Cascade observer design for a class of uncertain nonlinear systems with delayed outputs [a]

(*Automatica, Vol. 89, March 2018, pp. 125-134*)

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[a] Jeudi 15 novembre 2018

Problem formulation

- Class of systems diffeomorphic to

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) + B\varepsilon(t) \\ y_d(t) = Cx(t-d) = x^{(1)}(t-d) \end{cases}$$

- $x = \begin{pmatrix} x^{(1)} \\ \vdots \\ x^{(q-1)} \\ x^{(q)} \end{pmatrix} \in \mathbb{R}^n,\ x^{(i)} \in \mathbb{R}^p,\ i = 1, \ldots, q,$

- $A = \begin{pmatrix} 0_{(q-1)p,p} & I_{(q-1)p} \\ 0_p & 0_{p,(q-1)p} \end{pmatrix},$

- $B = \begin{pmatrix} 0_p & 0_p & \ldots & I_p \end{pmatrix}^T,\ C = \begin{pmatrix} I_p & 0_p & \ldots & 0_p \end{pmatrix},$

- $\varphi(u,x) = \begin{pmatrix} \varphi^{(1)}(u, x^{(1)}) \\ \varphi^{(2)}(u, x^{(1)}, x^{(2)}) \\ \vdots \\ \varphi^{(q-1)}(u, x^{(1)}, \ldots, x^{(q-1)}) \\ \varphi^{(q)}(u, x) \end{pmatrix}$ triangular nonlinearity,
- The input $u \in U$ a compact subset of $\mathbb{R}^m$ and the delayed output $y_d \in \mathbb{R}^p$,
- $d > 0$ is the constant (known) measurement delay,
- $\varepsilon : [-d, +\infty[\; \mapsto \; \mathbb{R}^p$ the system uncertainties.

Objective

• To design a cascade observer providing an estimation of the actual state by using the delayed output

• Two main obstacles have to be handled simultaneously

1. The presence of a time delay in the output measurements,

2. The presence of the uncertainties in the state equations.

• A third obstacle will also be considered when the outputs are available only at (not equally spaced) sampling instants.

Assumptions

- The state $x(t)$ and the control $u(t)$ are bounded, i.e. $x(t) \in X$ and $u(t) \in U$ for all $t \geq 0$ where $X \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^s$ are compact sets.

- The function φ is Lipschitz with respect to x uniformly in u, i.e.

$$\forall \rho > 0;\ \exists L_\varphi > 0;\ \forall u\ s.t.\ \|u\| \leq \rho; \forall (x, \bar{x}) \in X \times X,$$
$$\|\varphi^{(i)}(u, x) - \varphi^{(i)}(u, \bar{x})\| \leq L_\varphi \|x - \bar{x}\|.$$

- The unknown function ε is essentially bounded, i.e.

$$\exists \delta_\varepsilon > 0\ ;\ ess \sup_{t \geq -d} \|\ \varepsilon(t)\| \leq \delta_\varepsilon.$$

Notations

- For $j = 0, \ldots, m$ and $t \geq -\frac{j}{m}d$ where m is a positive integer,

$$x_j(t) = x\left(t - d + \frac{j}{m}d\right),\ u_j(t) = u\left(t - d + \frac{j}{m}d\right),\ \varepsilon_j(t) = \varepsilon\left(t - d + \frac{j}{m}d\right),$$

The following property is to be emphasized (the rational behind the cascade structure of the observer)

$$x_j\left(t - \frac{d}{m}\right) = x_{j-1}(t) \text{ and } u_j\left(t - \frac{d}{m}\right) = u_{j-1}(t),\ j = 1, \ldots, m.$$

- $\Delta_\theta = diag\left(I_p, \frac{1}{\theta}I_p, \ldots, \frac{1}{\theta^{q-1}}I_p\right)$, $\theta > 0$ a positif real.

Cascade observer equations

$$\left\{\begin{array}{rcl} \dot{\hat{x}}_j(t) & = & A\hat{x}_j(t) + \varphi(u_j(t), \hat{x}_j(t)) - G_j(t),\ \ j = 0, \ldots, m \\ G_0(t) & = & \theta\Delta_\theta^{-1} KC(\hat{x}_0(t) - y_d(t)) \text{ and for } j = 1, \ldots, m, \\ G_j(t) & = & e^{\bar{A}\frac{d}{m}} \left(G_{j-1}(t) + \left(A - \bar{A}\right)\left(\hat{x}_j\left(t - \frac{d}{m}\right) - \hat{x}_{j-1}(t)\right)\right. \\ & & \left. + \varphi\left(u_{j-1}(t), \hat{x}_j\left(t - \frac{d}{m}\right)\right) - \varphi(u_{j-1}(t), \hat{x}_{j-1}(t))\right), \end{array}\right.$$

