Invariance and symbolic control on monotone systems
application to intelligent buildings

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Outline

1. Temperature model and monotonicity
2. Invariance (CDC13)
3. Application (BuildSys13, ECC14)
4. Stabilization
5. Symbolic control
Underfloor Air Distribution

- Underfloor air cooled down
- Sent into the rooms by fans
- Air excess pushed through the ceiling exhausts
- Returned to the underfloor
- Disturbances: heat sources; opening of doors
Model

Temperature variations in room $i$:

- energy conservation;
- mass conservation.
Temperature variations in room $i$:

\[
\frac{dT_i}{dt} = \sum_j a_{i,j} (T_j - T_i) + b_i u_i (T_u - T_i) + \sum_j \delta_{dij} c_{i,j} * h(T_j - T_i) + \delta_{s_i} d_i (T_{s_i} - T_i^4) \]

Conduction through walls

Controlled fan air flow $u_i$

Open doors (flow hot→cold)

Radiation from heat sources
Model

Temperature variations in room $i$:

\[
\frac{dT_i}{dt} = \sum_j a_{i,j} (T_j - T_i) + b_i u_i (T_u - T_i) + \sum_j \delta_{d_{ij}} c_{i,j} \ast h(T_j - T_i) + \delta_{s_i} d_i (T_{s_i}^4 - T_i^4)
\]

- $a, b, c, d > 0$
- $\delta_s, \delta_d$: discrete state of the disturbances (heat sources and doors);
- \[
\begin{cases}
  h(x \leq 0) = 0 \\
  h(x > 0) = x^{3/2}
\end{cases}
\] : door heat transfer only in the colder room.
Generic system $\dot{x} = f(x, v)$ with trajectories $\Phi(t, x, v)$.

**Definition (Monotonicity)**

The system $\Phi$ is monotone if its trajectories preserve some partial orders:

\[ v \preceq_v v', \ x \preceq_x x' \Rightarrow \forall t \geq 0, \ \Phi(t, x, v) \preceq_x \Phi(t, x', v') \]
Monotonicity

Generic system $\dot{x} = f(x, v)$ with trajectories $\Phi(t, x, v)$.

**Definition (Partial order)**

$x \succeq_x x' \iff \forall i, (-1)^{\varepsilon_i} (x_i - x'_i) \geq 0$, with $\varepsilon_i \in \{0, 1\}$
Monotonicity

Generic system $\dot{x} = f(x, \nu)$ with trajectories $\Phi(t, x, \nu)$.

**Definition (Partial order)**

$x \succeq x' \iff \forall i, (-1)^{\varepsilon_i} (x_i - x'_i) \geq 0$, with $\varepsilon_i \in \{0, 1\}$

**Proposition (Angeli and Sontag, 2003)**

The system defined by $\dot{x} = f(x, \nu)$ is monotone if and only if,

$$
\forall x \in \mathbb{R}^n, \forall \nu \in \mathbb{R}^m, \left\{ \begin{array}{l}
(-1)^{\varepsilon_i + \varepsilon_j} \frac{\partial f_i}{\partial x_j}(x, \nu) \geq 0, \hspace{1cm} \forall i, \forall j \neq i, \\
(-1)^{\varepsilon_i + \gamma_k} \frac{\partial f_i}{\partial \nu_k}(x, \nu) \geq 0, \hspace{1cm} \forall i, \forall k.
\end{array} \right.
$$

Where $\varepsilon \in \{0, 1\}^n$ and $\gamma \in \{0, 1\}^m$ define the partial orders for $x$ and $\nu$. 
Monotonicity

Our model: \( \dot{T} = f(T, u, w, \delta) \)
- \( T \): state (temperature);
- \( u \): controlled input (fan air flow);
- \( w \): exogenous input (other temperatures);
- \( \delta \): discrete disturbance embedded in a continuous space.
Monotonicity

Our model: \( \dot{T} = f(T, u, w, \delta) \)
- \( T \): state (temperature);
- \( u \): controlled input (fan air flow);
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- \( \delta \): discrete disturbance embedded in a continuous space.

\[
T \succeq_T T' \iff \forall i, T_i \geq T'_i \\
u \succeq_u u' \iff \forall t \geq 0, \forall k, u_k(t) \leq u'_k(t) \\
w \succeq_w w' \iff \forall t \geq 0, \forall k, w_k(t) \geq w'_k(t) \\
\delta \succeq_\delta \delta' \iff \forall t \geq 0, \forall k, \delta_k(t) \geq \delta'_k(t)
\]

\[
\Phi(t, T, u, w, \delta) \succeq_T \Phi(t, T', u', w', \delta')
\]
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Pierre-Jean Meyer (Grenoble)
Robust Invariance

Definition (Robust Invariance)

The system is *Robust Invariant* in an interval \([T_r, \overline{T}_r]\) if,

\[
\forall T_0 \in [T_r, \overline{T}_r], \forall w \in [\underline{w}, \overline{w}], \forall \delta \in [\underline{\delta}, \overline{\delta}], \forall u \in [\underline{u}, \overline{u}], \\
\forall t \geq 0, \Phi(t, T_0, u, w, \delta) \in [T_r, \overline{T}_r].
\]

Proposition

*The minimal Robust Invariant interval* \([T_r, \overline{T}_r]\) *is given by*

\[
\begin{align*}
\left\{ f(T_r, u, \overline{w}, \delta) &= 0 \\
f(T_r, \overline{u}, w, \delta) &= 0
\right.
\end{align*}
\]
Robust Controlled Invariance

