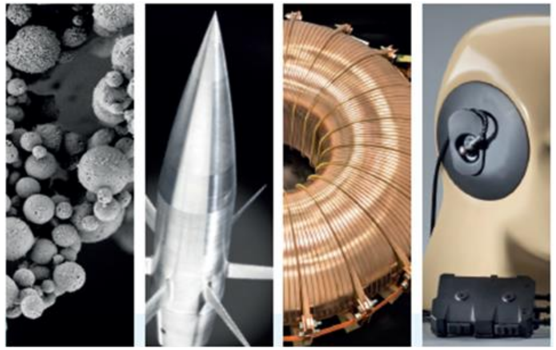

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Uneven Grid-based Linear Parameter-Varying Controller Design for Guided Projectiles

Société d'Automatique, de Génie Industriel & de Productique

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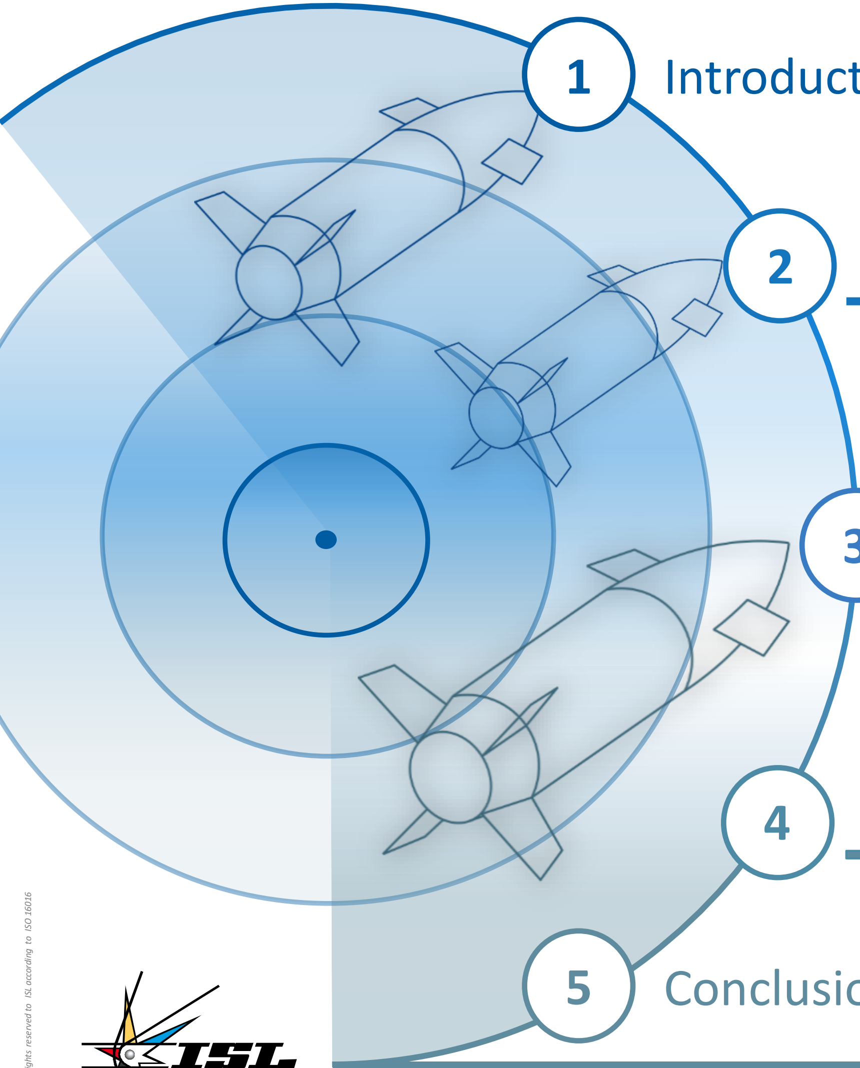

1. Introduction
2. LPV Modeling Process:
   - Nonlinear Model
   - quasi-LPV Model
   - LPV Grid-based Approach
3. Grid Design Analysis:
   - Projectile Stability Conditions
   - Reachable Flight Envelope
   - Grid Dimension & Basis Selection
4. LPV Control Design:
   - Grid-based Controller
   - 6-DoF Simulator
   - Trajectory Tracking Simulation
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Flight Control Framework

- Traditionally based on LTI gain-scheduling design;
- Controller obtained as the interpolation of a collection of local controllers designed at specific flight points;
- Lack of a priori stability and performance guarantees at any intermediate flight conditions.

Linear Parameter Varying Framework

- Increasing number of applications concerning aircraft and missiles LPV modeling and control design;
- Stronger stability and performance guarantees across the flight envelope considered during the design;
- Limited applications on guided projectiles design.

Long Range Guided Projectile (LRGP)

Objectives

- LPV grid-based modeling & control design.

