# LMI CONDITIONS FOR STABILITY AND $\mathcal{H}_\infty$ CONTROL OF DISCRETE-TIME MULTI-MODE MULTI-DIMENSIONAL SYSTEMS

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# Motivation

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Personal reasons of interest in this topic:

- Safe and stable transition between ADAS-Autonomous mode in vehicle applications.
- Standard switching control theory not directly applicable due to inconsistency between matrix dimensions.
- Need to develop methods to deal with this problematic.

# Objectives

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Extend well known LMI based results to cover $M^3D$ systems for:

- Proof of Stability
- Computation of the $\mathcal{H}_\infty$ norm
- Synthesis of State-Feedback controllers

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$M^3D$[1] $\equiv$ Multi-Mode Multi-Dimensional

[1] Verriest, Erik I. "Multi-mode multi-dimensional systems." Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems. 2006.

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$M^3D$[1] $\equiv$ Multi-Mode Multi-Dimensional
In this work we consider the active mode $i$ given as:

$$M^{(i)} = \begin{cases} x_{k+1}^{(i)} = \mathcal{A}^{(i)} x_k^{(i)} + \mathcal{B}^{(i)} w_k \\ z_k = \mathcal{C}^{(i)} x_k^{(i)} + \mathcal{D}^{(i)} w_k \end{cases} \tag{1}$$

[1] Verriest, Erik I. "Multi-mode multi-dimensional systems." Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems. 2006.

# Discrete-Time $M^3D$ Systems II

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The $M^3D$ system mode transition from mode $i$ to mode $j$ is defined by introducing the state mapping $T_{ji}$ as:

$$x^{(j)} = T_{ji} x^{(i)}, \quad T_{ji} \in \mathbb{R}^{n_j \times n_i} \tag{2}$$

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The $M^3D$ system mode transition from mode $i$ to mode $j$ is defined by introducing the state mapping $T_{ji}$ as:

$$x^{(j)} = T_{ji} x^{(i)}, \quad T_{ji} \in \mathbb{R}^{n_j \times n_i} \tag{2}$$

The system dynamics during the $M^3D$ transition from mode $i$ to $j$ are then given as:

$$M^{(ji)} = \begin{cases} x_{k+1}^{(j)} = T_{ji} x_{k+1}^{(i)} = T_{ji} \mathcal{A}^{(i)} x_k^{(i)} + T_{ji} \mathcal{B}^{(i)} w_k \\ z_k = \mathcal{C}^{(i)} x_k^{(i)} + \mathcal{D}^{(i)} w_k \end{cases} \tag{3}$$

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Each mode has associated a poly-quadratic energy function of the type:

$$V^{(i)}(x_k^{(i)}) = x_k^{(i)^T} X^{(i)} x_k^{(i)}, \tag{4}$$

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Each mode has associated a poly-quadratic energy function of the type:

$$V^{(i)}(x_k^{(i)}) = x_k^{(i)^T} X^{(i)} x_k^{(i)}, \tag{4}$$

For stability, the energy function must fulfill at the switching instance:

$$V^{(j)}(x_{k+1}^{(j)}) = V^{(j)}(T_{ji} x_{k+1}^{(i)}) \leq V^{(i)}(x_k^{(i)}) \tag{5}$$

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## Theorem

*A $M^3D$ discrete-time autonomous system $M$, is stable if, for each mode $i = 1, \ldots, m$ of $M$ there exist matrices $Q^{(i)} = Q^{(i)^T} > 0$, with $Q^{(i)} \in \mathbb{R}^{n_i \times n_i}$, and $G^{(i)} \in \mathbb{R}^{n_i \times n_i}$ such that the following conditions are satisfied:*

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$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T} \mathcal{A}^{(i)^T} \\ * & Q^{(i)} \end{bmatrix} \geq 0 \qquad (6)$$

$$\forall i \text{ mode}$$

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$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T} \mathcal{A}^{(i)^T} \\ * & Q^{(i)} \end{bmatrix} \geq 0 \qquad (6)$$

$\forall i$ *mode*

$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T} \mathcal{A}^{(i)^T} T_{ij} \\ * & Q^{(j)} \end{bmatrix} \geq 0 \qquad (7)$$

