Invariance and symbolic control on monotone systems application to intelligent buildings

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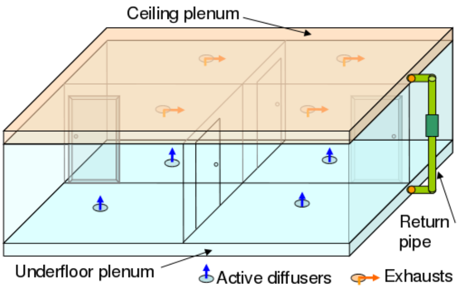
Outline

UnderFloor Air Distribution



- Underfloor air cooled down
- Sent into the rooms by fans
- Air excess pushed through the ceiling exhausts
- Returned to the underfloor

- Disturbances: heat sources; opening of doors

Model

Temperature variations in room i:

- energy conservation;
- mass conservation.

Model

Temperature variations in room i:

$$\frac{dT_i}{dt} = \sum_j a_{i,j}(T_j - T_i) \quad \text{Conduction through walls}$$
$$+ b_i u_i (T_u - T_i) \quad \text{Controlled fan air flow } u_i$$
$$+ \sum_j \delta_{d_{ij}} c_{i,j} * h(T_j - T_i) \quad \text{Open doors (flow hot}\rightarrow\text{cold)}$$
$$+ \delta_{s_i} d_i (T_{s_i}^4 - T_i^4) \quad \text{Radiation from heat sources}$$

Model

Temperature variations in room i:

$$\begin{aligned} \frac{dT_i}{dt} = \quad & \sum_j a_{i,j}(T_j - T_i) && \text{Conduction through walls} \\ & + b_i u_i (T_u - T_i) && \text{Controlled fan air flow } u_i \\ & + \sum_j \delta_{d_{ij}} c_{i,j} * h(T_j - T_i) && \text{Open doors (flow hot}\rightarrow\text{cold)} \\ & + \delta_{s_i} d_i (T_{s_i}^4 - T_i^4) && \text{Radiation from heat sources} \end{aligned}$$

- $a,\ b,\ c,\ d > 0$;
- $\delta_s,\ \delta_d$: discrete state of the disturbances (heat sources and doors);
- $\begin{cases} h(x \leq 0) = 0 \\ h(x > 0) = x^{3/2} \end{cases}$: door heat transfer only in the colder room.

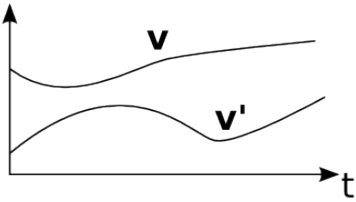
Monotonicity

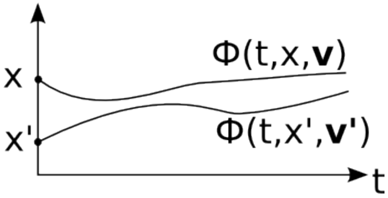
Generic system $\dot{x} = f(x, v)$ with trajectories $\Phi(t, x, \mathbf{v})$.

Definition (Monotonicity)

The system Φ is monotone if its trajectories preserve some partial orders:

$$\mathbf{v} \succeq_v \mathbf{v}', \; x \succeq_x x' \; \Rightarrow \; \forall t \geq 0, \; \Phi(t, x, \mathbf{v}) \succeq_x \Phi(t, x', \mathbf{v}')$$





Monotonicity

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Definition (Partial order)

$$x \succeq_x x' \;\Leftrightarrow\; \forall i,\; (-1)^{\varepsilon_i}(x_i - x'_i) \geq 0, \qquad \text{with } \varepsilon_i \in \{0, 1\}$$

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Proposition (Angeli and Sontag, 2003)

The system defined by $\dot{x} = f(x, v)$ is monotone if and only if,

$$\forall x \in \mathbb{R}^n,\; \forall v \in \mathbb{R}^m,\; \begin{cases} (-1)^{\varepsilon_i + \varepsilon_j} \dfrac{\partial f_i}{\partial x_j}(x, v) \geq 0, & \forall i,\; \forall j \neq i, \\ (-1)^{\varepsilon_i + \gamma_k} \dfrac{\partial f_i}{\partial v_k}(x, v) \geq 0, & \forall i,\; \forall k. \end{cases}$$

Where $\varepsilon \in \{0, 1\}^n$ and $\gamma \in \{0, 1\}^m$ define the partial orders for x and v.

Monotonicity

Our model: $\dot{T} = f(T, u, w, \delta)$

- T: state (temperature);
- u: controlled input (fan air flow);
- w: exogenous input (other temperatures);
- δ: discrete disturbance embedded in a continuous space.

Monotonicity

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$$\begin{aligned} T \succeq_T T' &\Leftrightarrow \quad \forall i,\ T_i \geq T'_i \\ \mathbf{u} \succeq_u \mathbf{u}' &\Leftrightarrow \forall t \geq 0,\ \forall k,\ \mathbf{u_k}(t) \leq \mathbf{u'_k}(t) \\ \mathbf{w} \succeq_w \mathbf{w}' &\Leftrightarrow \forall t \geq 0,\ \forall k,\ \mathbf{w_k}(t) \geq \mathbf{w'_k}(t) \\ \boldsymbol{\delta} \succeq_\delta \boldsymbol{\delta}' &\Leftrightarrow \forall t \geq 0,\ \forall k,\ \boldsymbol{\delta}_\mathbf{k}(t) \geq \boldsymbol{\delta}'_\mathbf{k}(t) \end{aligned}$$

$$\Phi(t, T, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}) \succeq_T \Phi(t, T', \mathbf{u}', \mathbf{w}', \boldsymbol{\delta}')$$

Outline

Robust Invariance

Definition (Robust Invariance)

The system is *Robust Invariant* in an interval $[\underline{T_r}, \overline{T_r}]$ if,

$$\forall T_0 \in [\underline{T_r}, \overline{T_r}],\ \forall \mathbf{w} \in [\underline{w}, \overline{w}],\ \forall \boldsymbol{\delta} \in [\underline{\delta}, \overline{\delta}],\ \forall \mathbf{u} \in [\underline{u}, \overline{u}],$$
$$\forall t \geq 0,\ \Phi(t, T_0, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}) \in [\underline{T_r}, \overline{T_r}].$$

Proposition

The minimal Robust Invariant interval $[\underline{T_r}, \overline{T_r}]$ *is given by*

$$\begin{cases} f(\overline{T_r}, \underline{u}, \overline{w}, \overline{\delta}) = 0 \\ f(\underline{T_r}, \overline{u}, \underline{w}, \underline{\delta}) = 0 \end{cases}$$

Robust Controlled Invariance

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The system is *Robust Controlled Invariant* in $[\underline{T}, \overline{T}]$ if,

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$$\exists \mathbf{u} \in [\underline{u}, \overline{u}] \mid \forall t \geq 0,\ \Phi(t, T_0, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}) \in [\underline{T}, \overline{T}].$$