Characteristics

- 155 mm fin-stabilized guided projectile;
- 2 front control canards, 4 symmetrical rear fins;
- Limited control authority/unstable behavior;

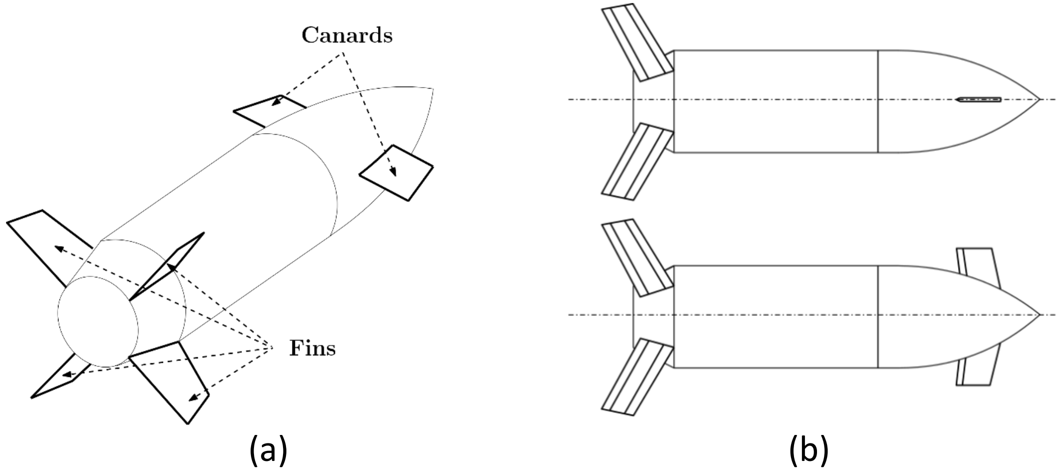


Figure 1. LRGP concept: (a) aerodynamic surfaces; (b) side-top view.

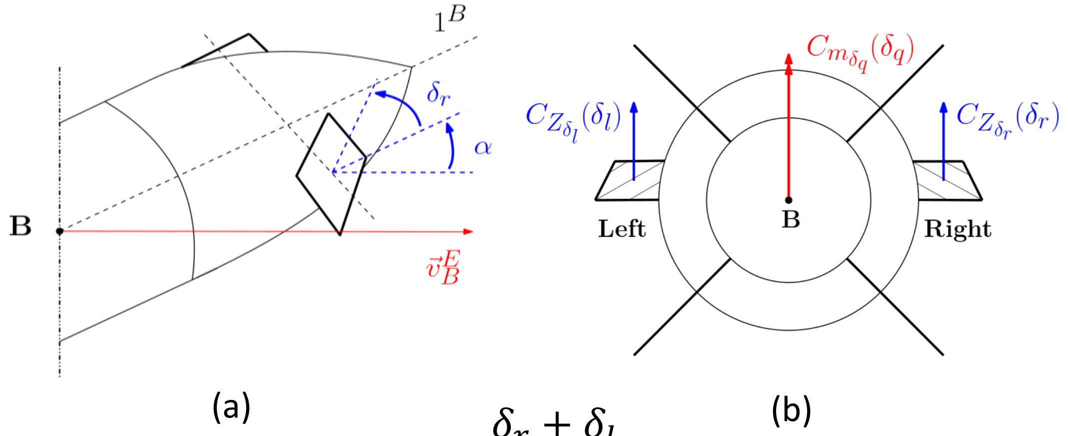
Pitch Channel Dynamics [1]

$$\begin{cases} \dot{\alpha} = \left(\dfrac{1}{mV}\right)\left(-X\sin\alpha + Z\cos\alpha + mg(\sin\alpha\sin\vartheta + \cos\alpha\cos\vartheta)\right) + q \\ \dot{q} = \dfrac{1}{I_{yy}}(M) \end{cases} \tag{1}$$

$$\eta_z = \frac{Z}{mg}$$

Aerodynamics [2]

$$\begin{cases} X = \bar{q}S\left(C_{X_{\alpha 0}} + C_{X_{\alpha 2}}\sin^2\alpha + C_{X_{\alpha 4}}\sin^4\alpha\right) \\ Z = \bar{q}S\left(C_{Z_{\alpha 1}}\sin\alpha + \left(\dfrac{d}{V}\right)C_{Z_D}q + C_{Z_{\delta_q}}\delta_q\right) \\ M = \bar{q}dS\left(C_{m_{\alpha 1}}\sin\alpha + C_{m_{\alpha 3}}\sin^3\alpha + C_{m_{\alpha 5}}\sin^5\alpha + \left(\dfrac{d}{V}\right)C_{m_D}q + C_{m_{\delta_q}}\delta_q\right) \end{cases} \tag{2}$$



$$\delta_q = \frac{\delta_r + \delta_l}{2}$$

Figure 2. Canards deflection: (a) local right deflection, δ_r; (b) virtual pitch deflection, δ_q.

NOTE: First-order aerodynamics approximation[2] to obtain an affine model-input dependence (δ_q).

[1] Vinco, G.M., Theodoulis, S., and Sename, O. (2022). "**Flight dynamics modeling and simulator design for a new class of long-range guided projectiles**".
[2] Vinco, G.M., Theodoulis, S., Sename, O., and Strub, G. (2022). "**Quasi-LPV Modeling of Guided Projectile Pitch Dynamics through State Transformation Technique**".