• $K = \begin{pmatrix} k_1 I_p \\ \vdots \\ k_q I_p \end{pmatrix}$, $k_i > 0$, $i = 1, \ldots, q$, $s.t.$ $\tilde{A} \triangleq A - KC$ is Hurwitz, i.e. there exist a positive constant ν and a SDP matrix P such that

$$\tilde{A}^T P + P\tilde{A} \leq -2\nu I_n.$$

- $\bar{A}$, $n \times n$ Hurwitz matrix,

- $\bar{A}$, K and θ are the observer design parameters,

- Observer initialization

$$\hat{x}_0(0) = \hat{x}(-d) \; and \; \hat{x}_j(s) = \hat{x}(s - d + \frac{j}{m}d), \;\; s \in [-\frac{j}{m}d, 0], \;\; j = 1, \ldots, m.$$

$\hat{x}(s)$, $s \in [-d, 0]$, any *a priori* selected estimate of the state vector.

Some remarks

- The cascade observer is composed by $m+1$ chained subsystems

1. The first subsystem is a high gain observer for the delayed state $x(t-d)$
2. Each one of the m remaining subsystems predicts the state of the preceding subsystem over an horizon of $\frac{d}{m} \Longrightarrow$ the state of the $m'th$ predictor is an estimate of the system actual state.
3. The rational behind the cascade observer design is based upon the following properties

$$x_j\left(t-\frac{d}{m}\right) = x_{j-1}(t) \text{ and } u_j\left(t-\frac{d}{m}\right) = u_{j-1}(t),\ j=1,\ldots,m.$$

• Observation error, $\tilde{x}_j(t) \stackrel{\Delta}{=} \hat{x}_j(t) - x_j(t)$, related to the predictor at rank $1 \leq j \leq m$,

$$\begin{aligned} \dot{x}_j(t) &= Ax_j(t) + \varphi(u_j(t), x_j(t)) + B\varepsilon_j(t) \\ &= \bar{A}x_j(t) + \varphi(u_j(t), x_j(t)) + (A - \bar{A})x_j(t) + B\varepsilon_j(t), \end{aligned}$$

$\bar{A}$ a design matrix parameter, to be chosen Hurwitz.

Hence,

$$x_j(t) = e^{\bar{A}\frac{d}{m}} x_{j-1}(t) + \int_{t-\frac{d}{m}}^{t} e^{\bar{A}(t-s)} \big(\varphi(u_j(s), x_j(s)) + (A - \bar{A})x_j(s) + B\varepsilon_j(s)\big)\, ds,$$

since $x_j(t - \dfrac{d}{m}) = x_{j-1}(t)$.

The state of the predictor $\hat{x}_j$

$$\begin{aligned}\dot{\hat{x}}_j(t) &= A\hat{x}_j(t) + \varphi(u_j(t), \hat{x}_j(t)) - G_j(t), \\ &= \bar{A}\hat{x}_j(t) + \varphi(u_j(t), \hat{x}_j(t)) + (A - \bar{A})\hat{x}_j(t) - G_j(t).\end{aligned}$$

Hence,

$$\hat{x}_j(t) = e^{\bar{A}\frac{d}{m}}\hat{x}_j\left(t - \frac{d}{m}\right) + e^{\bar{A}t}\int_{t-\frac{d}{m}}^{t} e^{-\bar{A}s}\left(\varphi(u_j(s), \hat{x}_j(s)) + (A - \bar{A})\hat{x}_j(s) - G_j(s)\right) ds.$$

Miming the relationship between the states $x_j(t)$ and $x_{j-1}(t)$, one imposes a similar relationship between $\hat{x}_j$ and $\hat{x}_{j-1}$, $j = 1, \ldots, m$

$$\hat{x}_j(t) = e^{\bar{A}\frac{d}{m}}\hat{x}_{j-1}(t) + r_j(t) + e^{\bar{A}t}\int_{t-\frac{d}{m}}^{t} e^{-\bar{A}s}\left(\varphi(u_j(s), \hat{x}_j(s)) + (A - \bar{A})\hat{x}_j(s)\right) ds,$$

the r_j's, $j = 1, \ldots, m$, vector functions, shall be determined simultaneously with the correction terms G_j's. This is achieved by equating the above two equations

$\Longrightarrow$

$$e^{\bar{A}\frac{d}{m}}\left(\hat{x}_j\left(t-\frac{d}{m}\right)-\hat{x}_{j-1}(t)\right)-r_j(t)=e^{\bar{A}t}\int_{t-\frac{d}{m}}^{t}e^{-\bar{A}s}G_j(s)ds.$$