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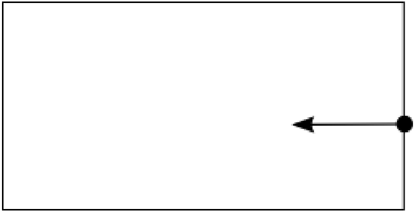

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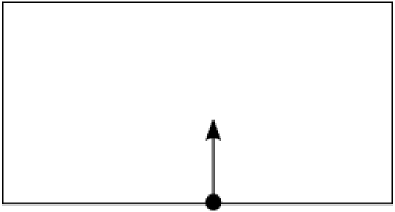

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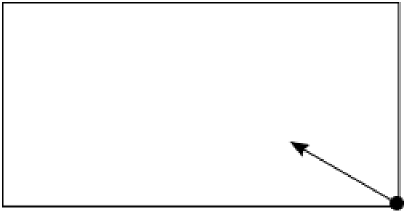

Robust Controlled Invariance

Proposition

The system is Robust Controlled Invariant in $[\underline{T}, \overline{T}]$ if and only if

$$\forall i, \quad \begin{cases} f_i(\overline{T}, \overline{u_i}, \overline{w}, \overline{\delta}) \leq 0 \\ f_i(\underline{T}, \underline{u_i}, \underline{w}, \underline{\delta}) \geq 0 \end{cases}$$

Robust Controlled Invariance

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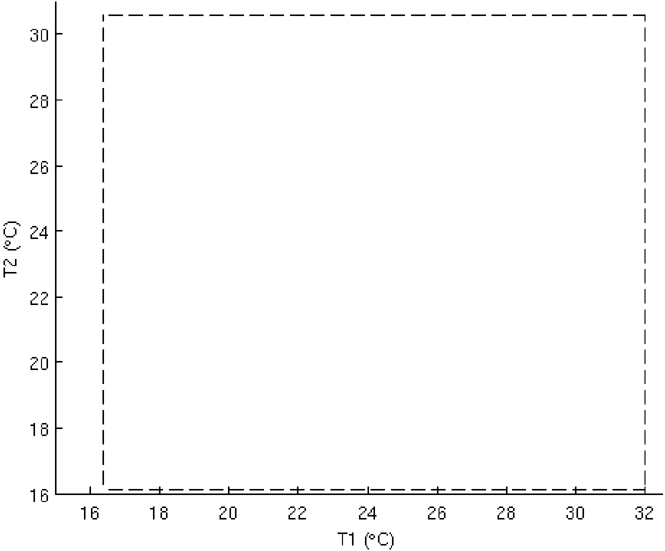
Definition (Decentralized Bang-Bang Controller)

$$\forall i, \quad \begin{cases} T_i \geq \overline{T_i} \Rightarrow u_i = \overline{u_i} \\ T_i \leq \underline{T_i} \Rightarrow u_i = \underline{u_i} \end{cases}$$

Controllable Spaces (2-room example)



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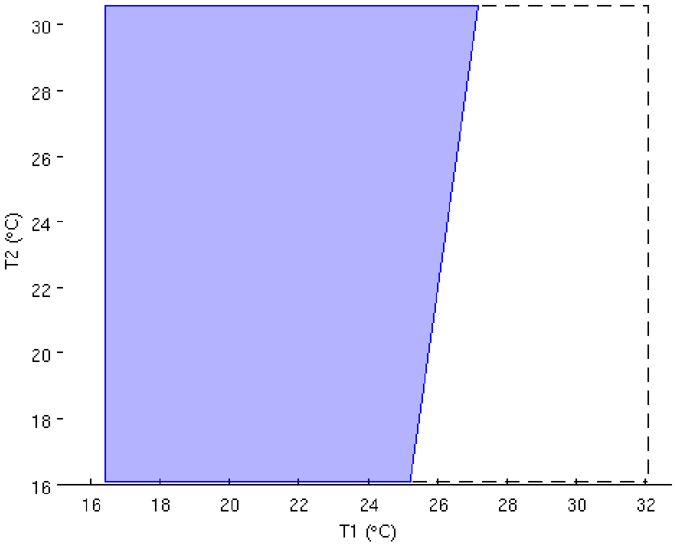

$f_1(\underline{T}, \underline{u_1}, \underline{w}, \underline{\delta}) \geq 0$

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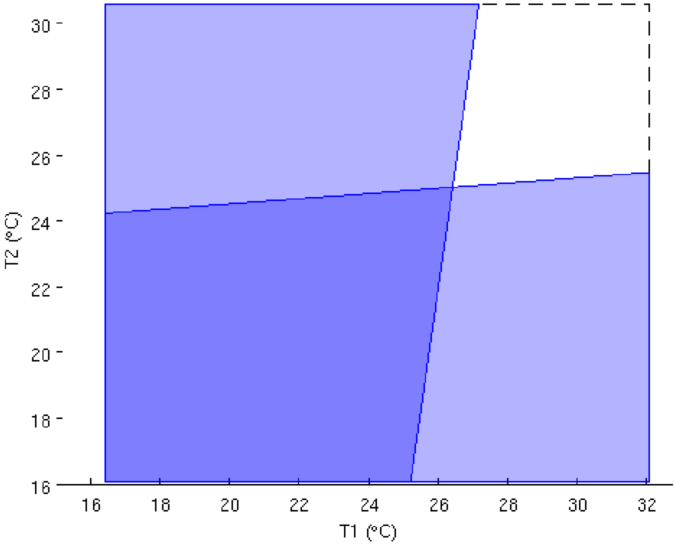

$$f_1(\underline{T}, \underline{u_1}, \underline{w}, \underline{\delta}) \geq 0$$

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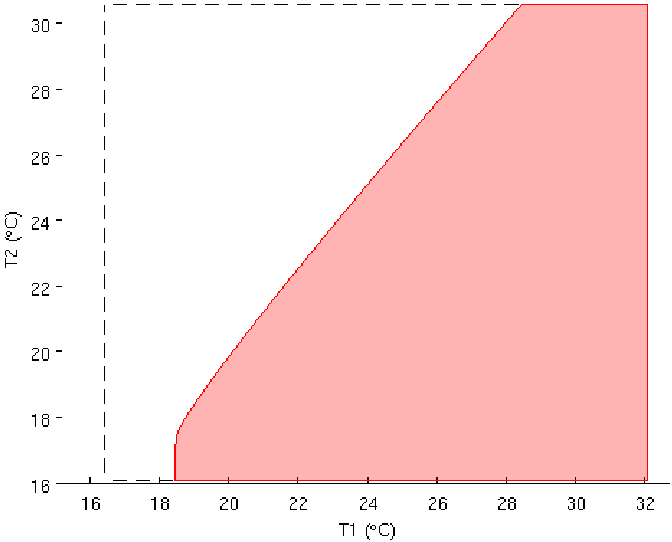