LPV State Transformation Approach [1]

- Select the set of scheduling variables: $\rho(t) = [\alpha, V, h]^T$;
- Reformulate the nonlinear system (1)-(2) as an output nonlinear model;
- Define the off-equilibrium state, input, and output ($q_{\mathrm{dev}}, \delta_{q,\mathrm{dev}}, \eta_{z,\mathrm{dev}}$), to hide the nonlinearities:
- Integrate the input to avoid any parameter dependence:

$$\begin{cases} q_{\mathrm{dev}} = q - q_{eq}(\alpha) \\ \delta_{q,\mathrm{dev}} = \delta_q - \delta_{q,eq}(\alpha) \\ \eta_{z,\mathrm{dev}} = \eta_z - \eta_{z,eq}(\alpha) \end{cases}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q}_{\mathrm{dev}}(\alpha) \\ \dot{\delta}_{q,\mathrm{dev}}(\alpha) \end{bmatrix} = \begin{bmatrix} 0 & 1 + \frac{\bar{q}SdC_{Z_{\mathrm{D}}}\cos\alpha}{2mV^2} & \frac{\bar{q}SC_{Z_{\delta q}}\cos\alpha}{mV} \\ 0 & \frac{\bar{q}Sd^2C_{m_{\mathrm{D}}}}{2I_{yy}V} - \frac{\partial q_{eq}}{\partial\alpha}\left(1 + \frac{\bar{q}SdC_{Z_{\mathrm{D}}}\cos\alpha}{2mV^2}\right) & \frac{\bar{q}SdC_{m_{\delta q}}}{I_{yy}} - \frac{\partial q_{eq}}{\partial\alpha}\left(\frac{\bar{q}SC_{Z_{\delta q}}\cos\alpha}{mV}\right) \\ 0 & -\frac{\partial\delta_{q,eq}}{\partial\alpha}\left(1 + \frac{\bar{q}SdC_{Z_{\mathrm{D}}}\cos\alpha}{2mV^2}\right) & -\frac{\partial\delta_{q,eq}}{\partial\alpha}\left(\frac{\bar{q}SC_{Z_{\delta q}}\cos\alpha}{mV}\right) \end{bmatrix} \begin{bmatrix} \alpha \\ q_{\mathrm{dev}}(\alpha) \\ \delta_{q,\mathrm{dev}}(\alpha) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \sigma$$

$$\begin{bmatrix} \alpha \\ q_{\mathrm{dev}}(\alpha) \\ \eta_{z,\mathrm{dev}}(\alpha) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{\bar{q}SdC_{Z_{\mathrm{D}}}}{mgV} & \frac{\bar{q}SC_{Z_{\delta q}}}{mg} \end{bmatrix} \begin{bmatrix} \alpha \\ q_{\mathrm{dev}}(\alpha) \\ \delta_{q,\mathrm{dev}}(\alpha) \end{bmatrix} \tag{3}$$

[1] Shamma, J. S., and Cloutier, J. R. (1993). "**Gain-scheduled missile autopilot design using linear parameter varying transformations**".

Grid-based Model

- Arbitrary model-parameter dependency;
- Interpolation of a set of LTI local realizations (A_i, B_i, C_i, D_i);
- No restrictions on the interpolation method, a_i:

$$\left[\begin{array}{c|c} A_{Gr}(\rho) & B_{Gr}(\rho) \\ C_{Gr}(\rho) & D_{Gr}(\rho) \end{array}\right] = \sum_{i=j}^{j+1} a_i(\rho) \left[\begin{array}{c|c} A_i & B_i \\ C_i & D_i \end{array}\right]; \quad j = [1, \dots, n_g] \qquad (4)$$

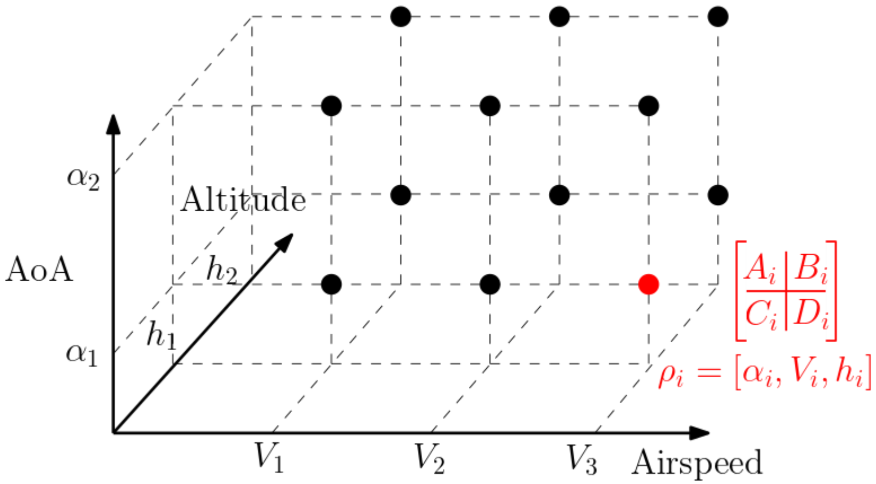


Figure 3. 3D grid-based LPV system definition.

Grid-based Control Design

- Resolution of a set of LMIs evaluated at all n_g grid points;
- Identification of a set of parameter-dependent Lyapunov functions $(X(\rho), Y(\rho))$;
- Fixed parameter dependency expressed through the selection of differentiable scalar functions (f_i, g_i):

$$X(\rho) = X_0 + \sum_{i=1}^{N} f_i(\rho) X_i \quad ; \quad Y(\rho) = Y_0 + \sum_{i=1}^{N} g_i(\rho) Y_i \qquad (5)$$

- LPV controller defined by the interpolation of the obtained LTI local controllers on the grid.

Critical aspects:

- Design grid points selection;
- Basis functions selection.

→ Trade-off Performance/Complexity.