Differentiating with respect to time each side of the above equation

$$\begin{aligned} G_j(t) &= e^{\bar{A}\frac{d}{m}}\left(G_{j-1}(t)+(A-\bar{A})\left(\hat{x}_j\left(t-\frac{d}{m}\right)-\hat{x}_{j-1}(t)\right)\right. \\ &\quad \left.+\varphi(u_{j-1},\hat{x}_j\left(t-\frac{d}{m}\right))-\varphi(u_{j-1},\hat{x}_{j-1}(t))\right)-\left(\dot{r}_j(t)-\bar{A}r_j(t)\right). \end{aligned}$$

Hence, if one chooses r_j such that

$$\dot{r}_j(t)=\bar{A}r_j(t),$$

then, the expression of the correction term G_j specializes

$$\begin{aligned} G_j(t) &= e^{\bar{A}\frac{d}{m}}\left(G_{j-1}(t)+\left(A-\bar{A}\right)\left(\hat{x}_j\left(t-\frac{d}{m}\right)-\hat{x}_{j-1}(t)\right)\right. \\ &\quad \left.+\,\varphi\left(u_{j-1}(t),\hat{x}_j\left(t-\frac{d}{m}\right)\right)-\varphi(u_{j-1}(t),\hat{x}_{j-1}(t))\right),\ 1\le j\le m \\ G_0(t) &= \theta\Delta_\theta^{-1}KC(\hat{x}_0(t)-y_d(t))\ \text{(High Gain Observer)} \end{aligned}$$

- Observation error, $\tilde{x}_0(t) \stackrel{\Delta}{=} \hat{x}_0(t) - x_0(t)$, related to the first subsystem (high gain observer)

$$\|\tilde{x}_0(t)\| \leq \mu(\theta) e^{-a_\theta t} \|\tilde{x}_0(0)\| + \frac{M}{\theta}\delta_\varepsilon,$$

  - $\mu(\theta)$, polynomial in θ,
  - $a_\theta = \dfrac{\theta\nu}{2\lambda_M(P)}$, $\quad (\tilde{A}^T P + P\tilde{A} \leq -2\nu I_n,\ \tilde{A} = A - KC)$
  - $M = 2\dfrac{\lambda_M(P)\sigma(P)}{\nu}$ with $\sigma(P) = \sqrt{\lambda_M(P)/\lambda_m(P)}$,
  - δ_ε, essential bound of the uncertainties.

- Since the matrix $\bar{A}$ is Hurwitz, there exists a positive number $\beta \geq 1$ such that

$$\forall t \geq 0 : \ \|e^{\bar{A}t}\| \leq \beta e^{-\bar{a}t},$$

$\bar{a} = \min_{i \in \{1,\ldots,n\}} \left|\Re(\lambda_i(\bar{A}))\right|$, $\lambda_i(\bar{A})$, $i = 1, \ldots, n$, the n eigenvalues of $\bar{A}$ (with negative real parts).

Theorem 1

If the matrix $\bar{A}$ is chosen such that $\bar{a} \leq a_\theta$ and if the number m is selected such that

$$\eta \frac{d}{m} < 1, \quad with \;\; \eta = \beta \left(L_\varphi + \|\bar{A} - A\| \right),$$

L_φ is the Lipschitz constant of φ, then one has for $j = 1, \ldots, m$,

$$\|\tilde{x}_j(t)\| \leq \rho_j e^{-\bar{a}t} + M_j \delta_\varepsilon, \;\; t \geq 0,$$

$$\rho_j = \frac{\eta}{1 - \eta \frac{d}{m}} \int_{-\frac{d}{m}}^{0} \|\tilde{x}_j(s)\| ds + \beta \chi_m^j \mu(\theta) \|\tilde{x}_0(0)\| + \frac{\beta}{1 - \eta \frac{d}{m}} \sum_{k=0}^{j-1} \chi_m^k \|r_{j-k}(0)\|,$$

$$M_j = \beta \chi_m^j \frac{M}{\theta} + \frac{\beta \frac{d}{m}}{1 - \eta \frac{d}{m}} \sum_{i=0}^{j-1} \chi_m^i, \qquad \chi_m = \frac{e^{-\bar{a}\frac{d}{m}}}{1 - \eta \frac{d}{m}}.$$

Remarks

- The convergence of the cascade observer is closely related to the observer dynamics of the first subsystem as well as the prediction dynamics of the remaining subsystems.

  - The delayed state observer dynamics can be appropriately assigned by the observer design parameters θ and K,

  - The prediction dominant dynamics can be tuned by the prediction design parameter $\bar{A}$ and the number of subsystems in the cascade.

