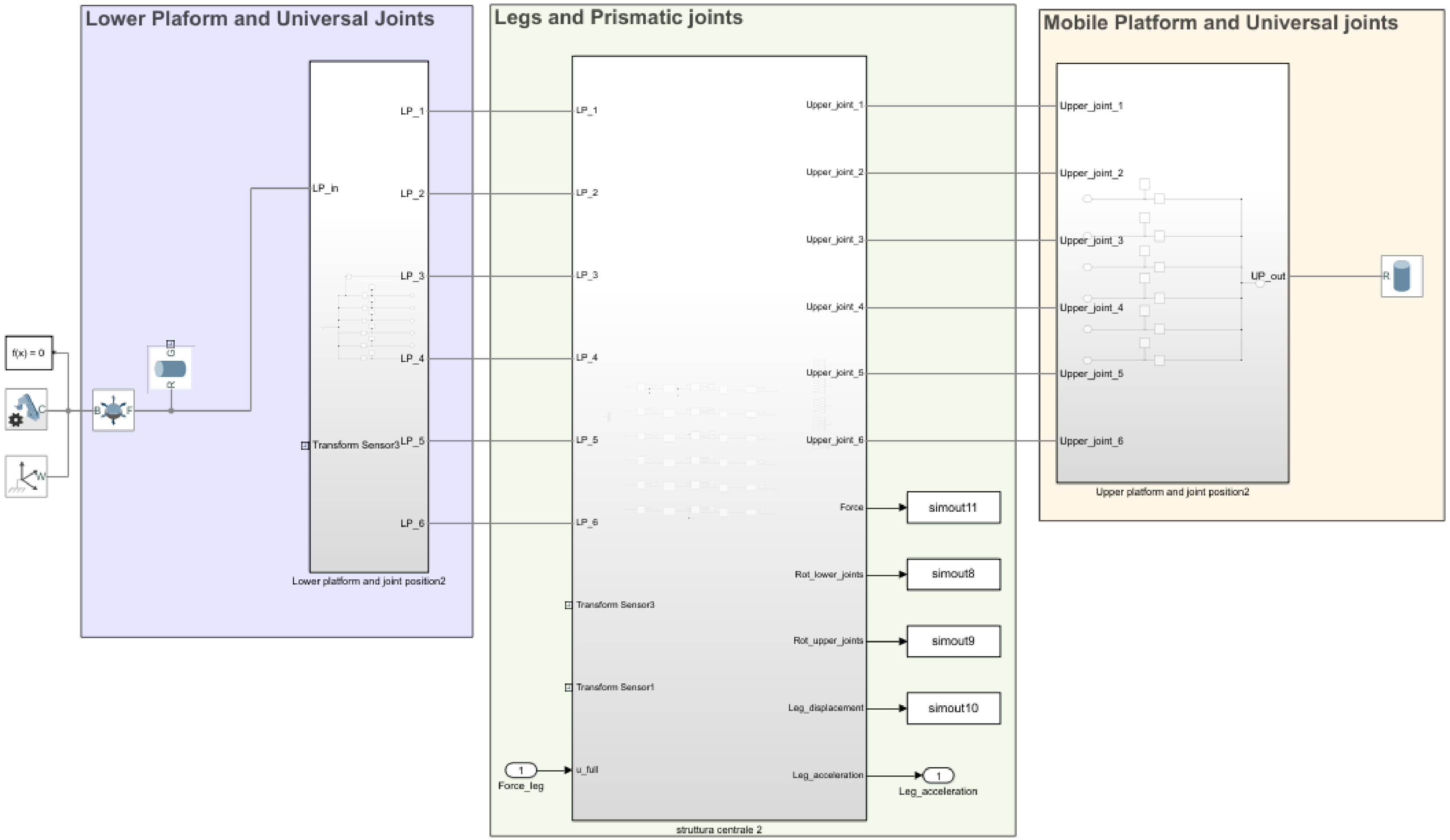

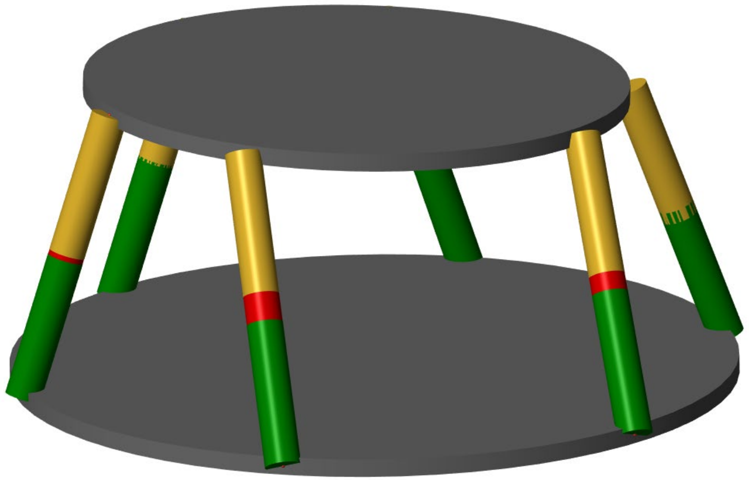

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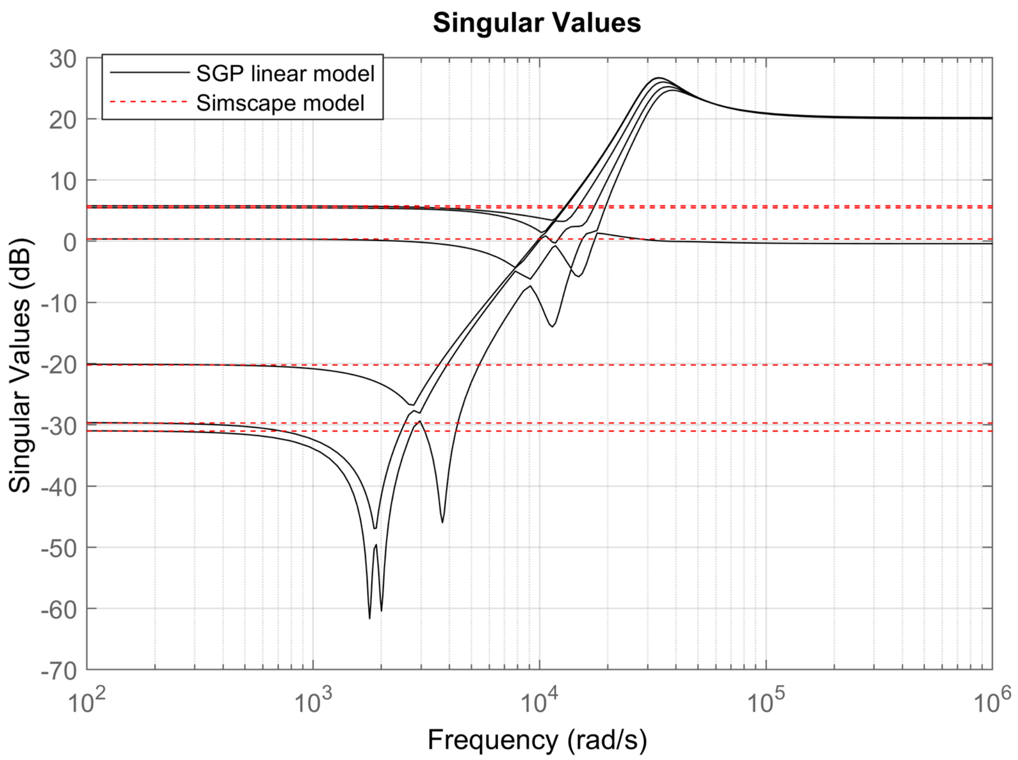
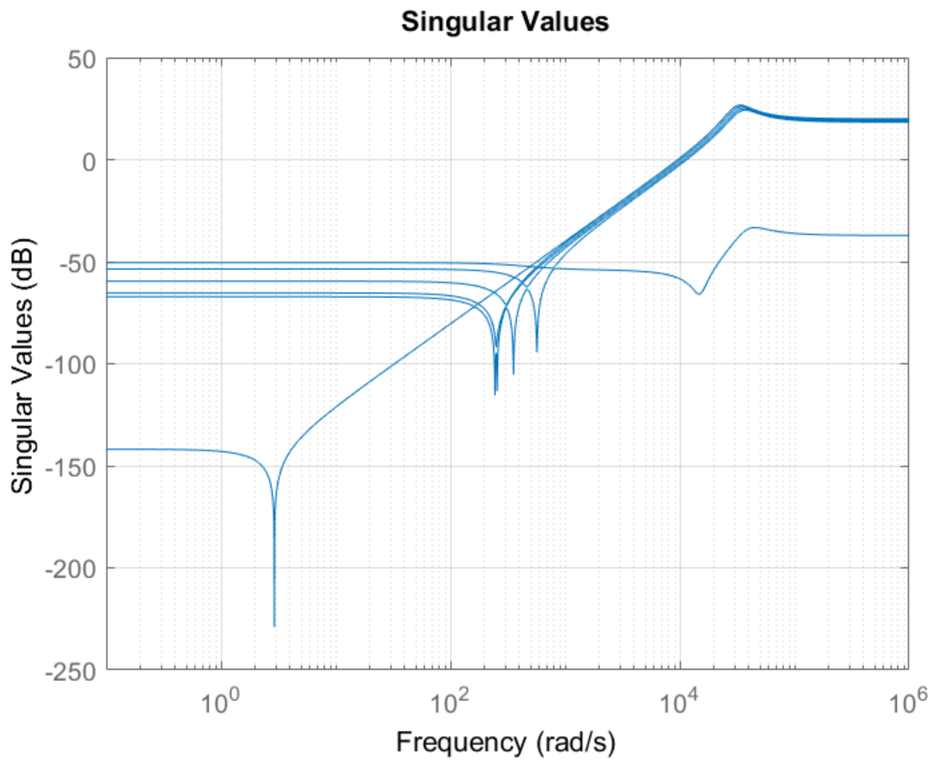
Validation of the nominal joint rotation model



Comparison of singular value plot between the linearized Simscape model and the developed SGP linear model



Singular value plot of the difference the linearized Simscape model and the developed SGP linear model

The work has investigated how to address the modelling of closed-loop kinematic multibody systems in a LFT framework by:

- **Analysing and solving (via some approximations) the loop closure equations** for parallel robots (potential extension to other classes of parallel robots i.e. Delta robot, other configurations of the SGP)
- Contributing in the **definition of some elementary blocks** to model such complex systems in a sub-structured way

Future developments

- Investigate **the range of validity of the approximations** (especially the nominal joint rotations)
- Further **reduce the number of repetitions** of the parameters in the model
- Inclusion of **joint flexibility**
- Inclusion of the **flexibility of the upper platform** (allowing channel inversion and elimination of the local spring-dampers)

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Thank you for your attention

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