$\forall (i,j)$ *connected pair of modes,* $i \neq j$

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Consider the active mode restricted to the autonomous dynamics:

$$M^{(i)} = \left\{ x_{k+1}^{(i)} = \mathcal{A}^{(i)} x_k^{(i)} \right. \quad (8)$$

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$$M^{(i)} = \left\{ x_{k+1}^{(i)} = \mathcal{A}^{(i)} x_k^{(i)} \right. \quad (8)$$

During the $M^3D$ transition the dynamics then are:

$$M^{(ji)} = \left\{ x_{k+1}^{(j)} = T_{ji} \mathcal{A}^{(i)} x_k^{(i)} \right. \quad (9)$$

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During the $M^3D$ transition the dynamics then are:

$$M^{(ji)} = \left\{ x_{k+1}^{(j)} = T_{ji} \mathcal{A}^{(i)} x_k^{(i)} \right. \tag{9}$$

According to the energy limited condition during the switching instance we have:

$$x_{k+1}^{(j)^T} X^{(j)} x_{k+1}^{(j)} - x_k^{(i)^T} X^{(i)} x_k^{(i)} \leq 0 \tag{10}$$

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According to the energy limited condition during the switching instance we have:

$$x_{k+1}^{(j)^T} X^{(j)} x_{k+1}^{(j)} - x_k^{(i)^T} X^{(i)} x_k^{(i)} \leq 0 \tag{10}$$

which leads to

$$x_k^{(i)^T} \left[ \mathcal{A}^{(i)^T} T_{ji}^T X^{(j)} T_{ji} \mathcal{A}^{(i)} - X^{(i)} \right] x_k^{(i)} \leq 0 \tag{11}$$

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Condition (11) can be eventually be modified into the following equivalent condition:

$$\begin{bmatrix} G^{(i)^T} X^{(i)} G^{(i)} & G^{(i)^T} \mathcal{A}^{(i)^T} T_{ji}^T \\ * & X^{(j)^{-1}} \end{bmatrix} \geq 0. \tag{12}$$

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Now, assume that condition (7) from the Theorem is true, with $X^{-1} \equiv Q$

$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - X^{(i)^{-1}} & G^{(i)^T} \mathcal{A}^{(i)^T} T_{ji}^T \\ * & X^{(j)^{-1}} \end{bmatrix} \geq 0, \tag{13}$$

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using the simplified Young's relation:

$$G^{(i)^T} X^{(i)} G^{(i)} \geq G^{(i)^T} + G^{(i)} - X^{(i)^{-1}}$$

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using the simplified Young's relation:

$$G^{(i)^T} X^{(i)} G^{(i)} \geq G^{(i)^T} + G^{(i)} - X^{(i)^{-1}}$$

then (13) is a sufficient condition for (12), and thus for the stability of $M$ during a $M^3D$ transition. ∎

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## Remark

*Condition (6) recovers the stability proof for discrete-time LTI system in:*

- *De Oliveira, M. C., Bernussou, J., & Geromel, J. C. (1999). A new discrete-time robust stability condition. Systems & control letters.*

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- *De Oliveira, M. C., Bernussou, J., & Geromel, J. C. (1999). A new discrete-time robust stability condition. Systems & control letters.*

## Remark

*Considering $T_{ji} = I$, the previous theorem gives sufficient conditions with dwell-time $\Delta_* = 1$ for results presented in:*

- *Geromel, J. C., Colaneri, P. (2006). Stability and stabilization of discrete time switched systems. International Journal of Control.*

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# $M^3D$ System Recall

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Discrete-time $M^3D$ system $M$ given as:

$$M^{(i)} = \begin{cases} x_{k+1}^{(i)} = \mathcal{A}^{(i)} x_k^{(i)} + \mathcal{B}^{(i)} w_k \\ z_k = \mathcal{C}^{(i)} x_k^{(i)} + \mathcal{D}^{(i)} w_k \end{cases} \tag{14}$$

# $M^3D$ System Recall


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Discrete-time $M^3D$ system $M$ given as:

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During the $M^3D$ transition, the dynamics are given as:

$$M^{(ji)} = \begin{cases} x_{k+1}^{(j)} = T_{ji} \mathcal{A}^{(i)} x_k^{(i)} + T_{ji} \mathcal{B}^{(i)} w_k \\ z_k = \mathcal{C}^{(i)} x_k^{(i)} + \mathcal{D}^{(i)} w_k \end{cases} \tag{15}$$