- In the uncertainty-free case, the estimation error related to each predictor at the rank j and in particular to the last one i.e. at rank m, converges exponentially to zero. In the presence of uncertainties, the estimation error remains bounded and the underlying ultimate bound is proportional to the uncertainties essential bound δ_ε.

• **Lemma.** *Let* A *be the* $n \times n$ *anti-shift block matrix and let* $\bar{A}$ *be a* $n \times n$ *Hurwitz matrix with eigenvalues* $\lambda_1, \ldots, \lambda_n$ *ordered such that*

$$0 < -\Re(\lambda_1) \leq \ldots \leq -\Re(\lambda_n),$$

$\Re(\cdot)$ *is the real part of the complex number* $(\cdot)$*. Then, one has*

$$|\Re(\lambda_1)| \leq \|A - \bar{A}\|.$$

• **Proposition.** *Let* $M_m \delta_\varepsilon$ *be the ultimate bound of the error between the actual state* $x(t)$ *and the state of the last subsystem* $\hat{x}_m(t)$ *in the cascade observer, i.e.* $\lim\sup_{t\to\infty} \|\hat{x}_m(t) - x(t)\| \leq M_m \delta_\varepsilon$*. Then, the sequence* $(M_m)_{m\in\mathbb{N}^\star}$ *is non increasing with*

$$\lim_{m\to\infty} M_m = \beta \frac{M}{\theta} e^{(\eta-\bar{a})d} + \beta \frac{e^{(\eta-\bar{a})d} - 1}{\eta - \bar{a}},$$

$$\lim_{m\to\infty} M_m = \beta\frac{M}{\theta}e^{(\eta-\bar{a})d} + \beta\frac{e^{(\eta-\bar{a})d}-1}{\eta-\bar{a}},$$

• The right hand side of the above equation is constituted by two terms

- The first, $\beta\frac{M}{\theta}e^{(\eta-\bar{a})d}$ can be made as small as desired by choosing values of θ high enough.
- The second, $\beta\frac{e^{(\eta-\bar{a})d}-1}{\eta-\bar{a}}$, is fixed and is the limit of the ultimate bound when the length of the cascade, i.e. m, is chosen sufficiently high.

• The term $\eta-\bar{a}$ appearing in the expression of the ultimate bound is directly related to the Lipschitz constant of the system nonlinearities

$$\begin{aligned} \eta-\bar{a} &= \beta\left(L_\varphi + \|A-\bar{A}\|\right) - \bar{a} \\ &\geq \beta L_\varphi \text{ since } \beta \geq 1 \text{ and } \ \|A-\bar{A}\| - \bar{a} \geq 0 \text{ according to the lemma.} \end{aligned}$$

$$\lim_{m\to\infty} M_m = \beta\frac{M}{\theta}e^{(\eta-\bar{a})d} + \beta\frac{e^{(\eta-\bar{a})d}-1}{\eta-\bar{a}}, \text{with } \eta-\bar{a}\geq\beta L_\varphi$$

• Since the function $\alpha \mapsto \dfrac{e^\alpha - 1}{\alpha}$ is increasing for $\alpha \geq 0$, one has

$$\lim_{m\to\infty} M_m \geq \beta\frac{M}{\theta}e^{\beta L_\varphi d} + \frac{e^{\beta L_\varphi d}-1}{L_\varphi},$$

i.e. the lower bound of the limit is an increasing function of the Lipschitz constant of the system nonlinearities.

• The cascade observer provides an estimate of the delayed state (first subsystem of cascade), as well as an estimate of the actual state (last subsystem):

- The ultimate bound of the observation error related to the delayed state can be made as small as desired (by choosing values of θ sufficiently high).
- This property is no longer true with for the actual state. Nevertheless, the smallest values of this bound can be reached by choosing values of m sufficiently high.

The sampled output case

- The outputs are available at the sampling instants $0 \leq t_0 < \ldots < t_l < \ldots$ with $\lim_{l \to +\infty} t_l = +\infty$

- There exist $0 < \tau_m \leq \tau_M < +\infty$ such that

$$0 < \tau_m \leq \tau_k = t_{k+1} - t_k \leq \tau_M, \quad \forall k \geq 0.$$

$$\begin{cases} \dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) + B\varepsilon(t) \\ y_d(t_k) = Cx(t_k - d) = x^{(1)}(t_k - d) \end{cases}$$

Some recalls

In the delay-free case ($y_d(t_k) = y(t_k)$), a continuous-discrete time high gain observer has been proposed (*Automatica 55, pp. 78-87,2015*)