**Definition (Robust Controlled Invariance)**

The system is *Robust Controlled Invariant* in $[\underline{T}, \overline{T}]$ if,

$$\forall T_0 \in [\underline{T}, \overline{T}], \forall w \in [\underline{w}, \overline{w}], \forall \delta \in [\underline{\delta}, \overline{\delta}],$$

$$\exists u \in [\underline{u}, \overline{u}] \mid \forall t \geq 0, \Phi(t, T_0, u, w, \delta) \in [\underline{T}, \overline{T}].$$
Robust Controlled Invariance

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The system is *Robust Controlled Invariant* in \([T, \overline{T}]\) if,

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\forall T_0 \in [T, \overline{T}], \ \forall w \in [\underline{w}, \overline{w}], \ \forall \delta \in [\underline{\delta}, \overline{\delta}],
\exists u \in [\underline{u}, \overline{u}] \mid \forall t \geq 0, \ \Phi(t, T_0, u, w, \delta) \in [T, \overline{T}].
\]
Definition (Robust Controlled Invariance)

The system is *Robust Controlled Invariant* in $[T, \overline{T}]$ if,

$$\forall T_0 \in [T, \overline{T}], \forall w \in [w, \overline{w}], \forall \delta \in [\delta, \overline{\delta}],$$

$$\exists u \in [u, \overline{u}] \mid \forall t \geq 0, \Phi(t, T_0, u, w, \delta) \in [T, \overline{T}].$$
Robust Controlled Invariance

Definition (Robust Controlled Invariance)

The system is Robust Controlled Invariant in \([T, \overline{T}]\) if,

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\forall T_0 \in [T, \overline{T}], \ \forall w \in [w, \overline{w}], \ \forall \delta \in [\delta, \overline{\delta}],
\exists u \in [u, \overline{u}] \mid \forall t \geq 0, \ \Phi(t, T_0, u, w, \delta) \in [T, \overline{T}].
\]
Proposition

The system is Robust Controlled Invariant in $[T, \overline{T}]$ if and only if

$$\forall i, \begin{cases} f_i(T, \bar{u}_i, \bar{w}, \delta) \leq 0 \\ f_i(T, \underline{u}_i, \underline{w}, \delta) \geq 0 \end{cases}$$
Proposition

The system is Robust Controlled Invariant in $[\underline{T}, \overline{T}]$ if and only if

$$\forall i, \begin{cases} f_i(\overline{T}, \overline{u_i}, \overline{w}, \delta) \leq 0 \\ f_i(\underline{T}, \underline{u_i}, \overline{w}, \delta) \geq 0 \end{cases}$$

Definition (Decentralized Bang-Bang Controller)

$$\forall i, \begin{cases} \underline{T_i} \geq \overline{T_i} \Rightarrow u_i = \overline{u_i} \\ \underline{T_i} \leq \overline{T_i} \Rightarrow u_i = \underline{u_i} \end{cases}$$
Controllable Spaces (2-room example)
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\[ f_1(T, u_1, w, \delta) \leq 0 \]

\[ f_2(T, u_2, w, \delta) \leq 0 \]

\[ f_1(T, u_1, w, \delta) \geq 0 \]

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Control simulation

3 discrete disturbances:
- heat source in room 1
- heat source in room 2
- door

8 possible combinations
Control simulation

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8 possible combinations
Conclusion

Criterion for Robust Controlled Invariance

- for a class of monotone systems,
- with local control,
- and bounded disturbances.

- Independent of the feedback control strategy.
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Experimental building

- $\approx 1m^3$
- 3 Peltier coolers
- Heat sources: lamps
- CompactRIO
- LabVIEW
Identification (least-squares) over 57079 data points ($\approx 16h$)

Evaluation on another scenario:

- Room 1
- Room 2
- Room 3
- Room 4

Graphs showing measured data and identified model for each room.
Control

- Linear saturated controller
- Interval satisfying the *Robust Controlled Invariance*:
1 Temperature model and monotonicity

2 Invariance (CDC13)

3 Application (BuildSys13, ECC14)

4 Stabilization

5 Symbolic control
Stabilization
Stabilization
Stabilization

Room 1

Room 2

Room 3

Room 4

Controlled experiment
Stabilization intervals

Temperature (°C)

Time (minutes)
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Symbolic model

Discretization of the state space
Symbols: sets defined by the grid
Symbolic model

Next state of the symbol, given $u$: over-approximation
Automaton: intersection between over-approximation and symbols
Symbolic model

17th state “outside”: unsafe
Symbolic model

Increased memory [Moor and Raisch 2002]
More accuracy, bigger complexity
Symbolic model

Increased memory [Moor and Raisch 2002]
More accuracy, bigger complexity
Symbolic control

Choice of the control interval: not RCI
Symbolic control

Memory span 3
No memory, $4 \times 4$ symbols
Symbolic control

No memory, $10 \times 10$ symbols
Symbolic control

No memory, $15 \times 15$ symbols
No memory, $20 \times 20$ symbols
Symbolic control

No memory, $30 \times 30$ symbols
Symbolic control

No memory, $40 \times 40$ symbols
Conclusion and perspectives

Method

- Discretize the state space
- Generate the automaton for a chosen memory span
- Remove unsafe states to obtain the safe automaton
- Bigger complexity by increasing the memory than the discretization

Perspectives

- Improve efficiency of the algorithm
- Separate controllers for each disturbance condition
- Safe automaton non-deterministic controller: optimization over several future steps
Conclusion and perspectives

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  - controller: optimization over several future steps
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