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## Theorem

*Given a discrete $M^3D$ system $M$ and positive scalar $\gamma_\infty$, if for each mode $i = 1, \dots, m$ of $M$ there exist matrices $Q^{(i)} = {Q^{(i)}}^T > 0$, with $Q^{(i)} \in \mathbb{R}^{n_i \times n_i}$, and $G^{(i)} \in \mathbb{R}^{n_i \times n_i}$ such that the following LMI problem is feasible:*

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$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T}\mathcal{A}^{(i)^T} & G^{(i)^T}\mathcal{C}^{(i)^T} & 0 \\ * & Q^{(i)} & 0 & \mathcal{B} \\ * & * & \gamma_\infty I & \mathcal{D} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \qquad (16)$$

$\forall i$ *mode*

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## Theorem

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$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T}\mathcal{A}^{(i)^T} & G^{(i)^T}\mathcal{C}^{(i)^T} & 0 \\ * & Q^{(i)} & 0 & \mathcal{B} \\ * & * & \gamma_\infty I & \mathcal{D} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \qquad (16)$$

$\forall i$ *mode*

$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & G^{(i)^T}\mathcal{A}^{(i)^T}T_{ji}^T & G^{(i)^T}\mathcal{C}^{(i)^T} & 0 \\ * & Q^{(j)} & 0 & T_{ji}\mathcal{B} \\ * & * & \gamma_\infty I & \mathcal{D} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \qquad (17)$$

$\forall (i,j)$ *connected pair of modes,* $i \neq j$

# $\mathcal{H}_\infty$ Norm Computation


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## Theorem

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$$\begin{bmatrix} {G^{(i)}}^T + G^{(i)} - Q^{(i)} & {G^{(i)}}^T {\mathcal{A}^{(i)}}^T & {G^{(i)}}^T {\mathcal{C}^{(i)}}^T & 0 \\ * & Q^{(i)} & 0 & \mathcal{B} \\ * & * & \gamma_\infty I & \mathcal{D} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \qquad (16)$$

$$\forall i \text{ mode}$$

$$\begin{bmatrix} {G^{(i)}}^T + G^{(i)} - Q^{(i)} & {G^{(i)}}^T {\mathcal{A}^{(i)}}^T T_{ji}^T & {G^{(i)}}^T {\mathcal{C}^{(i)}}^T & 0 \\ * & Q^{(j)} & 0 & T_{ji}\mathcal{B} \\ * & * & \gamma_\infty I & \mathcal{D} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \qquad (17)$$

$$\forall (i, j) \text{ connected pair of modes, } i \neq j$$

*The given positive scalar $\gamma_\infty$ is an upper bound of the $\mathcal{H}_\infty$ norm of $M$, such that $\|M\|_\infty \leq \gamma_\infty$. If the optimal $\gamma_\infty$ is required, the LMI minimization problem for $\gamma_\infty$ is still an LMI problem with variables $\gamma_\infty$, $Q$ and $G$.*

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Introduce the state-feedback control law

$$u_k = K^{(i)} x_k^{(i)}, \tag{18}$$

Let us consider the discrete-time $M^3D$ system $N$, where the dynamics of the active mode $i$ are:

$$N^{(i)} = \begin{cases} x_{k+1}^{(i)} = A^{(i)} x_k^{(i)} + B_u^{(i)} u_k + B_w^{(i)} w_k \\ z_k = C_z^{(i)} x_k^{(i)} + D_u^{(i)} u_k + D_w^{(i)} w_k \end{cases} \tag{19}$$

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## Theorem

*Given a discrete $M^3D$ system $N$ and positive scalar $\gamma_\infty$, if for each mode $i = 1, \ldots, m$ of $N$ there exist matrices $Q^{(i)} = Q^{(i)^T} > 0$, with $Q^{(i)} \in \mathbb{R}^{n_i \times n_i}$, $G^{(i)} \in \mathbb{R}^{n_i \times n_i}$ and $Y^{(i)} \in \mathbb{R}^{n_u \times n_i}$ such that the following LMI conditions are satisfied:*

$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & \Psi^{(i)}_{\mathbf{1},\mathbf{2}} & \Psi^{(i)}_{\mathbf{1},\mathbf{3}} & 0 \\ * & Q^{(i)} & 0 & B_w^{(i)} \\ * & * & \gamma_\infty I & D_w^{(i)} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \tag{20}$$