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \varphi(u(t), \hat{x}(t)) - \theta\Delta_\theta^{-1} K e^{-k_1\theta(t-t_k)}(C\hat{x}(t_k) - y(t_k)),$$

The upper bound of the sampling partition diameter, τ_M, has to satisfy

$$\tau_M \chi(\theta) < 1, \; with \; \chi(\theta) = \frac{\nu\sqrt{\lambda_m(P)}}{2(L_\varphi + \theta)\|K\|\lambda_M^{3/2}(P)},$$

The underlying estimation error satisfies

$$\|\hat{x}(t) - x(t)\| \leq \mu(\theta) e^{-\eta_\theta(\tau_M) t} \|\hat{x}(0) - x(0)\| + N_\theta(\tau_m, \tau_M) \frac{\delta_\varepsilon}{\theta},$$

$$\begin{aligned}
\eta_\theta(\tau_M) &= a_\theta e^{-a_\theta \tau_M} - \frac{(1 - e^{-a_\theta \tau_M})}{\chi_\theta}, \qquad a_\theta = \frac{\theta \nu}{2\lambda_M}, \\
N_\theta(\tau_m, \tau_M) &= \sqrt{\frac{\lambda_M}{\lambda_m}} \theta \tau_M \frac{2 - e^{-\eta_\theta(\tau_M)\tau_m}}{1 - e^{-\eta_\theta(\tau_M)\tau_m}},
\end{aligned}$$

τ_m and τ_M, the lower and upper bounds of the sampling partition diameter.

If the sampling period is constant, i.e. $\tau_m = \tau_M = T_s$, then $\eta_\theta(T_s)$ and $N_\theta(T_s)$ are respectively a decreasing and non decreasing functions of T_s and one has

$$\lim_{T_s \to 0} \eta_\theta(T_s) = a_\theta \quad \text{and} \quad \lim_{T_s \to 0} N_\theta(T_s) = M,$$

Cascade observer equations - Sampled outputs

$$
\left\{
\begin{array}{rcl}
\dot{\hat{z}}_j(t) & = & A\hat{z}_j(t) + \varphi(u_j(t), \hat{z}_j(t)) - H_j(t),\ j = 0, \ldots, m, \\
\\
H_0(t) & = & \theta \Delta_\theta^{-1} K e^{-k_1 \theta (t - t_k)} (C\hat{z}_0(t_k) - y_d(t_k)) \text{ for } t \in [t_k,\ t_{k+1}[, \\
\\
\text{and for } j & = & 1, \ldots, m, \\
\\
H_j(t) & = & e^{\bar{A}\frac{d}{m}} \left(H_{j-1}(t) + \left(A - \bar{A}\right) \left(\hat{z}_j \left(t - \frac{d}{m} \right) - \hat{z}_{j-1}(t) \right) \right. \\
 & & \left. + \varphi \left(u_{j-1}(t), \hat{z}_j \left(t - \frac{d}{m} \right) \right) - \varphi(u_{j-1}(t), \hat{z}_{j-1}(t)) \right).
\end{array}
\right.
$$

Theorem 2

If

- the upper bound of the sampling partition diameter τ_M satisfies

$$\tau_M \chi(\theta) < 1 \text{ with } \chi(\theta) = \frac{\nu\sqrt{\lambda_m}}{2(L_\varphi + \theta)\|K\|\lambda_M^{3/2}},$$

- the matrix $\bar{A}$ is chosen such that $\bar{a} \leq \eta_\theta(\tau_M)$,

- the number m of the cascaded systems is chosen such that $\eta\frac{d}{m} < 1$, $(\eta = \beta\left(L_\varphi + \|\bar{A} - A\|\right))$,

then,

one has for $j = 1, \ldots, m$,

$$\|\tilde{z}_j(t)\| \stackrel{\Delta}{=} \|\hat{z}_j(t) - x_j(t)\| \leq \bar{\rho}_j e^{-\bar{a}t} + \bar{M}_j \delta_\varepsilon, \ t \geq 0,$$

$$\begin{aligned} \bar{\rho}_j &= \frac{\eta}{1 - \eta\frac{d}{m}} \int_{-\frac{d}{m}}^{0} \|\tilde{z}_j(s)\| ds + \beta \chi_m^j \mu(\theta) \|\tilde{z}_0(0)\| + \frac{\beta}{1 - \eta\frac{d}{m}} \sum_{k=0}^{j-1} \chi_m^k \|r_{j-k}(0)\|, \\ \bar{M}_j &= \beta \chi_m^j \frac{N(\tau_m, \tau_M)}{\theta} + \frac{\beta\frac{d}{m}}{1 - \eta\frac{d}{m}} \sum_{i=0}^{j-1} \chi_m^i, \qquad \left(\chi_m = \frac{e^{-\bar{a}\frac{d}{m}}}{1 - \eta\frac{d}{m}} \right). \end{aligned}$$

