Output Injection Filtering Redesign in High-Gain Observers

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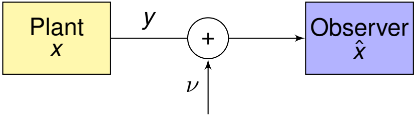
Outline

- Motivation
- The framework of high-gain observers
- General construction of filters
- Cascade structure
- Numerical example
- Conclusion and Perspectives

Motivation



x : plant state
$\hat{x}$: estimate
y : measured output
ν : measurment noise

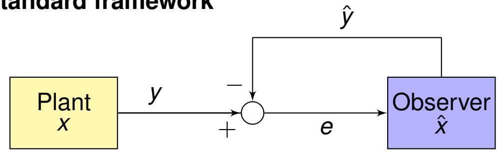
Objective :
Design an observer so that

- the estimation error $|x - \hat{x}|$ converges asymptotically to zero when $\nu = 0$
- guarantee good sensitivity properties in presence of noise (ISS/BIBS)

Problem :
How to reduce the effect of the noise in a nonlinear context

Filter pre-loop

Standard framework



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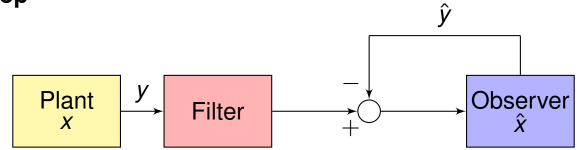
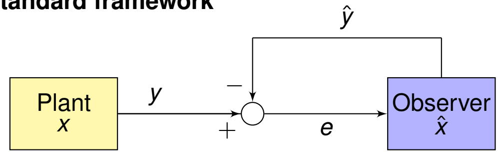


Problem : convergence is not ensured

Solution : the observer dynamics needs to copy also the dynamics of the filter

Filter pre/post-loop

Standard framework



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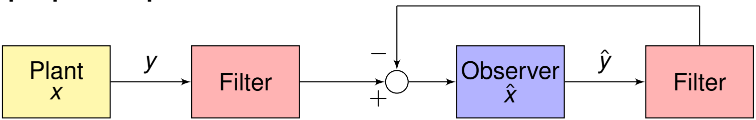
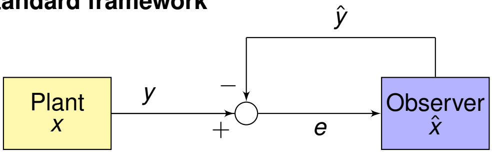


Inconvenient : we need to replicate the filter dynamics twice

Filter in-the-loop

Standard framework



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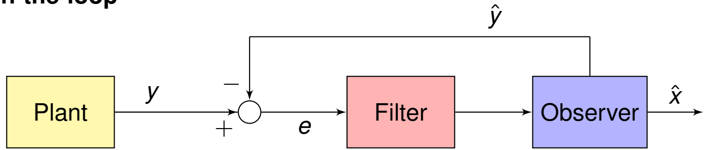

Open questions : how to design the filter ?

The framework of high-gain observers

We focus on autonomous systems in the *observability canonical form*

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= \varphi(x) \\ y &= x_1 + \nu(t) \end{cases} \qquad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Assumptions :

- $x(t) \in X \subset \mathbb{R}^n$ for all $t \geq 0$;
- φ : locally Lipschitz function;
- $\nu(t)$: bounded measurement noise;

Goal : design a semi-global *tunable* observer

✓ asymptotic estimate in nominal conditions : $\nu(t) \equiv 0$;

✓ ISS w.r.t. measurement noise;

✓ the rate of convergence may be made arbitrarily fast;

✗ focus on sensitivity to high-frequency measurement noise.

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- $\hat{x}$: estimate of x,
- $\boldsymbol{k} \geq 1$: high-gain parameter,
- $|\varphi_s(\hat{x}) - \varphi(x)| \leq L_\varphi |\hat{x} - x|$ for all $(\hat{x}, x) \in \mathbb{R}^n \times X$

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Theorem

There exists $\mathbf{k}^* \geq 1$ such that for all $\mathbf{k} > \mathbf{k}^*$

$$\|\hat{x}(t) - x(t)\| \leq a_0 \mathbf{k}^{n-1} \exp(-a_1 \mathbf{k} t)\|\hat{x}(0) - x(0)\| + a_2 \mathbf{k}^{n-1} \sup_{t \geq 0} \|\nu(t)\|$$

Pros.

✓ Easy to design and to tune

✓ Rate of convergence arbitrarily fast

✓ Robust to model uncertainties on φ

Cons.

