ON MODELING & ROBUST LPV/$\mathcal{H}_\infty$ BASED OBSERVATION OF FUEL SLOSH DYNAMICS

Application to spacecraft attitude control

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Anthony Bourdelle  
3rd yr. PhD Student

Jean-Marc Biannic  
Research Director

http://w3.onera.fr/smac
1. Introduction to Sloshing in Spacecraft

2. From CFD to LPV models

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5. Observer and Closed-Loop Stability Analysis

6. Conclusion and Future Work
Introduction to Sloshing in Spacecraft
Key Elements [1/2]

- Sloshing: liquid free surface movement inside tanks or containers\(^1\)
- Low frequency and badly damped phenomenon
- Spacecraft carry lifespan-defining mass of liquid propellant
  - e.g. 4% (DEMETER, 2004) to 38% (DAWN, 2007 & Astra 2A, 1998) of launch mass
- Coupled fluid-structure dynamics $\rightarrow$ disruptive forces and torques
- Alteration of spacecraft pointing accuracy
- Compromises system perf. and stability $\rightarrow$ more complex controller design\(^2\)

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Space application → surface tension effects has to be considered

Microgravity conditions are difficult to reproduce in laboratories
e.g. 0G flights or drop towers (short duration ~ 20 s)

Very complex analytical descriptions → Computational Fluid Dynamics

In-situ experiments: Sloshsat-FLEVO (ESA), Spheres (NASA) and Fluidics (ESA)

Flight data have been used to adjust and validate CFD models
e.g. DIVA (IMFT) and COMFLO (University of Groningen)

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Illustration: $\mu$-satellite DEMETER (CNES)
Usual methods and their drawbacks [1/2]

- Baffles and bladders in propellant tanks\(^7\)
  - \(\uparrow\) Increases sloshing frequency and reduces its amplitude
  - \(\downarrow\) Heavier satellite and more expensive mission

- Time margins between aggressive maneuvers to let propellant settle down
  - \(\uparrow\) Avoid propellant over-excitation
  - \(\downarrow\) Reduces mission availability

- Smoothed angular velocity references profiles
  - \(\uparrow\) Reduces propellant excitation
  - \(\downarrow\) Whole satellite agility may no longer be exploited

Usual methods and their drawbacks [2/2]

- **Notch filters**
  - Mitigates sloshing influence
  - Reduces satellites bandwidth (sloshing frequencies are uncertain)

- **Linear Time Invariant Models** (will be further detailed later)
  - Suitable for model-based control
  - Valid only for specific cases and small amplitude motion

- **Infinite-Dimensional Models**
  - More representative
  - Unsuitable for 2D/3D coupled motion in microgravity

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Problem Statement and Proposed Solution

- **Problem Statement**
  - Always more stringent attitude pointing accuracy and stability requirements
  - Need for very effective Attitude Control Systems

- **Proposed Solution**
  - Development of a new model of propellant sloshing torque
  - Observer design to enhance attitude control by compensating torque
From CFD to LPV models
Equivalent Mechanical Models

Approximation of the liquid with a mechanical system\textsuperscript{10}
e.g. spring-mass, pendulum, free-mass or mass constrained on a surface

- Successfully used for decades, for launchers and satellites\textsuperscript{11}
- Can be addressed like flexible modes\textsuperscript{12}
- Model-based controller design\textsuperscript{13}
- Based on linearized fluid dynamics models
- Often valid only for axisymmetric problems with small amplitude motion
- Not dependent on inertial forces acting on the fluid during attitude maneuver

Characterization of Sloshing Torque

Example: IMFT study for several bang-off maneuvers (square shape acc. profile)

Figure 1: Torque $\Gamma_Z$ along the $Z$-axis for a $4.72 \times 10^{-2} \, \text{rad/s}$ steady-state velocity

System: spherical tank, diameter - 0.585 m, filling ratio - 50%, lever arm - 0.4 m

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Parameters affecting sloshing

