Sliding mode control application on UAVs under external disturbances

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RT-MaG Project
Real Time-Marseille Grenoble Project
www.gipsa-lab.fr/projet/RT-MaG
Based on article: *Altitude and attitude sliding mode control of UAV under wind disturbances*. G.Perozzi, D.Efimov, JM.Biannic, L.Planckaert, P.Coton. Submitted to IFAC 2017 Toulouse.

- Action in urban areas (eg: earthquakes like in Italy).
- Fluid obstacle.
- Unpredictable turbulent airflow pattern.
- Aerodynamic model, which takes into account wind disturbances directly inside of UAV dynamics equations;
- Nonlinear control law which considers realistic assumptions on external disturbances of quadrotors.

**Why sliding mode control?**
- SMC is an efficient tool to design robust controllers for nonlinear systems operating under uncertainty conditions
Rotational matrix

\[
R = \begin{bmatrix}
C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\phi S_\psi + C_\psi S_\theta C_\phi \\
S_\psi C_\theta & C_\psi C_\phi + S_\psi S_\theta S_\phi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\
-S_\theta & C_\theta S_\phi & C_\theta C_\phi
\end{bmatrix}
\]

Passage from earth frame (\(\mathcal{R}_0\)) to body frame (\(\mathcal{R}\))

\[
[X^T]_{\mathcal{R}} = [X^T]_{\mathcal{R}_0} \cdot R
\]
Dynamics

- **Traslational dynamics in the body frame**

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} + m
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times \begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = \begin{bmatrix}
F_{X_{\text{aero}}} \\
F_{Y_{\text{aero}}} \\
F_{Z_{\text{aero}}}
\end{bmatrix} + m
\begin{bmatrix}
-g \sin \theta \\
g \cos \theta \sin \phi \\
g \cos \theta \cos \phi
\end{bmatrix}
\]

- **Rotational dynamics with respect to inertial earth frame**

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = - \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} + \begin{bmatrix}
L_{\text{aero}} \\
M_{\text{aero}} \\
N_{\text{aero}}
\end{bmatrix}
\]

- **Relationship between angular velocities and euler angles**

\[
\begin{align*}
\dot{\phi} &= p + \tan \theta (q \sin \phi + r \cos \phi) \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= \frac{q \sin \phi + r \cos \phi}{\cos \theta}
\end{align*}
\]
UAV desired movements are obtained changing rotors speed in a proper way (altitude and attitude).
Aerodynamic forces and momenta for each rotor

\[ F_{Xj} = -\rho AR^2 \frac{u_j - u_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Hj}\omega_j^2 \]

\[ F_{Yj} = -\rho AR^2 \frac{v_j - v_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Hj}\omega_j^2 \]

\[ F_{Zj} = -\rho AR^2 C_{Tj}\omega_j^2 \]

\[ L_j = -\text{sign}\ \omega_j \rho AR^3 \frac{u_j - u_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Rm_j}\omega_j^2 \]

\[ M_j = -\text{sign}\ \omega_j \rho AR^3 \frac{v_j - v_w}{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}} C_{Rm_j}\omega_j^2 \]

\[ N_j = -\text{sign}\ \omega_j \rho AR^3 C_{Qj}\omega_j^2 \]
• Total aerodynamic forces

\[ F_{X_{\text{aero}}} = \sum_{j=1}^{4} F_{X_j}, \quad F_{Y_{\text{aero}}} = \sum_{j=1}^{4} F_{Y_j}, \quad F_{Z_{\text{aero}}} = \sum_{j=1}^{4} F_{Z_j} \]

• Total aerodynamic momenta

\[ L_{\text{aero}} = \sum_{j=1}^{4} (L_j + F_{Z_j}l_{s_j} - hF_{Y_j}) \]

\[ M_{\text{aero}} = \sum_{j=1}^{4} (M_j - F_{Z_j}l_{c_j} + hF_{X_j}) \]

\[ N_{\text{aero}} = \sum_{j=1}^{4} (N_j + F_{Y_j}l_{c_j} - F_{X_j}l_{s_j}) \]

\[ c_j = \cos \left( \frac{\pi}{2} (j - 1) + \epsilon \right) \]

\[ s_j = \sin \left( \frac{\pi}{2} (j - 1) + \epsilon \right) \]
• Aerodynamic coefficients from blade element momentum theory

\[ \mu_j = \frac{\sqrt{(u_j - u_w)^2 + (v_j - v_w)^2}}{R|\omega_j|} \]

• Simplified coefficients

\[ \lambda_j = \lambda_{stat} - \frac{4}{\sigma a} K_z \frac{w_j - w_w}{R|\omega_j|} \]

\[ C_{Tj} = C_{T stat} + K_z \frac{w_j - w_w}{R|\omega_j|} \]

\[ C_{Hj} = K_D \mu_j \]
SMC design is composed of two steps:

- **Design of a surface.** While on the sliding surface, the dynamics is restricted to the equations of the surface and is robust against external disturbances.