✗ Sensitivity to measurement noise

✗ Peaking phenomena

✗ Numerical implementation

The framework of high-gain observers

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad\vdots \\ \dot{x}_n = \varphi(x) \\ y = x_1 + \nu(t) \end{cases} \qquad \begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \ell_1 \mathbf{k}\,(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + \ell_2 \mathbf{k^2}\,(y - \hat{x}_1) \\ \quad\vdots \\ \dot{\hat{x}}_n = \varphi_s(\hat{x}) + \ell_n \mathbf{k^n}\,(y - \hat{x}_1) \end{cases}$$

Let us use the following compact notation

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & & & & 1 \\ 0 & 0 & & \cdots & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \; C^\top = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \; L = \begin{pmatrix} \ell_1 \\ \vdots \\ \ell_n \end{pmatrix}, \; D_{\mathbf{k}} = \mathrm{diag}(\mathbf{k}, \cdots, \mathbf{k}^n)$$

to obtain the compact equations :

$$\begin{cases} \dot{x}(t) = Ax(t) + B\varphi(x(t)) \\ y(t) = Cx(t) + \nu(t) \end{cases} \qquad \dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi_s(\hat{x}(t)) + D_{\mathbf{k}} L(y(t) - C\hat{x}(t))$$

Background (not exhaustive)

First appearances (1988-1994)

- Basic motivations : systematic design, separation principle
  - A. Tornambé
  - J.P. Gauthier, I. Kupka, H. Hammouri, G. Besançon, F. Deza, G. Bornard, ...
  - H. K. Khalil, A. Saberi, P. Sannuti, ...
  - A. Teel, L. Praly
  - S.V. Emel'yanov, S.K. Korovin, S.V. Nikitin, ...
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Further developments : adaptive techniques (around 2010)

- Adaptive drawbacks (peaking, sensitivity to noise)
  - J. H. Aherns and H.K. Khalil (2009)
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Recent results : low-power approaches (from 2015)

- Based on dynamic extension and low-power implementations of k
  - D. Astolfi and L. Marconi (2015)
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High-frequency measurement noise

Consider the case of noise generated as $\nu(t) = \sum_{j=1}^{N} \nu_j \sin\left(\frac{\omega_j}{\epsilon} t + \phi_j\right)$.

When $\epsilon \to 0$, the measurement noise contains only high-frequency components.

We consider the effect of the measurement noise on the i-th component of the estimation error.

Standard Lyapunov analysis : $\limsup_{t\to\infty} \|\hat{x}_i(t) - x_i(t)\| \leq c_1 \boldsymbol{k}^{i-1} \sup_{t\geq 0} \|\nu(t)\|$.

Recent results : $\limsup_{t\to\infty} \|\hat{x}_i(t) - x_i(t)\| \leq c_2 \epsilon \boldsymbol{k}^{i} \sup_{t\geq 0} \|\nu(t)\|$.

[D. Astolfi, L. Marconi, L. Praly, A. Teel. NOLCOS 2016]

We focus on the design of low-pass filter :
the relative degree should improve the second bound.

Low-pass filter strategy

Consider the case of noise generated as $\nu(t) = \sum_{j=1}^{N} \nu_j \sin\left(\frac{\omega_j}{\epsilon} t + \phi_j\right)$.

Ultimate bound : $\limsup_{t\to\infty} \|\hat{x}_i(t) - x_i(t)\| \leq c_2 \epsilon \boldsymbol{k}^i \sup_{t\geq 0} \|\nu(t)\|$.

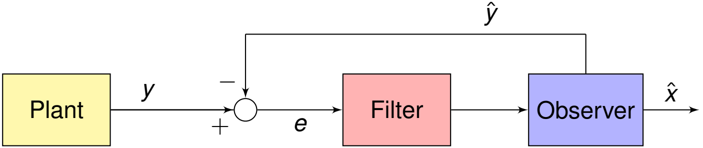
We focus on the design of low-pass filter : the relative degree should improve the second bound.

Recent results in [D. Astolfi, L. Marconi, L. Praly, A. Teel, Automatica 2018] suggests that if the relative degree between the measurement noise and the i-th estimation error is r then we recover :

$$\limsup_{t\to\infty} \|\hat{x}_i(t) - x_i(t)\| \leq c_2 \epsilon^r \boldsymbol{k}^i \sup_{t\geq 0} \|\nu(t)\|$$

General construction of the filter

Main idea : augment the relative degree between y and $\hat{x}$



$$\dot{x} = Ax + B\varphi(x)$$
$$y = Cx + \nu$$

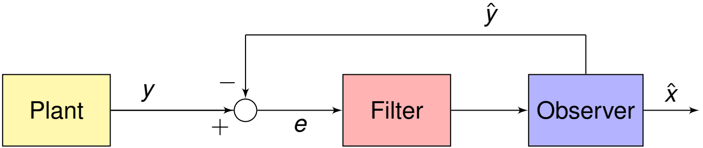
$$\dot{\eta} = \mathbf{k} F\eta + \mathbf{k} Ge$$
$$\eta(0) = 0$$

$$\dot{\hat{x}} = A\hat{x} + B\varphi_s(\hat{x}) + D_{\mathbf{k}} L\eta$$

How can we design the filter ?