*with*

$$\Psi^{(i)}_{\mathbf{1},\mathbf{2}} = G^{(i)^T} A^{(i)^T} + Y^{(i)^T} B_u^{(i)^T}, \; \Psi^{(i)}_{\mathbf{1},\mathbf{3}} = G^{(i)^T} C_z^{(i)^T} + Y^{(i)^T} D_u^{(i)^T}$$

$\forall i$ *mode*

$$\begin{bmatrix} G^{(i)^T} + G^{(i)} - Q^{(i)} & \Psi^{(ji)}_{\mathbf{1},\mathbf{2}} & \Psi^{(ji)}_{\mathbf{1},\mathbf{3}} & 0 \\ * & Q^{(j)} & 0 & T_{ji} B_w^{(i)} \\ * & * & \gamma_\infty I & D_w^{(i)} \\ * & * & * & \gamma_\infty I \end{bmatrix} \geq 0 \tag{21}$$

*with*

$$\Psi^{(ji)}_{\mathbf{1},\mathbf{2}} = G^{(i)^T} A^{(i)^T} T_{ji}^T + Y^{(i)^T} B_u^{(i)^T} T_{ji}^T, \; \Psi^{(ji)}_{\mathbf{1},\mathbf{3}} = G^{(i)^T} C_z^{(i)^T} + Y^{(i)^T} D_u^{(i)^T}$$

$\forall (i,j)$ *connected pair of modes,* $i \neq j$

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In the previous theorem we use the relations:

$$\begin{aligned} \mathcal{A}^{(i)} &= A^{(i)} + B_u^{(i)} K^{(i)} & \mathcal{B}^{(i)} &= B_w^{(i)} \\ \mathcal{C}^{(i)} &= C_z^{(i)} + D_u^{(i)} K^{(i)} & \mathcal{D}^{(i)} &= D_w^{(i)} \end{aligned} \tag{22}$$

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With the introduction of the linearizing change of variables

$$Y^{(i)} = K^{(i)} G^{(i)} \tag{23}$$

# State-Feedback III


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With the introduction of the linearizing change of variables

$$Y^{(i)} = K^{(i)} G^{(i)} \tag{23}$$

If the LMI problem is feasible, the controller is then reconstructed according to:

$$K^{(i)} = Y^{(i)} {G^{(i)}}^{-1} \tag{24}$$

# Some Remarks

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## Remark

*Condition (20) recovers the synthesis LMI for discrete-time LTI system in:*

- *De Oliveira, M. C., Geromel, J. C., & Bernussou, J. (1999, December). An LMI optimization approach to multiobjective controller design for discrete-time systems. In Proceedings of the 38th IEEE Conference on Decision and Control.*

# Some Remarks

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## Remark

*To reduce conservatism, a new slack variable can be introduced* $Y^{(ji)} = K^{(ji)}G^{(ji)}$ *in LMI condition (21), such that the state-feedback controller* $K^{(ji)} = Y^{(ji)}G^{(ji)^{-1}}$ *is active during the transition from mode* $i$ *to mode* $j$.

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A system with two modes considered:

1. Autonomous Vehicle Steering Problem
2. Driver with ADAS controller for lateral assistance

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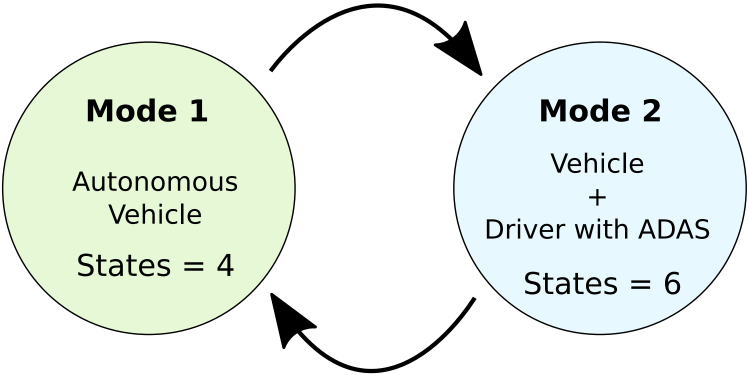
A system with two modes considered:

1. Autonomous Vehicle Steering Problem
2. Driver with ADAS controller for lateral assistance



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States of Mode 1:

$$x(k)^{(1)} = [v_y \; r \; x_e \; x_u]^T \tag{25}$$

# Problem Introduction II

States of Mode 1:

$$x(k)^{(1)} = [v_y \ r \ x_e \ x_u]^T \tag{25}$$

States of Mode 2:

$$x(k)^{(2)} = [v_y \ r \ x_e \ \hat{x}_{u1} \ \hat{x}_{u1} \ x_d]^T \tag{26}$$

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States of Mode 1:

$$x(k)^{(1)} = [v_y \; r \; x_e \; x_u]^T \tag{25}$$

States of Mode 2:

$$x(k)^{(2)} = [v_y \; r \; x_e \; \hat{x}_{u1} \; \hat{x}_{u1} \; x_d]^T \tag{26}$$

State transition map from mode 2 to mode 1:

$$T_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

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States of Mode 1:

$$x(k)^{(1)} = [v_y \ r \ x_e \ x_u]^T \tag{25}$$

States of Mode 2:

$$x(k)^{(2)} = [v_y \ r \ x_e \ \hat{x}_{u1} \ \hat{x}_{u1} \ x_d]^T \tag{26}$$

State transition map from mode 2 to mode 1:

$$T_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

and in the other direction

$$T_{21} = T_{12}^T \tag{28}$$

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1. Each mode has an independent controller

# Control Scenarios

1. Each mode has an independent controller
2. Co-design of controllers using the given SF Theorem

# Control Scenarios

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1. Each mode has an independent controller
2. Co-design of controllers using the given SF Theorem
3. Controller for mode 2 given, controller for mode 1 designed using the given SF Theorem using the information of the mode 2 controller

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1. Each mode has an independent controller
2. Co-design of controllers using the given SF Theorem
3. Controller for mode 2 given, controller for mode 1 designed using the given SF Theorem using the information of the mode 2 controller

*For solutions 2 and 3, we used transition specific slack variables $Y^{(ji)}$ and $G^{(ji)}$.

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- Full car dynamics from Renault Megane car model simulation

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- Full car dynamics from Renault Megane car model simulation
- Human driver simulated using a driver model

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- Full car dynamics from Renault Megane car model simulation
- Human driver simulated using a driver model
- First section of Montmelò circuit

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- Full car dynamics from Renault Megane car model simulation
- Human driver simulated using a driver model
- First section of Montmelò circuit
- The vehicle starts in Autonomous mode (mode 1),

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- Full car dynamics from Renault Megane car model simulation
- Human driver simulated using a driver model
- First section of Montmelò circuit
- The vehicle starts in Autonomous mode (mode 1), switch to Driver steering (mode 2) to deal with the chicane,

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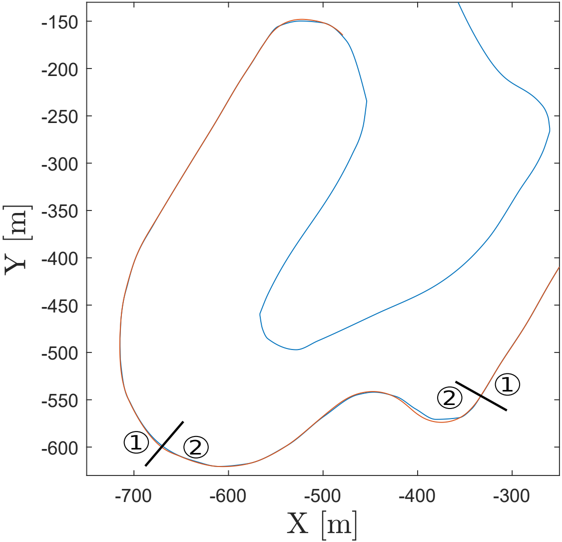
- Full car dynamics from Renault Megane car model simulation
- Human driver simulated using a driver model
- First section of Montmelò circuit
- The vehicle starts in Autonomous mode (mode 1), switch to Driver steering (mode 2) to deal with the chicane, back to Autonomous mode after
- Constant speed $v_x = 20m/s$, $72km/h$

