Convex design of a controller

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Introduction

Principle: solving a convex optimization problem, whose minimization objective and constraints directly correspond to time- and frequency-domain specifications on an LTI or LFT plant model.

Structure:

- Convex design of a Youla parameter for the control of an LTI plant model.
- Convex design of a Youla parameter for the gain-scheduled control of an LFT plant model.

Convex design of a Youla parameter for an LTI plant

Principle:

- Design of an initial controller $K_0(s)$, typically under an observed state feedback form, for the augmented open loop plant model $P(s)$.
- Closed loop transfer matrix $= T_1(s) + T_2(s)Q(s)T_3(s)$, affine w.r.t. the Youla parameter $Q(s)$. The $T_i(s)$ depend on $P(s)$ and $K_0(s)$.
- Convex design of $Q(s)$.
- $K(s)$ is deduced from $K_0(s)$ and the optimal value of $Q(s)$.

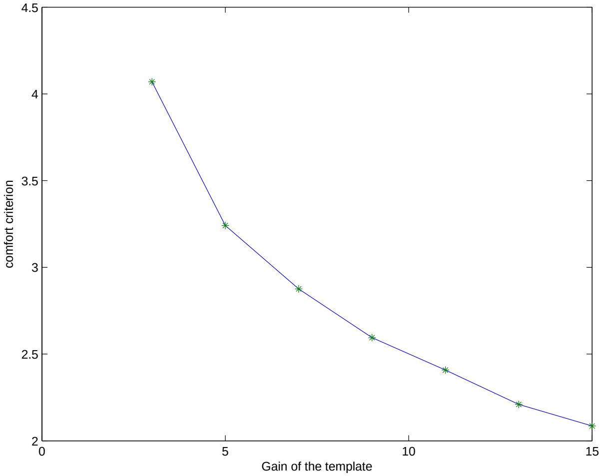
Convex design of a Youla parameter for an LTI plant

The feasibility of design spec. can be checked:

- Parameterization of the whole set of stabilizing controllers.
- Does there exist a controller, without constraint on its order, that satisfies a set of design spec. ?
- Study of the trade-off between conflicting design objectives, e.g. minimization of a comfort criterion VS actuator activity.
- Impossible to conclude if a non-convex optimization problem: "locally unfeasible" constraints, but may be "globally feasible".
- Computation of a reference performance obtained with an optimal (very) high order controller: if the same performance can be obtained with a low-order one, it is validated as a quasi-optimal controller.

Convex design of a Youla parameter for an LTI plant

Minimization of the peaks of the closed loop frequency domain response of a flexible aircraft VS actuator activity:



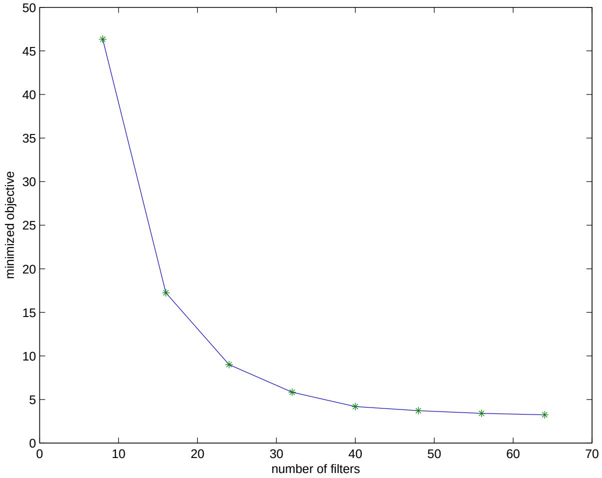
Convex design of a Youla parameter for an LTI plant

Solving an infinite- / approximate finite-dimensional optimization problem:

- Use of a basis of filters $Q(s) = \sum_i \theta_i Q_i(s)$, with fixed filters $Q_i(s)$. Convex optimization w.r.t. θ_i.
- Choice of the dynamics (poles) of the basis using the knowledge of the open or closed loop plant.
- Use of an orthonormal basis for the numerical conditioning of the optimization problem.
- Ideally, an infinite dimensional basis should be used to cover the whole set of asymptotically stable transfer matrices $Q(s)$ / asymptotically stabilizing controllers.
- Progressive design of $Q(s)$: stop when adding more filters does not further minimize the objective.
- Other theoretically more proper methods exist to deal with the infinite-dimensional aspect $(\ldots)$.

Convex design of a Youla parameter for an LTI plant

Progressive design with 8, 16, . . . , 64 filters, when minimizing the peaks of the closed loop frequency domain response of a flexible aircraft for a given actuator activity:



Convex design of a Youla parameter for an LTI plant

In practice:

- For a given (augmented) open loop plant, design of the initial controller directly under an observed state-feedback form: modal design of the state feedback and observer gains, loop shaping H_∞ design technique with I/O spec (...).
- Choice of the poles of the orthonormal basis of filters.
- Definition of the spec:
  - Shaping the time-domain response to given (un)measured exogenous input signals: overshoot, settling time...
  - Shaping the freq. domain response over finite freq. intervals, for several channels: extended H_∞, H_2 spec.
- Use of a frequency domain cutting planes solver to compute the optimal value of the Youla parameter.
- Tested on a flexible A/C problem: necessary use of a dynamic feedback controller (a static one for a rigid aircraft). Solver tested with about 100 states in the flexible model.