Sufficient (not constructive) result

Main idea : augment the relative degree between y and $\hat{x}$



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$$\eta(0) = 0$$

$$\dot{\hat{x}} = A\hat{x} + B\varphi_s(\hat{x}) + D_{\mathbf{k}} L\eta$$

Theorem

Let (F, G, L) be such that $\begin{pmatrix} A & L \\ -GC & F \end{pmatrix}$ is Hurwitz.

Then, there exists $\mathbf{k}^* \geq 1$ such that for all $\mathbf{k} > \mathbf{k}^*$

$$\|\hat{x}(t) - x(t)\| \leq a_0 \mathbf{k}^{\mathbf{n-1}} \exp(-a_1 \mathbf{k} t)\|\hat{x}(0) - x(0)\| + a_2 \mathbf{k}^{\mathbf{n-1}} \sup_{t\geq 0} \|\nu(t)\|$$

Sketch of the proof

Plant :

$$\dot{x} = Ax + B\varphi(x), \qquad y = Cx + \nu$$

Observer :

$$\dot{\hat{x}} = A\hat{x} + B\varphi_s(x) + D_{\boldsymbol{k}} L\eta, \qquad \dot{\eta} = \boldsymbol{k} F\eta + \boldsymbol{k} G(y - C\hat{x}), \ \eta(0) = 0.$$

Change of coordinates : $\hat{x} \mapsto \tilde{x} = \boldsymbol{k} D_{\boldsymbol{k}}^{-1}(\hat{x} - x)$.
Note that $D_{\boldsymbol{k}}^{-1} A = \boldsymbol{k} A D_{\boldsymbol{k}}^{-1}$.
Error dynamics :

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\eta} \end{pmatrix} = \boldsymbol{k} \begin{pmatrix} A & L \\ -GC & F \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \eta \end{pmatrix} + \frac{1}{\boldsymbol{k}^{(n-1)}} \begin{pmatrix} B \\ 0 \end{pmatrix} \left[\varphi(x + \frac{1}{\boldsymbol{k}} D_{\boldsymbol{k}} \tilde{x}) - \varphi(x) \right].$$

Stability of $\begin{pmatrix} A & L \\ -GC & F \end{pmatrix}$ and Lyapunov analysis allow to conclude the result of the theorem :

$$\|\hat{x}(t) - x(t)\| \leq a_0 \boldsymbol{k}^{n-1} \exp(-a_1 \boldsymbol{k} t) \|\hat{x}(0) - x(0)\| + a_2 \boldsymbol{k}^{n-1} \sup_{t \geq 0} \|\nu(t)\|.$$

Cascade structure of the low-pass filter

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \ell_1 \, \boldsymbol{k} \boxed{(y - \hat{x}_1)} \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \ell_2 \, \boldsymbol{k^2} \boxed{(y - \hat{x}_1)} \\
&\vdots \\
\dot{\hat{x}}_n &= \varphi_s(\hat{x}) + \ell_n \, \boldsymbol{k^n} \boxed{(y - \hat{x}_1)}
\end{aligned}
$$

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\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \ell_1 \mathbf{k} \boxed{\eta_1} \\
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&\vdots \\
\dot{\hat{x}}_n &= \varphi_s(\hat{x}) + \ell_n \mathbf{k^n} \boxed{\eta_n}
\end{aligned}$$

where the η_i are extra variables generated by a bank of filters

Cascade structure of the low-pass filter (2)

Standard High-Gain Observer

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + \ell_1\, \mathbf{k}(y - \hat{x}_1)\\ \dot{\hat{x}}_2 &= \hat{x}_3 + \ell_2\, \mathbf{k^2}(y - \hat{x}_1)\\ &\vdots\\ \dot{\hat{x}}_n &= \varphi_s(\hat{x}) + \ell_n\, \mathbf{k^n}(y - \hat{x}_1)\end{aligned}$$