$f_1(\overline{T}, \overline{u_1}, \overline{w}, \overline{\delta}) \leq 0$

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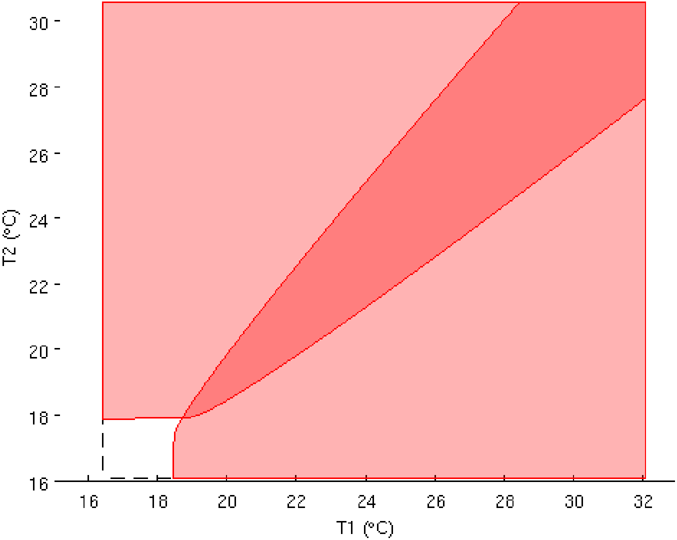

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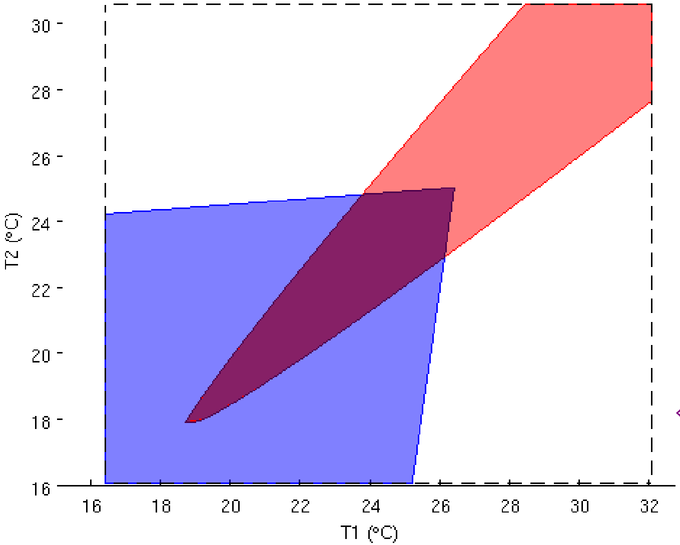

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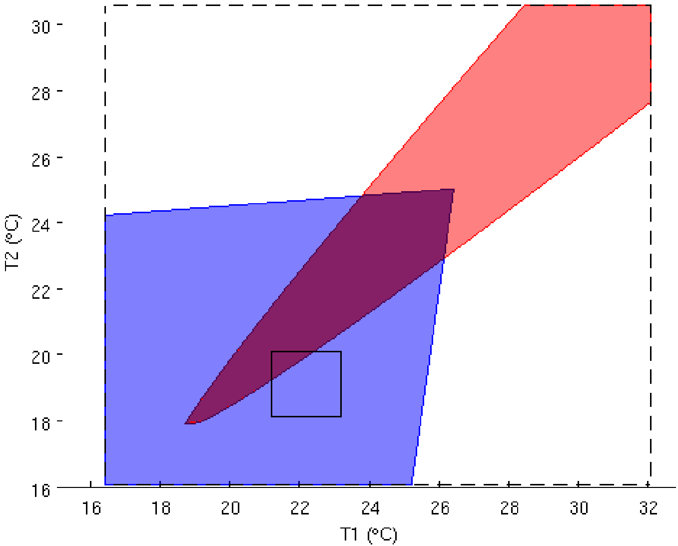

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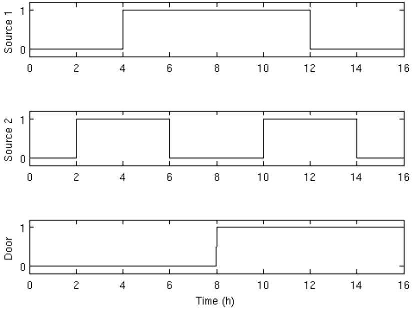
Control simulation

3 discrete disturbances:

- heat source in room 1
- heat source in room 2
- door

8 possible combinations



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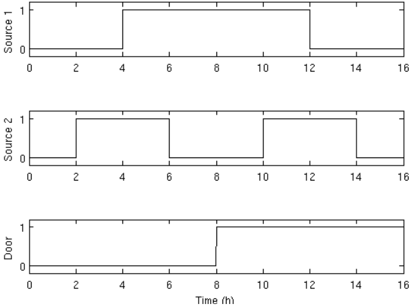
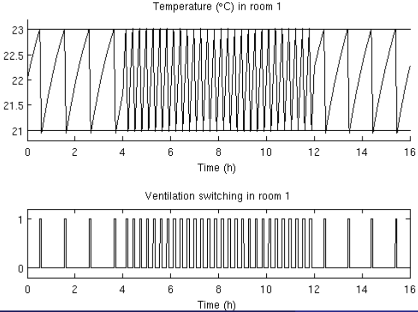
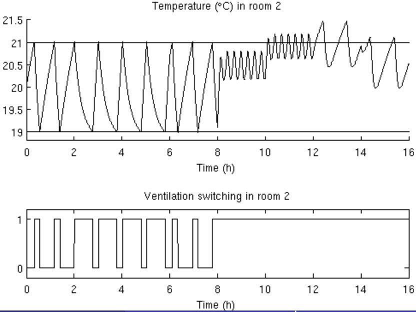






Conclusion

Criterion for *Robust Controlled Invariance*

- for a class of monotone systems,
- with local control,
- and bounded disturbances.

- Independent of the feedback control strategy.

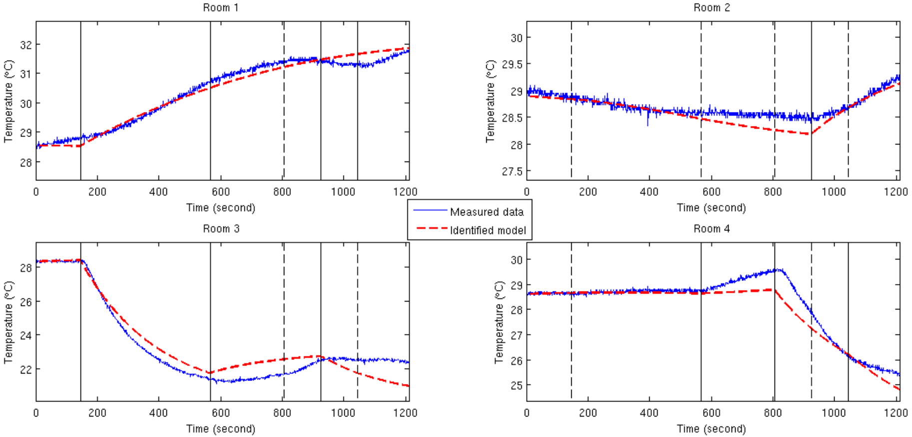
Outline

Experimental building

- $\approx 1m^3$
- 3 Peltier coolers
- Heat sources: lamps
- CompactRIO
- LabVIEW