Projectile Stability Conditions

Objectives

- Verify the projectile dynamics across the flight envelope;
- Identify the most critical conditions;
- Allocate more design points of the grid in the most critical areas.

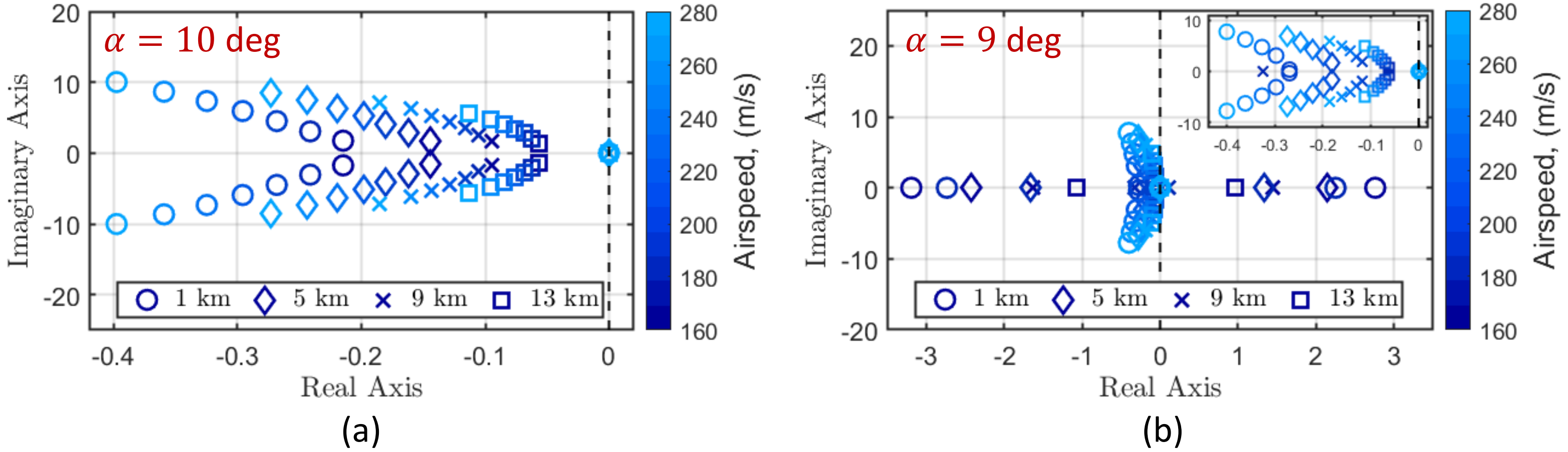
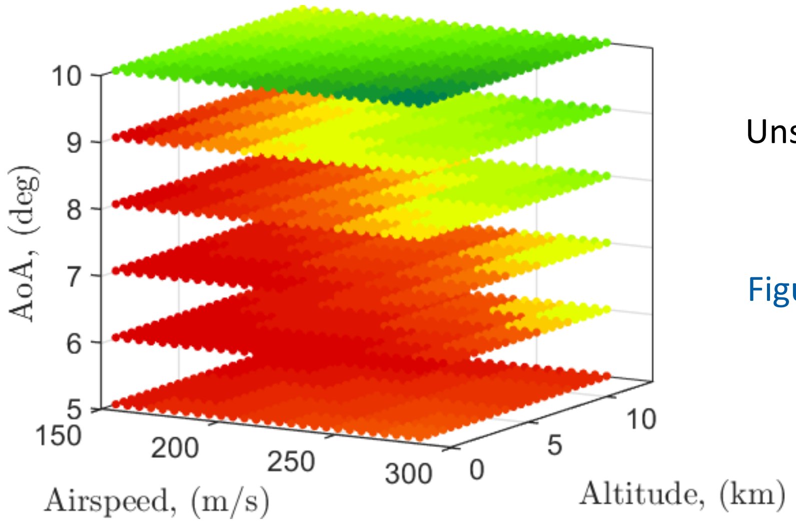


Figure 4. Pole-zero maps: (a) stable flight conditions; (b) stable/unstable flight conditions.

Observations

- System dynamics more unstable for :
  - Low AoA (stable $\forall\ \alpha \geq 10$ deg);
  - Low Airspeed;
  - Low Altitude.
- Stall conditions for $\alpha \geq 25$ deg.



Projectile Stability Map

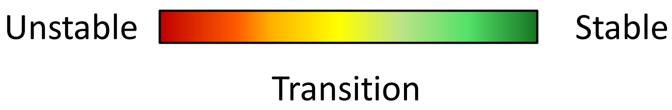


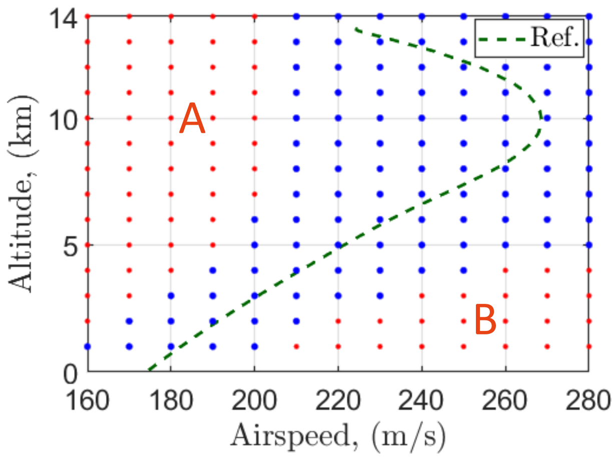
Figure 5. Stability analysis results: 3D stability grid.