# Solution 1 - No critical transition





# Solution 1 - Critical transition

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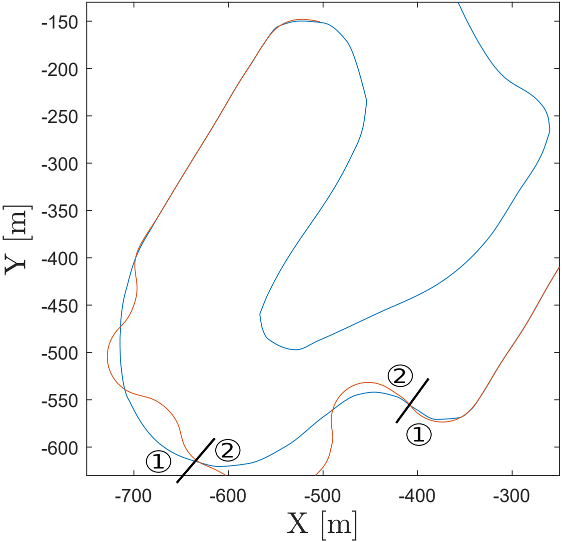

# Solution 2 - Critical transition

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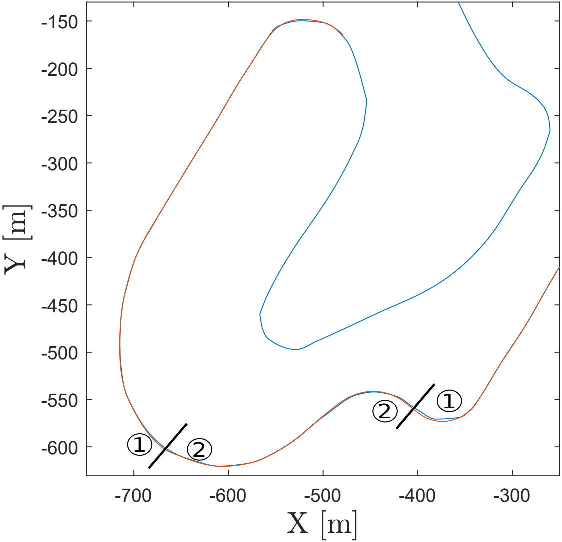

# Solution 3 - Critical transition


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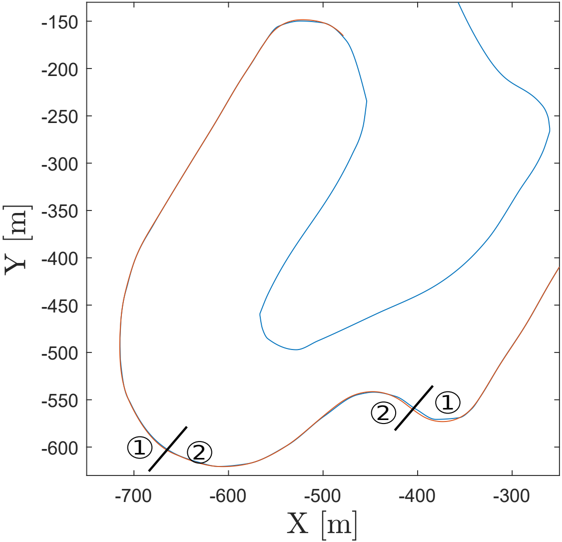

# Solution 2 - Control Action

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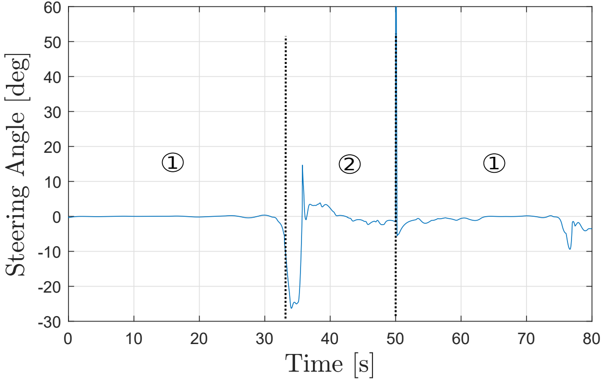


- Should be noted that $K^{(ji)}$ was not implemented

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- Provided theory allows for stable $M^3D$ transitions, even in critical scenarios.
- Improvements required for actuator smoothness during the transition.

# Perspectives

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- Work submitted to CDC with option to publish in the L-CSS journal.
- In the future we plan to deal with the state jump issue
- Extend to LPV systems
- This work opens the door to study multiple problems: Fault Tolerant Control in case of actuator loss, platoon control, etc ...

Thanks for your attention!