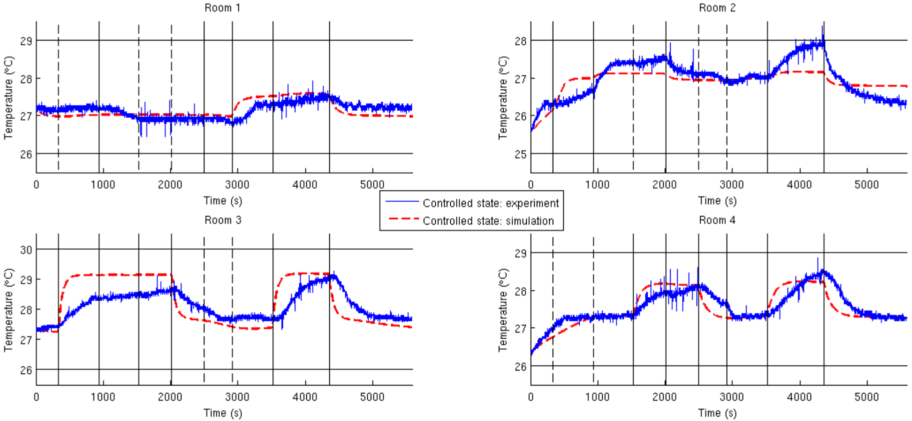
Identification

- Identification (least-squares) over 57079 data points ($\approx 16h$)
- Evaluation on another scenario:



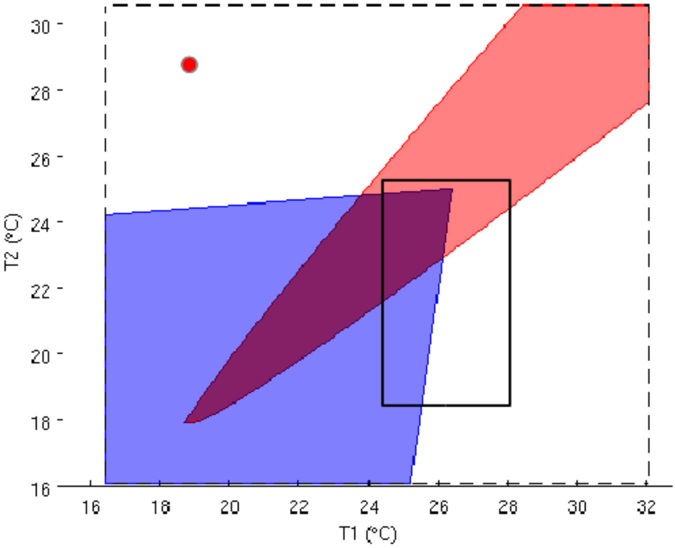
Control

- Linear saturated controller
- Interval satisfying the *Robust Controlled Invariance*:



Outline

Stabilization



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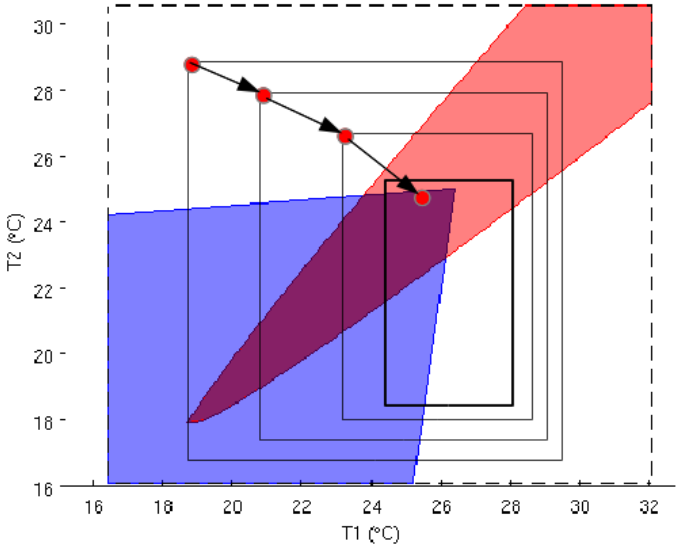

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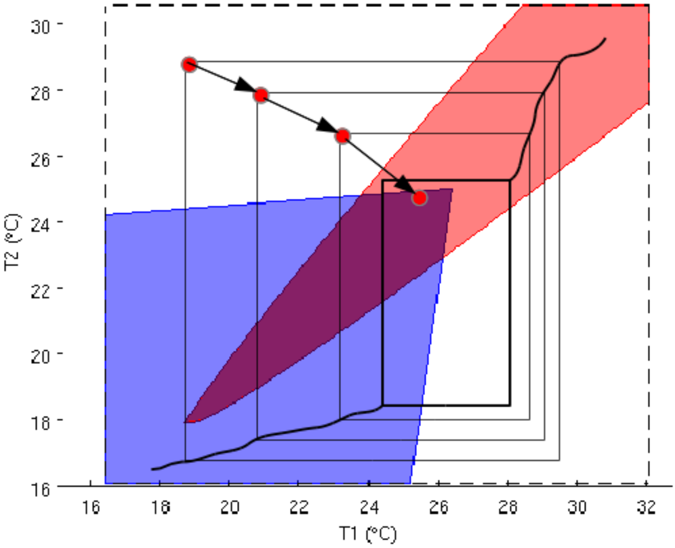

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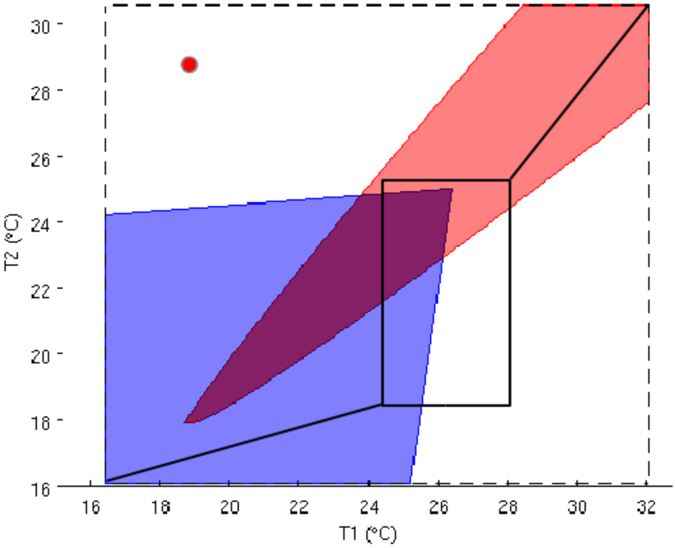