Objectives

- Baseline gliding phase Altitude/Airspeed trajectory relation;
- Identify the feasible areas of the flight envelope, based on the variation of the initial firing conditions;
- Select the design grid points inside those areas.

Observations

- Neglect the unfeasible areas from the design:
  - A: High h - low V;
  - B: Low h - high V.
- Improve the accuracy of the grid points selection.



• Unfeasible
• Feasible

Figure 6. Reference h/V gliding trajectory constraints.

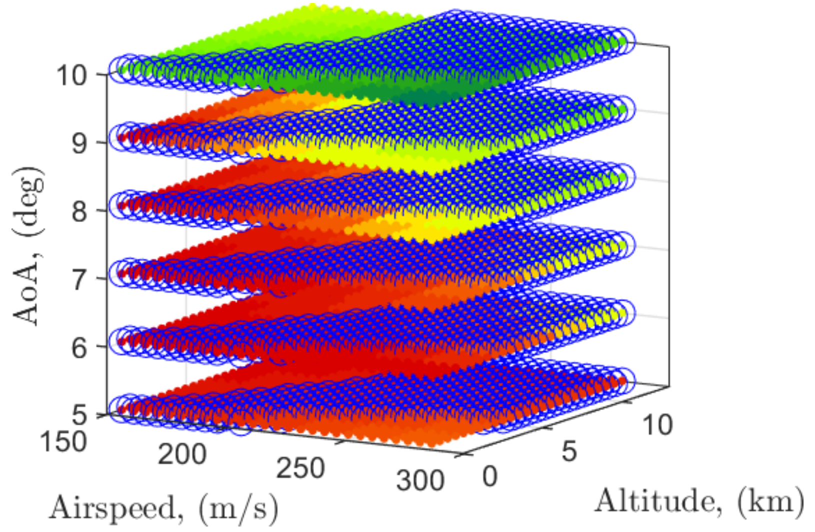


Figure 7. Analysis results: stability/reachable flight areas superposition.

Grid Dimension & Basis Selection

Objectives

- Assess the relevance of the n_g selection for each individual parameter range;
- Identify the best basis functions $(f_i,\ g_i)$ for each parameter variation: trade-off complexity/performance;
- Iterative $\mathcal{H}_\infty$ design, assuming the γ_∞ as design quality index for the comparison.

Analysis

select one parameter $\rho \in (\alpha, V, h)$
$range = [\rho_{min}, \rho_{max}]$
for $i \in (3, \ldots, \max n_g)$
  $range = [range, randi(range)]$
  for $j \in$ investigated basis functions
    γ_∞ = LPV controller synthesis $(i, j, range)$
  end
end

Design Scheme

- LPV $\mathcal{H}_\infty$ mixed sensitivity design (output feedback architecture);
- Reliable tracking capability (W_e);
- Limited control effort (W_u);
- Reasonable input/output disturbance rejection (W_{d_i}, W_{d_o}).

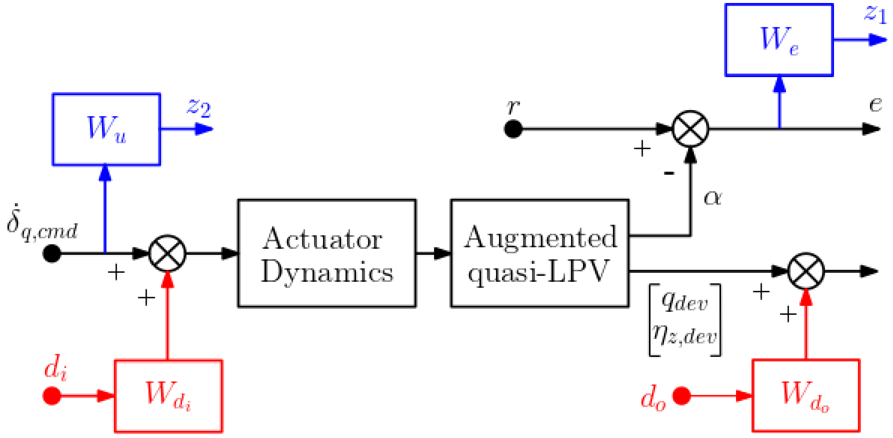


Figure 8. Mixed sensitivity design scheme.

Grid Dimension & Basis Selection

Analysis

- Basis functions selected based on the "mimic" principle:

$$\alpha \begin{cases} f_{a1} = a \\ f_{a2} = \sin(a) \\ f_{a3} = \cos(a) \end{cases} \quad V \begin{cases} f_{V1} = V \\ f_{V2} = 1/V \\ f_{V3} = V^2 \end{cases} \quad h \begin{cases} f_{h1} = h \\ f_{h2} = 1/h \\ f_{h3} = h^2 \end{cases}$$

- Considered rages of variations:

$\alpha \in [0, 15]$ deg ; $V \in [150, 280]$ m/s ; $h \in [1, 13]$ km ;

$\dot{\alpha} \in [-10, 10]$ deg/s ; $\dot{V} \in [-5, 5]$ m/s^2 ; $\dot{h} \in [-50, 50]$ m/s ;

Observations

- Linear parameter-dependence: uniform trend, lower performance;
- Nonlinear dependence: incoherent trend, average higher performance;
- Generally negligible relevance of n_g on the design performance.