Example

- $q = 3$ and $p = 2$, i.e. $x = \begin{pmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{pmatrix} \in \mathbb{R}^6$, $x^{(1)} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $x^{(2)} = \begin{pmatrix} z_3 \\ z_4 \end{pmatrix}$,

$x^{(3)} = \begin{pmatrix} z_5 \\ z_6 \end{pmatrix}$, $z_i \in \mathbb{R}$,

- $u = \begin{pmatrix} u_1 = 10\cos((t) \\ u_2 = 10\sin(t) \end{pmatrix}$, $u_1, u_2 \in \mathbb{R}$, $y = x^{(1)} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \in \mathbb{R}^2$,
- $\varphi^{(1)}(u,x) = \begin{pmatrix} -0.2z_1 + u_1 \\ -0.2z_2 + u_2 \end{pmatrix}$, $\varphi^{(2)}(u,x) = \begin{pmatrix} -0.05z_3^3 + u_2 z_3 \\ -0.05z_4^3 + u_1 z_4 \end{pmatrix}$,
- $\varphi^{(3)}(u,x) = \begin{pmatrix} -0.1z_5 + 2tanh(z_5) \\ -0.1z_6 + 2tanh(z_6) \end{pmatrix}$, $\varepsilon(t) = \begin{pmatrix} \varepsilon_1(t) = 0.5\cos(0.1t) \\ \varepsilon_2(t) = 0.5\sin(0.1t) \end{pmatrix}$

- The objective is to estimate the actual state components $z_k(t), k = 1, \ldots, 6$, from the delayed sampled outputs $z_1(t_k - d)$ and $z_2(t_k - d)$,

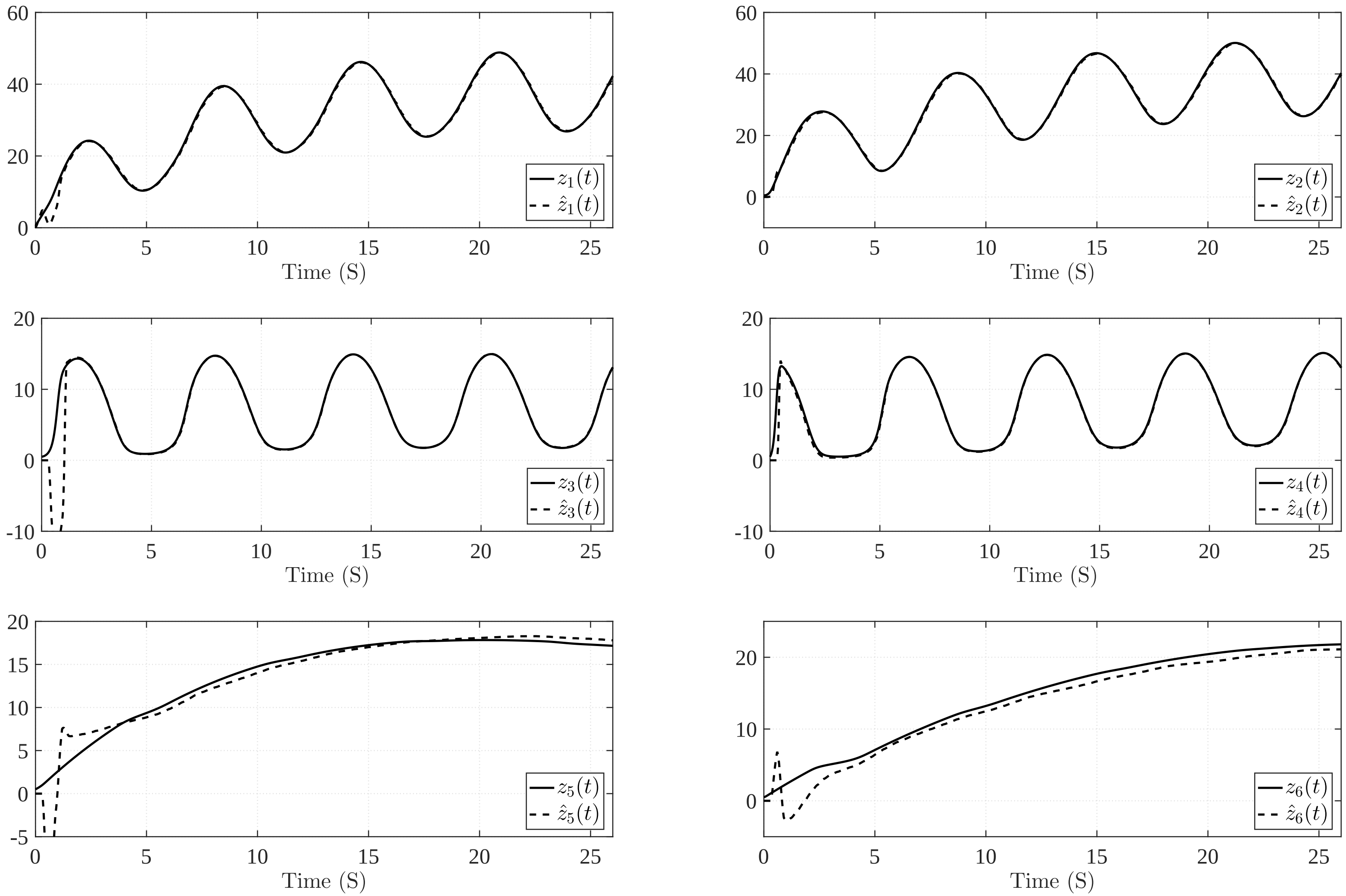
- This shall be achieved by using a continuous-discrete time cascade observer,