Cascade structure

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + \ell_1 \mathbf{k}\eta_1 & \dot{\eta}_1 &= -\alpha\mathbf{k}\eta_1 + \mathbf{k}\beta\,(y - \hat{x}_1)\\ \dot{\hat{x}}_2 &= \hat{x}_3 + \ell_2 \mathbf{k^2}\eta_2 & \dot{\eta}_2 &= -\alpha\mathbf{k}\eta_2 + \mathbf{k}\beta\,\eta_1\\ &\vdots & &\vdots\\ \dot{\hat{x}}_n &= \varphi_s(\hat{x}) + \ell_n \mathbf{k^n}\eta_n & \dot{\eta}_n &= -\alpha\mathbf{k}\eta_n + \mathbf{k}\eta_{n-1}\end{aligned}$$

Main motivation :

- steady-state behaviour of η_1 : $\frac{\beta}{\alpha}(y - \hat{x}_1)$
- stead-state behaviour of η_2 : $\frac{\beta}{\alpha}\eta_1 = \frac{\beta^2}{\alpha^2}(y - \hat{x}_1)$
- stead-state behaviour of η_n : $\frac{\beta}{\alpha}\eta_{n-1} = \cdots = \frac{\beta^n}{\alpha^n}(y - \hat{x}_1)$

Compact notation

Let us use the following compact notation

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \\ 0 & & & & 1 \\ 0 & 0 & & \cdots & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \quad L = \begin{pmatrix} \ell_1 \\ \vdots \\ \ell_n \end{pmatrix}$$

by which the two observers are written as

$$\dot{\hat{x}} = A\hat{x} + B\varphi_s(\hat{x}) + D_{\boldsymbol{k}} L(y - C\hat{x})$$

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B\varphi_s(\hat{x}) + D_{\boldsymbol{k}} \mathrm{diag}(L)\eta \\ \dot{\eta} &= -\boldsymbol{k}(\alpha I - \beta A^\top)\eta + \beta \boldsymbol{k} C^\top (y - C\hat{x}) \end{aligned}$$

Convergence proof for the revised HGO

- Similarly to standard high-gain observer, we can prove the convergence of the observer by changing coordinates in a similar way
- The proof is more involved because now we need to select $\ell_1, \dots, \ell_n\ \alpha, \beta$ such that M is Hurwitz

$$M = \left(\begin{array}{c|c} A & \mathrm{diag}(L) \\ \hline -\beta C^T C & -\alpha I + \beta A^T \end{array}\right) = \left(\begin{array}{ccccc|ccccc} 0 & 1 & 0 & \cdots & 0 & \ell_1 & 0 & \cdots & & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & \ell_2 & & & \\ & & & \ddots & & \vdots & & \ddots & & \\ 0 & & & & 1 & & & & \ell_{n-1} & \\ 0 & & & & 0 & 0 & & & & \ell_n \\ \hline -\beta & 0 & & \cdots & 0 & -\alpha & & & & \\ 0 & & & & & \beta & -\alpha & & & \\ \vdots & & & & \vdots & & \ddots & \ddots & & \\ & & & & & & & \beta & -\alpha & \\ 0 & & & \cdots & 0 & & & & \beta & -\alpha \end{array}\right)$$

- Convergence is ensured for $\boldsymbol{k}$ large enough

A technical lemma

Let $\mathcal{S}_{\alpha,\beta} \subset \mathbb{C}_{<0}$ be the following set

$$\mathcal{S}_{\alpha,\beta} := \left\{ (x+iy) \in \mathbb{C} \;:\; \alpha^2 x + \beta y^2 < 0 \right\},$$

and recall the following definitions

$$F = A - LC, \qquad M = \left(\begin{array}{c|c} A & \operatorname{diag}(L) \\ \hline -\beta C^T C & -\alpha I + \beta A^T \end{array} \right).$$

Then we have the following result.

Stability of M

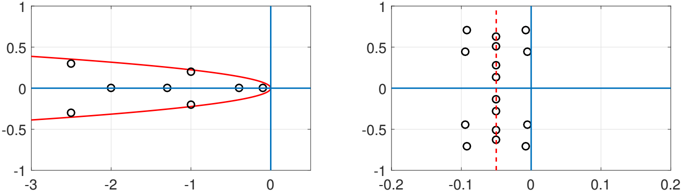
Let two positive reals α, β be fixed and consider the following matrix
The following holds :

(i) The characteristic polynomial of M satisfies

$$p_M(\lambda) = \beta^n p_F\left(\frac{\lambda(\lambda+\alpha)}{\beta}\right).$$

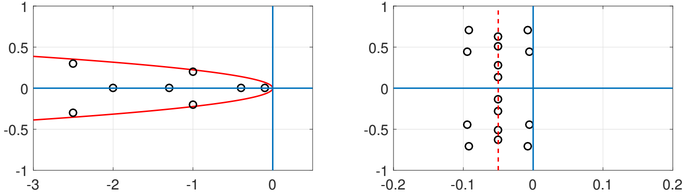
(ii) If L is chosen such that $\sigma(F) \subset \mathcal{S}_{\alpha,\beta}$, then both F and M are Hurwitz.