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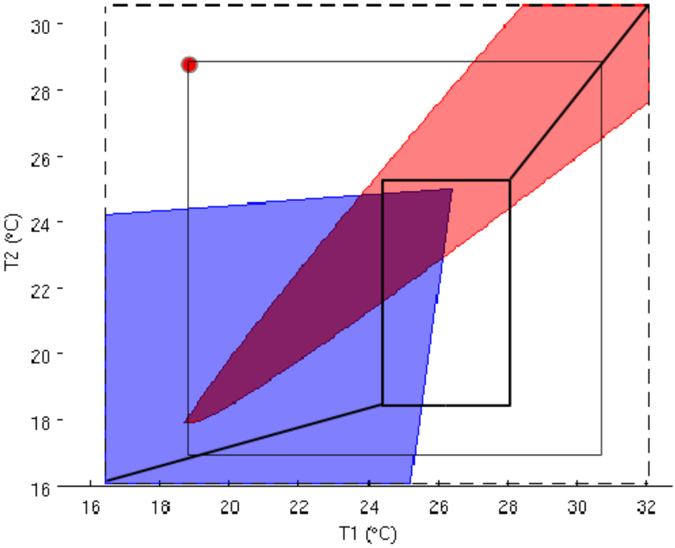

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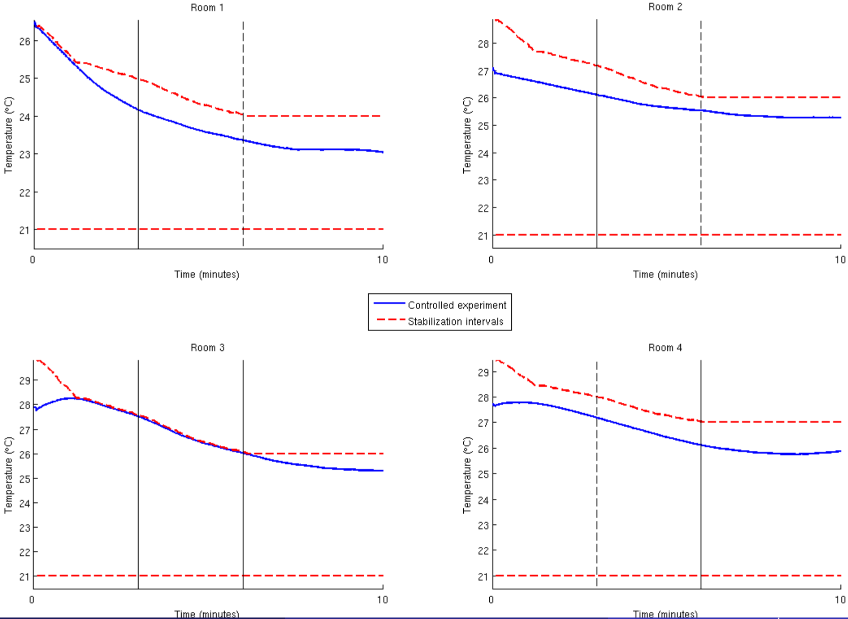
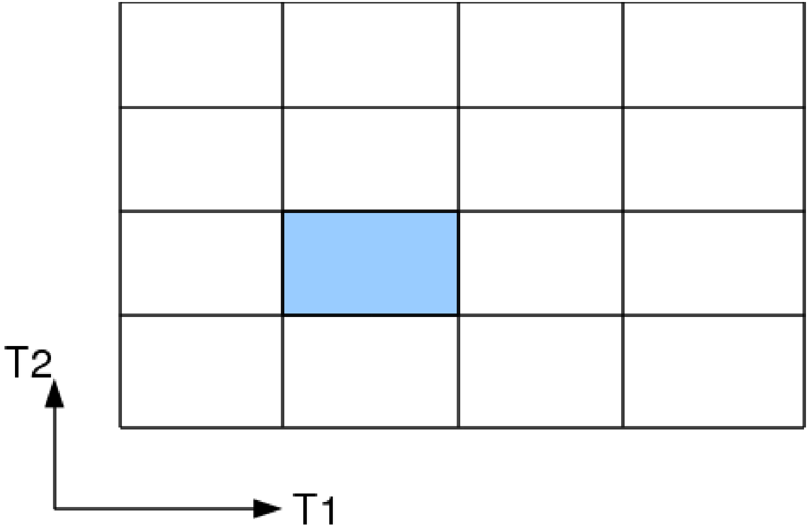


Outline

Symbolic model

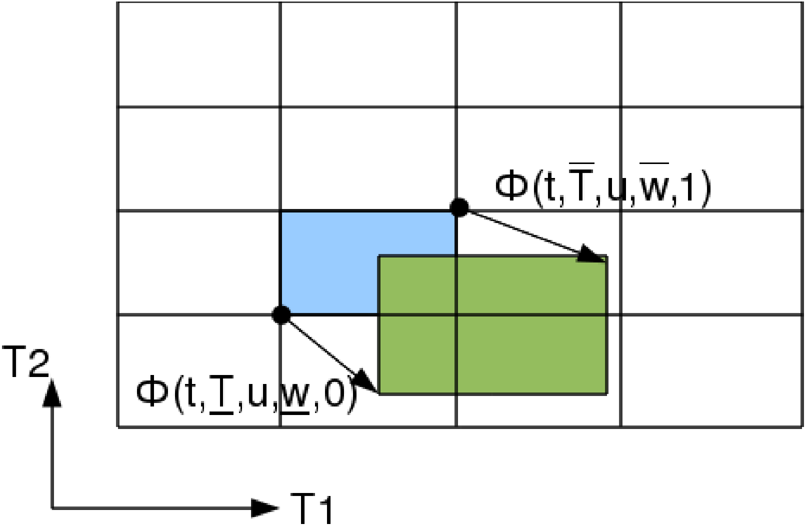
Discretization of the state space
Symbols: sets defined by the grid