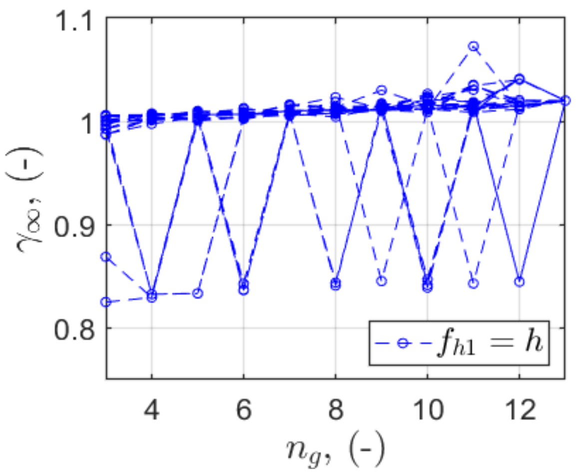


Figure 9. Grid analysis: γ_∞ index for $f_{h1} = h$.

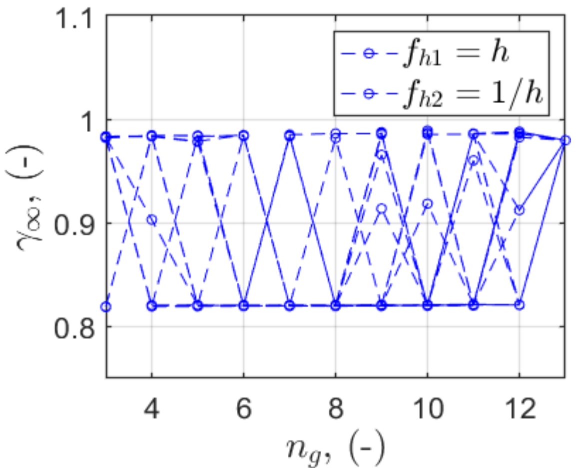


Figure 10. Grid analysis: γ_∞ index for $f_{h1} = h, f_{h2} = 1/h$.

Analysis Results

- Superpose the reachable flight points with the stability information.
- Select the flight points to cover the most critically feasible conditions.

Grid $n_g = 80$

$$\begin{cases} \alpha = [3,7,9,12]\ \text{deg} & \dot{\alpha} \in [-10,10]\ \text{deg/s} \\ V = [180,200,230,270]\ \text{m/s} & \dot{V} \in [-5,5]\ \text{m/s}^2 \\ h = [1,3,6,9,13]\ \text{km} & \dot{h} \in [-50,50]\ \text{m/s} \end{cases}$$

Lyapunov Functions

$$X(\rho) = Y(\rho) = X_0 + \sin(\alpha)X_{\alpha 1} + \cos(\alpha)X_{\alpha 2} + VX_{V1} + hX_{h1}$$

Frequency Design Results

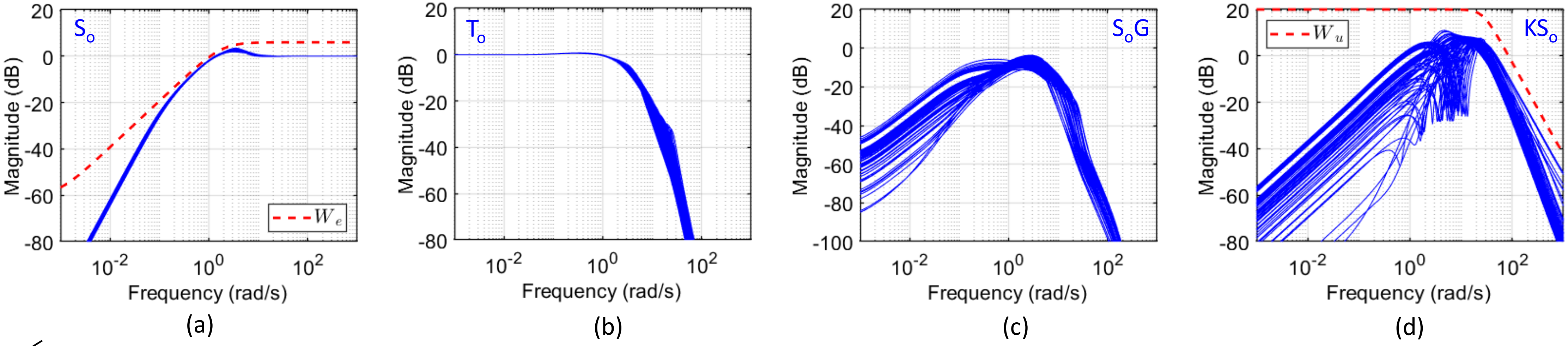


Figure 11. Design results: (a) sensitivity functions; (b) complementary sensitivity; (c) plant sensitivity; (d) controller sensitivity.

6-DoF Simulator

Simulation Objectives

- Tracking of a reference angle-of-attack guidance trajectory;
- Limit the control effort to avoid the saturation of the canards.

Simulation Environment

- Guidance: angle-of-attack reference signal based on a Lift-to-Drag optimization law [1];
- Atmosphere model: International Standard Atmosphere (ISA) 1975, ISO 2533;
- Aerodynamic model: multivariable regression model based on CFD dataset [2];
- Output deviation: measurements adjustment to comply with the State Transformation formulation.