- **Design a feedback control law to provide convergence of the system trajectory to the sliding surface, and to obtain a finite time convergence.**
- Reaching phase: the trajectory, starting from a nonzero initial conditions, reaches the sliding surface.
- Sliding surface: the trajectory remains and evolves according to the dynamics specified by the sliding surface.
Chattering issue:

- In theory the trajectory slides along the surface.
- In practice there is high frequency switching called chattering.

Solutions have been developed to reduce the chattering so that the trajectory remains in a small neighborhood of the surface (High order SMC, saturation function).
- **Errors**

\[ e_z = z - z_{des} \]
\[ e_\phi = \phi - \phi_{des} \]
\[ e_\theta = \theta - \theta_{des} \]
\[ e_\psi = \psi - \psi_{des} \]

- **Sliding surface**

\[ S_i = \dot{e}_i + \alpha_i e_i, \quad \alpha_i > 0 \]

- **Lyapunov function**

\[ V_i = \frac{1}{2} S_i^2 \]
System

\[ \dot{X} = f(X, U, d) \]

State

\[ X = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z} \ \phi \ \theta \ \psi \ p \ q \ r]^T \]

Control

\[
U = \begin{bmatrix}
U_z \\
U_\theta \\
U_\phi \\
U_\psi
\end{bmatrix} = \begin{bmatrix}
K_f & K_f & K_f & K_f \\
K_f l c_j & K_f l c_j & K_f l c_j & K_f l c_j \\
-K_f l s_j & -K_f l s_j & -K_f l s_j & -K_f l s_j \\
K_m & -K_m & K_m & -K_m
\end{bmatrix} \begin{bmatrix}
\omega_1^2 \\
\omega_2^2 \\
\omega_3^2 \\
\omega_4^2
\end{bmatrix}
\]

where \( K_f = \rho A R^2 C_{T_{stat}} \), \( K_m = \rho A R^3 \left( \frac{\sigma C_{D_0}}{8} + \lambda_{stat} \sigma a \left( \frac{\theta_0}{6} - \frac{\lambda_{stat}}{4} \right) \right) \). Control inputs are proportional to the terms with \( \omega_j^2 \). The other terms dependent linearly on \( \omega_j \) and wind velocities are considered as disturbances. Since we do not know in advance the wind perturbations, then we cannot use these terms in controls.
Upper-bound of the control equation from Jensen’s inequality

\[ \sum_{j=1}^{4} |\omega_j| \leq K \sqrt{|U_z|}, \quad K = \frac{2}{\sqrt{K_f}} \]

Disturbance upper-bounds after substitutions

\[
\begin{align*}
|d_x| & \leq \bar{K}_D (|X| + D_x) \sqrt{|U_z|} \\
|d_y| & \leq \bar{K}_D (|X| + D_y) \sqrt{|U_z|} \\
|d_z| & \leq \bar{K}_z (|X| + D_z) \sqrt{|U_z|} \\
|d_\phi| & \leq \tilde{K}_\phi (f_\phi_1 (X) + D_\phi_1) \sqrt{|U_z|} + \bar{K}_\phi (f_\phi_2 (X) + D_\phi_2) \\
|d_\theta| & \leq \tilde{K}_\theta (f_\theta_1 (X) + D_\theta_1) \sqrt{|U_z|} + \bar{K}_\theta (f_\theta_2 (X) + D_\theta_2) \\
|d_\psi| & \leq \tilde{K}_\psi (f_\psi_1 (X) + D_\psi_1) \sqrt{|U_z|} + \bar{K}_\psi (f_\psi_2 (X) + D_\psi_2)
\end{align*}
\]
Steps to design the altitude control:

- **System in compact form**
  \[
  \ddot{z} = g - (\cos \phi \cos \theta) \frac{1}{m} (U_z + d_z)
  \]

- **Error between reference signal and state value**
  \[
  e_z = z - z_{des}
  \]

- **Derivative of the sliding surface**
  \[
  \dot{S}_z = \ddot{z} + \alpha_z \dot{z} = g - \frac{\cos \theta \cos \phi}{m} (U_z + d_z) + \alpha_z \dot{z}
  \]

- **Control equation**
  \[
  U_z = \frac{m}{\cos \theta \cos \phi} (g - \dot{u}_z + \alpha_z \dot{z}) \leq \frac{m}{\gamma} (|g + \alpha_z \dot{z}| + |\dot{u}_z|)
  \]