- **Tank filling ratio**: Thrusters saved for orbital maneuvers \(\rightarrow\) constant filling ratio
- **Gravity vector w.r.t. the spacecraft, linked to the attitude** \(\theta\)
  - Gravity effects can be neglected (microgravity)
- **Liquid properties**, e.g. density, viscosity, surface tension
  - Propellant properties do not change
- **Tank geometry and position inside the spacecraft**
  - Rigid tank with fixed position
- **Angular speed** \(\Omega\) and acceleration \(\dot{\Omega}\)
  - Linked to inertial forces acting on the fluid during attitude maneuvers
We will consider a satellite *bang-off-bang* attitude maneuver around a single axis. Our reasoning can be generalized to any maneuver given appropriate CFD data. Model sloshing disruptive torque instead of propellant behavior. Sloshing torque $\Gamma_F$ as the output of a nonlinear 2$^{nd}$ order system with varying frequency $\omega$ and damping ratio $\epsilon$:

$$\ddot{\Gamma}_F + C_s(\Omega, \dot{\Omega})\dot{\Gamma}_F + K_s(\Omega, \dot{\Omega})\Gamma_F = -A_s(\Omega, \dot{\Omega})\Omega - B_s(\Omega, \dot{\Omega})\dot{\Omega}$$

(1)

$$C_s(\Omega, \dot{\Omega}) = 2\xi(\Omega, \dot{\Omega})\omega(\Omega, \dot{\Omega})$$

(2)

$$K_s(\Omega, \dot{\Omega}) = \omega(\Omega, \dot{\Omega})^2$$

(3)

- Generalization/abstraction of Equivalent Mechanical Models
- Nonlinearity results from the dependence of $A_S$, $B_S$, $C_S$ and $K_S$ to $(\Omega, \dot{\Omega})$
\( A_S, B_S, C_S \) and \( K_S \) can be identified by using CFD results:

- Definition of \( N \) small time intervals
- On each interval \( \Omega \) and \( \dot{\Omega} \) are assumed constant
- On each interval the nonlinear model becomes Linear Time Invariant
- \( \omega \) and \( \epsilon \) can be bounded by analyzing CFD results
- Use a Constrained Least Squares method (Matlab\textsuperscript{TM} \texttt{lsqlin} routine)
- Result: sets \( \{ C_{si}, K_{si}, A_{si}, B_{si} \}_{i \leq N} \) associated to \( \{ \Omega_i, \dot{\Omega}_i \}_{i \leq N} \)
- Note that better results (relative error \( \leq 10\% \)) are obtained by proceeding on two different submodels, one for each side of the acceleration discontinuity
(a) submodel before the discontinuity  (b) submodel after the discontinuity

**Figure 2:** Identification results examples
Sloshing state-space representation:

\[
\begin{pmatrix}
\dot{x}_F \\
\ddot{x}_F
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
-K_S & -C_S
\end{pmatrix}
\begin{pmatrix}
\dot{x}_F \\
\ddot{x}_F
\end{pmatrix} +
\begin{pmatrix}
0 & 0 \\
-A_S & -B_S
\end{pmatrix}
\begin{pmatrix}
\dot{\Omega} \\
\ddot{\Omega}
\end{pmatrix} =
\begin{pmatrix}
0 & 1
\end{pmatrix}
\begin{pmatrix}
x_F
\end{pmatrix} +
\begin{pmatrix}
0 & 0 \\
-A_S & -B_S
\end{pmatrix}
\begin{pmatrix}
\dot{x}_F \\
\ddot{x}_F
\end{pmatrix} +
\begin{pmatrix}
0 & 0 \\
-A_S & -B_S
\end{pmatrix}
\begin{pmatrix}
\dot{\Omega} \\
\ddot{\Omega}
\end{pmatrix}
\begin{pmatrix}
\dot{\Omega}
\end{pmatrix}
\]

(4)

\[\Gamma_F = \begin{pmatrix}
0 & 1
\end{pmatrix} x_F \] (5)

Uncertainties arise from numerical simulation, identification and modeling errors

Poorly known uncertainties → useless to develop accurate model (e.g. LFT based)

Bounded disturbance \( w \) such that \( ||w||_2 \leq \bar{w} \) is introduced:

\[
\dot{x}_F = A_F(K_S, C_S) x_F + B_F(A_S, B_S) \begin{pmatrix}
\dot{\Omega} \\
\ddot{\Omega}
\end{pmatrix} + \begin{pmatrix}
0 \\
1
\end{pmatrix} w
\]

(6)
Single-axis dynamics of an actuated satellite:

\[
\dot{x}_{SAT} = A_{SAT} x_{SAT} + B_{SAT}(\Gamma_F + \Gamma_P + \Gamma_C) \quad (7)
\]

\[
\theta = C_\theta x_{SAT} \quad (8)
\]

\(\Gamma_P\) is a non-sloshing disturbing torque, \(\Gamma_C\) is the control torque.