Symbolic model

Next state of the symbol, given *u*: over-approximation

Automaton: intersection between over-approximation and symbols



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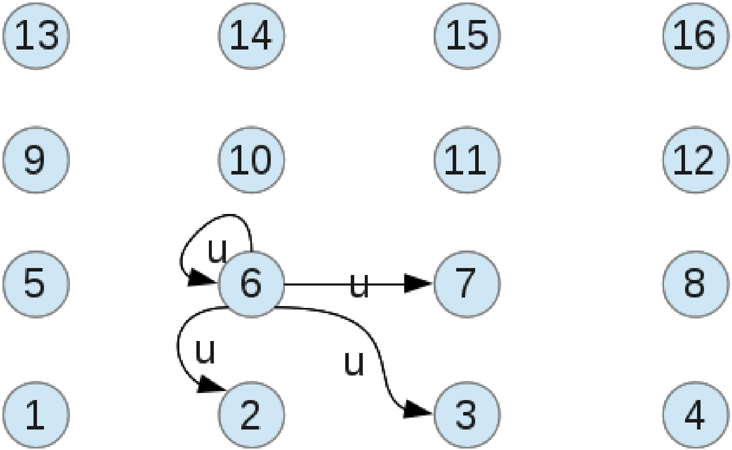
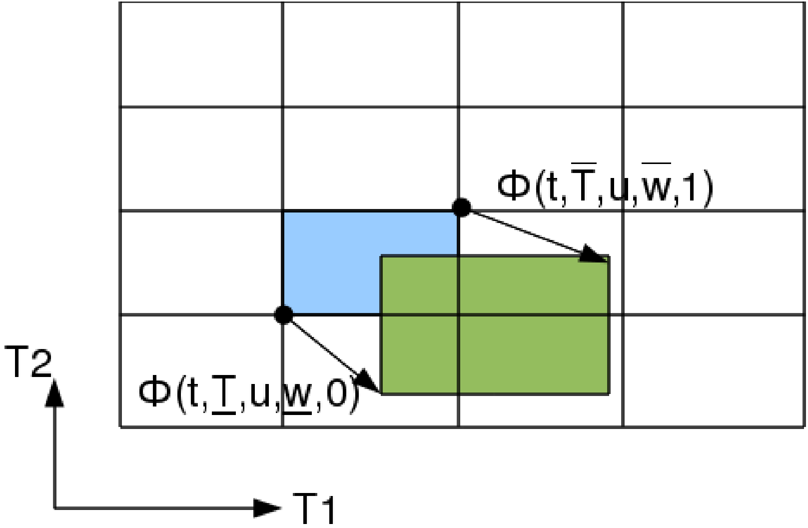


17^{th} state "outside": unsafe

Symbolic model

Increased memory [Moor and Raisch 2002]

More accuracy, bigger complexity



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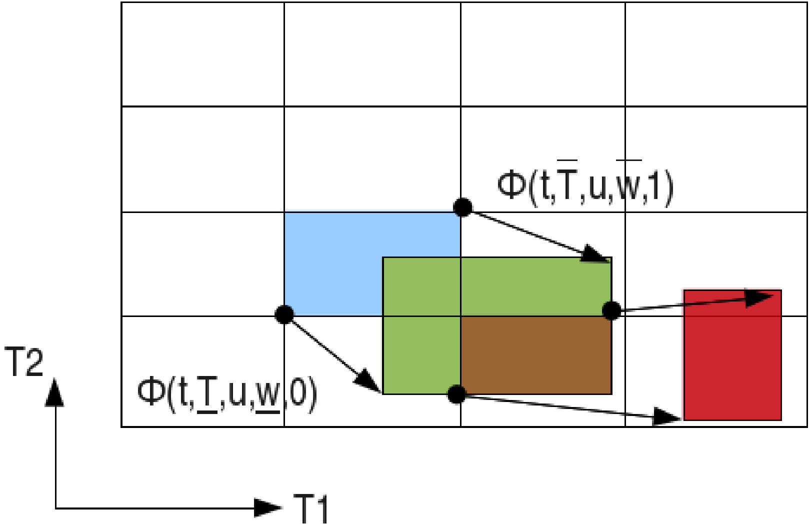
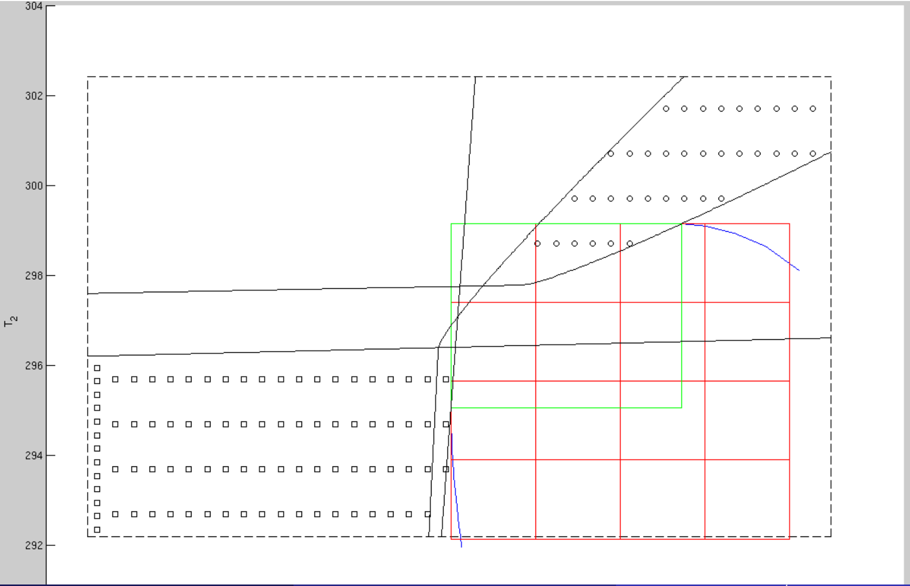