- **Derivative of Lyapunov function**
  \[
  \dot{V} = S_z \dot{S}_z \leq S_z \dot{u}_z + |S_z| d_z \frac{1}{m} = S_z \dot{u}_z + |S_z| \frac{1}{m} K_z (|X| + D_z) \sqrt{|U_z|}
  \]
Altitude control

\[
\dot{V} \leq S_z \tilde{u}_z + |S_z| \left( \varrho(X) + \nu(X) \sqrt{|\tilde{u}_z|} \right)
\]

\[
\varrho(X) = \frac{1}{m} \sqrt{\frac{m}{\gamma}} \bar{K}_z (|X| + D_z) \sqrt{|g + \alpha_z \dot{z}|}
\]

\[
\nu(X) = \frac{1}{m} \sqrt{\frac{m}{\gamma}} \bar{K}_z (|X| + D_z)
\]

- Auxiliary control

\[
\tilde{u}_z = -\beta(X) \text{sign}(S_z)
\]

\[
\beta(X) = \frac{1}{2} \left( \nu(X)^2 + 2\varrho(X) + \nu(X) \sqrt{\nu^2(X) + 4\varrho(X)} \right) + \delta
\]

- Finite time stability proved

\[
\dot{V} < -\sqrt{2\delta} \sqrt{V}
\]
- Attitude is equivalent to a control of linear acceleration so it leads to stabilizing the linear speed.

Steps to design the roll control:
- System in compact form

\[
\dot{\phi} = \dot{\theta} \dot{\psi} \frac{l_{yy} - l_{zz}}{l_{xx}} + \frac{1}{l_{xx}} (U_\phi + d_\phi)
\]

- Error between reference signal and state value

\[
e_\phi = \phi - \phi_{des}
\]

- Derivative of the sliding surface

\[
\dot{S}_\phi = \ddot{\phi} + \alpha_\phi \dot{\phi} = \dot{\theta} \dot{\psi} \frac{l_{yy} - l_{zz}}{l_{xx}} + \frac{1}{l_{xx}} (U_\phi + d_\phi) + \alpha_\phi \dot{\phi}
\]

- Control equation

\[
U_\phi = l_{xx} \left( -\dot{\theta} \dot{\psi} \frac{l_{yy} - l_{zz}}{l_{xx}} + \ddot{u}_\phi - \alpha_\phi \dot{\phi} \right)
\]
Derivative of Lyapunov function

\[ \dot{V} = S_\phi \dot{S}_\phi \leq S_\phi \tilde{u}_\phi + |S_\phi| d_\phi \frac{1}{I_{xx}} \]

\[ \leq S_\phi \tilde{u}_\phi + \frac{|S_\phi|}{I_{xx}} \left( \tilde{K}_\phi (f_{\phi 1}(X) + D_{\phi 1}) \sqrt{|U_z|} + \tilde{K}_\phi (f_{\phi 2}(X) + D_{\phi 2}) \right) \]

Auxiliary control

\[ \tilde{u}_\phi = - \frac{1}{I_{xx}} \text{sign} S_\phi \left( \tilde{K}_\phi (f_{\phi 1}(X) + D_{\phi 1}) \sqrt{|U_z|} \right. \]

\[ \left. + \tilde{K}_\phi (f_{\phi 2}(X) + D_{\phi 2}) \right) \]
Simulation data and constraints:

- wind signal
- mass UAV 0.47 Kg
- max rotor speed 400 rad/s
- max thrust rotors 5.6 N
- rotors dynamics $\frac{1}{1+0.03s}$
For a sufficiently small $\phi$, if for a sign function all trajectories converge to an equilibrium, then with a saturation all trajectories converge in a compact set around that equilibrium.

Saturation function:

$$\text{sat}_\phi(x) = \begin{cases} \text{sign}(x) & \text{if } |x| > 1 \\ \arctan \left( \frac{1}{\phi} x \right) & \text{otherwise} \end{cases}$$
• System response to desired input with no wind disturbances

(i) $z$ correction and altitude control

(j) pitch correction and control

(k) roll correction and control

(l) yaw correction and control
Robustness of the proposed control under wind disturbances and convergence

- z correction and altitude control
- Pitch correction and control
- Roll correction and control
- Yaw correction and control
Summarizing:

- Choice of UAV physical model influenced by wind disturbance;
- SMC applied to UAV problems (attitude and altitude) with simplified coefficients equations;
- Robustness of the proposed SMC method with respect to:
  - wind disturbances;
  - uncertainty of identified model parameters;
  - unmodeled rotor dynamics;
- Further work in trajectory considering also $x_{des}, y_{des}, \dot{x}_{des}, \dot{y}_{des}$.

Thank you for your attention!