To also estimated \(\Gamma_P\) the state vector is extended and \(\dot{\Gamma}_P = 0\) is considered.

Further analysis of the identif. results highlights a link between parameters:

\[
B_S = \alpha_{AB} A_S + \beta_{AB} \quad (9)
\]

\[
C_S = \alpha_{KC} K_S + \beta_{KC} \quad (10)
\]
Combining equations → uncertain LPV model of the liquid-filled satellite:

\[ \dot{x} = A(\alpha(t))x + B_u \Gamma_C + \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \begin{bmatrix} \Gamma_C \end{bmatrix} w \]  \hspace{1cm} (11)

\[ \begin{pmatrix} \theta \\ \Gamma_D \end{pmatrix} = C_m x \]  \hspace{1cm} (12)

where:

\[ \alpha(t) = (\alpha_A(t), \alpha_K(t)) \]

\[ = (A_S[\Omega(t), \dot{\Omega}(t)], K_S[\Omega(t), \dot{\Omega}(t)]) \]  \hspace{1cm} (13)

\[ \Gamma_D = \Gamma_F + \Gamma_P \]  \hspace{1cm} (14)

\[ x = [x_F \ x_{SAT} \ \Gamma_P]^T \]  \hspace{1cm} (15)

Filtering effect of the low-pass actuators → param. variations only in the \( A \) matrix
Using $\alpha$ as parameter, instead of $(\Omega, \dot{\Omega})$, has the following advantages:
- $A(\alpha)$ is a linear function of $\alpha$ (simplifies observer design and stab. analysis)
- $A_S, B_S, C_S$ and $K_S$ do not need to be explicitly written as functions of $(\Omega, \dot{\Omega})$

Reactions wheels limitation:
- Bounded control torque capacity
- Restricted variations of $(\Omega(t), \dot{\Omega}(t))$

This permits to characterize a narrowed definition domain for $A_S$ and $K_S$

$\alpha(t)$ takes its values in a polytope $\mathcal{P}$ of 9 vertices $\mathcal{P}_i$, $i \in \{1, 2, \ldots, 9\}$, i.e.

$$\alpha(t) \in \mathcal{P} := \text{Co}\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_9\} \quad (16)$$
$\mathcal{H}_\infty$-based Observer Design
Aim is to enhance attitude control independently of any existing controller

Decoupling of the satellite from sloshing dynamics obtained by canceling the disturbing torques estimate from the control input

Solution: design of a reliable LPV observer

Estimated torque has to be accurate in spite of model disturbances $w$

Observer has to compensate the small delay induced by actuators dynamics

Observer state-space representation:

$$\dot{x} = A(\alpha(t)) \dot{x} + B_u \Gamma_C + L(\alpha(t))(\theta - \hat{\theta})$$

$$= (A(\alpha) - L(\alpha) C_m) \dot{x} + [B_u L(\alpha)][\Gamma_C \theta]^T$$

$$\hat{\Gamma}_D = C_z x + [0 0][\Gamma_C \theta]^T$$

where $\dot{x}$ and $\hat{\Gamma}_D$ are $x$ and $\Gamma_D$ estimates, $L(\alpha)$ is the observer gain
Dynamics of the state error:

\[
\begin{align*}
\dot{\epsilon} &= A_{\text{obs}} x + B_w w, \quad \epsilon = x - \hat{x} \\
(S) &\quad z = C_z \epsilon \\
&\quad = \Gamma_D - \hat{\Gamma}_D
\end{align*}
\tag{20}
\tag{21}
\tag{22}
\]