Numerical examples for $n = 8$



Left figure : $\sigma(F) = \sigma(A - LC)$, with $\sigma(F) \in \mathcal{S}_{\alpha,\beta}$; Right figure : $\sigma(M)$; Dotted red line : $-\alpha/2$

Numerical examples for $n = 8$



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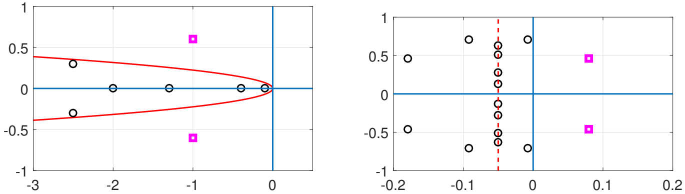


Left figure : $\sigma(F) = \sigma(A - LC)$, with $\sigma(F) \notin \mathcal{S}_{\alpha,\beta}$ in purple : Right figure : $\sigma(M)$; Dotted red line : $-\alpha/2$.

Main result

System : $$\begin{cases} \dot{x} & = & Ax + B\varphi(x), \\ y & = & Cx + \nu \end{cases}$$

Observer : $$\begin{cases} \dot{\hat{x}} & = & A\hat{x} + B\varphi_s(\hat{x}) + D_{\mathbf{k}} \mathrm{diag}(L)\eta \\ \dot{\eta} & = & -\mathbf{k}(\alpha I - \beta A^\top)\eta + \beta \mathbf{k} C^\top (y - C\hat{x}) \end{cases}$$

Theorem

Let $\alpha, \beta > 0$ be fixed and let L be chosen such that $\sigma(F) \subset \mathcal{S}_{\alpha,\beta}$, $F = A - LC$. Then, there exists a real number $\mathbf{k}^* \geq 1$, such that, for any $\mathbf{k} > \mathbf{k}^*$, the following bound holds

$$|\hat{x}(t) - x(t)| \leq b_1 \mathbf{k}^{n-1} \exp(-b_2 \mathbf{k} t)|\hat{x}(0) - x(0)| + b_3 \mathbf{k}^{n-1} \sup_{t\geq 0} |\nu(t)|$$

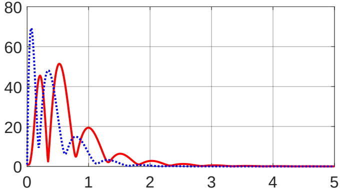
for all $t \geq 0$, for all initial conditions $(x(0), \hat{x}(0), \eta(0)) \in X \times \mathbb{R}^n \times \{0\}$ and for some constants $b_i > 0$, $i = 1, 2, 3$ independent of $\mathbf{k}$.

Simulation results

Convergence of the estimation error $|x(t) - \hat{x}(t)|$

Red line : filtered HGO

Dotted blue line : standard HGO



Normalized asymptotic estimation errors with measurement noise

$$\nu(t) = \bar{\nu} \ \sin(\omega\, t)$$

filtered HGO

	$\omega = 50$	$\omega = 100$	$\omega = 300$
$\|\|\tilde{x}_1\|\|_a$	2	0.6	0.07
$\|\|\tilde{x}_2\|\|_a$	10	1.4	0.07
$\|\|\tilde{x}_3\|\|_a$	22	1.6	0.02
$\|\|\tilde{x}_4\|\|_a$	18	0.8	0.01

standard HGO

	$\omega = 50$	$\omega = 100$	$\omega = 300$
$\|\|\tilde{x}_1\|\|_a$	5	2.5	0.9
$\|\|\tilde{x}_2\|\|_a$	50	25	8
$\|\|\tilde{x}_3\|\|_a$	230	120	40
$\|\|\tilde{x}_4\|\|_a$	500	260	90

Conclusion

Contribution :

- We proposed a new structure for the design of high-gain observers
- Dimension : $2n$
- The relative degree between y and $\hat{x}_i$ larger than 2 for any i
- Retains standard features of HGO while improving sensitivity

Future works :

- Characterize the sensitivity to measurement noise in the nonlinear framework
- Study alternative design for the filter

Thank you for your attention.