- In all the simulations,

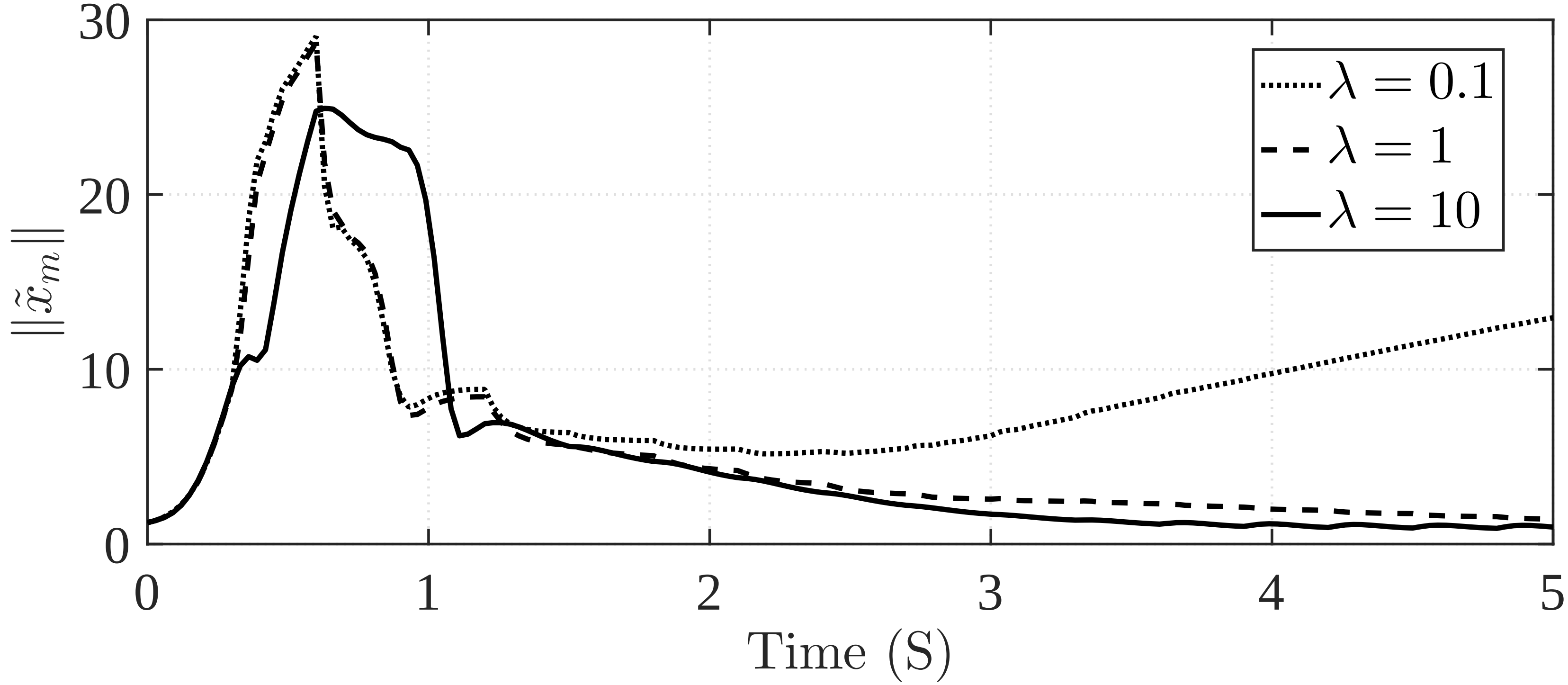
  - $\theta = 5$, $K = [3I_2 \ 3I_2 \ I_2]^T$,

  - $\bar{A} = A - \lambda I_6$, A is the 6×6 anti-shift matrix and $\lambda > 0$ is a positive real.