\(A(\alpha)\) is a linear function of \(\alpha\):

\[
A(\alpha) = A_0 + \alpha_A A_A + \alpha_K A_K
\tag{23}
\]

Thus we propose to search a structured observer gain:

\[
L(\alpha) = L_0 + \alpha_A L_A + \alpha_K L_K
\tag{24}
\]

The system has then an affine LPV structure
Characterization as a multi-model $\mathcal{H}_\infty$ design problem [2/4]

- Recall: $\alpha(t) \in \mathcal{P} := \text{Co}\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_9\}$
- Affine LPV structure $\rightarrow$ a polytopic model can be easily deduced:

\[
\alpha = \sum_{i=1}^{9} \beta_i \mathcal{P}_i, \quad \beta_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{9} \beta_i = 1
\]  

(25)

\[
S(\alpha) = \sum_{i=1}^{9} \beta_i S(\mathcal{P}_i)
\]  

(26)

Figure 3: Vertices of the polytopic model
Characterization as a multi-model $\mathcal{H}_\infty$ design problem [3/4]

- Approach suitable to be addressed by a $\mathcal{H}_\infty$ multi-model robust design techniques on the 9 LTI models ($S_{i\leq 9}$) (LPV system frozen at the vertices $P_{i\leq 9}$)\(^{15}\)
- With **systune** Matlab\textsuperscript{TM} routine\(^{16,17}\) it is possible:
  - to compute bounded gains $L_0$, $L_A$ and $L_K$
  - to minimize the estimation error
  - to constrain the observer/error dynamics
- Remark: A resolution is also possible via extended LMI-based LPV techniques to be proposed for LPVS 2019

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Figure 4: Design model block diagram
The following constraints have been defined for the $\mathcal{H}_\infty$ problem:

- Minimum decay rate: $0.001$ rad/s
- Minimum damping ratio: $0.7$
- Maximum observer frequency: $5$ rad/s
- Absolute value of gains < 2

Error signal is weighted by a low-pass transfer function $W_z(s)$ to minimize the steady-state estimated torque error:

$$W_z(s) = 2 \frac{0.01}{s + 0.01}$$  \hspace{1cm} (27)

The model disturbance $w$ is weighted by a constant filter $W_w(s) = 0.01$

Actuators induced delays compensated by augmenting $z$ with a derivative term:

$$z = (\Gamma_D - \hat{\Gamma}_D) + E(\hat{\Gamma}_F - \dot{\hat{\Gamma}}_F)$$  \hspace{1cm} (28)

where the gain $E$ is tuned according to the characteristics of the actuator.
Illustration
Required attitude control perf. inspired by DEMETER satellite bus benchmark\textsuperscript{18}:

- Pointing steady-state error $< 0.04$ deg
- Pointing rate steady-state error $< 0.1$ deg/s
- Angular momentum $< 0.12$ Nms
- Control torque $< 0.005$ Nm

Satellite inertia $I_z = 30$ kg.m$^2$

Satellite controlled by a PD controller satisfying in the absence of sloshing:

$$\Gamma_C = 0.3553\delta_\theta + 6.2845\delta_\Omega$$ \hbox{(29)}

The actuator is a reaction wheel modeled by the following transfer function:

$$RWS(s) = \frac{1.2s + 0.76}{s^2 + 2.4s + 0.76}$$ \hbox{(30)}

To get faster responses, a guidance torque $\Gamma_d$ is added in a feed-forward path.

Figure 5: Parameter-varying closed-loop model block diagram
Simulation results - Disturbing torque

![Graph showing disturbing torque over time](image)

- $\Gamma_d$ est. & filt.
- $\Gamma_d$

Disturbing torque (Nm)