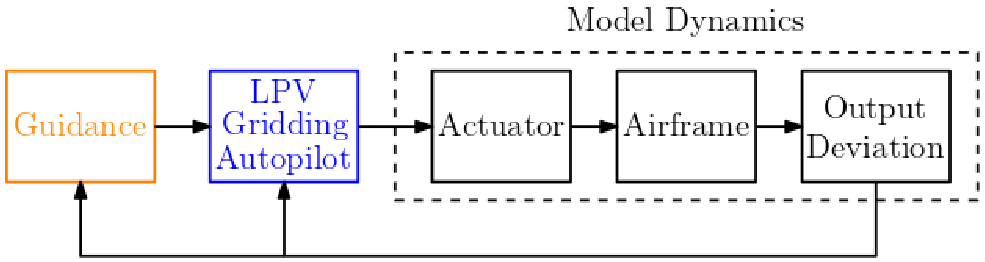


Figure 12. 6-DoF nonlinear simulator environment.

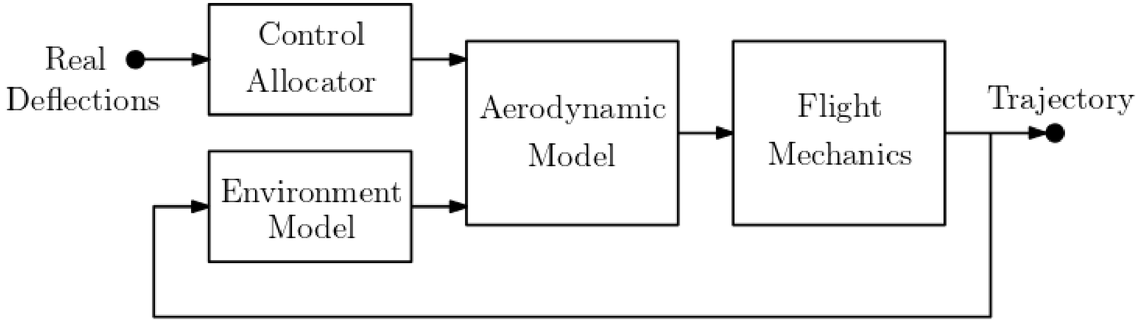


Figure 13. 6-DoF simulator: Airframe architecture.

[1] Kelley, H. J., Cliff, E. M., and Lutze, F. H. (1982). **"Boost–glide range-optimal guidance".**
[2] Vinco, G.M., Theodoulis, S., and Sename, O. (2022). "**Flight dynamics modeling and simulator design for a new class of long-range guided projectiles**".

Simulation Results

- ✓ Reliable tracking performances affected by a reasonable steady state error (< 1%);
- ✓ Total canards pitch deflection far below from the saturation limits (< 25 deg);
- ✓ Stability ensured at all the flight conditions across the investigated gliding phase trajectory.

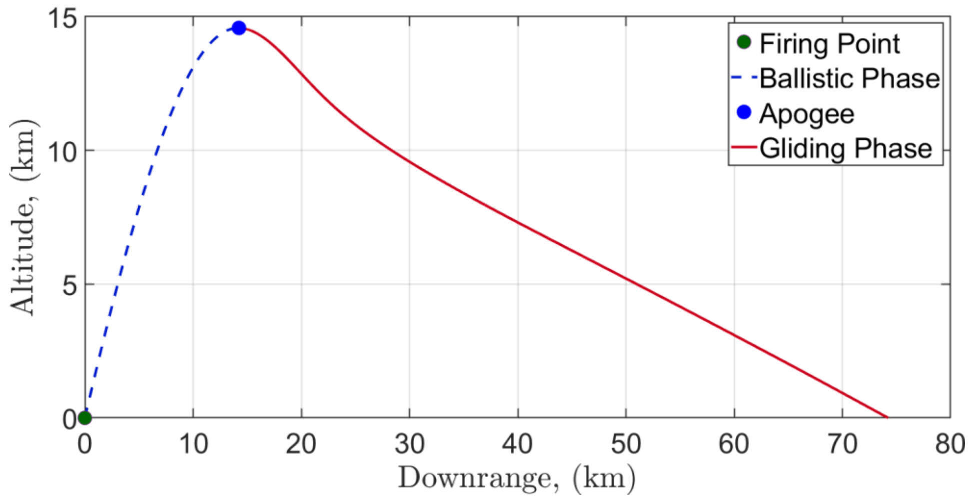


Figure 14. Simulation results: projectile gliding trajectory.