- Continuous estimates of the system actual states: $d = 0.3s, \lambda = 10, m = 5.$

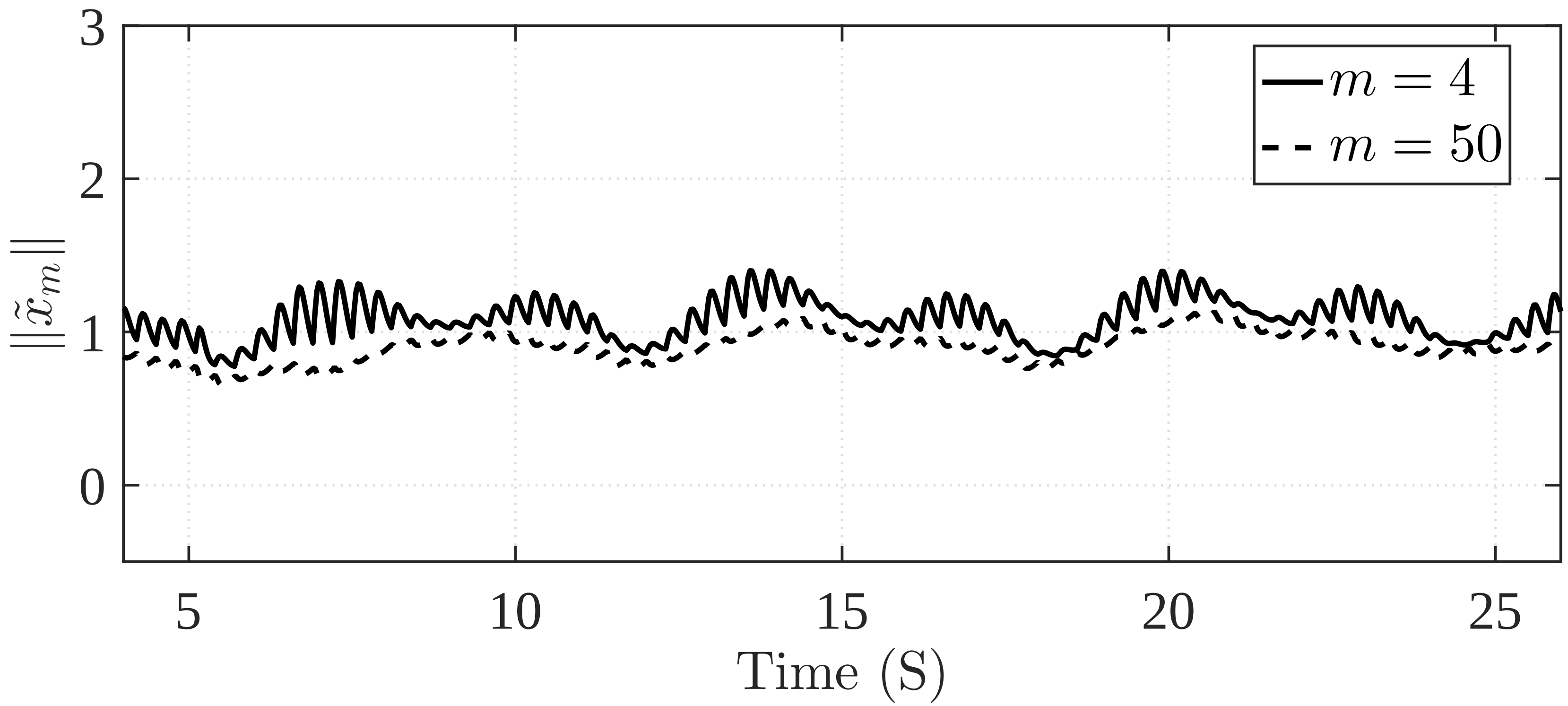


• Time evolution of the estimation error obtained with three values of $\bar{A} = A - \lambda I_6$



• Relatively high values for λ have to be avoided $\Longrightarrow$ high values for $\|A - \bar{A}\| \Longrightarrow$ the condition $\beta \left(L_\varphi + \|\bar{A} - A\|\right) \dfrac{d}{m} < 1$ may be violated if m is not augmented.

• Zoom on the ultimate bound of the estimation error obtained with two values of m



• Time evolution of the estimation error obtained with two values of d $(\lambda = 10, m = 50)$