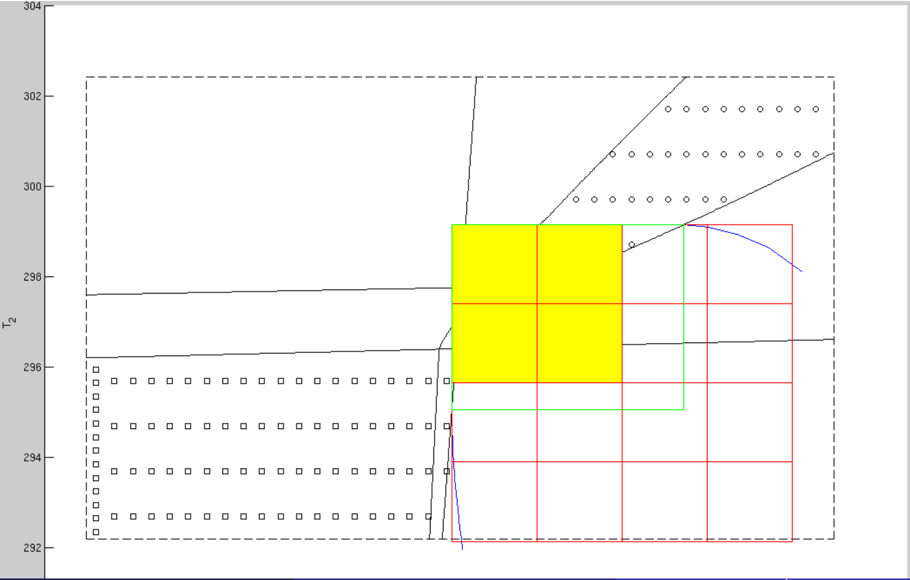
Symbolic control

Choice of the control interval: not RCI



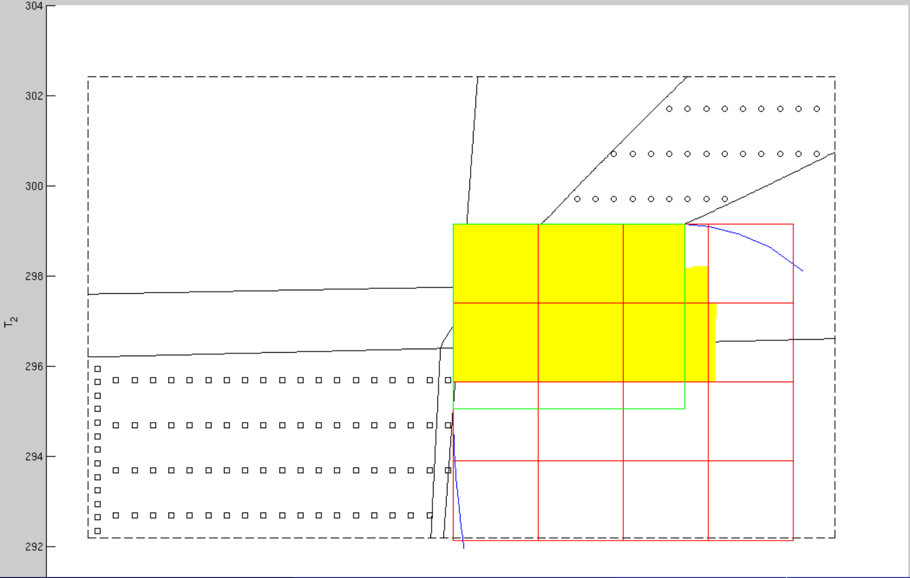
Symbolic control

Memory span 1



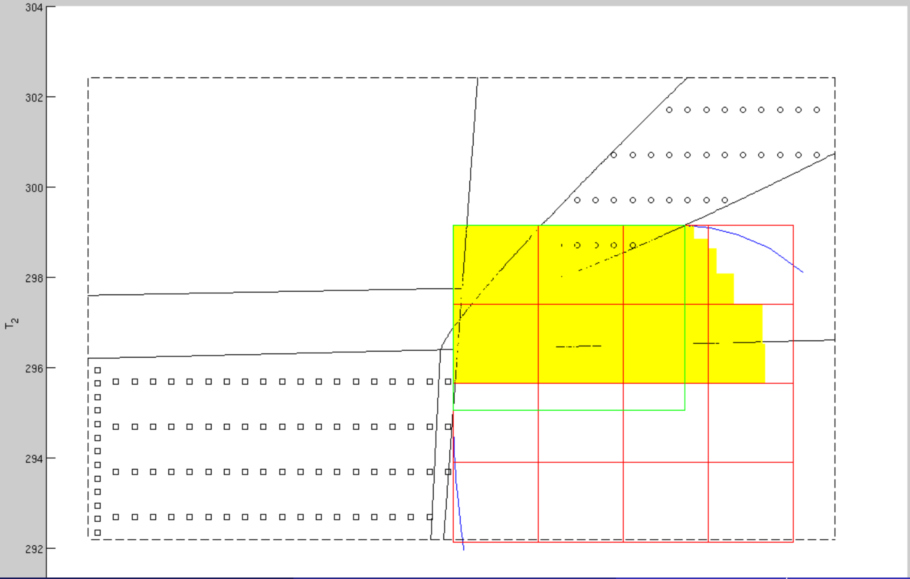
Symbolic control

Memory span 2



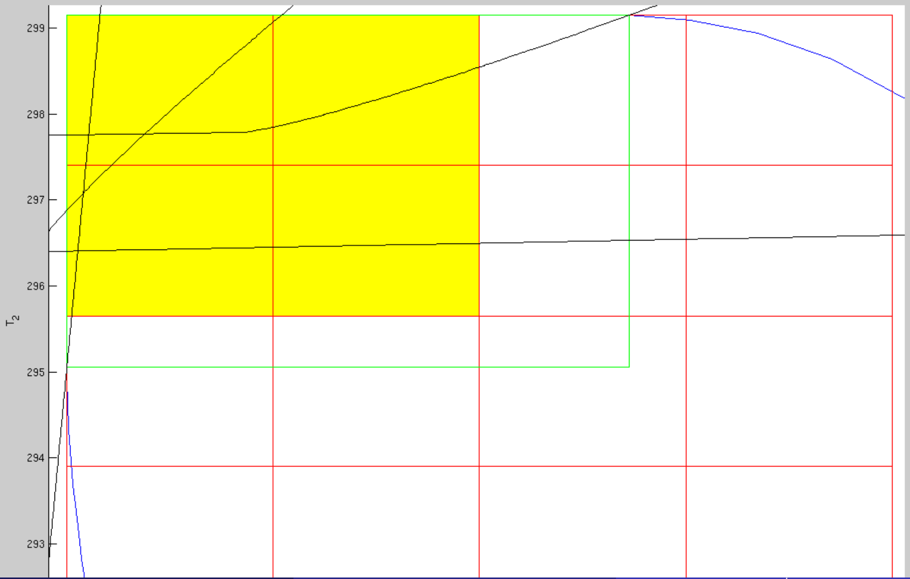
Symbolic control

Memory span 3



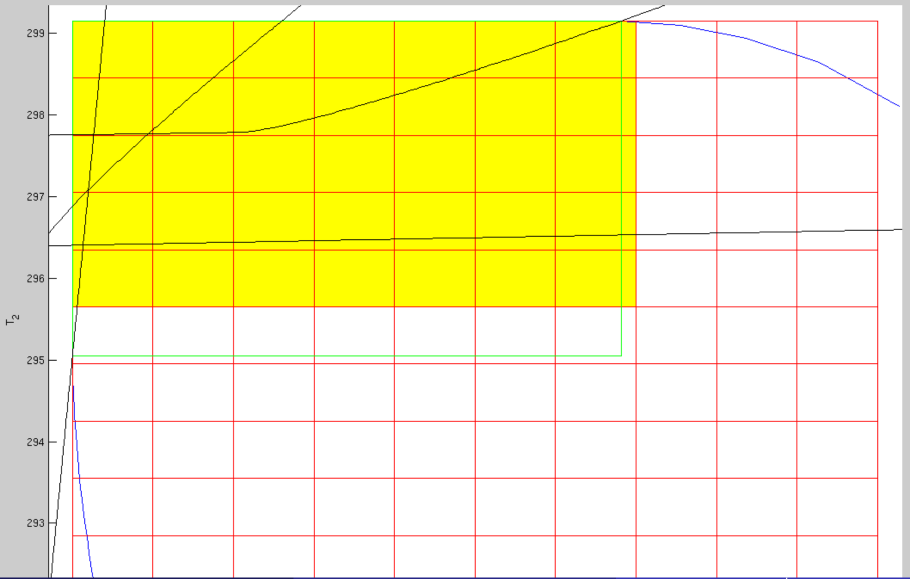
Symbolic control

No memory, 4×4 symbols



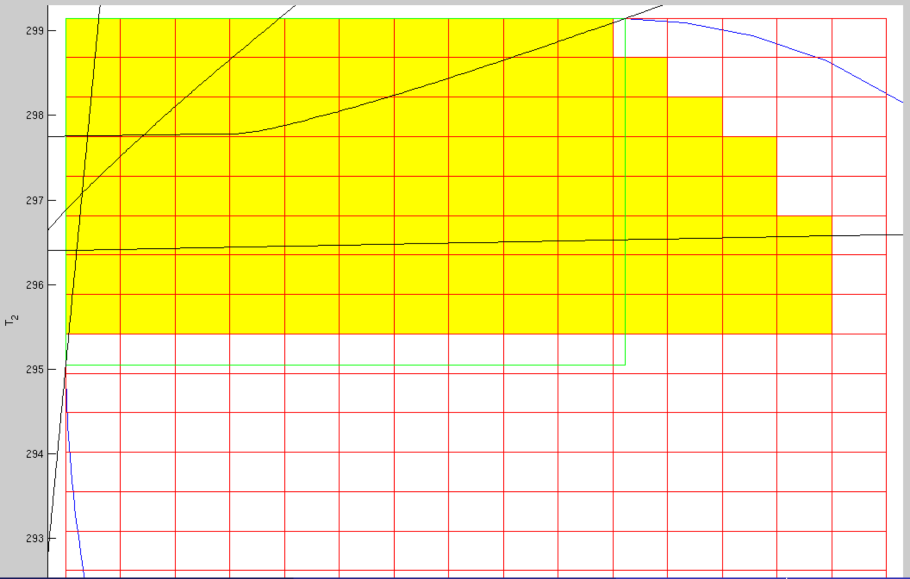
Symbolic control

No memory, 10×10 symbols



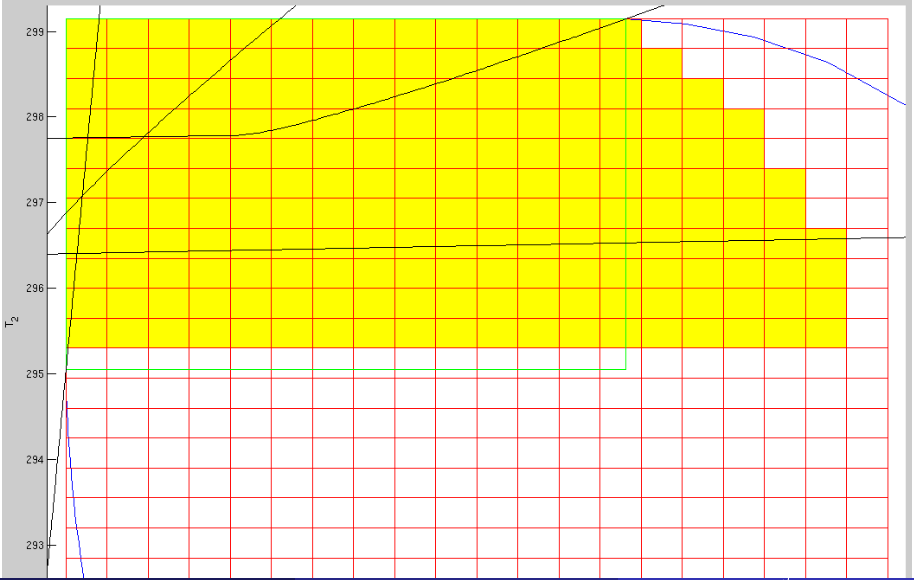
Symbolic control

No memory, 15×15 symbols



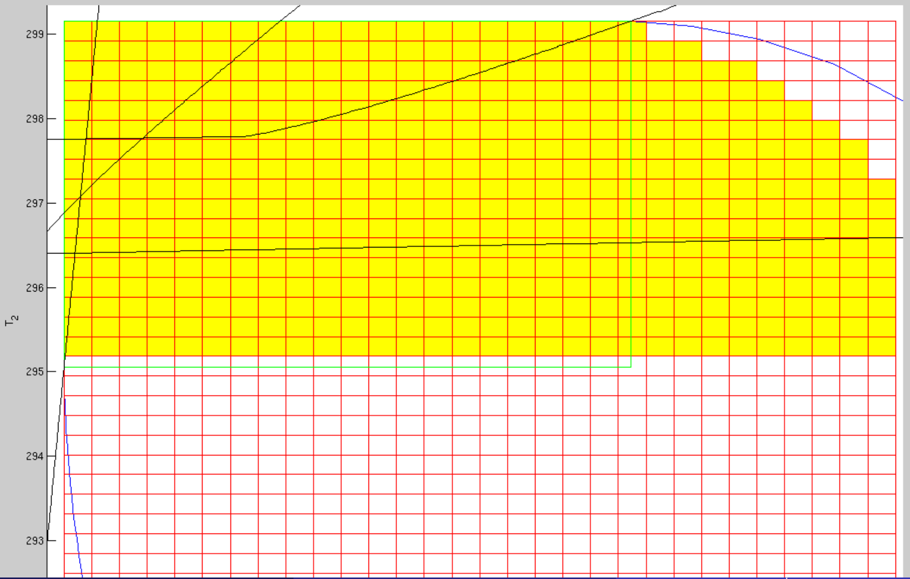
Symbolic control

No memory, 20 × 20 symbols



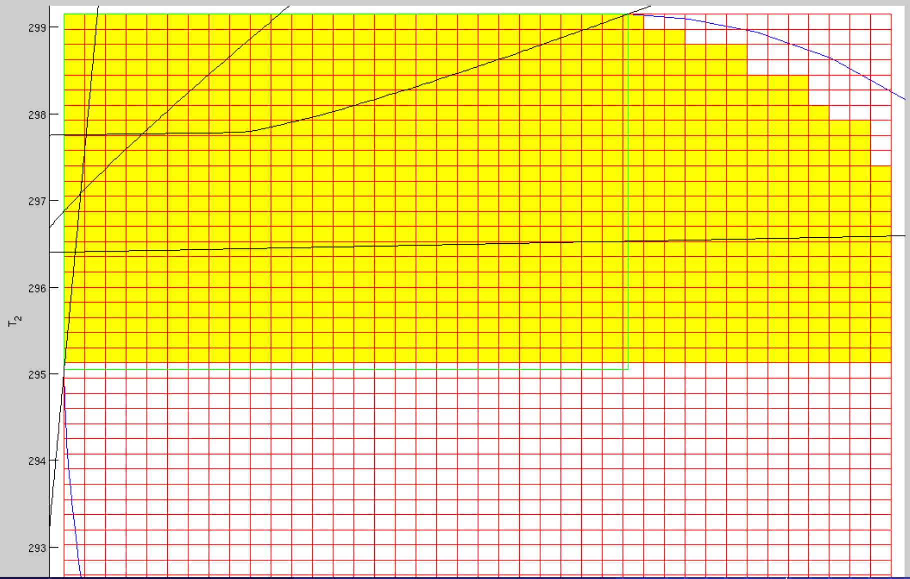
Symbolic control

No memory, 30×30 symbols



Symbolic control

No memory, 40×40 symbols



Conclusion and perspectives

Method

- Discretize the state space
- Generate the automaton for a chosen memory span
- Remove unsafe states to obtain the safe automaton

- Bigger complexity by increasing the memory than the discretization

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Perspectives

- Improve efficiency of the algorithm
- Separate controllers for each disturbance condition
- Safe automaton non-deterministic
  - controller: optimization over several future steps

Invariance and symbolic control on monotone systems application to intelligent buildings

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