Simulation Trajectories

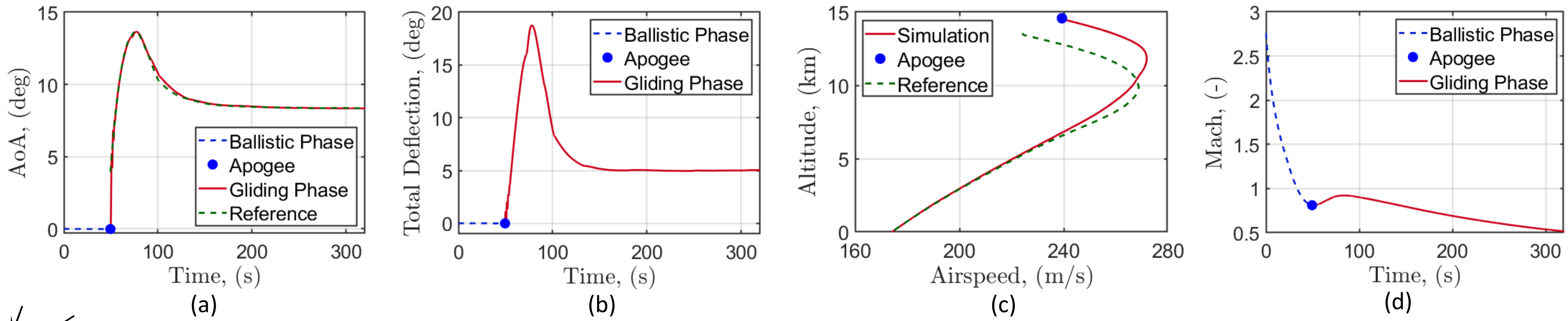


Figure 15. Simulation trajectories: (a) angle-of-attack; (b) total pitch deflection; (c) h/V relation; (d) Mach number.

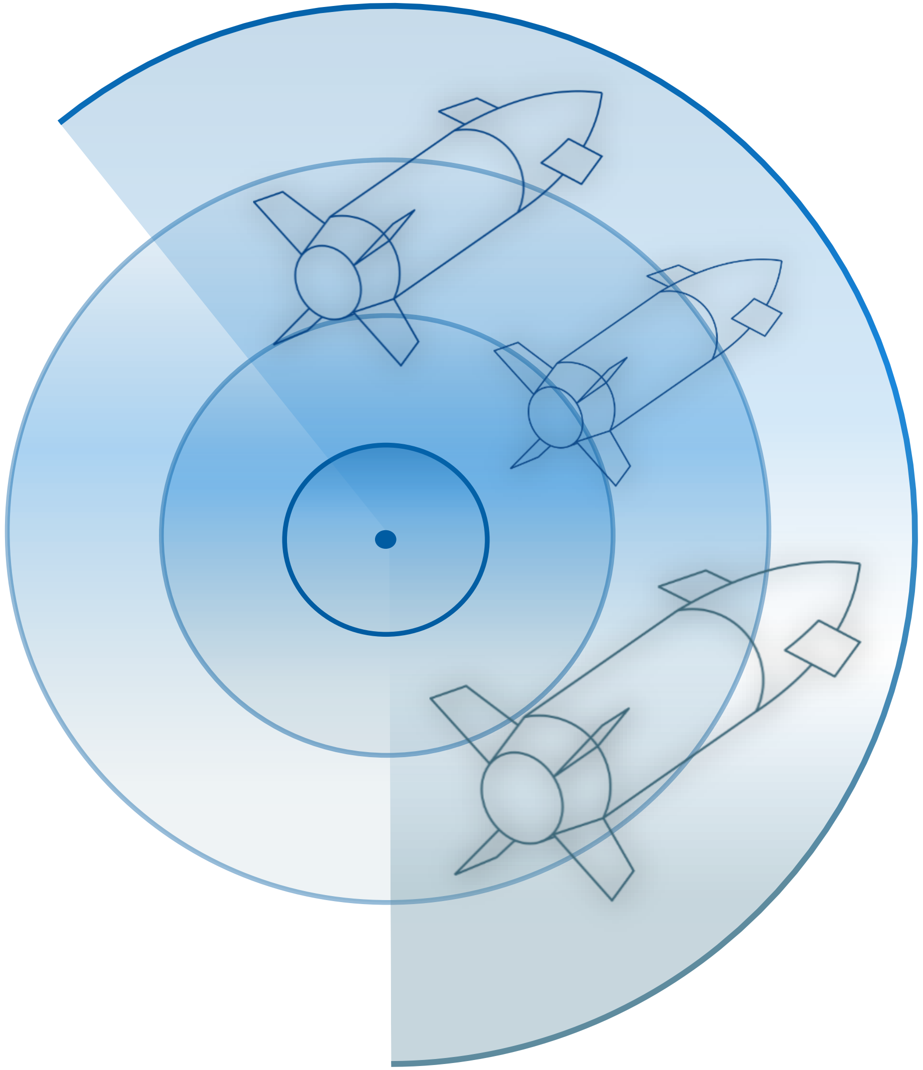
Conclusions

Achievements

- Suitable LPV grid-based model formulation of the projectile dynamics;
- Reduction of the grid-based design computational complexity;
- Reliable controller design performances.

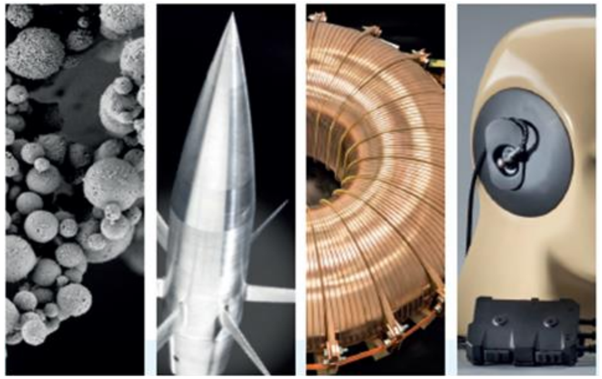
Future works

- Controller robustness assessment on a dense grid of flight conditions;
- Performance comparison with alternative LPV design methods.



Thank you for your kind attention!

Any questions ?



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