Time (sec)
Simulation results - Attitude

**Figure 6**: Start of the maneuver

**Figure 7**: Reach of steady-state

**Figure 8**: Error
Simulation results - Angular velocity

**Figure 9:** Start of the maneuver

**Figure 10:** Reach of steady-state

**Figure 11:** Error
Simulation results - Control torque and angular momentum requirements

**Figure 12:** Control torque

**Figure 13:** Angular momentum
Simulation results - Robustness to Parameters Errors $P \rightarrow P + \Delta_P$

Figure 14: Est. Dist. Torque - $\Delta = 0\%$

Figure 15: Est. Dist. Torque - $\Delta = 30\%$

Figure 16: Attitude Error - Comparison
Observer and Closed-Loop Stability Analysis
- No theoretical guarantee regarding time-varying stability
- Stability has then be checked \textit{a posteriori}
- Achieved with quadratic and Parameter-Dependent Lyapunov functions (PDLF)
Stability verified, independently of the rate of variation of the parameters, if a symmetric positive definite matrix $P_{Obs} > 0$ can be found such that:

$$A_{Obs}(\alpha)^T P_{Obs} + P_{Obs} A_{Obs}(\alpha) < 0, \ \forall \alpha \in \mathcal{P}$$  \hspace{1cm} (31)

- Polytopic approach → condition reduces to 9 Linear Matrix Inequalities (LMI):

$$A_{Obs}(\mathcal{P}_i)^T P_{Obs} + P_{Obs} A_{Obs}(\mathcal{P}_i) < 0, \ \ i = 1, \ldots, 9$$  \hspace{1cm} (32)

- The observer has 7 states → $7 \times 8/2 = 28$ decision variable

- Problem solved using the feasp Matlab \textsuperscript{T} M LMI solver

- Observer is quadratically stable
The closed-loop plant dynamics can be described by a matrix $A_{CL}(\alpha) \in \mathbb{R}^{13 \times 13}$

This matrix has the same properties as the observer $A$ matrix, thus:

$$A_{CL}(\alpha) \in \mathcal{O}\{A_{CL}(\mathcal{P}_1), \ldots, A_{CL}(\mathcal{P}_9)\} \quad (33)$$

Quadratic stability is too conservative in this case and could not be established.

PDLF $P(\alpha)$ taking into account the parameters variation rate is needed:

$$P(\alpha) = P_0 + \alpha_A P_A + \alpha_K P_K + \alpha_A \alpha_K P_{AK} + \alpha_A^2 P_{A_2} + \alpha_K^2 P_{K_2} \quad (34)$$

With $|\dot{\alpha}_A| < \rho_A$ and $|\dot{\alpha}_K| < \rho_K$, new stability conditions are obtained as, $\forall \alpha \in \mathcal{P}$:

$$A(\alpha)^T P(\alpha) + P(\alpha) A(\alpha)$$

$$\pm \rho_A (P_A + \alpha_K P_{AK} + 2\alpha_A P_{A_2}) \quad (35)$$

$$\pm \rho_K (P_K + \alpha_A P_{AK} + 2\alpha_K P_{K_2}) < 0$$

$$P(\alpha) > 0 \quad (36)$$
Both inequalities are nonlinear functions (second-order polynomial)

A finite set of LMIs is obtained by searching $P_0, P_A, \ldots, P_{K_2}$ on a given grid

*a posteriori* verif. that constraints are satisfied everywhere inside the polytope

Test be performed by computing $\mu$ upper and lower bounds

A grid with 84 points has been considered:
- $5 \times 84 = 420$ LMIs and $6 \times 13 \times 14/2 = 546$ decision variables
- $\rho_A = 5.6 \times 10^{-4}$ and $\rho_K = 2.5 \times 10^{-3}$

Solution has been found and validated with $\mu$ test in less than 5 min

Closed-loop is asymptotically stable

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Conclusion and Future Work
New way to model sloshing disturbing torque as an LPV system
Model successfully exploited to design an LPV torque observer
Pert. compensation to enhance existing controller designed without sloshing
Observer quadratic stability over the parametric domain
Closed-loop asymptotic stability with PDLF
Proposed for ACA 2019
Study case corresponds to a tank larger than the one fitted to DEMETER satellite

This tank is wall less and half-filled (worst case scenario)

Despite such conditions our approach succeeded in reducing attitude error

Likely the use of this approach could permit to reduce tank complexity and mass

Control torque and angular momentum max. values are sometimes exceeded

Future work: address this issue with reference governors\textsuperscript{20} to adapt the reference

Proposed for EUCASS 2019

Thank you for your attention!
Questions and comments are welcomed!
→ Anthony.Bourdelle@onera.fr