Convex design of a Youla parameter for an LTI plant

Use of a frequency domain cutting planes solver:

- Dedicated to the control of a high order state-space model.
- Use of an initial small size design frequency gridding, on which the spec. should be satisfied. Validation on a fine gridding, iterative refinement of the design frequency gridding.
- The use of a dedicated solver enables to save much computational time when progressively introducing the filters in the design and when exploring trade-offs:
  - A series of optimization problems to be solved.
  - With interior point LMI/sdp methods, each design is independently performed. Difficulty to introduce an initial pt.
  - With the frequency domain cutting planes solver, introduction of a feasible initial point + the subgradients which approximate the minimization objective and constraints are kept from an optimization to another.

Convex design of a Youla parameter for an LFT plant

Case considered:

- The TI scheduling parameters δ_i of the open loop LFT plant model are measured.
- The open loop LFT plant model should describe the continuum of linearized models about trim points or around a trajectory.
- LTI (modal and H_∞) spec. to be satisfied on a continuum.

Other cases:

- TV parameters inside the LPV/LFT model, nonlinear model.
- Main points: parameterizing the whole set of stabilizing controllers + reducing the control design problem to a convex optimization problem.

Convex design of a Youla parameter for an LFT plant

Principle:

- Design of an initial controller under an LFT observed state feedback form = embedded LFT open loop plant model + fixed/gain-scheduled state-feedback and observer gains.
- Computation of $T_1(s,\delta) + T_2(s,\delta)Q(s,\delta)T_3(s,\delta)$, by connecting the LFT plant model and initial controller.
- Design of a δ-scheduled Youla parameter $Q(s,\delta)$ = design of an augmented LTI Youla parameter $\mathcal{Q}(s)$.
- Multi-model design of $\mathcal{Q}(s)$ with the frequency-domain cutting planes solver + validation on a fine parameter gridding, or on the continuum with μ analysis.
- To a large extent, guaranteed convergence of this iterative scheme due to the convex nature of the optimization problem.

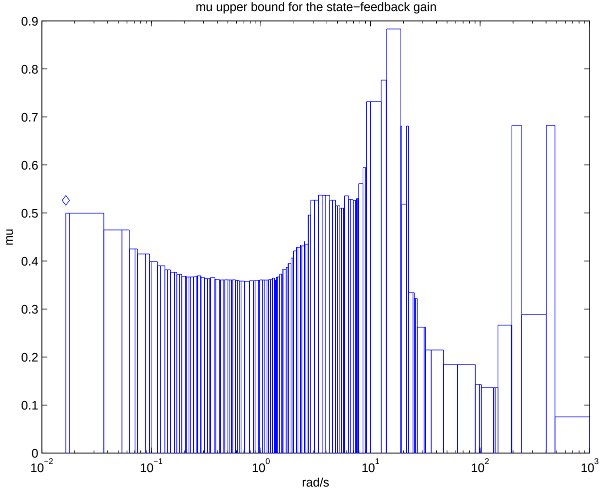
Convex design of a Youla parameter for an LFT plant

Application to an LFT missile model:

- Reichert's model.
- A rather low complexity LFT model: angle of attack repeated 4 times, Mach 6 times, 4 states.
- Modal design of the state feedback and observer gains using a multi-model polytopic technique, extended to the gain-scheduled case.
- Validation with μ analysis, to check the robustness of the pole placement inside a truncated sector.

Convex design of a Youla parameter for an LFT plant

μ analysis for the validation of the state-feedback gain:



Convex design of a Youla parameter for an LFT plant

Design of the Youla parameter $Q(s)$ for the LFT missile model:

- Minimization of the frequency-domain peak on the angle of attack under a constraint on the actuator rate output.
- Multi-model design of the Youla parameter + the values of the minimized objective and of the constraint are validated over a fine parameter gridding and over the continuum.

Minimization of $\max_{\delta \in D} \|H_1(s, \delta, Q(s))\|_\infty$ under the constraint $\max_{\delta \in D} \|H_2(s, \delta, Q(s))\|_\infty \leq C$:

- $H_1(s, \delta, Q(s))$ = TF between an additive disturbance δu on the plant input and the angle of attack, affine w.r.t. $Q(s)$.
- $H_2(s, \delta, Q(s))$ = TF between δu and the actuator rate output.
- Computation of $\max_{\delta \in D} \|H_i(s, \delta, Q(s))\|_\infty$ for a given $Q(s)$:
  - Upper bound provided by skew μ upper bound.
  - Lower bound provided by the validation over a fine parameter gridding.